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EFFECTS OF THE PION-PION RESONANCE AND THE THREE-PION RESONANCE
OR BOUND STATE ON NEUTRAL PION DECAY

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Recently Fraser and Fulco¹ and Chew² have proposed that a two-pion P-wave resonance and a three-pion resonance or bound state may account respectively for the isotopic vector and scalar components of nucleon electromagnetic structure. The purpose of this note is to investigate the effects of such resonances on neutral-pion decay.

The dispersion analysis of neutral-pion decay was first considered by Goldberger and Treiman,³ but they assumed nucleon-antinucleon pairs to be the most important intermediate states and neglected multipion states. Here we adopt a different approach and consider the contributions of the least massive states. This can be done if we extend a photon variable q^2 into the complex plane instead of the meson variable p^2 used by Goldberger and Treiman.

Following the standard method, one has⁴ (see Fig. 1)

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† A preliminary account of this work was given at the 1959 Thanksgiving meeting of the American Physical Society, November 27-29, 1959, Cleveland, Ohio, [Hou-sen Wong, Bull. Am. Phys. Soc. 4, 407 (1959)].

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$$\langle q(\mu), k(\nu) | T | p(5) \rangle = \frac{i(2\pi)^4 \delta^4(p - q - k) F_2(-q^2; -k^2; -p^2) \epsilon_\mu^\nu}{(8 q_0 k_0 p_0)^{1/2}}, \quad (1)$$

where we have

$$F_2 = (4 p_0 q_0)^{1/2} \langle q(\mu) | S_2(0) | p(5) \rangle,$$

and p is the pion four-momentum. The indices μ and ν refer to the polarization of the photons of momenta q and k , respectively. The number "5" inside the matrix element represents a neutral pion in the initial state.

From general invariance arguments, the F function can be written in the form

$$F(-q^2; -k^2; -p^2) = \epsilon_{\alpha\beta\mu\nu} q_\alpha k_\beta \epsilon_\mu^\nu f(-q^2; -k^2; -p^2). \quad (2)$$

We assume that, with both p^2 and k^2 on the mass shell, the scalar function $f(-q^2)$ satisfies the following dispersion relation without subtraction:

$$f(-q^2) = \frac{1}{\pi} \int_4^{\infty} \frac{\text{Im} f(t) dt}{t + q^2}. \quad (3)$$

The lifetime of π^0 is then given by

$$\tau = 64 \pi [f(0)]^{-2}. \quad (4)$$

Using the unitary condition, we can express the absorptive part of F as

$$\text{Im } F = \pi e_{\mu} e_{\nu}^* \sqrt{2g_0} \sum_n \delta^4(q - p_n) \langle 0 | J_{\mu}(0) | n \rangle \langle n | J_{\nu}(0) | p(3) \rangle .$$

Since our approach is to assume that the function is determined by near-by singularities, no intermediate state except the 2π and 3π states are considered here. Let us first investigate the effect from the two-pion P-wave resonance alone and leave the 3π contribution to be discussed later; then we can show

$$\begin{aligned} \text{Im } F(-q^2) &= \frac{e_r}{48\pi} e_{(\alpha\mu)} e_{\nu} k_{\beta} e_{\mu} e_{\nu}^* \\ &\times \frac{(-q^2 - 4)^{3/2}}{(-q^2)^{1/2}} F_{\pi}^{\dagger}(-q^2) M_1(-q^2) , \end{aligned} \tag{5}$$

where F_{π}^{\dagger} is the Hermitian conjugate of F_{π} , the pion form factor, and M_1 is the P-wave amplitude for photopion production from pions. It has been shown by the author that the M_1 function is linearly related to a real constant Λ and the F_{π} function.⁵ Comparing Eqs. (2) and (5), we obtain

$$r_{2\pi}(-q^2) = \frac{e_r}{48\pi^2} \int_4^{\infty} \frac{(t-4)^{3/2}}{\sqrt{t}(t+q^2)} F_{\pi}^{\dagger}(t) M_1(t) dt . \tag{6}$$

If the Frazer-Fulco form-factor function is used for a resonance at $t_{2\pi} = 10$ we find 2×10^{-16} sec $< \tau < 4 \times 10^{-16}$ sec for $1.8 e \geq |\Lambda| \geq 1.3 e$.⁶ Thus we have shown that the contribution of two-pion resonance is capable of producing a large effect in the

neutral pion decay.

We note that the f function in Eq. (6) is function in the virtual-photon mass variable. Thus $-x \equiv q^2 = (p_+ + p_-)^2$ represents the square of total four momentum of the electron-positron pair in the process $\pi^0 \rightarrow \gamma + e^+ + e^-$. Since x is less than 1, we can write

$$f(x) = f_{2\pi}(0)(1 + \alpha x) \approx f_{2\pi}(0) \left(1 + \frac{x}{t_{2\pi}}\right) \quad (7)$$

for small x . Thus, we see that α is always positive in this approximation.

The calculation so far is based on the assumption that only the 2π state contributes to the dispersion integral, but there is no good reason to expect the 3π contribution to be negligible. Since no one has succeeded in treating the matrix element $\langle \pi | 3\pi \rangle$, we are not able to handle this part. However, we may observe that if the three-pion $I = 0$ and $J = 1$ state is resonant or bound at energy $\sqrt{t_{3\pi}}$, as suggested by Chew, then

$$f_{3\pi}(-q^2) = \frac{\text{constant}}{q^2 + t_{3\pi}},$$

so that

$$f(-q^2) = f_{2\pi}(-q^2) + f_{3\pi}(-q^2) = \frac{f_{2\pi}(0)}{1 + \frac{q^2}{t_{2\pi}}} + \frac{f_{3\pi}(0)}{1 + \frac{q^2}{t_{3\pi}}}$$

Thus, if we define

$$\beta = \frac{f_{3\pi}(0)}{f_{2\pi}(0)},$$

then for small $x(\equiv -q^2)$,

$$r(x) \approx r(0) \left[1 + \left(\frac{\alpha}{1+\beta} \right) \left(\frac{1}{r_0} + \frac{\beta}{r_0} \right) x \right]. \quad (8)$$

Thus, if the 2π and 3π states make comparable contributions (as they do in the nucleon charge structure) and β has a negative sign, it is possible that the parameter α defined in Eq. (7) to be negative.⁸ An attempt is in progress to determine β from other experimental data involving the 3π state.

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(no date). These authors have also studied the f function from
a different approach and obtained $\alpha \approx 0.06$.
8. The author is indebted to Dr. N. Samios of Brookhaven National
Laboratory for advance communication of his preliminary
experimental result that $\alpha = -0.24 \pm 0.16$.

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FIGURE CAPTION

Fig. 1. Neutral pion decay. Wavy lines are photons; broken line, pion.

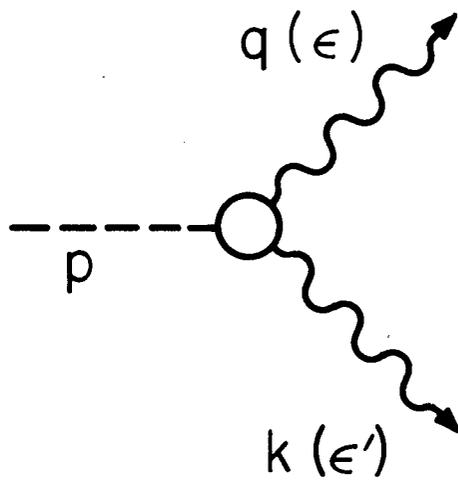


Fig 1
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