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Radiation Laboratory

TWO PROBLEMS OF STRUCTURE IN THE THEORY OF WEAK INTERACTIONS

PART I: RADIATIVE PION DECAY INTO ELECTRONS

PART II: ELECTROMAGNETIC PROPERTIES OF A CHARGED VECTOR MESON INTERMEDIARY IN WEAK INTERACTIONS

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James Allan Young

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PART I: RADIATIVE PION DECAY INTO ELECTRONS

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PART I: RADIATIVE PION DECAY INTO ELECTRONS

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February 8, 1961

ABSTRACT

The possibility of distinguishing the pion structure-dependent radiation from the conventional inner bremsstrahlung radiation in the radiative decay of pions into electrons is discussed. Calculation of the photon energy spectrum and angular correlation shows that evidence for pion structure would be obtained if any photons of energy less than 70 Mev were detected in 180° coincidence with π-decay electrons. The probability of such events per unit solid angle is $\geqslant 0.2\times10^{-7}$ relative to ordinary $\pi \rightarrow \mu + \nu$ decay, if the assumption of a conserved vector current is made to relate the rate of radiative decay through the weak V-interaction to the rate of $\pi^0 \rightarrow 2\gamma$ decay.

I. INTRODUCTION

The universal V-A form of the Fermi interaction has in recent years been suggested by the evidence in β and μ decay. The other weak interactions are then, in principle, consequences of strong couplings together with the universal Fermi interaction. In the decay of π mesons into electrons, where the momentum transfer is large, evidence on the decay mechanism can be obtained,[1,2] in principle, by observing the associated radiative decay $\pi \to e + \nu + \gamma$. In this paper we amplify the calculation by Vaks and Ioffe[1] and discuss the possibility of distinguishing structure-dependent effects from less interesting structure-independent effects. We supplement the electron spectrum already presented[1,3] by calculating the photon spectrum, which may be more easily observed experimentally.

The diagrams for the radiative decay are given in Fig. 1. Diagrams (a) and (b), when defined in a gauge-invariant way, give rise to the inner bremsstrahlung by a decelerated or accelerated charge or magnetic moment. The matrix element for this is proportional to $eGm/\sqrt{k}$, where e and G are the electromagnetic and Fermi coupling constants, m is the electron (or muon) mass, and k the photon energy. Diagrams (c) and (d) of Fig. 1 are structure-dependent, since here the emission of a photon depends on the nature of the "black box." The matrix elements for these diagrams are proportional to $eG\sqrt{k}\,(\mu/M)$, where μ is the pion mass and M a mass or energy typical of the intermediate states involved in the "black box." The two processes—inner bremsstrahlung and "black box" (or structure-dependent) radiation—are coherent, but the interference termis negligible in $\pi \to e + \nu + \gamma$ decay. (In $\pi \to \mu + \nu + \gamma$ the reverse is the case: because of the small momentum transfer involved, the structure-dependent radiation is small compared with the inner bremsstrahlung, and the interference term dominates the square of the structure-dependent matrix element. For this reason radiative $\pi \to \mu$ decay, although more frequent by several orders of magnitude than radiative $\pi \to e$ decay, reveals nothing indicative of

the pion decay structure.) The interesting question is not whether radiative $\pi^{\pm}$ decay occurs, but whether the interesting structure-dependent effects can be disentangled from the ordinary quasiclassical bremsstrahlung. We find that a unique proof of structure to the π-decay mechanism can be obtained if any photons of energy less than $k_{max} = 70$ Mev are detected in 180° correlation to the direction of the decay electron. The probability of such a decay per unit solid angle per π decay is, however, approximately 0.2×10^{-7}.

II. INNER BREMSSTRAHLUNG (IB)

The matrix element for the inner bremsstrahlung is defined as the gauge-invariant part of diagrams (a) and (b) of Fig. 1. On invariance grounds this is of the form

$$M_{IB} = e\,(m/\sqrt{k})\,f_A(\mathcal{P}^2)\,\bar{\psi}_e\,[\,(p\cdot\epsilon/p\cdot k - \mathcal{P}\cdot\epsilon/\mathcal{P}\cdot k) + i\,\sigma_{\mu\nu}F_{\mu\nu}/4p\cdot k]\psi_\nu,$$

where $f_A(\mathcal{P}^2)$ is the amplitude for the nonradiative decay, $\mathcal{P}$ is the pion four-momentum, p the electron four-momentum, ϵ the photon polarization four-vector, $F_{\mu\nu} = \epsilon_\mu k_\nu - \epsilon_\nu k_\mu$, and ψ_e and ψ_ν are, respectively, the electron (or muon) and neutrino field operators. The two terms in M_{IB} correspond to emission of radiation by the accelerated charge and magnetic moment, respectively.

This matrix element leads to the differential transition probability,

$$d^3W_{IB} = W_{e\nu}\,\frac{\alpha}{(2\pi)^3}\,\frac{1}{\mu^2}\left\{\mu^2[\vec{p}^{\,2} - (\vec{p}\cdot\vec{k})^2/k^2] + 2(Ek - \vec{p}\cdot\vec{k})(\mu k - Ek + \vec{p}\cdot\vec{k})\right\} \times \frac{d^3p\,d^3k}{(Ek - \vec{k}\cdot\vec{p})^2}\;\frac{\delta(\mu - E - E_\nu - k)}{EE_\nu k}, \qquad (1)$$

where $\alpha = e^2/4\pi$ is the fine-structure constant, E the electron energy, E_ν the neutrino energy, and $W_{e\nu}$ the nonradiative decay rate. The electron energy spectrum resulting from this expression has been given previously[1,3] and is not repeated here.

We suspect that, because of the overwhelming background of $\pi \to e + \nu$ and $\pi \to \mu \to e$ electrons, the photon radiation (or at least the hard component in which we are interested) may be more easily distinguished than the spectrum of electrons. The spectrum of photons into solid angle $d\Omega = 2\pi \sin\theta d\theta$, where θ is the angle between the photon

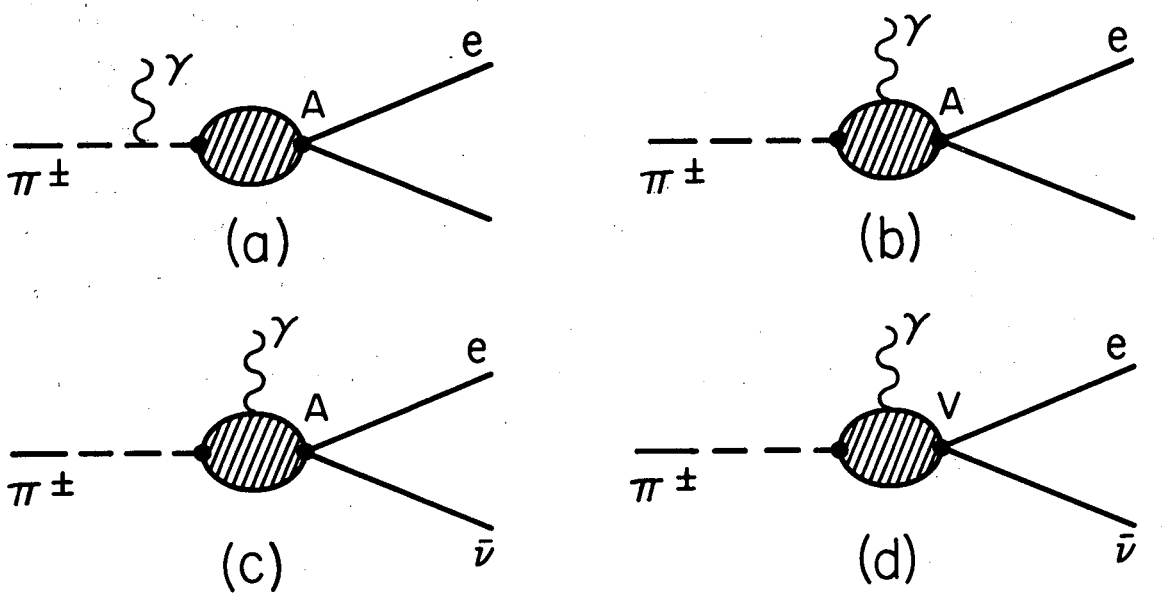

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Fig. 1. The possible diagrams for the radiative electron decay of the pion.

and the electron, is obtained by integrating Eq. (1) over electron energies. This relation is generally complicated by the energy condition $\mu = E + k + |\vec{k} + \vec{p}|$.

