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AN EFFECTIVE-RANGE FORMULA FOR PHOTOPION PRODUCTION FROM PIONS

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In the Chew-Mandelstam[1] approach to the strong-interaction problem at low energies, the amplitude for $\gamma + \pi \rightarrow 2\pi$ plays a central role in any phenomena involving photons. Recently, Ball[2] has found that this process produces an additive correction to the formulas of Chew et al.[3] for photopion production from nucleons. Compton scattering by pions and by nucleons also involves the $\gamma + \pi \rightarrow 2\pi$ reaction. A simple and reasonably accurate formula for the $\gamma + \pi \rightarrow 2\pi$ amplitude which can be used in any of these applications is given here.

We assume that the simple invariant transition amplitude, $M(s,t,u)$ has the Mandelstam representation[4]

$$M(s,t,u) = \frac{1}{\pi^2} \int_4^\infty \int_4^\infty \rho(x,y) \left[\frac{1}{(x-s)(y-t)} + \frac{1}{(x-t)(y-u)} + \frac{1}{(x-u)(y-s)} \right] dx\, dy. \tag{1}$$

It is defined through the relation,

$$T_{fi} = \frac{i(2\pi)^4 \delta^4 (K + p_1 - p_2 - p_3) \sum \left[\left(-\frac{i}{\sqrt{2}}\right) \epsilon_{\alpha\beta\gamma}\, \epsilon_{\lambda\sigma\mu\nu} (p_1)_\lambda (p_2)_\sigma K_\mu \epsilon_\nu \right] M(s,t,u)}{(16\, p_{10}\, p_{20}\, p_{30}\, K)^{1/2}} \tag{2}$$

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where α, β, and γ are the isoptopic indices of the pions, and $\epsilon_{\alpha\beta\gamma}$ and $\epsilon_{\lambda\sigma\mu\nu}$ are the conventional antisymmetric tensors of third and foruth rank, respectively. Here s, t, and u are defined[5] as (see Fig. 1)

$$s = -(K + p_1)^2 ,$$

$$t = -(K - p_2)^2 , \qquad (3)$$

and

$$u = -(K - p_3)^2 ,$$

and are related by the condition

$$s + t + u = 3.$$

When the photon K and meson p_1 are the incoming particles, the variables s, t, and u are related to barycentric system variables

$$s = 4(1 + p^2) ,$$

$$t = 1 - 2k E + 2kp \cos\theta ,$$

and

$$u = 1 - 2kE - 2kp \cos\theta ,$$

where p and E are the magnitudes of the outgoing pion momenta and energy, respectively, k is the energy of the photon, and we have $\cos\theta = (\vec{p} \cdot \vec{k})/pk$. The formula for the differential cross section in the barycentric system is

$$\frac{d\sigma}{d\Omega} = \sum \frac{p}{k} \left| \frac{1}{8E} \quad \frac{T_{fi}}{2\pi} \right|^2 .$$

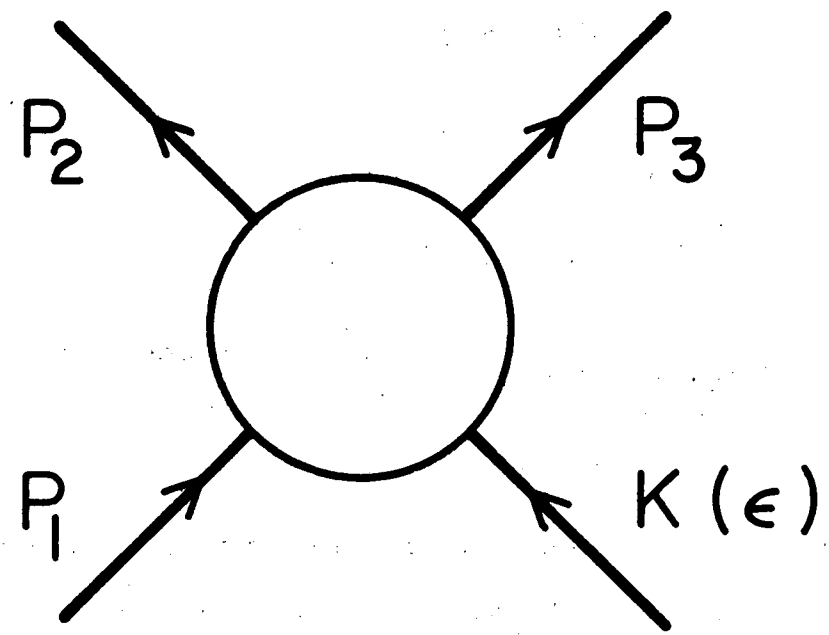

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Fig. 1. Diagram of $\gamma + \pi \rightarrow 2\pi$.

From the Mandelstam representation, we can locate the singularities of the partial-wave amplitudes, defined through the relation

$$M(s,z) = \sum_{\text{odd}\,\ell} M_\ell(s)\, P'_\ell(z) ,$$

where $z = \cos\theta$. The M'_ℓ s are analytic in the whole complex s plane except for cuts on the real axis. The right-hand cut runs from 4, the physical threshold for two pions, to ∞; the left cut runs from 0 to $-\infty$. Using these analytic properties, the unitary condition, and the Omnés[6]-Frazer-Fulco[7] method, we find

$$M_\ell(s) = \frac{1}{\pi D_\ell(s)} \int_{-\infty}^{0} \frac{D_\ell(s') \operatorname{Im} M_\ell(s')}{s' - s}\, ds' , \qquad (4)$$

where $D_\ell(s)$ is the π-π denominator function introduced by Chew and Mandelstam.[1]

