UCRL -9189

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Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

MUON ABSORPTION IN LIQUID HYDROGEN

Steven Weinberg

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The long-awaited measurement of the rate of μ-absorption on protons could conceivably use hydrogen in any form, but there are technical advantages in working with liquids, as in bubble chambers. However many physicists have expressed pessimism about the utility of such an experiment, because in liquid H_2 muons form $p - \mu - p$ molecular ions ("mulecules") in addition to $p - \mu$ atoms. The purpose of this note is to show how it is nevertheless possible to interpret the observable rate of muon absorption in liquid hydrogen in terms of the basic $pn\mu\nu$ interaction. The main problem is to calculate the rate for the process

$$p - \mu - p \rightarrow n + \nu + p \tag{1}$$

in terms of the muon absorption in atoms; because of the extreme spin dependence of the V - A interaction,[1] this requires knowledge of the orientation of proton and muon spins as well as the overlap of their wave functions at the instant of absorption. The problem is more complicated than has been realized. Almost all of our remarks will apply to solid as well as to liquid hydrogen, and we will confine ourselves to the case of isotopically pure $(H_1^1)_2$.

* This work was performed under the auspices of the United States Atomic Energy Commission.

The only two bound orbital states of the $p - \mu - p$ system[2] are a para $1s\sigma g$ ground state with rotational angular momentum $L = 0$ and binding energy 2771 ev, and a $1s\sigma g$ ortho state with $L = 1$ which is $\Delta E = 148$ ev higher. The formation of these states by electron ejection in a collision of a $p - \mu$ atom with an H_2 molecule is respectively an E0 or E1 process, and hence the ratio of the formation rates is of order[3]

$$\text{PARA/ORTHO} \sim k_e^2 a_\mu^2 \sim 2 \cdot 10^{-4} \tag{2}$$

where k_e is the wave number of the ejected electron. Actually, detailed calculations[2] show this ratio to be less than $3 \cdot 10^{-5}$, so virtually all mulecules are formed in the ortho state.

It seems furthermore that the mulecules formed in the ortho state stay there during the few microseconds of muon lifetime. If we define the ortho → para electric dipole transition matrix element to be

$$|\langle P | \int \rho \underset{\sim}{R} | O \rangle| \equiv e\, a_\mu D \tag{3}$$

then the rate for de-excitation accompanied by ejection of an electron from a hydrogen molecule in a collision is

$$\begin{aligned} \omega_e(O \to P) &= 16\pi\, n_e\, m_e\, a_\mu^2\, e^4\, D^2/3k_e \\ &= 2.7 \cdot 10^{10}\, D^2\, \text{sec}^{-1} \ . \end{aligned} \tag{4}$$

(We have taken n_e to be the number density of electrons in a bubble chamber, $n_e = 3.5 \cdot 10^{22}\ \text{cm}^{-3}$, and set $k_e = \sqrt{2\, m_e(\Delta E - E_H)}$, where $\Delta E = 148$ ev, and $E_H = 15.6$ ev is the electron separation energy

in H_2.) It is also possible that the mulecule forms "ordinary" atoms or molecules with electrons and protons from the liquid.[4] The rate ω_A for Auger ortho → para conversion in such a system is again given by (4), with n_e replaced by the density n_e' of bound electrons at the $p - \mu - p$. If, for example, we take $n_e' = 1/\pi a_e^3$ then

$$\omega_A(0 \to P) = 61\, \omega_e(0 \to P) = 1.6 \cdot 10^{12} D^2 \sec^{-1} . \tag{5}$$

Of course, both ω_e and ω_A are much larger than the radiative conversion rate,

$$\omega_\gamma(0 \to P) = 4(\Delta E)^3 a_\mu^2 e^2 D^2/3 = 8.1 \cdot 10^7 D^2 \sec^{-1} . \tag{6}$$

According to (4), (5), and (6), we would need $D^2 \gtrsim 10^{-7}$ for any significant ortho → para conversion to occur during the muon lifetime, even if electrons were bound to all mulecules. It is unlikely that the E1 transition moment is this large. The ortho and para states have total proton spins $S_p = 1$ and $S_p = 0$, and the only effects that can give rise to an E1 matrix element between such states are impurities in the states,[5] or the electric moments generated by moving magnetic dipoles.[6] The first effect would give at most a D of the order of the amplitude of the $1s\sigma u$ ($L = 0$, $S_p = 1$) admixture in the para state, which can be estimated roughly as a typical magnetic interaction energy, $\alpha^4 m_\mu \sim 0.3$ ev, divided by the minimum energy difference between $1s\sigma g$ and $1s\sigma u$ states whose wave functions overlap appreciably, roughly 3 Kev, giving an amplitude

of about 10^{-4}. Of course D is much smaller than this, since the $1s\sigma u$ para impunity will not have perfect overlap with the $1s\sigma g$ ortho wave function. The second effect mentioned would contribute to D an amount of order $2.79\ \Delta E/m_p \sim 4 \cdot 10^{-7}$. If D were this small, E1 conversion would take as long as would be expected for conversion by M2 transitions.

