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The possibility of pion-hyperon resonances associated with "bound states" of the $\bar{K}$-nucleon system was pointed out in some detail several years ago.[1] In view of the clear evidence, recently reported,[2] of a π-Λ resonance at 1385 Mev, it may be of interest to present a simple physical model which may clarify the meaning of the description "bound-state resonance" and which allows a simple picture of its production process qualitatively consistent with all the evidence at present available.

We consider wave functions[3] ψ, ϕ representing the $\bar{K}$-N and π-Y channels respectively (Y denotes Λ, Σ hyperons), and satisfying the boundary conditions

$$(r\psi)_{r=0} = \alpha \frac{d}{dr}(r\psi)\Big|_0 + \beta \frac{d}{dr}(r\phi)\Big|_0 , \tag{1a}$$

$$(r\phi)_{r=0} = \beta^{+} \frac{d}{dr}(r\psi)\Big|_0 + \gamma \frac{d}{dr}(r\phi)\Big|_0 . \tag{1b}$$

These boundary conditions represent a zero-range approximation to the interactions within and between these channels. Their form is dictated by the linearity and hermiticity of the Schrodinger equation. In the $I = 1$ channels, both π-Λ and π-Σ systems participate and ϕ is a 1x2 column matrix, β

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β a 2x1 row matrix, and γ a 2x2 square matrix; α, β, and γ are real if time-reversal invariance holds.

For $\bar{K}$-N elastic scattering, the π-Y channels have outgoing waves, so

$$r\phi = (2m_Y)^{-1/2} \; c \, e^{iqr} \quad . \tag{2}$$

Substituting this dependence in (1b), we may eliminate ϕ from these boundary conditions, with the result

$$(r\psi)_0 = [\alpha + i\,(\beta q\,(1 - i\gamma q)^{-1}\,\beta^{\dagger})]$$

$$= (a + ib)\,\frac{d}{dr}\,(r\psi)\Big|_0 \quad . \tag{3}$$

This is the boundary condition introduced by Jackson et al.[4] for discussion of $\bar{K}$-N scattering.

When the imaginary part b is small--i.e., the cross-channel interaction parameter β is small (we argue below that γ is not large)--we may approximate the wave function of the system by first neglecting β. If α is negative, Eq. (1a) then allows an eigensolution

$$\psi = \frac{1}{\sqrt{4\pi\mu_K\,|\alpha|}}\;\frac{\exp(-r/|\alpha|)}{r} \;, \tag{4}$$

representing an $S_{1/2}$ $\bar{K}$-N bound state of mass $M^* \approx M_N + m_K - \alpha^2/2\mu_K$, where μ_K denotes the $\bar{K}$-N reduced mass. However, this state is linked with the energetically available π-Y channels. From (1b), the wave function ϕ

corresponding to the eigensolution ψ of Eq. (4) can be obtained and the rate of outflow in the π-Y channels can then be computed. This outflow leads to a finite lifetime for the $\bar{K}$-N bound state, corresponding to a half-width

$$\Gamma/2 = 2\pi\, C^{\dagger} q\, C = \frac{1}{\mu_K\, |\alpha|^3} \left\{ \beta (1 + i\gamma)^{-1}\, q\, (1 - i\gamma q)^{-1}\, \beta^{\dagger} \right\} . \tag{5}$$

From (3), this bracket is equal to the imaginary part b of the $\bar{K}$-N scattering length, and we have the simple result[5],

$$\Gamma/2 = b/(\mu_K\, |\alpha|^3). \tag{6}$$

As is physically reasonable, this width will be small if b is small (i.e., the reaction $\bar{K} + N \rightarrow Y + \pi$ has a low rate) or if $|\alpha|$ is large (corresponding to a diffuse structure for the $\bar{K}$-N bound state).

The set of $\bar{K}$-N scattering parameters which corresponds to the possibility of an I = 1 bound state is the (a-) set of Dalitz and Tuan,[6] for which b_1 is small and a_1 is large and negative. The present K^--p data at low energies lead to $a_1 + ib_1 = -1.08\ (\pm 0.2) + i\ 0.20\ (\pm 0.06)$ fermi.[7] With neglect of the energy dependence of these parameters,[8] we then have for the $\bar{K}$-N bound-state parameters (with $\alpha \approx a$, neglecting γ),

$$E_r = M_N + m_K - 1/(2\mu_K a^2) = 1382\ (\pm 20)\ \text{Mev},$$
$$\Gamma/2 = b/\mu_K a^3 \approx 18\ \text{Mev}. \tag{7}$$

The radius parameter of the $\bar{K}$-N state (to be compared with 4.3 fermis for the deuteron) is then $|a| = 1.08$ fermis, so that it is indeed quite diffuse.