If we assume that for $s \geq 4$, we have $\operatorname{Im} M \approx \operatorname{Im} M_1$, we can show that

$$M(s,t,u) \approx \frac{1}{\pi} \int_4^\infty \operatorname{Im} M_1(x) \left[\frac{1}{x-s} + \frac{1}{x-t} + \frac{1}{x-u} \right] dx . \qquad (5)$$

We see that all partial waves for $\ell > 1$ can be obtained once M_1 is known. Formula (5) is the basis of an independent calculation that has been carried out by Gourdin and Martin.[8]

Using crossing symmetry, we can obtain a formula for $\operatorname{Im} M_1(s)$ for $s < 0$ in terms of $\operatorname{Im} M_1(s)$ for $s > 4$. We do not give this formula here because the effect-range approach does not require such detailed

information. In any event, it is clear that for $\ell = 1$, the integral of Eq. (4) is homogeneous, so that its solution is not unique, at least with respect to a multiplicative factor. Therefore an arbitrary parameter must be introduced. In the calculation below, we shall fix this parameter Λ as the value of $M_1(s)$ at $s = 1$. At present, we do not know how to relate Λ to fundamental constants, and for the time being it must be determined from experiment.

To proceed further, we need the denominator function, D_1, for p-wave pion-pion scattering. The pion-pion calculations of Chew and Mandelstam have not yet reached a conclusive stage but these authors have given an approximate form for D_1 which corresponds to the replacement of the unphysical branch cut in the pion-pion amplitude by a finite number of poles.[9] Further, they have shown that two poles lead to an accurate approximation in the physical region up to $s \sim 40$. Once the parameters of the two-pole formula have been determined, it will be a straightforward problem to incorporate the information into the amplitude $\gamma + \pi \rightarrow 2\pi$. The determination of the pion-pion parameters is still in progress,[10] but we outline here, for future use, the form of solution of our problem that corresponds to the pole approximation of Chew and Mandelstam.[9] We shall illustrate the method with the one-pole pion-pion amplitude, for which parameters have been given by Frazer and Fulco.[7]

We propose to replace the left-hand cut of the amplitude M_1 by poles with appropriate positions and residues. The "n-pole formula" thus obtained contains $2n$ parameters. One of these may be identified with Λ; the rest will be determined from crossing relations.

By representing Im M_1 on the left-hand cut by one or two delta functions, we obtain from Eq. (4) the one-pole formula

$$M_1(s) = \frac{\Lambda (1 + a) D_1(1)}{(s + a) D_1(s)} \tag{6}$$

or the two-pole formula

$$M_1(s) = \frac{\Lambda D_1(1)}{D_1(s)} \left[\frac{D_1(-b)}{s + b} + \frac{\Lambda_1 D_1(-c)}{s + c}\right] \left[\frac{D_1(-b)}{1 + b} + \frac{\Lambda_1 D_1(-c)}{1 + c}\right]^{-1} \tag{7}$$

where Λ and Λ_1 are real, and a, b, and c are real and positive. The parameters Λ_1, a, b, and c may be determined from the following crossing relations which are derived from Eq. (5):

$$M_1(1) - \frac{3}{2} M_1^L(1) = 0 \tag{8a}$$

$$\frac{\partial}{\partial s} M_1(1) = 0 \tag{8b}$$

$$\frac{\partial^2}{\partial s^2} M_1(1) - 6 \frac{\partial^2}{\partial s^2} M_1^L(1) = 0, \tag{8c}$$

where

$$M_1^L(s) = \frac{1}{\pi} \int_{-\infty}^{0} \frac{\operatorname{Im} M_1(s')\, ds'}{s' - s}$$

$$= \frac{\Lambda (1 + a) D_1 (1)}{(s + a) D_1 (-a)}$$

for the one-pole formula, and

$$= \Lambda D_1 (1) \left[\frac{1}{s+b} + \frac{\Lambda_1}{s+c} \right] \left[\frac{D_1(-b)}{1+b} + \frac{\Lambda_1 D_1 (-c)}{1+c} \right]^{-1}$$

for the two-pole formula.

If the Frazer-Fulco one-pole D_1 (s) function is used for a resonance at $s = 10$, we find $a \cong 5.7$ from the crossing condition (8a). The parameters Λ_1, b, and c in the two-pole formula may be determined from (8a) to (8c). It turns out that no choice of Λ_1, b, and c can satisfy all three conditions (8a) to (8c) if the Frazer-Fulco one-pole formula is used. However, the first two conditions of Eqs. (9) can be satisfied with the pair of poles in the range between 0 and -4.93. The p-wave amplitude in the physical region not only is insensitive to the positions of the two poles so long as (8a) and (8b) are satisfied, but also is not much different from the one-pole solutions, which satisifies (8a) alone. Thus we may be confident of the accuracy of the two-pole results once the parameters of the two-pole π-π scattering formula are known. The one-pole formula already obtained is probably adequate for most purposes.

We can estimate the order of magnitude of Λ by considering the decay of neutral pions, assuming that the $\gamma + \pi \rightarrow 2\pi$ reaction plays a prominent role in the intermediate state. We find $|\Lambda| \approx 1.3\, e$ ($e^2 = 1/137$) if the π^0 lifetime is 4×10^{-16} sec.[11] Our calculation is based on the assumption that only the least massive (two-pion) intermediate state contributes to the dispersion integral, but there is no good reason to

neglect the 3π contribution, especially if there is a resonance or a bound state at roughly the same energy as the two-pion resonance.[12] A better estimate of Λ may come from photopion production on nucleons, where Ball has shown that $\gamma + \pi \rightarrow 2\pi$ makes an important and characteristic contribution.[2]

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