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ABSTRACT

A variation of the Adair analysis permitting the simultaneous utilization of data at all production angles in S- and P-wave production of spin-3/2 resonances is given. Also the existence is noted of decay angular distributions that, except for normalization, are independent of the production angle, without limitation in the production states.

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The spin of resonances narrow enough to be considered quasi-stable particles can be determined by an analysis of the decay angular distribution of such of those particles as are produced in the forward direction, in the manner proposed by Adair.[1] The method suffers, however, because the range of directions that may be considered forward is not precisely defined and because only a fraction of the available data can be used, often rendering the statistical uncertainties large. When the production state is pure S-wave all production angles can be used and these drawbacks do not occur. Formulas for the decay angular distribution of spin-1/2 and spin-3/2 particles that include all production states have previously been given[2] and their applications to spin determination have been recently discussed.[3] However, these latter methods suffer from the fact that different production angles are treated separately: although averages over production angles can be made, no simple theoretical correlation between different productions angles is specified.

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The purpose of this note is to point out, for the case of S- and P-wave production, which seems to occur frequently, that one may obtain relationships permitting data at all production angles to be simultaneously used in identifying spin-3/2 resonances. The case considered is the joint production of a spin-zero and a spin-3/2 particle in the collision of a spin-zero and an unpolarized spin-1/2 particle, and the subsequent decay of the spin-3/2 particle into a spin-zero and a spin-1/2 particle. (Parity is assumed conserved, as we are interested in strong interactions. The intrinsic parities of the various particles turn out to be irrelevant.) If the decay angular distribution is given by $W(\Theta;\ \Theta,\ \Phi)$, and the production angular distribution by $I_0(\Theta)$, then[2]

$$
\begin{aligned}
4\pi\, I_0(\Theta)\, W\,(\Theta;\Theta,\Phi) = {} & I_0(\Theta) \\
& + \{\alpha + (\beta + \delta)\cos\Theta + (\gamma + \epsilon + \rho)\cos^2\Theta\}\,(3\cos^2\Theta_z - 1) \\
& + \{\delta + (\epsilon + 2\rho)\cos\Theta\}\,(3\cos\Theta_z\cos\Theta_x)\sin\Theta \\
& + \rho\sin^2\Theta\,(3\cos\Theta_x - 1),
\end{aligned}
$$

where Θ_z and Θ_x are the polar decay angles with respect to polar axes along the z and x directions, respectively. The z axis is along the incident production direction and the y axis is the production normal. The constant coefficients $\alpha, \ldots, \rho$ are given in terms of the combinations of production matrix elements

$$g_0 = \frac{\lambda}{4\pi} \left[\frac{1}{2} R_{11}^{1/2} + \sqrt{\frac{5}{2}} R_{11}^{3/2} \right],$$

$$g_3 = \frac{\lambda}{4\pi} \left[R_{11}^{1/2} - \sqrt{\frac{2}{5}} R_{11}^{3/2} + 2\sqrt{\frac{3}{5}} R_{13}^{5/2} \right],$$

$$r_0 = \frac{\lambda}{4\pi} \left(-\sqrt{2} R_{02}^{3/2} \right),$$

$$r_1 = \frac{\lambda}{4\pi} \left(-\sqrt{15} R_{13}^{5/2} \right)$$

by

$$\alpha = \frac{3}{8} | g_3 |^2 + \frac{1}{2} | r_0 |^2 - \frac{1}{2} \operatorname{Re} g_0 g_3^* - \frac{1}{2} | g_0 |^2,$$

$$\beta = \operatorname{Re} r_0 r_1^* + \frac{1}{2} \operatorname{Re} g_3 r_0^* - \operatorname{Re} g_0 r_0^*,$$

$$\gamma = \frac{1}{2} | r_1 |^2 + \frac{1}{2} \operatorname{Re} g_3 r_1^* - \operatorname{Re} g_0 r_1^*,$$

$$\delta = \frac{1}{2} \operatorname{Re} g_3 r_0^* + \operatorname{Re} g_0 r_0^*,$$

$$\epsilon = \frac{1}{2} \operatorname{Re} g_3 r_1^* + \operatorname{Re} g_0 r_1^* - \frac{1}{4} | g_3 |^2 + | g_0 |^2,$$

$$\rho = \frac{3}{8} | g_3 |^2 + \frac{1}{2} \operatorname{Re} g_0 g_3^* - \frac{1}{2} | g_0 |^2,$$

and the production angular distribution is given by

$$I_0(\theta) = A + B \cos \theta + C \cos^2 \theta,$$

where

$$A = | g_0 |^2 + \frac{3}{4} | g_3 |^2 + | r_0 |^2 ,$$

$$B = + 2 \text{ Re } r_0 r_1^* + 2 \text{ Re } g_3 r_0^* ,$$

$$C = - | g_0 |^2 + \frac{1}{4} | g_3 |^2 + | r_1 |^2 + 2 \text{ Re } g_3 r_1^* .$$