If the photon and electron emerge in opposite directions so that $\theta = 180^\circ$ and $|\vec{p}+\vec{k}|^2 = (p-k)^2$, the energy condition becomes $(\mu - E - p)$ $(\mu - E + p - 2k) = 0$. The photon spectrum obtained from Eq. (1) is then

$$\left(\frac{d^2 W_{IB}}{dk d\Omega}\right)_{\theta = 180^\circ} = W_{e\nu} \frac{\alpha p^2 k}{\mu^2} \frac{(\mu - E - p)}{(E + p)[p\mu - k(p+E)]}, \qquad (2)$$

where p and E are determined by the energy condition. Now $\mu - E - p = 0$ unless $k = (\mu - E + p)/2$ which lies between $(\mu - m_e)/2$ and $k_{max} = (\mu^2 - m_e^2)/2\mu$. Thus according to Eq. (2), the only inner bremsstrahlung photons at $\theta = 180^\circ$ are those of essentially maximum energy k_{max}. The probability for the emission of such photons is, according to Eq. (2) (setting $k = k_{max}$),

$$\left(\frac{d^2 W_{IB}}{dx d\Omega}\right)_{\theta = 180^\circ,\ k = k_{max}} = W_{e\nu} \frac{\alpha}{(2\pi)^2} \left(1 - \frac{m_e^2}{\mu^2}\right)^3 = 1 \text{ sec}^{-1},$$

where $k = xk_{max}$. The inner bremsstrahlung spectrum at 180° vanishes then except for photons of very near maximum energy.

For angles other than 180° (or 0°) the photon spectrum will be adequately described by setting $m_e = 0$. In this approximation, we obtain for any angle θ between photon and electron

$$\frac{d^2 W_{IB}}{dx d\Omega} = W_{e\nu} \frac{\alpha}{2\pi^2} \left(\frac{1 + \cos\theta}{1 - \cos\theta}\right) \frac{(x - 1)^2 + 1}{[2 + x(\cos\theta - 1)]^2} \frac{1}{x}. \qquad (2a)$$

Equation (2a) is thus applicable to all angles except 0° and 180° provided $k \leq (\mu - m_e)/2 = 69.6$ Mev.

Integrating Eq. (2a) for photon energies greater than some low-energy cutoff δ, we obtain the electron-photon angular correlation,

$$dW_{IB}/d\Omega = W_{e\nu}(\alpha/2)\,(2\pi)^{-2}\,\lambda^{-3}$$

$$\times\{\lambda + (1-\lambda)(1-2\lambda^2)\ln(1-\lambda)$$

$$+2\lambda^2(1-\lambda)[\ln(1/x_{min} - 1]\}\ , \qquad (3)$$

where $\lambda = \sin^2\theta/2$ and $x_{min} = 2\delta/\mu$. The rate of $\pi \to e + \nu + \gamma$ decay per unit solid angle with e and γ at 180° to each other is

$$\frac{(dW_{IB}/d\Omega)\,\theta = 180^\circ}{W_{\mu\nu}} = 1.2\times10^{-8}/\text{steradian}. \qquad (3a)$$

Equation (3) agrees with Eq. (21) of Vaks and Ioffe, since when the minimum photon energy is δ the maximum electron energy is approximately $\mu/2 - \delta(1-\lambda)$, so that y_{max} in Vaks and Ioffe equals $1 - (1-\lambda)x_{max}$ above.

III. STRUCTURE-DEPENDENT (SD) RADIATION

Out of the pseudoscalar pion field operator ϕ and the electromagnetic field operator A_μ only two vectors $a\phi\tilde{F}_{\mu\nu}\mathcal{P}_\nu$ and $b\phi F_{\mu\nu}\mathcal{P}_\nu$ can be constructed in a gauge-invariant manner. Here $F_{\mu\nu} = A_\mu k_\nu - A_\nu k_\mu$, and

$$\tilde{F}_{\mu\nu} = (i/2)\epsilon_{\mu\nu\lambda\rho}F_{\lambda\rho}$$

is the tensor dual to $F_{\mu\nu}$, and a and b are functions of $\mathcal{P}\cdot k$, which must be, assuming PC invariance, relatively real. The gauge-invariant contribution of diagrams (c) and (d) of Fig. 1 must therefore be of the form,

$$M_V = i(\alpha)^{1/2}G_V a\phi\tilde{F}_{\mu\nu}\mathcal{P}_\nu\bar{\psi}_e\gamma_\mu 1/2\,(1+\gamma_5)\psi_\nu,$$
$$M_A = i(\alpha)^{1/2}G_A b\phi F_{\mu\nu}\mathcal{P}_\nu\bar{\psi}_e\gamma_\mu 1/2\,(1+\gamma_5)\psi_\nu\,. \tag{4}$$

Writing

$$q_\mu = p_\mu + p_\mu{}^\nu\,,\quad B_\mu = \bar{\psi}_e\gamma_\mu 1/2\,(1+\gamma_5)\psi_\nu,\quad G_{\mu\nu} = q_\mu B_\nu - q_\nu B_\mu\,,$$

one finds that Eqs. (4) take the form

$$M_V = -(i/2)(\alpha)^{1/2}G_V a\phi\tilde{F}_{\mu\nu}G_{\mu\nu},$$
$$M_A = (i/2)(\alpha)^{1/2}G_A b,\, F_{\mu\nu}G_{\mu\nu}. \tag{5}$$

The matrix element for the gamma decay of pseudo-scalar π^0 mesons is, on invariance grounds,

$$M_{\pi^0} = -(i/2)ac\phi\tilde{F}_{\mu\nu}F_{\mu\nu}\,, \tag{6}$$

from which the rate of π^0-decay is

$$W_{\pi^0} = (a^2/4)(2\pi)^{-8}\mu^3c^2\,, \tag{7}$$

where c is a constant depending on the pion decay structure. In lowest-order perturbation theory (where the "black box" in Fig. 1 stands for a nucleon-antinucleon loop, each of mass M, coupled to the pion field via

via $(g/\sqrt{2})\bar{\psi}_N \gamma_5 \tau \psi N \cdot \phi)$, we have[4]

$$c = a = 4(\pi)^{5/2} g/M . \tag{8}$$

This relation between the electromagnetic decay of the π^0 and the vector radiative decay of the $\pi^\pm$ holds, for photons of near maximum energy, to all orders if the Feynman-Gell-Mann principle of conservation of the weak vector current[5] is assumed. (Generally a is a function of k which equals c strictly only when $k = \mu/2$, the photon energy in π^0 decay. We are neglecting this possible energy dependence and setting $a = c =$ constant.) This assumption, which was also made in reference 1, determines the over-all rate of $\pi^\pm$ decay, and will be made in the remainder of this paper.

A. Photon Spectrum and Angular Correlation

The differential transition probability obtained from Eqs. (4) or Eqs. (5) is

$$\begin{aligned} d^3 W_{SD} = {} & (2G_V{}^2/\alpha)\,(2\pi)^{-5}\mu^{-2} W_{\pi^0} k \\ & \times [\,(1+\gamma^2)\,(1 - \beta\cos\theta\cos\phi) + 2\gamma(\cos\phi - \cos\theta)] \\ & \times \delta(E + E_\nu + k - \mu)\, d^3p\, d^3k, \end{aligned} \tag{9}$$

where β is the electron velocity, ϕ the angle between neutrino and photon and $\gamma = bG_A/aG_V$.[6]

From Eq. (9) we obtain the photon spectrum angular distribution

$$\begin{aligned} \frac{d^2 W_{SD}}{dx\,d\Omega} = {} & (G_V{}^2/4\alpha)\,(2\pi)^{-4}\mu^4 W_{\pi^0} \frac{x^3(1-x)^2}{[\,2 + x(\cos\theta - 1)]^4} \\ & \times \{\, 2[\,(1+\gamma^2)\,(1+\cos^2\theta) - 4\gamma\cos\theta] \\ & + x(x-2)\,(1+\gamma)^2(1-\cos\theta)^2\} . \end{aligned} \tag{10}$$

In the case of $\theta = 180^{\circ}$ Eq. (10) reduces to

$$\left(\frac{d^2 W_{SD}}{dx d\Omega}\right)_{\theta = 180^{\circ}} = (G_V^{\ 2}/a)\ (4\pi)^{-4}\mu^4 W_{\pi^0}\ (1 + \gamma)^2 x^3. \tag{11}$$

These equations show that, at least for $\gamma \sim 1$, structure-dependent radiation is predominantly hard and in the backwards direction. This is to be contrasted with the inner bremsstrahlung which is predominantly soft and in the forward direction. In lowest order perturbation theory b = a, and Eq. (10) gives for the angular distribution of photon and electron:

$$\begin{aligned} dW_{SD}/d\Omega = {} & (G^2/4a)\ (2\pi)^{-4}\mu^4 W_{\pi^0} \\ & \times\{ \lambda^2/4 + (1 - \lambda)\ \lambda^{-6}(-\lambda^3 + 15\lambda^2 \\ & - 45\lambda + 35)\ \ln\ (1 - \lambda) + (1 - \lambda)\ (12\lambda^5)^{-1} \\ & \times [\ 3\lambda^3(\lambda^3 + \lambda^2 + \lambda + 1) + 10\ (5\lambda^2 - 33\lambda + 42)\ ]\}\ . \end{aligned} \tag{12}$$

