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NEUTRON-PROTON SCATTERING AND THE DETERMINATION OF THE PION-NUCLEON COUPLING CONSTANT

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This article reports a determination of f^2, the pion-nucleon coupling constant, from the neutron-proton differential charge-exchange cross section, using a method proposed by Chew[1]. For this purpose, the neutron-proton differential cross section at 710-Mev has been measured in hydrogen at nine angles within the range 180 to 160 deg (center-of-mass system) and in deuterium at two angles within the same range.

The method of the experiment was to focus the 740-Mev proton beam of the Berkeley synchrocyclotron upon a liquid deuterium target. The neutrons ejected from the deuterium at an angle of 7-deg 8-min relative to the incident proton beam passed through a sweeping magnet, were collimated, passed through another sweeping magnet, and impinged upon a liquid hydrogen target. A counter telescope consisting of four scintillators and a velocity-selecting Cherenkov counter[2] detected the elastic protons ejected from the hydrogen with an efficiency of about 60%, rejected all π-mesons and in conjunction with a γ-ray converter, rejected a large γ-ray component in the neutral beam with an efficiency of greater than 99%. The efficiency of the counter telescope was measured as a function of angle and velocity by performing a proton-proton scattering experiment in which both the scattered and recoil protons were detected. The prpton beam incident on the deuterium target was monitored by a secondary emission chamber[3] which was calibrated, in a low intensity beam, against an argon-filled ionization chamber.

*This work was done under the auspices of the U.S. Atomic Energy Commission.

The method used to acquire information about the energy spectrum of the neutron beam was to study experimentally the velocity spectrum of the protons emerging from the deuterium at the same angle as the neutron beam. The effective energy spectrum of the neutron beam was deduced from these measurements and could be represented by a Gaussian distribution with a standard deviation of 8 Mev and a most probable energy of 710 ± 8 Mev.[4]

The use of deuterium as the neutron source provided a direct method of measuring the absolute neutron intensity. Both targets were filled with deuterium and the counter telescope set at an angle of 7 deg 8 min. By charge symmetry the scattering amplitudes are identical for the two successive scatterings (proton-neutron at the first target and neutron-proton at the second target). Therefore the ratio of the number of protons from the second target to the number of protons incident on the first target is proportional to the square of the neutron-proton cross section in deuterium at 7 deg 8 min (laboratory angle). Once this cross section was known it was possible to predict the absolute neutron flux in terms of the incident proton flux.

The results of the measurements are listed in Table I. The angular distribution was measured to an accuracy of 5%. The errors of the listed data include those due to the normalization and the uncertainty in the neutron transmission of the γ-ray converter which was used for angles less than 5 deg. The cross section in deuterium at 180 deg is 0.48 ± 0.08 of the cross section in hydrogen. This decrease is a manifestation of the Pauli principle. Neutron-proton scattering at 180-deg in deuterium leaves behind two protons in the final state and some of these states are forbidden by the exclusion principle.[5]

Chew[1] proposes that the neutron-proton differential cross section be multiplied by $x^2 = (1+\cos\phi + \frac{\mu^2}{MT})^2$ and extrapolated to $x = 0$, which is the position of the

one-pion-exchange pole. The residue of the pole is given by

$$a_0 = \frac{4(f^2)^2}{T^2(\frac{T}{2M} + 1)} , \qquad (1)$$

where T is the laboratory-system kinetic energy, ϕ is the center-of-mass scattering angle, μ is the charged pion rest mass and M is the nucleon mass. If the neutron-proton cross section[7] is multiplied by x^2 the result is

$$x^2 \frac{d\sigma}{d\omega} = y(x) = \frac{g^4}{4E^2} \left(x - \frac{\mu^2}{MT}\right)^2 + xA(x) + x^2 B(x) \qquad (2)$$

where E is the total energy of a nucleon and A and B represent the contributions of intermediate states other than the one-pion-exchange.

Figure 1 is a plot of $x^2 \frac{d\sigma}{d\omega}$ versus x for the data obtained at 710 Mev. A least-square fit in polynomials in x was made to the data on the IBM 704 computer. Polynomials up to, and including fifth order were sought. The criteria for the selection of the best fit are contained in a paper by Cziffra.[6] The values of q/M, where q is the sum of the squares of the residues divided by the errors, and M is the number of degrees of freedom, are listed in Table II. The minimum of q/M is a criterion for determining the best fit. The values of q/M are consistent with the expected values.