These formulas are given for the case in which the intrinsic parity difference is even, but the other case is obtained by replacing $L = J \pm 1/2$ by $L = J \mp 1/2$ in the production matrix elements $R_{L'L}^{J}$.

Inserting the easily verified relations

$$\beta + \delta = \frac{1}{2} B,$$

$$\gamma + \epsilon + \rho = \frac{1}{2} (A + C) - \alpha,$$

one obtains, after averaging over Φ_z, the z azimuthal angle,

$$\langle 4\pi \, I_0 \, W \rangle_{\Phi_z} =$$

$$I_0(\theta) \left(\frac{3}{2} \cos^2 \theta_z + \frac{1}{2} \right) - (A - 2\alpha + \rho) \sin^2 \theta \left(\frac{3}{2} \cos^2 \theta_z - \frac{1}{2} \right),$$

where

$$(A - 2\alpha + \rho) = \frac{3}{2} \left| g_0 + \frac{1}{2} g_3 \right|^2 .$$

Since the production distribution can be considered known, this gives the Φ_z-averaged decay distribution in terms of a single (positive) parameter. By likelihood methods the most likely value of the parameter $(A - 2\alpha + \rho)$ can be found, and the likelihood that the process is consistent with the assumption of S- and P-wave production of spin-3/2 particle can be determined.

Also, one can average instead over Φ_x, the azimuthal angle about the x axis. Then one finds

$$\langle 4\pi\, I_0(\theta)\, W\rangle_{\Phi_x} = I_0(\theta)\left(\frac{5}{4} - \frac{3}{4}\cos^2\theta_x\right) + \left(\frac{A}{2} - \alpha + 2\rho\right)\sin^2\theta\left(\frac{3}{2}\cos^2\theta_x - \frac{1}{2}\right),$$

which is again a one-parameter formula.

The reason for performing the azimuthal average is to bring the formulas to one-parameter forms; a likelihood analysis using the general six-parameter expression would not be expected to yield significant results when data are scarce. However, it is not necessary to average always with a weighting factor of unity. Averaging with the weighting factor $\cos 2\Phi$ gives experimentally independent information and also leads to one-parameter forms. One finds

$$\langle 4\pi\, I_0\, W \cos 2\Phi_z\rangle_{\Phi_z} = (3/4)\,\rho \sin^2\theta \sin^2\theta_z,$$

$$\langle 4\pi\, I_0\, W \cos 2\Phi_x\rangle_{\Phi_x} = (3/8)\,\{I_0 + (2\alpha - A)\sin^2\theta\}\sin^2\theta_x\,.$$

The parameters appearing in these formulas, which can also be determined by likelihood methods, are the same as those in the first two formulas.

The first of the two formulas above is particularly simple and restrictive. If ρ is large enough the distribution will show, in a folded two-dimensional histogram, a peaking at the corner where $\theta = \theta_z = 0$. Except for the over-all normalization, the dependence on both variables is completely specified, no parameter appearing. The second formula is similar to the Adair formula in that the decay distribution is completely specified for forward production, but it is simpler because with increasing production angle only the normalization changes, not the form.

If one averages rather with $\cos \Phi_x$ or $\cos \Phi_y$ the result, for either case, is

$$\langle 4\pi I_0 W \cos \Phi \rangle_\Phi = (3/2) \{\delta + (\epsilon + 2\rho) \cos \theta\} \sin \theta \sin \theta \cos \theta.$$

Here again, except for normalization, the dependence on θ is completely specified and is independent of the production angle θ. The production-angle dependence is determined by two parameters. However, the same function enters for the average over both Φ_x and Φ_y.

With the inclusion of higher partial waves the coefficients α, $\beta, \cdots, \rho$ and A, B, C become functions of the production angle, θ, but this does not alter the uniquely specified decay angular distributions given in the three above equations.

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