Equations (3) and (12) are plotted in Fig. 2, with the lower limit[7] assumed to be $0.5 \times 10^{16}\ \mathrm{sec}^{-1}$ for W_{π^0}. The electron-photon angular distribution is the superposition of two noninterfering mechanisms: (a) the inner bremsstrahlung, Eq. (3), and (b) the photon emission by the intermediate states in $\pi^{\pm}$ decay, Eq. (12). In calculation of the latter the rate of radiative decay through the V interaction has been related to the rate W_{π^0} of π^0 decay by the assumption of a conserved vector current. The decay through the A interaction can be related to that through the V interaction by choosing $\gamma = (b G_A)/(a G_V) = 1$ as is suggested by lowest order perturbation theory.[8]

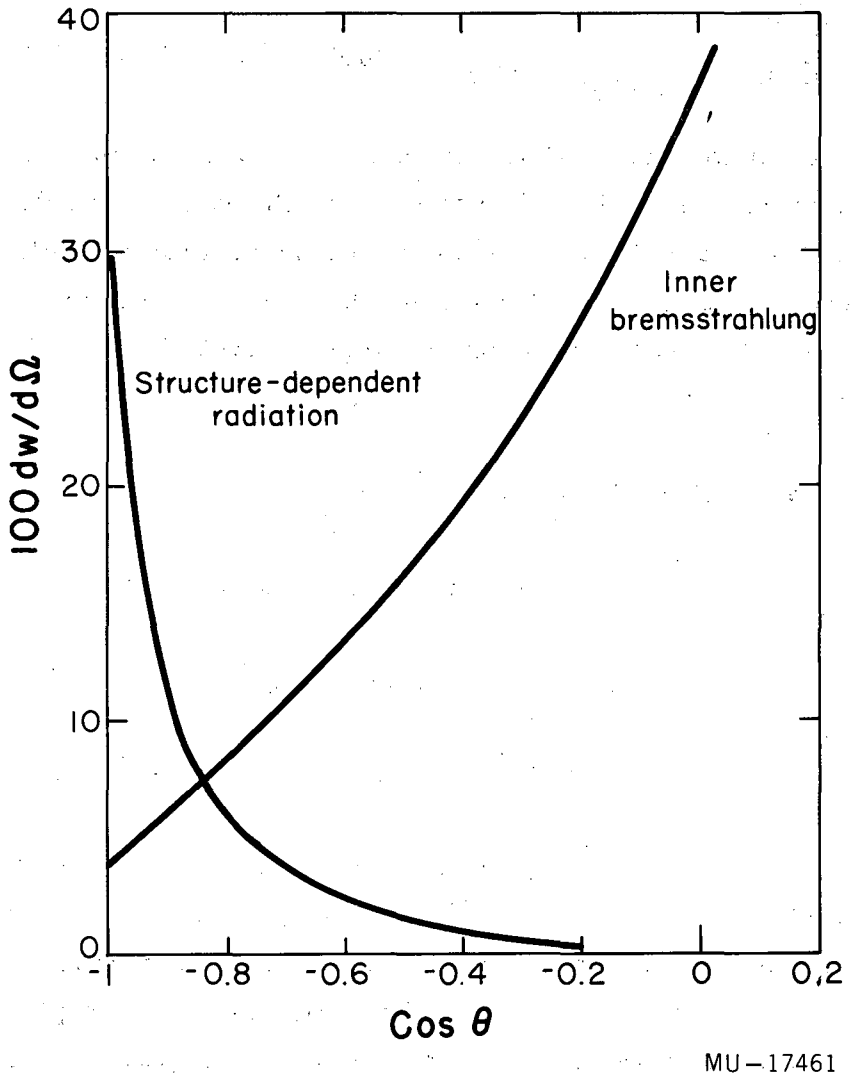

Fig. 2. Plot of the angular distribution between the photon and electron for inner bremmstrahlung and for structure-dependent radiation.

IV. DISCUSSION

We have seen that at $\theta = 180^{\circ}$ no inner bremsstrahlung photons occur of energy less than $(\mu - m_e)/2$, while structure-dependent photons of all energies occur. Thus if any photons are detected of energy less than 69.6 Mev when photon and electron are in anticoincidence there is unambiguous proof of structure mediating the $\pi^{\pm}$ decay. The number of such decays per unit solid angle about $\theta = 180^{\circ}$ per $\pi \rightarrow \mu + \nu$ decay is calculated, according to Eq. (12), to be

$$\frac{(dW_{SD}/d\Omega)_{\theta = 180^{\circ}}}{W_{\mu\nu}} \geqslant 0.2 \times 10^{-7}/\text{steradian} .$$

This conclusion rests on the assumption relating the V interaction $\pi \rightarrow e\nu\gamma$ matrix element to the rate of π^0 decay but is insensitive to the particular choice of the parameter b in the A interaction matrix element provided $b \neq -a$.

The lowest published upper limit on the rate of radiative $\pi \rightarrow e$ decay is that obtained by Cassels and collaborators.[8,9] This group measured the rate of electron-gamma production at $\theta = 180^{\circ}$ and used a calculated electron-photon angular correlation function to convert these measurements into a total rate of radiative $\pi \rightarrow e$ decay relative to normal $\pi \rightarrow \mu$ decay:

$$W_{e\nu\gamma}/W_{\mu\nu} \cong (3 \pm 5) \times 10^{-6} . \tag{13}$$

In the earlier work[9] the angular correlation function was calculated assuming an ST β-decay interaction, while in the later work[7] the V-A interaction was used but the interference term between V and A was neglected. From the rate (13) quoted in reference 9 and the fact that the relative probability for emission into the backwards direction $(1/W)dW/d(\cos\theta)$ was taken to be 3.0, we can work backwards to find

$$\frac{(dW_{e\nu\gamma}/d\Omega)_{\theta = 180^\circ}}{W_{\mu\nu}} = \frac{3.0}{2\pi} \quad (3 \pm 5) \times 10^{-6}$$

$$= (1.5 \pm 2.5) \times 10^{-6}/\text{steradian.}$$

Since we calculated

$$\frac{(dW_{IB}/d\Omega)_{\theta = 180^\circ}}{W_{\mu\nu}} = 1.2 \times 10^{-8} ,$$

and

$$\frac{(dW_{SD}/d\Omega)_{\theta = 180^\circ}}{W_{\mu\nu}} \gtrsim 2 \times 10^{-8}/\text{steradian,} \tag{13a}$$

the experimental sensitivity would have to be improved by two orders of magnitude to detect the interesting structure-dependent radiation. In looking for the structure-dependent radiation one should discriminate against the 70-Mev inner bremsstrahlung "line spectrum." If these photons are not observed one will have to conclude either (a) that the vector current is not conserved, and that if the pion decay structure involves baryons at all, the typical energies of the intermediate states involved probably exceed the nucleon rest mass, or (b) that the pion decay should be regarded as primary. In either case applying the idea of the universal Fermi interaction to other weak decays will have practically lost its attractiveness.

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$$dW_{SD}/dy = (G^2/3a)\,(2\pi)^{-3}2^{-5}\mu^4 W_{\pi^o} y^2 \times [\,6(1-2y)(1-\gamma)^2 + y^2(7(1+\gamma^2) - 10\gamma)\,],$$

where $y = E/E_{max}$.

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PART II. ELECTROMAGNETIC PROPERTIES OF A CHARGED VECTOR MESON INTERMEDIARY IN WEAK INTERACTIONS

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ABSTRACT

A systematic study is made of the electromagnetic properties of charged vector mesons (B mesons). The various formalisms used to describe charged particles of spin 1 are compared, and a new first-order formulation of the Stuckelberg formalism is given. For the most general first-order Proca Lagrangian, subject to the usual symmetry requirements we eliminate redundant components to obtain a Hamiltonian formulation. The theory is interpreted in the non-relativistic limit, and the terms corresponding to spin-orbit coupling and electric quadrupole-moment interaction are identified. The analogy to spin 1/2 theory has led us to consider classical spin equations of motion which agree with the quantum mechanical equations to order m^{-2}.

This general form for the electromagnetic interaction is applied to a recalculation of the $\mu \rightarrow e + \gamma$ decay rate through a vector meson intermediary. We conclude, on the basis of $\mu \rightarrow e$ conversion alone, that it is not necessary to abandon the intermediary B-meson hypothesis in weak interactions.

As a means of producing B-mesons, we propose searching for their pair production in the Coulomb field of a nucleus. By using the Weizsacker-Williams approximation, the pair-production cross section is calculated in the high-energy limit for vector mesons with gyromagnetic ratios unity and zero. This method of production is compared and contrasted with the alternative high-energy neutrino method of production.