Another test that can be applied to the polynomials in an effort to determine the best fit is the Fisher F test. The results of this test are listed in Table II where $P(n)$ is to be interpreted as the probability of being correct if the polynomial of order n is selected to represent the data. The $P(n)$ reveal an unsatisfactory situation. If the data represent a low-order polynomial, then $P(n)$ should peak at a particular n. The indication is that the data do not represent a polynomial of order $n \leq 5$.

The contribution of multiple-meson exchange to the amplitude can be represented analytically by

$$A(x) \propto \int_0^\infty \frac{a(x')\,dx'}{(x' + x + 3\,\frac{\mu^2}{MT})} \qquad (3)$$

We assume that this branch cut can be approximated by a simple pole (as would be the case if there were a strong π-π resonance) located at an unknown value of x. We then define the function u(x) by

$$u(x) = (x + a)^2\, y(x) \qquad (4)$$

where a is the value of x at which the pole occurs. At x = 0 the residue of u(x) is a^2 times the residue of y(x). We choose a value for a and make a least-square fit to $(x+a)^2\, x^2\, \frac{d\sigma}{d\omega}$. The values of a used corresponded to tene values of the total energy of the multiple-meson intermediate state between 1 and 5 meson masses. The values of a corresponding to energies less than two meson masses have no physical justification. For each case, the minimum of q/M was noted and the behavior of P(n) studied. Figure 2 shows the results of this analysis. The second and third order polynomials in the neighborhood of $m_i = 2$ definitely exhibited the behavior one expects if the data represent a second order polynomial. The residue (and all other coefficients) showed a distinct plateau as a function of n. (This is to be compared with the results in Table I where no such plateau is present). The values of f^2 obtained for each m_i are shown in Table III. The errors on f^2 includes the errors of the input data and a factor representing the goodness-of-fit.

The inclusion of the multiple-pion exchange by the preceding method definitely removes the ambiguity in the choice of the best fit. The value of f^2 in which one can have the most confidence, based on this analysis, is $f^2 = 0.085 \pm 0.011$. Cziffra[7] has used Chew's proposal to analyze data at 90 Mev and 400 Mev and finds f^2 approx 0.06 with about 10% error.

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Table I. Neutron-proton differential cross section in hydrogen and deuterium at 710 Mev

Cross section (10^{-27} cm^2/sr) center-of-mass system	ϕ, Center-of-mass system angle (deg)
Hydrogen	
6.15 ± .54	180.00
5.04 ± .51	175.89
4.39 ± .39	172.94
3.97 ± .38	170.60
3.84 ± .31	168.25
3.12 ± .25	165.90
3.35 ± .28	163.25
2.71 ± .24	161.37
2.65 ± .23	158.90
Deuterium	
2.97 ± .42	180.00
2.85 ± .14	163.25

Table II. Results of least-square fits to $k^2 \frac{d\sigma}{d\omega}$ at 710 Mev

Order of polynomial n	q/M	P(n)	Coupling constant f^2
1	1.447	0.01	imaginary
2	0.652	0.30	0.032 ± .012
3	0.658	0.60	0.059 ± .022
4	0.748	0.85	0.096 ± .050
5	0.988		0.128 ± .169