To discuss the shape of the π-Λ resonance and the Σ/Λ ratio at resonance, we return to Eqs. (1a, b). For $E < M_N + m_K$, we have

$$r\,\psi = F(2\mu_K)^{-1/2}\, e^{-\kappa r} \tag{8}$$

where $\kappa = \left\{2\mu_K(M_N + m_K - E)\right\}^{1/2}$. Just as we previously eliminated ϕ, ψ can now be eliminated from the boundary conditions, leading to the boundary condition for ϕ,

$$(r\phi)_0 = \Gamma \frac{d}{dr}(r\phi)\,|_0 = \left(\gamma - \frac{\kappa\,(\beta\beta^{\dagger})}{1 + \kappa\alpha}\right) \frac{d}{dr}\,(r\phi)\,|_0 . \tag{9}$$

We note that, at the resonance energy, the coefficient Γ becomes infinite, as is physically appropriate. The first term of Γ represents "potential scattering": if γ were large, the resonance would have an asymmetrical form of the type well known in neutron-nucleus resonance scattering.[9] Since this effect is not apparent in the data, we believe that γ is not large (as appears very reasonable for a pion-hyperon state with $j = \frac{1}{2}$), and we will neglect it here compared with the resonance term. The pion-hyperon scattering phases may be obtained by diagonalizing $\Gamma' = q^{1/2}\,\Gamma\,q^{1/2}$, with the result

$$\tan\,\delta_r = \frac{\kappa(\beta_\Sigma^{\,2}\,q_\Sigma + \beta_\Lambda^{\,2}\,q_\Lambda)}{1 + \kappa\alpha}, \qquad \tan\,\delta_{nr} = 0, \tag{10}$$

where δ_r, δ_{nr} denote the resonant and nonresonant phase shifts, respectively.

The resonant state has the form

$$| \pi Y, \text{ resonant} \rangle = \left\{ \beta_\Lambda \, q_\Lambda^{1/2} \, | \Lambda\pi \rangle + \beta_\Sigma \, q_\Sigma^{1/2} \, | \Sigma\pi \rangle \right\} , \quad (11)$$

and the total π-Λ cross section is

$$\sigma_{tot}(\pi\Lambda) = \frac{4\pi}{q_\Lambda^2} \frac{(\pi b_\Lambda)^2}{(1 + \pi a)^2 + (\pi b)^2} , \quad (12)$$

where

$$b = b_\Sigma + b_\Lambda = \beta_\Sigma^2 \, q_\Sigma + \beta_\Lambda^2 \, q_\Lambda$$

is generally energy-dependent.

Experimentally,[2] the Σ/Λ ratio at resonance is believed to be less than 10%. From (11), this ratio is given by $R_r = (\beta_\Sigma/\beta_\Lambda)_r^2 (q_\Sigma/q_\Lambda)_r$. In the present model, an estimate of this ratio depends on a normalization in terms of the Σ/Λ ratio, $R_t = (\beta_\Sigma/\beta_\Lambda)_t^2 (q_\Sigma/q_\Lambda)_t$, for the $I = 1$ channels at the $\bar{K}$-N threshold. The measured value[10] is $R_t = 1.0 \pm 0.8$; a lower limit, $R_t > 0.30 \pm 0.06$, may be obtained from the Σ^-/Σ^+ ratio and the Λ-π^0 rate in K^--p capture at rest. If ℓ_Σ, ℓ_Λ denote the orbital angular momenta in the π-Σ and π-Λ channels, the resonance ratio will be $R_t (q_\Sigma/q_{\Sigma t})^{2\ell_\Sigma+1} / (q_\Lambda/q_{\Lambda t})^{2\ell_\Lambda+1}$ if the energy-dependence of β_Σ, β_Λ is controlled by the centrifugal barriers, that is, $0.8 \, R_t$ with odd parities for K-Σ and K-Λ, $0.5 \, R_t$ for even parities. We see that it is difficult to obtain a ratio as small as that observed unless the amplitudes β_Σ and β_Λ have some energy dependence beyond that due to the centrifugal barrier effect.

With the $\bar{K}$-N bound-state interpretation, the production of the π-Λ resonant state,

$$\bar{K} + N \rightarrow (\bar{K}N) + \pi, \qquad (\bar{K}N) \rightarrow \Lambda + \pi, \tag{13}$$

is closely analogous with the well-known process of deuteron formation in nucleon-nucleon collisions,

$$N + N \rightarrow (NN) + \pi . \tag{14}$$

We note that, for the K^- energies of interest, the pion produced in Reaction (13) travels a distance of approx. 6 fermis in one mean lifetime of the $(\bar{K}N)$ bound state. For the same reason that p-wave pion production is dominant[11] in Reaction (14) above final c.m. momenta ≈ 50 Mev/c, we expect that p-wave pion production will be strong in Reaction (13). This appears consistent with the excitation curve observed.