I. INTRODUCTION

A charged vector meson has been proposed as an intermediary field (B field) in the weak interactions.[1] This meson must, if it exists, have a mass greater than that of the K meson and a very short lifetime. Against such an intermediary field, it has been argued that, provided the two neutrinos in μ decay are capable of annihilating each other, such a B field would allow the decay $\mu \rightarrow e + \gamma$ in first order in the μ-decay coupling constant G with a rate considerably larger than the upper limit experimentally observed. Here we wish to show, by a systematic study of charged-vector meson theory, how this rate actually depends on the electromagnetic interactions assumed for the vector meson, and to see if the absence of $\mu \rightarrow e$ conversion need exclude the intermediate-meson hypothesis.

An important and definitive test for the B meson is the possibility of detecting its production. Lee and Yang[2] have proposed searching for the B meson by looking for the onset of the semiweak process $\nu \rightarrow e + B$. We wish to propose another experiment for the production of B mesons not requiring high-energy neutrinos and whose cross section, instead of being semiweak (10^{-37} cm^2), is typically electromagnetic (10^{-31} cm^2). It is suggested that the pair production of $B^{\pm}$ mesons by high-energy photons in the Coulomb field of a nucleus is a possible method of observing the production of vector mesons, if such exist.

II. ELECTROMAGNETIC INTERACTIONS OF A CHARGED VECTOR MESON

A. Comparison of the Formulations of the Theory of Spin 1

1. First-Order Proca Equations

A first-order form of the Proca theory[3] is given by the Lagrangian

$$\mathcal{L} = \frac{1}{2} U^{+}_{\mu\nu}(\partial_\mu U_\nu - \partial_\nu U_\mu) + \frac{1}{2}(\partial_\mu U^{+}_\nu - \partial_\nu U^{+}_\mu) U_{\mu\nu} - \frac{1}{2} U^{+}_{\mu\nu} U_{\mu\nu} + m^2 U^{+}_\mu U_\mu \tag{2.1}$$

for the case of free fields. In Eq. (2.1) $U_\mu(x)$, $U_{\mu\nu}(x)$ are independent field variables, $U^{+}_\mu(x)$, $U^{+}_{\mu\nu}(x)$ are the Hermitian conjugate fields, and m is the mass. The above Lagrangian gives the free field equations

$$U_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu ,$$

$$\partial_\mu U_{\mu\nu} = m^2 U_\nu .$$

In the presence of an electromagnetic field we perform the usual gauge-invariant replacement[4] $\partial_\mu \to \pi_\mu = \partial_\mu - i e A_\mu$, where $A_\mu(x)$ is the electromagnetic four-potential, which yields the field equations

$$U_{\mu\nu} = \pi_\mu U_\nu - \pi_\nu U_\mu , \tag{2.2}$$

$$\pi_\mu U_{\mu\nu} = m^2 U_\nu \tag{2.3}$$

The second-order wave equation

$$(\pi^2 - m^2) U_\nu - \pi_\mu \pi_\nu U_\mu = 0 \tag{2.4}$$

is obtained by substituting Eq. (2.2) into Eq. (2.3). Since a four-vector field must actually possess only three independent components, a subsidiary condition eliminating the unwanted fourth component is needed. This is most easily obtained from Eq. (2.3),

$$\pi_\nu \pi_\mu U_{\mu\nu} = -\frac{1}{2}(\pi_\mu \pi_\nu - \pi_\nu \pi_\mu) U_{\mu\nu} = (ie/2) F_{\mu\nu} U_{\mu\nu} = m^2 \pi_\mu U_\mu$$

or

$$\pi_\nu U_\nu = (ie/2m^2) F_{\mu\nu} U_{\mu\nu} , \tag{2.5}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .$$

The second-order wave equation (2.4) then becomes

$$(\pi^2 - m^2) U_\nu - (ie/2m^2) \pi_\nu (F_{\mu\lambda} U_{\mu\lambda}) + ie F_{\mu\nu} U_\mu = 0 . \tag{2.6}$$

2. Duffin-Kemmer Formalism

The first-order Proca equations (2.2) and (2.3) may be written in the matrix form $(\beta_\mu \pi_\mu + m) \psi = 0$ by setting

$$\psi = \begin{bmatrix} -1/m\, U_{14} \\ -1/m\, U_{24} \\ -1/m\, U_{34} \\ -1/m\, U_{23} \\ -1/m\, U_{31} \\ -1/m\, U_{12} \\ U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} ;$$

$$\beta_1 = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}, \quad \beta_2 = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix},$$

$$\beta_3 = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}, \quad \beta_4 = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}$$

These β's satisfy the algebra-defining equations

$$\beta_\mu \beta_\nu \beta_\lambda + \beta_\lambda \beta_\nu \beta_\mu = \beta_\mu \delta_{\nu\lambda} + \beta_\lambda \delta_{\mu\nu} .$$

The first-order Proca equations are thus a realization of the Duffin-Kemmer formalism.

3. Discussion of Second-Order Field Equations

In a first-order formalism, the subsidiary condition eliminating the timelike vector mesons either is one of the equations of motion or can be derived from them. When the equations of motion are of second order, however, the subsidiary condition must be separately assumed. The second-order equations obtained by the substitution $\partial_\mu \to \pi_\mu$ will then generally not be mutually consistent without the addition of suitable $F_{\mu\nu}$ terms. For example, equations

$$(\Box^2 - m^2)\, U_\mu = 0 \qquad \text{and} \qquad \partial_\mu U_\mu = 0$$

on $\partial_\mu \to \pi_\mu$ become

$$(\pi^2 - m^2)\, U_\mu = 0 \,, \tag{2.7}$$

$$\pi_\mu U_\mu = 0 \,. \tag{2.8}$$

Since $[\pi_\nu, \pi^2] \neq 0$, Eq. (2.7) is inconsistent with Eq. (2.8). A similar difficulty arises with the conventional Stuckelberg formalism[4] in the case of electromagnetic interaction. For these reasons we have preferred to use a Lagrangian giving first-order equations of motion which after $\partial_\mu \to \pi_\mu$ can be iterated so as to yield the consistent second-order equations (2.5) and (2.6).

4. Stuckelberg Formalism

There is one other dynamical form of the vector meson theory introduced by Stuckelberg[4] which is well known in the neutral-meson case. There has apparently been, however, no consistent treatment of the electromagnetic interaction of charged mesons in the Stuckelberg formalism. The original Stuckelberg theory is a second-order formalism involving a four-vector field Z_μ and a scalar field B. In the absence of interaction, these fields are related to the Proca field U_μ by the equation $U_\mu = Z_\mu + m^{-1} \partial_\mu B$. By the subsidiary condition

$$\partial_\mu Z_\mu + m B = 0 \,,$$

the scalar field B cancels out the fourth component of the vector meson field. In the conventional formulation, when the electromagnetic interaction is introduced by the minimal substitution $\partial_\mu \to \pi_\mu$, this separately imposed subsidiary condition becomes inconsistent with the field equations. We will consider here a new first-order formulation of this theory which is internally consistent automatically and turns out to be identical with the Proca theory.

For free mesons consider the Lagrangian

$$\begin{aligned}\mathcal{L} = {} & 1/2\, Z^+_{\mu\nu} \left[\partial_\mu Z_\nu - \partial_\nu Z_\mu + m^{-1} (\partial_\mu \partial_\nu - \partial_\nu \partial_\mu) B \right] \\ & + 1/2 \left[\partial_\mu Z^+_\nu - \partial_\nu Z^+_\mu + m^{-1} (\partial_\mu \partial_\nu - \partial_\nu \partial_\mu) B^+ \right] Z_{\mu\nu} \\ & - 1/2\, Z^+_{\mu\nu} Z_{\mu\nu} + m^2 Z^+_\mu Z_\mu + m Z^+_\mu \partial_\mu B + m \partial_\mu B^+ Z_\mu \\ & + C^+_\mu \partial_\mu B + \partial_\mu B^+ C_\mu - C^+_\mu C_\mu ,\end{aligned} \tag{2.9}$$

where $Z_{\mu\nu}$, B, Z_μ, C_μ are independent field variables. On variation of $\mathcal{L}$ we obtain the equations

$$\partial_\nu Z_{\nu\mu} - m^2 Z_\mu - m \partial_\mu B = 0 , \tag{2.10}$$

$$Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu , \tag{2.11}$$

$$\partial_\nu Z_\nu + m^{-1} \partial_\mu C_\mu = 0 , \tag{2.12}$$

$$C_\mu = \partial_\mu B. \tag{2.13}$$

By operating on Eq. (2.10) with ∂_μ we obtain Eq. (2.12) on using Eq. (2.13). Substitute Eq. (2.11) into Eq. (2.10) to obtain

$$(\Box^2 - m^2) Z_\mu - \partial_\mu (\partial_\nu Z_\nu + mB) = 0 ,$$

and, using Eqs. (2.12) and (2.13), we find

$$(\Box^2 - m^2)(Z_\mu + m^{-1}\partial_\mu B) = 0 . \tag{2.14}$$

Set $U_\mu = Z_\mu + m^{-1}\partial_\mu B$ so that Eq. (2.14) along with the condition $\partial_\mu U_\mu = \partial_\mu Z_\mu + m^{-1}\Box^2 B = 0$ (which is identical to Eqs. (2.12) and (2.13) reduces to the Proca equations. Thus the internally consistent equations

$$\partial_\nu Z_{\nu\mu} - m^2 Z_\mu - m\,\partial_\mu B = 0 , \tag{2.15}$$

$$Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu , \tag{2.16}$$

together with (2.14), are equivalent to the Proca equations.