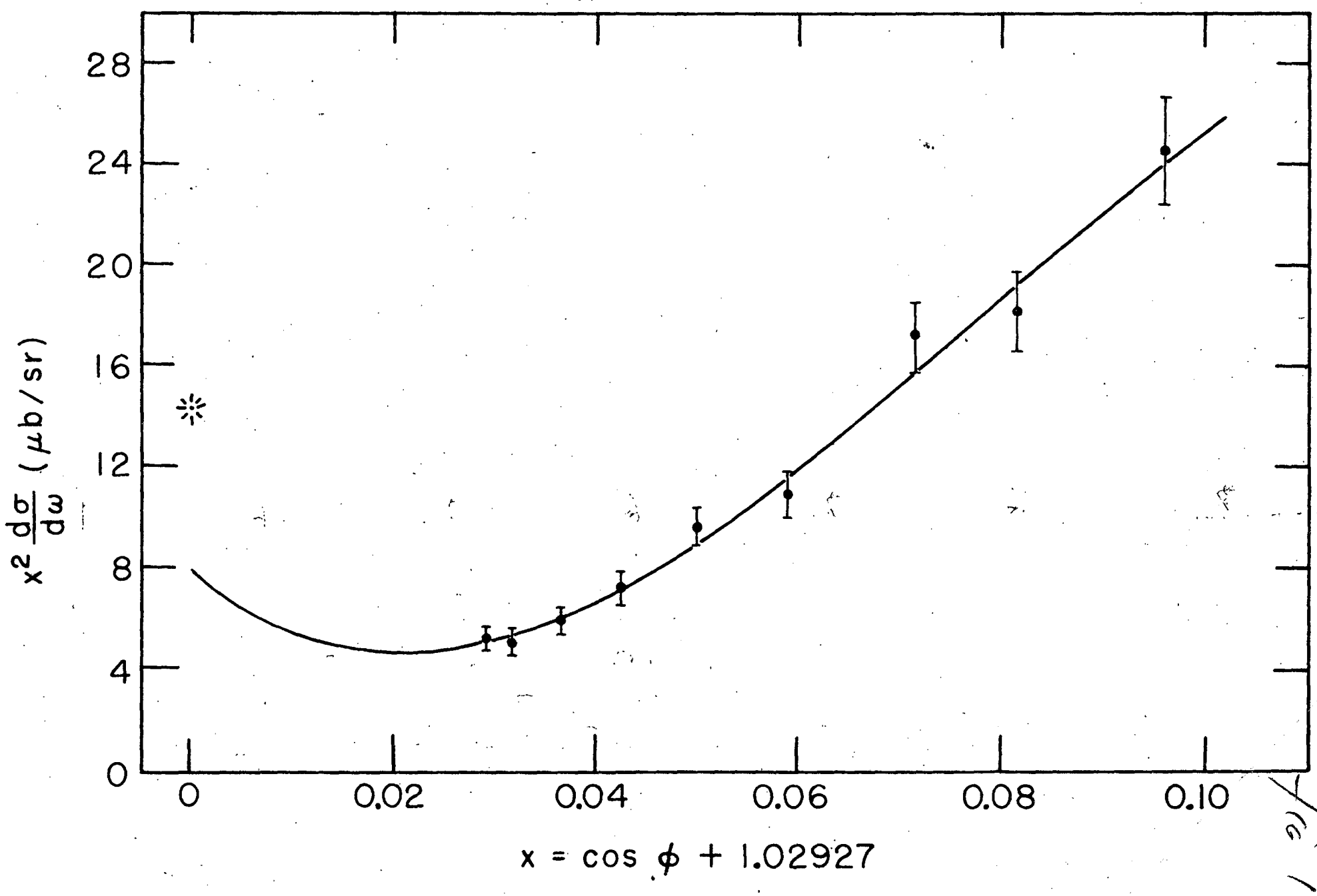
Table III. Results of least-square fit to $(x+a)^2 x^2 \frac{d\sigma}{d\pi}$ at 710 Mev

m_π (meson masses)	Order of best fit, n	$(q/M)_{min}$	P(n)	f^2
1	3	0.576	0.64	
1.5	2	0.562	0.55	0.154 ± .014
2	2	0.522	0.97	0.085 ± .011
2.3	2	0.527	0.76	0.070 ± .010
2.5	2	0.556	0.69	0.063 ± .011
3	2	0.575	0.64	0.053 ± .011
3.5	2	0.571	0.50	0.047 ± .011
4	2	0.600	0.46	0.043 ± .011
4.5	2	0.616	0.43	0.041 ± .012
5	2	0.626	0.38	0.038 ± .012

FIGURE LEGENDS

Fig. 1. Plot of $x^2 \frac{d\sigma}{d\omega}$ at 710-Mev versus $x = 1 + \cos\phi + \frac{\mu^2}{MT}$. The end of the physical region (180 deg) is $x = 0.02927$. The solid curve is the cubic fit to the data. The asterisk denotes the residue corresponding to $f^2 = 0.08$.

Fig. 2. Statistical parameters of the best fit for a given m_i, versus m_i. m_i is the total energy of the intermediate state in units of the charged meson mass. Circles are values of $(q/M)_{min}$, triangles are corresponding values of $P(n)$.