Further, Reaction (14) is known to take place predominantly in those states where the π-N (3,3) resonant interaction can be effective. It is reasonable to expect similarly for Reaction (13) that the (3,3) resonance will play in important role in determining the dominant production states. With this assumption, Y^* production must take place primarily from $\Pi = 1$ initial $\bar{K}$-N states, and we then have the following:

(a) $Y^{*+}/Y^{*-} \approx 1$ for the K^--p reaction, as is consistent with observation except at the highest energy (for 1150 Mev/c K^- mesons, an energy which is somewhat beyond the (3,3) resonance region); and

(b) since $I = 1$ holds for the K^--n and $\bar{K}^0$-p systems,

$$\sigma(K^- + n \to \pi^0 + Y^{*-})[= 2\sigma(K_2^0 + p \to \pi^0 + Y^{*+}] = \sigma(K^- + p \to \pi^- + Y^{*+})$$

$$+ \sigma(K^- + p \to \pi^+ + Y^{*-}). \qquad (15)$$

Further,

(c) characteristic correlations then predicted for the (E_+, E_-) plot of the $\Lambda + \pi^+ + \pi^-$ events appear to be qualitatively consistent with the observations.[13]

Also, as for the π^+-d system, the (3,3) resonance will lead,[14] on the present model, to a rather broad "resonance" in the π-Y^* system for total energy $\approx$ 1650 Mev.

In conclusion, we remark that the evidence in support of global symmetry from the study of hyperon-nucleon interactions is now quite strong.[15] As Amati et al. have emphasized recently, the occurrence of an $I = 1$, $j = \frac{3}{2}$ π-Λ resonance is then to be expected.[16] The location (1375 Mev), the half-width (approx. 27 Mev) and the Σ/Λ ratio predicted are all compatible with the Y^* observations. However, it must be remembered that the influence of the near-by $\bar{K}$-N channel on the location of this $j = \frac{3}{2}$ resonance may be considerable, and that this resonance may well be displaced to some higher energy value.

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3. For convenience in the later expressions, we normalize each wavefunction to $(2\omega)^{-1}$, where $\omega = \omega_1\omega_2/(\omega_1 + \omega_2)$ is the "reduced energy" in the corresponding two-particle channel.

4. Jackson, Ravenhall and Wyld, Nuovo cimento 9, 834 (1958).

5. This formula is essentially equivalent to that for the lifetime width of the K^--p Coulomb 1s state, after b is replaced by $\frac{1}{2}(b_0 + b_1)$ and α by the appropriate Bohr radius. However, the difference in shape between (4) and the Coulomb wave function gives rise to an additional factor of 2.

6. R. H. Dalitz and S. F. Tuan, Ann. Phys. 8, 100 (1959).

7. For a discussion of their derivation, see R. H. Dalitz, Revs. Modern Phys. (to be published), Appendix A.

8. We are anxious to emphasize explicitly that there is little physical reason to expect these parameters a, b to be energy-independent. On the contrary, as we note later, a decrease in b with decreasing energy may be expected from phase-space and centrifugal-barrier considerations. However, we have at present no clear guidance concerning what energy dependence to assume, since this will depend on the adoption of a specific model. Ross and Shaw (Ref. 1) have assumed an effective range expansion, but the present data are consistent with zero-effective range.

9. See, for example, J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (John Wiley and Sons, New York, 1952), pp. 401 and 468-9. We note that a small value of γ has quite a strong effect on the shape, and especially on the symmetry, of this resonance, but that it has little effect on its location or width.

10. L. Alvarez, The Interactions of Strange Particles, Lawrence Radiation Laboratory Report UCRL-9354, August 1960.

11. See, for example, M. Gell-Mann and K. M. Watson, Ann. Rev. Nucl. Sci. 4, 219 (1954).

12. Note that the $\bar{K}$ meson cannot emit a single pion, although it can scatter pions emitted from the nucleon. This can give rise to some s-wave production. Such scattering appears essential if (3,3) π-N resonant enhancement is to occur.

13. As shown by calculations at present being carried out jointly with Dr. D. H. Miller and which will be reported at a later time.

14. We note that, in the $\pi + (\bar{K}N)$ state with $I = 1$, $m_I = 0$, the π-N (3,3) resonance has a weighting factor only one-half of that in the π-d state. As a result, the "resonance" effect is not expected to be as marked as is observed in the latter case. Further, because of the large $\bar{K}$-N binding energy, this resonance will be greatly broadened relative to the (3,3) resonance, especially on the high-energy side.

15. R. H. Dalitz, Revs. Modern Phys., to be published (1961).

16. Amati, Vitale, and Stanghellini, Phys. Rev. Letters 5, 524 (1960).