The advantage of the above first-order formulation is the possibility of introducing the electromagnetic interaction consistently. Put $\partial_\mu \to \pi_\mu$ in Eq. (2.9) to obtain

$$\begin{aligned}\mathcal{L} = {} & 1/2\, Z^+_{\mu\nu}\,[\,\pi_\mu Z_\nu - \pi_\nu Z_\mu - ie/m\, F_{\mu\nu} B] \\ & + 1/2\,[\pi_\mu Z^+_\nu - \pi_\nu Z^+_\mu + ie/m\, F_{\mu\nu} B^+]\, Z_{\mu\nu} \\ & - 1/2\, Z^+_{\mu\nu} Z_{\mu\nu} + m^2 Z^+_\mu Z_\mu + m\, Z^+_\mu \pi_\mu B + m\, \pi_\mu B^+ Z_\mu \\ & + C^+_\mu \pi_\mu B + \pi_\mu B^+ C_\mu - C^+_\mu C_\mu .\end{aligned} \tag{2.17}$$

From Eq. (2.17) follow the equations

$$\pi_\nu Z_{\nu\mu} - m^2 Z_\mu - m\,\pi_\mu B = 0 , \tag{2.18}$$

$$Z_{\mu\nu} = \pi_\mu Z_\nu - \pi_\nu Z_\mu - ie/m\, F_{\mu\nu} B , \tag{2.19}$$

$$\pi_\nu Z_\nu + m^{-1}\pi_\mu C_\mu - ie/2m\, F_{\mu\nu} Z_{\mu\nu} = 0 , \tag{2.20}$$

$$C_\mu = \pi_\mu B, \tag{2.21}$$

as in the free-field case (if we use Eq. 2.21), operating on Eq. (2.18) with π_μ gives Eq. (2.20). Substitute Eq. (2.19) into Eq. (2.18) to find

$$(\pi^2 - m^2)\, Z_\mu - \pi_\nu\, \pi_\mu Z_\nu - m\, \pi_\mu B - ie/m\, \pi_\nu\, (F_{\nu\mu} B) = 0 .$$

When Eqs. (2.20) and (2.21) are used, this latter equation becomes

$$(\pi^2 - m^2)\,(Z_\mu + m^{-1}\, \pi_\mu B) + ie\, F_{\nu\mu}\,(Z_\nu + m^{-1}\, \pi_\nu B)$$

$$- ie/2m^2\, \pi_\mu\, (F_{\lambda\nu} Z_{\lambda\nu}) = 0 , \qquad (2.22)$$

on making use of the commutation relations

$$[\pi_\mu, \pi^2] = - ie\, \pi_\nu\, F_{\mu\nu} - ie\, F_{\mu\nu}\, \pi_\nu .$$

If we set $U_\mu = Z_\mu + m^{-1}\, \pi_\mu B$, then $Z_{\mu\nu} = U_{\mu\nu}$, and Eq. (2.22) becomes

$$(\pi^2 - m^2)\, U_\mu - ie/2m^2\, \pi_\mu (F_{\lambda\nu} U_{\lambda\nu}) + ie\, F_{\nu\mu} U_\nu = 0 ,$$

which is identical with Eq. (2.6) in the Proca theory. In addition, the subsidiary condition Eq. (2.5) in the Proca theory is readily seen to be identical to Eq. (2.20).

B. Most General Lagrangian for a Charged Vector Meson

1. Divergence Transformations

The theories we have just considered possess, as we shall see in Section D, a "normal" magnetic moment, i.e., their gyromagnetic ratio g is 1. The Lagrangians we have been using are not unique, however. In the Proca theory the divergence

$$\mathcal{L}' = \gamma\, \partial_\nu [\partial_\mu U^+_\nu U_\mu - \partial_\mu U^+_\mu U_\nu]$$

$$= \gamma [\partial_\mu U^+_\nu \partial_\nu U_\mu - \partial_\mu U^+_\mu \partial_\nu U_\nu] , \tag{2.23}$$

where γ is a dimensionless constant, may be added to the free-field Lagrangian (2.1) The divergence $\mathcal{L}'$ will not change the field equations derived from the Lagrangian. However, the Lagrangian $\mathcal{L} + \mathcal{L}'$ will have, as field equations in the presence of electromagnetic interaction,

$$U_{\mu\nu} = \pi_\mu U_\nu - \pi_\nu U_\mu , \tag{2.24}$$

$$\pi_\mu U_{\mu\nu} - m^2 U_\nu + ie\gamma F_{\mu\nu} U_\mu = 0 . \tag{2.25}$$

The term proportional to γ in Eq. (2.25) will correspond to an additional magnetic moment interaction.[4] We see then that there are infinitely many free-particle Lagrangians leading to the free-field equations but differing in the distribution of charge density. Thus the principle of minimal electromagnetic interaction does not define a "normal" magnetic moment unless the free-particle Lagrangian is specified. Since, for any choice of γ, the theory is nonrenormalizable,[5] this criterion too (as in the spin 1/2 case) is not usable to define a preferred electromagnetic interaction.

2. Electric Quadrupole Moment Interaction

Group theoretical considerations allow a particle of spin 1 to possess an electric quadrupole-moment in addition to a magnetic dipole moment. We now proceed to show how an electric quadrupole-moment interaction can be added to the first-order Proca Lagrangian. We require that such an interaction be bilinear in the meson field variables U_μ and $U_{\mu\nu}$, and linear in the electric charge e and the derivatives of the electromagnetic field $\partial_\lambda F_{\mu\nu}$. Since these derivatives are constrained by the Maxwell equations

$$\partial_\nu F_{\mu\lambda} - \partial_\mu F_{\nu\lambda} = \partial_\lambda F_{\mu\nu} ,$$

only the form

$$\mathcal{L}'' = a\, e\, U^+_{\mu\nu} U_\lambda \partial_\lambda F_{\mu\nu} + a^* e\, U^+_\lambda U_{\mu\nu} \partial_\lambda F_{\mu\nu} \tag{2.26}$$

satisfies these requirements along with the requirements of Lorentz and gauge invariance. The multiplication factor a is now determined by demanding invariance of this electromagnetic interaction under time reversal.

We define the time-reversed fields (apart from arbitrary phases, which are the same for all terms in the total Lagrangian) by

$$A_i^T = A_i(\vec{r}, -t) , \qquad A_0^T(\vec{r}, t) = -A_0(\vec{r}, -t) ,$$

$$U_i^T = U_i(\vec{r}, -t), \qquad U_0^T(\vec{r}, t) = -U_0(\vec{r}, -t) ,$$

$$\partial_i^T = \partial_i , \qquad \partial_4^T = -\partial_4, \quad a^T = a^* .$$

Applying these definitions to Eq. (2.26), we have

$$(\mathcal{L}'')^T = \mathcal{L}'' = a^* e\, U^+_{\mu\nu} U_\lambda \partial_\lambda F_{\mu\nu} + ae\, U_{\mu\nu} U^+_\lambda \partial_\lambda F_{\mu\nu} ,$$

and thus, in complete analogy to the β-decay Hamiltonian, all coupling constants must be relatively real, and a pure imaginary. Choosing $a = i\, q/4m^2$, where q is an arbitrary dimensionless constant, we obtain the electric quadrupole-moment interaction

$$\mathcal{L}'' = (ie\, q/4m^2)\, [\, U^+_{\mu\nu}\, U_\lambda - U^+_\lambda\, U_{\mu\nu}\,]\, \partial_\lambda F_{\mu\nu}\,. \tag{2.27}$$

We have been unable to introduce a term like (2.27) in a "normal" way by suitable choice of a free-particle Lagrangian without going to derivatives of third or higher order. The quadrupole moment is nevertheless subject to the same degree of ambiguity as the magnetic moment, since, as we shall see in Section D, the "normal" interaction (2.23) already implies a certain amount of quadrupole moment.