De-excitation by rearrangement collisions of the $p - \mu - p$ with H_2 molecules is not retarded by the necessity to flip spins, but is hopelessly slow because of the high Coulomb barrier. The cross section for ortho $\rightarrow$ para conversion by this process contains the familiar factor e^{-G} where $G = 2\pi/137\ \beta$ and βc is the initial $p - \mu - p$ velocity. In order that $G < 100$, the $p - \mu - p$ would have to have kinetic energy greater than 200 ev, and this is impossible as it is formed with 0.02 ev, and $kT < 0.003$ ev.

The fact that process (1) occurs from the ortho state would not complicate matters badly if it were correct (as seems to have been previously assumed) that ortho mulecules had definite total spin $(\underset{\sim}{S}_\mu + \underset{\sim}{S}_p)^2 = S(S + 1)$. It has been shown[7] that if $\omega(S)$ is the rate for (1) in such states, and ω_F is the rate for muon absorption in $p - \mu$ atoms with total spin F, then

$$\omega(1/2) = 2\,\gamma_0\left(\frac{3}{4}\,\omega_0 + \frac{1}{4}\,\omega_1\right) \tag{7}$$

$$\omega(3/2) = 2\,\gamma_0\,\omega_1 \tag{8}$$

which may be compared with the smaller absorption rate in para mulecules

$$\omega(\mathrm{PARA}) = 2\,\gamma_p\left(\frac{1}{4}\,\omega_0 + \frac{3}{4}\,\omega_1\right) . \tag{9}$$

Here γ_0, γ_p are the ratios of the muon density at one of the protons in an ortho or para mulecule (averaged over the separation between protons) to the muon density at the proton in a $p - \mu$ atom. Using the CJR wave functions, we have found that[8]

$$2\,\gamma_0 = 1.165 \qquad\qquad 2\,\gamma_p = 1.308\,. \tag{10}$$

These values are subject to corrections[9] of order m_μ/m_p due to admixture of higher orbitals than $1s\sigma g$.

Of course, S is a good quantum number only if the forces coupling $\underset{\sim}{S}_p$ and $\underset{\sim}{S}_\mu$ to $\underset{\sim}{L}$ are ignored. In this lowest approximation, the ortho state forms a degenerate quintuplet of states $|\,J, S\,\rangle$ which we can take with definite values of S and $(\underset{\sim}{S}_\mu + \underset{\sim}{S}_p + \underset{\sim}{L})^2 = J(J+1)$; the states are: $|\,\frac{1}{2}, \frac{1}{2}\,\rangle$, $|\,\frac{1}{2}, \frac{3}{2}\,\rangle$, $|\,\frac{3}{2}, \frac{1}{2}\,\rangle$, $|\,\frac{3}{2}, \frac{3}{2}\,\rangle$, $|\,\frac{5}{2}, \frac{3}{2}\,\rangle$. The degeneracy among them is actually lifted by magnetic and relativistic effects which split the quintuplet into five states $|\,\psi_n\,\rangle$ separated by energies of order $\alpha^4 m_\mu = 0.3$ ev, with total angular momentum $J_1 = J_2 = \frac{1}{2}$, $J_3 = J_4 = \frac{3}{2}$, $J_5 = \frac{5}{2}$. (Each of these five states still retains a $2\,J_n + 1$-fold degeneracy, which plays no role here. The field needed for a Paschen-Back effect is of order 10^9 gauss.)

It is easy to show that the $S = \frac{1}{2}$ and $S = \frac{3}{2}$ components of the true energy eigenstates $|\,\psi_n\,\rangle$ don't interfere in reaction (1) if no attempt is made to measure the momentum or polarization of the left-over proton. Furthermore there is no interference among the $|\,\psi_n\,\rangle$ states, since their energy separation is much larger than their natural width of $3 \cdot 10^{-10}$ ev. Hence the rate for (1) is given as the sum

$$\omega_{p\mu p} = \xi\, \omega(1/2) + (1 - \xi)\, \omega(3/2) \tag{11}$$

where

$$\xi \equiv \sum_n P_n \xi_n \tag{12}$$

$$\xi_n \equiv \left| \langle J_n, S = \tfrac{1}{2} \mid \psi_n \rangle \right|^2 \qquad (\xi_5 = 0) \tag{13}$$

and P_n is the probability of the ortho mulecule occupying $|\psi_n\rangle$ at the instant of absorption.