28
24
20
16
12
8
4
0
$x^2 \frac{d\sigma}{d\omega}$ (μb/sr)
0
0.02
0.04
0.06
0.08
0.10
x = cos ϕ + 1.02927

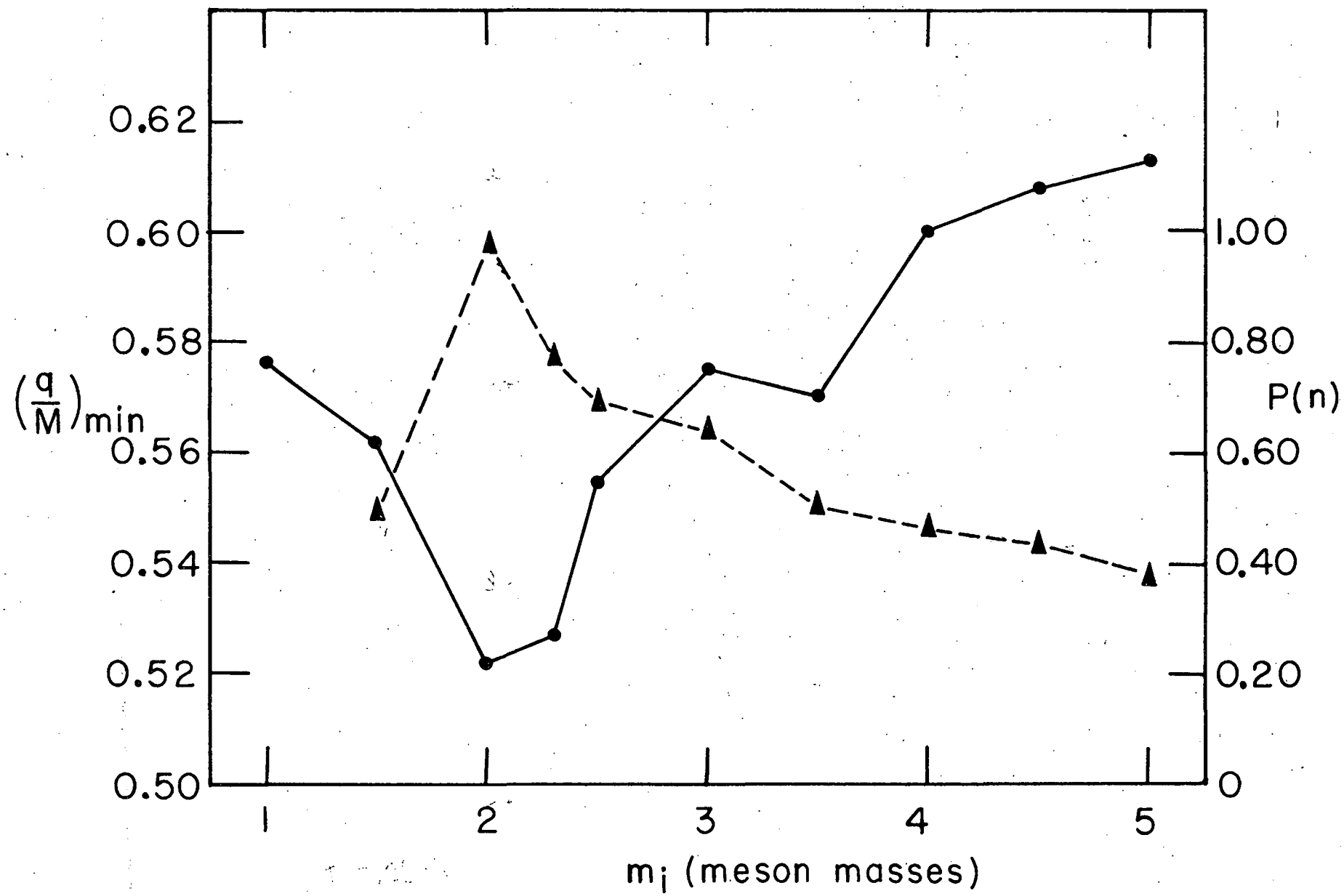

0.62
0.60
0.58
0.56
0.54
0.52
0.50
$\left(\frac{q}{M}\right)_{min}$
1.00
0.80
0.60
0.40
0.20
0
P(n)
1
2
3
4
5
m_i (meson masses)

Fig 3

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