Adding Eqs. (2.1), (2.23) (with $\partial_\mu \to \pi_\mu$), and Eq. (2.27), we now have as the total Lagrangian

$$\begin{aligned}\mathcal{L} &= 1/2\, U^+_{\mu\nu}\, (\pi_\mu U_\nu - \pi_\nu U_\mu) + 1/2\, (\pi_\mu U^+_\nu - \pi_\nu U^+_\mu) U_{\mu\nu} \\ &\quad - 1/2\, U^+_{\mu\nu} U_{\mu\nu} + m^2\, U^+_\mu U_\mu + (ie\,\gamma/2)\, (U^+_\mu U_\nu - U^+_\nu U_\mu)\, F_{\mu\nu} \\ &\quad + (ie\, q/4m^2)\, [\, U^+_{\mu\nu} U_\lambda - U^+_\lambda U_{\mu\nu}\,]\, \partial_\lambda F_{\mu\nu}\,. \end{aligned} \tag{2.28}$$

Except for the possibility of letting γ and q have form factor space-time dependence, this Lagrangian is the most general charged vector meson Lagrangian consistent with the ordinary invariance requirements. The vector-meson theory tacitly used in the original[6, 13] $\mu \to e + \gamma$ argument corresponded to the choice $\gamma = q = 0$. As discussed in Section B-1, we know of no physical criterion justifying a particular choice of γ.

In the next two sections we investigate more fully the physical content of this theory.

C. Generalized Sakata-Taketani Equation

1. Elimination of Redundant Components

The Lagrangian (2.28) furnishes the field equations

$$\pi_\mu U_{\mu\nu} - m^2 U_\nu + ie\,\gamma\, U_\mu F_{\mu\nu} + (ie\,q/4m^2)\, U_{\mu\lambda}\, \partial_\nu F_{\mu\lambda} = 0 \,, \tag{2.29}$$

$$U_{\mu\nu} = \pi_\mu U_\nu - \pi_\nu U_\mu + (ie\,q/2m^2)\, U_\lambda\, \partial_\lambda F_{\mu\nu} \,. \tag{2.30}$$

A meson field satisfying first-order wave equations is expected to have six dynamically independent components, corresponding to the three independent field variables and their time derivatives. Equations (2.29) and (2.30) must therefore contain four redundant components which we wish to eliminate. Since in Eqs. (2.29) and (2.31) U_{ij} $(i, j = 1, 2, 3)$ and U_4 do not contribute to the time development of the meson field, these are the four components to be eliminated. After this elimination we possess a Hamiltonian form of the theory. For simplicity, we consider the electromagnetic fields time-independent.

From Eq. (2.29) we have

$$U_4 = (1/m^2)\,(\pi_i U_{i4} + ie\,\gamma\, U_i\, F_{i4}) \,.$$

Let $m\phi_i \equiv U_{i4}$, so that

$$U_4 = 1/m\; \vec{\pi}\cdot\vec{\phi} + (e\,\gamma/m^2)\,\vec{U}\cdot\vec{E} \,,$$

where $\vec{E}$ is the electric field strength. Also from Eq. (2.29),

$$\pi_j U_{ji} - m^2 U_i + \pi_4 U_{4i} = -\,ie\,\gamma\, F_{ji}\, U_j - ie\,\gamma\, U_4\, F_{4i}$$

$$- (ie\,q/2m^2)\, U_{4j}\, \partial_i F_{4j} - (ie\,q/4m^2)\, U_{\ell m}\, \partial_i F_{\ell m} \,,$$

which becomes

$$i\,\frac{\partial \phi_i}{\partial t} = e\varphi\,\phi_i + m\,U_i + m^{-1}\,[\,\vec{\pi}\times(\vec{\pi}\times\vec{U})\,]_i + ie\gamma m^{-1}\,(\vec{U}\times\vec{H})_i$$

$$+ e\gamma m^{-2}\,\vec{E}\,(\vec{\pi}\cdot\vec{\phi}) + e^2\gamma^2 m^{-3}\,\vec{E}\,(\vec{U}\cdot\vec{E}) - e\,(q/2)m^{-2}\phi_j\,\partial_i E_j$$

$$- ie\,(q/2)\,m^{-3}\,(\vec{\pi}\times\vec{U})_j\,\partial_i H_j \,, \tag{2.31}$$

where φ is the scalar potential, and $\vec{H}$ is the magnetic field strength. We wish to write this last equation in matrix form. It is lengthy, but not difficult, to show that if one introduces the spin-1 matrices

$$S_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad S_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

Eq. (2.31) can be written as

$$\begin{aligned} i \frac{\partial \phi}{\partial t} = {} & e\varphi\phi + mU - m^{-1} (\vec{S} \cdot \vec{\pi})^2 U - e\gamma m^{-1} (\vec{S} \cdot \vec{H}) U - e\gamma m^{-2} S_i S_j E_j \pi_i \phi \\ & + e\gamma m^{-2} (\vec{E} \cdot \vec{\pi}) \phi - e^2\gamma^2 m^{-3} (\vec{S} \cdot \vec{E})^2 U + e^2\gamma^2 m^{-3} \vec{E}^2 U \\ & + e(q/2)m^{-2} S_i S_j \partial_j E_i \phi - e (q/2)m^{-2} (\vec{\nabla} \cdot \vec{E})\phi \\ & - ie (q/2)m^{-3} S_i S_j \partial_i (\vec{H} \times \vec{\pi})_j U + ie (q/2)m^{-3} \vec{\nabla} \cdot (\vec{H} \times \vec{\pi}) U , \end{aligned} \tag{2.32}$$

and $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$, $U = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix}$. Now Eq. (2.30) becomes

$$U_{4i} = \pi_4 U_i - \pi_i U_4 + ie (q/2)m^{-2} U_j \partial_j F_{4i} ,$$

which can also be written in matrix form:

$$\begin{aligned} i \frac{\partial U}{\partial t} = {} & e\varphi U + m \phi + m^{-1} (\vec{S} \cdot \vec{\pi})^2 \phi - (\vec{\pi}^2/m) \phi - em^{-1} (\vec{S} \cdot \vec{H}) \phi \\ & + e\gamma m^{-2} S_i S_j \pi_j E_i U - e\gamma m^{-2} (\vec{\pi} \cdot \vec{E}) U + e (q/2) m^{-2} S_i S_j \partial_i \mathbf{E}_j U \\ & - e (q/2)m^{-2} (\vec{\nabla} \cdot \vec{E}) U. \end{aligned} \tag{2.33}$$

We define now a six-component wave function $\psi = \frac{1}{\sqrt{2}}\begin{pmatrix} U + \phi \\ -U + \phi \end{pmatrix}$ so that Eq. (2.32) and (2.33) take the form

$$
\begin{aligned}
i \frac{\partial \psi}{\partial t} = \Big[& e\varphi + m\rho_3 - (ge/2m)\, \vec{S} \cdot \vec{H} - (\rho_3 + i\rho_2)\, \vec{\pi}^2/2m - (eQ/4) Q_{ij}\, \partial E_i / \partial x_j \\
& + ie(g-1)/4m^2 \, [\, \vec{S} \cdot (\vec{E} \times \vec{\pi}) - \vec{S} \cdot (\vec{\pi} \times \vec{E}) \,] + e\, Q/2 \; \vec{\nabla} \cdot \vec{E} \\
& + i\rho_2 (\vec{S} \cdot \vec{\pi})^2/m - (e/2m^2)\, (g-1)\rho_1 \, [\, (\vec{S} \cdot \vec{E})(\vec{S} \cdot \vec{\pi}) + (\vec{S} \cdot \vec{\pi})(\vec{S} \cdot \vec{E}) \\
& - 2\vec{E} \cdot \vec{\pi} - \vec{\nabla} \cdot \vec{E} \,] + ie\,(g-2)/2m\; \rho_2\, (\vec{S} \cdot \vec{H}) \\
& - e^2 (g-1)^2/2m^3\, (\rho_3 - i\rho_2)\big((\vec{S} \cdot \vec{E})^2 - \vec{E}^2\big) \\
& - ie\, q/2m^3\, (\rho_3 - i\rho_2)\, S_i S_j \partial_i\, (\vec{H} \times \vec{\pi})_j \\
& + ie\, q/2m^3\, (\rho_3 - i\rho_2) \;\; \vec{\nabla} \cdot (\vec{H} \times \vec{\pi}) \Big] \psi \,, \qquad (2.34)
\end{aligned}
$$

where $g = \gamma + 1$, $Q_{ij} = S_i S_j + S_j S_i$, and $Q = -(g - 1 + q)\,(\hbar/mc)^2$. For $g = 1$, $q = 0$, Eq. (2.34) reduces to the Sakata-Taketani[7] equation. The charge matrices ρ_1, ρ_2, ρ_3 are the usual 2×2 Pauli matrices:

$$\rho_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad i\rho_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \rho_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