It is easy to calculate the relative probabilities P_n^0 for formation of the five ortho states. Since the mulecule is formed from a proton and an $F = 0$ $p - \mu$ atom[10] it has $S = \frac{1}{2}$ initially, and hence

$$P_n^0 = \frac{2 J_n + 1}{6} \xi_n \,. \tag{14}$$

Furthermore, it seems reasonable to suppose that no significant number of transitions among the $|\psi_n\rangle$ occur during the muon lifetime. The Coulomb barrier prevents rearrangement collisions;[11] M1 radiative transitions would take a day; and probably M1 conversion in collisions with H_2 molecules is very slow because the energy available is insufficient for electron ejection. Assuming then that $P_n = P_n^0$, we obtain

$$\xi = \sum_n \frac{2 J_n + 1}{6} \xi_n^2 \tag{15}$$

and our problem reduces to that of computing the ξ_n. However, there is one important result that can be derived without any calculation. Since $\xi_1 + \xi_2 = \xi_3 + \xi_4 = 1$, it follows directly from (15) that

$$\frac{1}{2} \leq \xi \leq 1 \tag{16}$$

whereas in general we would only know that $0 \leq \xi \leq 1$. If we accept that $\omega_0 \sim 50\,\omega_1$,[7] then because we cannot have ξ near zero it is safe to approximate

$$x \equiv \omega_{p\mu p}/\omega_0 \cong (2\,\gamma_0)\,\tfrac{3}{4}\,\xi \tag{17}$$

and furthermore the neutrons emitted should have almost 100% longitudinal polarization.

The Hamiltonian H' that couples $\underset{\sim}{S}_p$, $\underset{\sim}{S}_\mu$, and $\underset{\sim}{L}$ takes the general form

$$\begin{aligned} H' = {} & E_1\,\underset{\sim}{S}_p\cdot\underset{\sim}{L} + E_2\,\underset{\sim}{S}_\mu\cdot\underset{\sim}{L} + E_3\,\underset{\sim}{S}_\mu\cdot\underset{\sim}{S}_p \\ & + E_4\left(\tfrac{1}{2}(\underset{\sim}{S}_\mu\cdot\underset{\sim}{L})(\underset{\sim}{S}_p\cdot\underset{\sim}{L}) + \tfrac{1}{2}(\underset{\sim}{S}_p\cdot\underset{\sim}{L})(\underset{\sim}{S}_\mu\cdot\underset{\sim}{L}) - \tfrac{2}{3}(\underset{\sim}{S}_\mu\cdot\underset{\sim}{S}_p)\right) \\ & + E_5\left((\underset{\sim}{S}_p\cdot\underset{\sim}{L})^2 + \tfrac{1}{2}(\underset{\sim}{S}_p\cdot\underset{\sim}{L}) - \tfrac{4}{3}\right) \end{aligned} \tag{18}$$

which yields for the ξ_n,

$$\xi_1 = 1 - \xi_2 = \frac{1}{2}\left\{1 + \left[1 + 2\left(\frac{-4E_1 + 4E_2 - 5E_4 + 10E_5}{2E_1 + 7E_2 - 9E_3 - 5E_4 - 13E_5}\right)^2\right]^{-1/2}\right\} \tag{19}$$

$$\xi_3 = 1 - \xi_4 = \frac{1}{2}\left\{1 + \left[1 + 5\left(\frac{-4E_1 + 4E_2 + E_4 - 2E_5}{8E_1 + E_2 - 9E_3 + 4E_4 + 4E_5}\right)^2\right]^{-1/2}\right\} . \tag{20}$$

The coefficients E_j May be calculated from the formulae:[12]

$$E_1 = \frac{e^2}{m_p^2}\left(\frac{2\mu_p - 1}{2}\left\langle \frac{r_{12}^2 + r_{1\mu}^2 - r_{2\mu}^2}{2\, r_{12}^2\, r_{1\mu}^3} - r_{12}^{-3}\right\rangle - \mu_p \quad r_{12}^{-3}\right) \tag{21}$$

$$E_2 = \frac{e^2}{m_\mu\, m_p}\left\langle \frac{r_{12}^2 + r_{1\mu}^2 - r_{2\mu}^2}{r_{12}^2\, r_{1\mu}^3}\right\rangle \tag{22}$$

$$E_3 = \frac{8\pi\, e^2\, \mu_p}{3\, m_\mu\, m_p}\left\langle \delta^3(r_{1\mu})\right\rangle \tag{23}$$

$$E_4 = \frac{3\, e^2\, \mu_p}{5\, m_\mu\, m_p}\left\langle r_{1\mu}^{-3} - \frac{3(r_{12}^2 + r_{1\mu}^2 - r_{2\mu}^2)^2}{4\, r_{12}^2\, r_{1\mu}^5}\right\rangle \tag{24}$$