2. <u>Operators and Expectation Values</u>

Since the charge is given by

$$Q = e \int d^3x \;\; \psi^+ \rho_3 \psi \equiv e(\psi, \rho_3 \psi) \,,$$

expectation values $\bar{A}$ of operators A must be defined relative to the indefinite charge metric ρ_3, i.e.,

$$\bar{A} = \int d^3 x\, \psi^+ \rho_3 A \psi \,.$$

In order that these expectation values be real, the operators must satisfy the condition of pseudo Hermiticity

$$A = \rho_3 A^+ \rho_3 , \tag{2.35}$$

where $A^+ = (A^T)$ is the ordinarily defined Hermitian adjoint. Note that H is pseudo-Hermitian ($H = \rho_3 H^+ \rho_3$), so that its interpretation as the energy is consistent. For the canonical transformations ($\psi = S\psi'$) between the same physical state in different representations, we require Q to be invariant, i.e., that

$$S^{-1} = \rho_3 S^+ \rho_3 . \tag{2.36}$$

Such transformations S are called pseudo-unitary transformations. We find, as in the nonrelativistic case ($\rho_3 = 1$),

$$\frac{d\bar{A}}{dt} = i \, [H, A]_- .$$

In the following discussion we shall omit the prefix "pseudo," always understanding Hermiticity and unitarity to be defined relative to the metric ρ_3 by Eqs. (2.35) and (2.36).

D. Nonrelativistic Limit of the Vector Meson Theory

To find the nonrelativistic limit of Eq. (2.34) we use the Foldy-Wouthuysen method[9] of successive unitary transformations. The free-particle Hamiltonian (e = 0 in Eq. 2.34 is diagonalized by the unitary transformation

$$\exp\left((1/2)\, i\rho_1 \phi\right),$$

where

$$\tan(\phi/2) = (2i/(E^2 + m^2))\ [\ \vec{P}^2/2 - (\vec{S}\cdot\vec{P})^2]\ ,$$

so that we have

$$U = \begin{bmatrix} \dfrac{E+m}{2\sqrt{mE}} & \dfrac{-(P^2/2 - (S\cdot P^2)}{(E+m)\sqrt{mE}} \\[2ex] \dfrac{-(P^2/2 - (S\cdot P^2)}{(E+m)\sqrt{mE}} & \dfrac{E+m}{2\sqrt{mE}} \end{bmatrix}$$

and $E = \sqrt{P^2 + m^2}$. Thus in the free-meson case $H' = U^{-1} HU = \rho_3 E$, so that each sign of the charge (energy) can be represented by a three-component wave function.

In the interacting Hamiltonian of Eq. (2.34) we define "even" operators as those containing ρ_3 or 1, and "odd" operators as those containing ρ_2 or ρ_1. For the nonrelativistic limit we require that H be free of odd operators up to some order in the inverse mass. Successive canonical transformations U, where

$$U = e^{iS},\quad S = i\rho_3 0/2m\ ,$$

and the 0 are odd operators of the Hamiltonian, will effect the elimination of 0 from the Hamiltonian. An example of such a 0 is $i\rho_2(\vec{S}\cdot\vec{\pi})^2/m$. The resulting wave equation is

$$i\ \partial\psi/\partial t = (H_0 + H_1)\ \psi; \tag{2.37}$$

and

$$H_0 = e\varphi + m + \vec{\pi}^2/2m - (\vec{\pi}^2)^2/8m^3 + 0\,(m^{-5}) ,$$

$$H_1 = -\frac{e}{2mC}\ \vec{S}\cdot[g\vec{H} + \frac{g-1}{2mC}(E\times\vec{\pi} - \vec{\pi}\times\vec{E})]$$

$$- eQ/4\ Q_{ij}\ \partial E_i/\partial X_j - i\,e\,Q/2\hbar\,(\vec{\pi}\cdot\vec{E}) + 0\,(m^{-3}) ,$$

where $\vec{\pi} = \vec{P} - e\vec{A}$. The three terms in H_1 are identified as a magnetic-moment spin-orbit coupling term, an electric-quadrupole coupling term, and a (non-Hermitian) Darwin term. Except for this last term, the same Hamiltonian $H_0 + H_1$ is also obtained for spin -0 ($S_i = Q_{ij} = 0$) and for spin - 1/2 ($S_i = \frac{1}{2}\sigma_i$, $Q_{ij} = 0$) particles of arbitrary gyromagnetic ratio. The Darwin term is zero for spin 0 and $\frac{-ie\hbar}{(2mC)^2}\vec{\pi}\cdot\vec{E}$ for spin 1/2. Except for these Darwin terms, which vanish in the classical ($\hbar = 0$) limit, particles of different spin are thus found to obey the same nonrelativistic wave equation (2.37), once allowance is made for the possibility of arbitrary magnetic dipole and electric quadrupole moments in the higher-spin cases. This result suggests that, except for the obscure and specifically quantum mechanical Darwin term, the nonrelativistic wave equation is actually spin-independent and that its form depends on classical invariance arguments only.

It is worth noting that a vector particle could have, except for $g = 1$, a quadrupole-moment interaction proportional to the "anomalous moment" $g - 1$, even if the specific form (2.27) had not been introduced. Unless there are reasons (unknown) for preferring $g = 1$ theory, a term (2.27) is not to be excluded. As we shall see later, such a q term apparently does not lead to any more divergent a form of electromagnetic interaction than does the γ term itself.

The factor 1/4 has been introduced before Q in H_1 in order to make our normalization of the quadrupole moment strength conform to that conventionalized by Ramsey.[10] Consider the meson to have its spin along the positive z axis, and also take as a very weak electric field

$$E_1 = -(K/2)x, \quad E_2 = -(K/2)y, \quad E_3 = KZ ,$$

where K is a small constant. For a meson with spin up $\psi = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\\ 0\end{pmatrix}$, so that

$$\left\langle \uparrow \left| \frac{-e}{4} \; Q\, Q_{ij} \; \frac{\partial E_i}{\partial X_j} \right| \uparrow \right\rangle = \frac{-eQ}{4} \; K .$$

Ramsey defines the energy E of an electric-quadrupole moment q as

$$E = -(q/4)\,(\partial E_3/\partial Z)_{Z=0}$$

for particles with spin along the positive z axis. The quadrupole moment is usually divided by the charge and given in units cm^2, and so the vector meson has quadrupole moment $Q = -(g - 1 + q)\,(\hbar/mc)^2 cm^2$. If we consider the spin projection along the z axis to be 0, then $\psi = \begin{pmatrix}0\\ 0\\ 1\end{pmatrix}$ and

$$\left\langle S_3 = 0 \left| -\frac{eQ}{4} \; \frac{\partial E_i}{\partial X_j} \; Q_{ij} \right| S_3 = 0 \right\rangle = \frac{eQ}{2} \; K ,$$

to give $Q'(S_3 = 0) = -2Q$, in agreement with the group theoretical result

$$Q(m) = \frac{3m^2 - S(S+1)}{S(2S-1)} \; Q ,$$

where S is the particle spin and m the projection of the spin along the z axis. The charge distribution can be considered as having the shape of an ellipsoid of revolution centered at the origin, and thus $Q = 4/5\ \eta R^2$, where $\eta = (C^2 - a^2)/(C^2 + a^2)$, $R = \frac{1}{2}(a^2 + C^2)$ is the mean square radius, C is the axis of the ellipsoid in the z direction, and a is the axis perpendicular to the z direction. A positive quadrupole moment corresponds to a cigar-shaped charge distribution, and a negative quadrupole moment corresponds to a pancake charge distribution.

For $g = 1$, $q = 0$, our result (2.37) reduces to that obtained by Case.[11]

E. Classical Spin Equations of Motion

In the preceding section we noted that spinning particles of the same gyromagnetic ratio have (except for the Darwin term) the same Hamiltonian, at least to order $1/m^2$. This suggests the possibility of a classical spin-independent description of the magnetic-moment precession. Bargmann, Michel, and Telegdi[12] have recently given such a description using a four-vector S_μ for the spin or magnetic moment. In quantum mechanics the spin has, however, more often been described as part of the angular momentum antisymmetric tensor $S_{\mu\nu}$. We will here derive covariant classical equations of motion in terms of the more familiar $S_{\mu\nu}$. While the equations (2.40) we obtain are apparently quite different from the equations (2.42) obtained by Bargmann, Michel, and Telegdi, the two sets of equations are actually the same when S_μ and $S_{\mu\nu}$ are related as they have to be. This will show then that covariant spin-precession equations equivalent to those of Bargmann, Michel, and Telegdi can be derived from classical invariance arguments by using the more familiar $S_{\mu\nu}$ formulation for the spin angular momentum.