$$E_5 = \frac{3\, e^2\, \mu_p^2}{5\, m_p^2}\left\langle r_{12}^{-3}\right\rangle \tag{25}$$

where $\mu_p = 2.79$, and $r_{ij} \equiv |\underset{\sim}{r}_i - \underset{\sim}{r}_j|$. The first term in E_1 represents the spin-orbit force on the protons; E_2 and the second term in E_1 come from the magnetic interaction between the proton momentum and the muon and proton spins; E_3 and E_4 come from the interaction between proton and muon magnetic moments; and E_5 arises from the interaction between proton magnetic moments. We have not carried out the necessary integrations to obtain the E_j numerically, as it seems most important at this stage to show that the task is straightforward.

Since some muon absorptions will occur with muons still in $F = 0$ $p - \mu$ atoms, the observed total rate of all absorptions in liquid hydrogen is

$$\omega_{abs} = \omega_0(1 + xy)/1 + y$$

where y is the product of the muon mean life times the rate per $p - \mu$ atom of $p - \mu - p$ formation. According to CJR, $y = 14.4$ at a proton number density $n_p = 3.5 \cdot 10^{22}$ cm^{-3}. Since x (as given by (17), (10), and (16)) must be between 0.44 and 0.88, the value of ω_0 determined by a measurement of ω_{abs} (and calculation of x) will be quite insensitive to the precise value of y, providing y isn't too small.

It is possible to test many of our assumptions by measuring ω_{abs} over a range of hydrogen densities. If CJR are correct, varying n_p from 1.8 to $4.5 \cdot 10^{22}$ cm^{-3} (possible in a counter experiment) would change y from 7.4 to 18.9, giving according to (17) a decrease in ω_{abs} of between 1% and 8%. If an earlier calculation[13], giving a y five times smaller than that of CJR, were correct, then the decrease in ω_{abs} would be between 2.5% and 16.5%. If our estimates are wrong, and ortho → para conversion becomes important at the higher densities, x and ω_{abs} can drop by almost a factor of 3, while if ortho → ortho transitions became important ω_{abs} could rise or fall.

The author has benefited from conversations on these matters with a great many physicists, and would particularly like to thank

R. H. Dalitz, R. Hildebrand , D. L. Judd, L. M. Lederman, R. J. Myers, W. Nierenberg, H. Primakoff, M. A. Ruderman, R. Sard, J. Steinberger, V. I. Telegdi, and S. B. Treiman.

FOOTNOTES

1. Bernstein, Lee, Yang, and Primakoff, Phys. Rev. 111, 313 (1958). Also reference 7.
2. Cohen, Judd, and Riddell, Phys. Rev. (to be published) and UCRL reports 8390, 8391, 8802. This work will be referred to below as CJR. See also reference 14.
3. In our notation $\hbar = c = 1$, $a_i \equiv 1/e^2 m_i$, $e^2 = \alpha = 1/137$.
4. This possibility was suggested to the author by J. Steinberger.
5. R. H. Dalitz (private communication).
6. See e.g. J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics, Wiley, New York (1952), Eq. XII 3.34.
7. H. Primakoff, Revs. Modern Phys. 31, 802 (1959). Also references 9 and 14.
8. I would like to thank Mr. V. Brady for performing the numerical integrations needed on the UCRL IBM-650.
9. T. Y. Wu (preprint) has stated that for protons fixed a distance r_{12} of 2 mesonic units, these corrections raise the value of 2γ from 1.317 to 1.422. However, Wu does not seem to perform the averaging over the nuclear separation needed to obtain $2\gamma_0$ and $2\gamma_p$. This averaging places the nucleons at an effective distance of 2.90 and 2.45 respectively. Apart from corrections due to higher mesonic orbitals, the CJR wave functions are essentially exact, agreeing at $r_{12} = 2$ and 4 to within 1% with those of Bates, Ladsham, and Stewart, Phil. Trans. Roy. Soc. A246, 215 (1953).

10. S. S. Gershtein, J. Exptl. Theoret. Phys. (U. S. S. R.) 34, 463, 993 (1958).

11. For neutral $p - \mu$ atoms it is rearrangement collisions that give rapid $F = 1 \rightarrow F = 0$ conversion, as shown in reference 10.

12. These have been derived using techniques described by H. A. Bethe and E. E. Salpeter, Handbuch der Physik Band XXXV, p. 256 ff. Spinger-Verlag, Berlin (1957). Care must be taken to follow their prescriptions in evaluating E_4 .

13. Ia. B. Zeldovick and S. S. Gershtein, J. Exptl. Theoret. Phys.(U.S.S.R.) 35, 649 (1958).

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