We wish to generalize to an arbitrary Lorentz frame the equation of spin precession

$$d\vec{s}/dt = (eg/2m)\, \vec{s} \times \vec{H}\,, \tag{2.38}$$

which holds in a rest frame, by using an antisymmetric tensor $S_{\mu\nu}$. The tensor $S_{\mu\nu}$ must have only three independent components, which in a rest frame are s_1, s_2, s_3. This condition is expressed covariantly by the constraint

$$S_{\mu\nu}\, U_\nu = 0\,, \tag{2.39}$$

where U_ν is the four-velocity ($U^2 = -1$). It is readily confirmed that the unique expression for the time variation of $S_{\mu\nu}$ consistent with the particle equation of motion $dU_\mu/d\tau = e/m\, F_{\mu\nu} U_\nu$ and reducing to the form (2.38) in a rest frame is

$$\begin{aligned} dS_{\mu\nu}/d\tau = &-(eg/2m)\,[S_{\mu a} F_{a\nu} - S_{\nu a} F_{a\mu}] \\ &-(e(g-2)/2m)\,[U_\mu S_{\beta\nu} - U_\nu S_{\beta\mu}]\, F_{\beta a} U_a\,. \end{aligned} \tag{2.40}$$

Here τ is the eighth-time.

Define a four-vector S_a by the relation

$$S_a = -\, i/2\ \epsilon_{a\mu\nu\beta}\, S_{\mu\nu}\, U_\beta\,, \tag{2.41}$$

which then also satisfies a supplementary condition

$$S_\mu U_\mu = 0\,.$$

The time variation of S_a can be obtained from Eqs. (2.40) and (2.21):

$$\begin{aligned} dS_a/d\tau &= -\, i/2\ \epsilon_{a\mu\nu\beta}\,[\,\dot{U}_\beta\, S_{\mu\nu} + U_\beta\, \dot{S}_{\mu\nu}] \\ &= ie/4m\ \epsilon_{a\mu\nu\beta}\,[\,gU_\beta\,(S_{\mu\lambda}\, F_{\lambda\nu} - S_{\nu\lambda}\, F_{\lambda\mu}) \\ &\qquad - (g-2)\, U_\lambda\, F_{\rho\lambda}\, U_\beta\,[\,U_\mu\, S_{\rho\nu} - U_\nu\, S_{\rho\mu}]\,] \\ &\qquad - ie/2m\ \epsilon\epsilon_{a\mu\nu\beta}\, S_{\mu\nu}\, F_{\beta\lambda}\, U_\lambda\,, \end{aligned}$$

where $\dot{A} \equiv dA/d\tau$. Now use the two relations

$$S_{\mu\nu} = i\ \epsilon_{\mu\nu a\beta}\, U_a\, S_\beta\,,$$

$$\begin{aligned} \epsilon_{\mu a\beta\nu}\ \epsilon_{\mu\lambda\rho\sigma} = [\,&\delta_{a\lambda}\,\delta_{\beta\rho}\,\delta_{\sigma\nu} - \delta_{a\lambda}\,\delta_{\beta\sigma}\,\delta_{\rho\nu} + \delta_{a\rho}\,\delta_{\nu\lambda}\,\delta_{\sigma\beta} \\ &- \delta_{a\rho}\,\delta_{\beta\lambda}\,\delta_{\sigma\nu} + \delta_{a\sigma}\,\delta_{\beta\lambda}\,\delta_{\nu\rho} - \delta_{a\sigma}\,\delta_{\beta\rho}\,\delta_{\lambda\nu}] \end{aligned}$$

to obtain

$$ds_a/d\tau = e/m\ [\,g/2\ F_{a\nu}\, S_\nu - (g/2 - 1)\, S_\nu\, F_{\nu\mu}\, U_\mu\, U_a\,]\,. \tag{2.42}$$

This is the result obtained by Bargmann, Michel, and Telegdi.[12]

We now show, in particular, that Eqs. (2.40) and (2.42) both lead to the same (spin-orbit coupling) coupling between spin and momentum in an electric field, and thus to order $1/m^2$. For this purpose we express both equations in three-vector form and keep terms linear in the velocity $\vec{V}$. From Eq. (2.40) we have

$$d\vec{s}/dt = - \; eg/2m \; [\; - \vec{s} \times \vec{H} + (\vec{s} \times \vec{V}) \times \vec{E} \;] - e\,(g-2)/2m \; [\; \vec{s}(\vec{V} \cdot \vec{E}) - \vec{E}\,(\vec{s} \cdot \vec{V}) \;]$$

$$= eg/2m \; \vec{s} \times \vec{H} + e(g - 2)/2m \; \vec{s} \times (\vec{E} \times \vec{V}) + e/m \; \vec{E} \times (\vec{s} \times \vec{V}) \; .$$

But

$$\vec{E} \times (\vec{s} \times \vec{V}) = \frac{1}{2} \vec{s} \times (\vec{E} \times \vec{V}) - m/2e \; d\vec{V}'/dt \; ,$$

where $\vec{V}' = \vec{s}\, V^2 - \vec{V}\,(\vec{s} \cdot \vec{V})$; and we have used $\dot{\vec{V}} = e/m\vec{E}$, so that we write

$$d\vec{s}/dt = eg/2m \; \vec{s} \times \vec{H} + e\,(g-1)/2m \; \vec{s} \times (\vec{E} \times \vec{V}) + m/2e \; d\vec{V}'/dt$$

to terms linear in $\vec{V}$. Now consider the case in which the spin changes slowly compared with the velocity, and the velocity periodically takes on the same values, so that we can drop the last term. The spin precession result to order m^{-2} then becomes

$$d\vec{s}/dt = eg/2m \; \vec{s} \times \vec{H} + e(g-1)/2m^2 \; \vec{s} \times (\vec{E} \times \vec{p}) \tag{2.43}$$

for particles with a positive charge. Equation (2.42) expressed in the same way becomes

$$ds/dt = e/m \; [\; g/2 \; \vec{s} \times \vec{H} + g/2 \; \vec{E}\,(\vec{s} \cdot \vec{V}) - (g/2 - 1) \; \vec{V}\,(\vec{S} \cdot \vec{E}) \;]$$

$$= eg/2m \; \vec{s} \times \vec{H} + e(g - 1)/2m \; \vec{s} \times (\vec{E} \times \vec{V}) + m/2e \; d\vec{V}''/dt \; ,$$

where $\vec{V}'' = - \vec{V}\,(\vec{s} \cdot \vec{V})$. Thus, by dropping the last term in exactly the same way as we arrived at Eq. (2.43), we obtain the same result. It is easily shown that (2.43) is identical with the result obtained from the Hamiltonian Eq. (2.37) though the relation $d\vec{s}/dt = i \; [\; H, \vec{s} \;]$.

III. APPLICATION TO DECAY: $\mu^{\pm} \to e^{\pm} + \gamma$

Feinberg[6] and Gell-Mann[13] have argued that if there were a B meson intermediary in weak interactions, the photodecay of the muon $\mu \to e + \gamma$ would be allowed with a rate considerably larger than that experimentally observed. The situation is far from clear, however, since the decay $\mu \to e + \gamma$ is actually strongly dependent on the electromagnetic properties of this B meson.

A. ($\mu \to e\gamma$) Matrix Element

The Feynman diagrams for the process $\mu \to e + \gamma$ are given in Fig. 1; the matrix element for the process $\mu \to e$ with emission of a real or virtual photon is given by the expression[14]

$$\mathcal{M} = ie\,\bar{\omega}_e (1 - \gamma_5)\Gamma_\nu \omega_\mu A_\nu , \tag{3.1}$$

where ω_e, ω_μ are the electron and muon spinors respectively, and

$$\Gamma_\mu = -i\,(2\pi)^{-3} \left\{ i f_0 \,(\gamma_\mu K_\nu - \gamma_\nu K_\mu)\, K^{-2} + f_1\, \sigma_{\mu\nu}/\mu \right\} K_\nu .$$

Thus:

$$i\Gamma_\mu A_\mu = (2\pi)^{-3} \left\{ f_0\, \gamma_\mu\, j_\mu^{\text{ext.}}/K^2 + (f_1/2\mu)\sigma_{\mu\nu}\, F_{\mu\nu} \right\}. \tag{3.2}$$

Here K is the photon momentum, μ the muon mass, and

$$F_{\mu\nu} = i(K_\nu A_\mu - K_\mu A_\nu) ,$$

$$j_\mu^{\text{ext}} = i\, K_\nu F_{\mu\nu} .$$

The form factors f_0 and f_1, which are functions of K^2, are responsible for monopole radiation (in the Coulomb field of a nucleus) and dipole radiation respectively. The rate for $\mu \to e + \gamma$ with emission of a real photon is proportional to $|f_1(0)|^2$, and the rate for the coherent process $\mu + n \to e + n$ is proportional to $\left\{ f_0(\mu^2) + f_1(\mu^2) \right\}^2$.

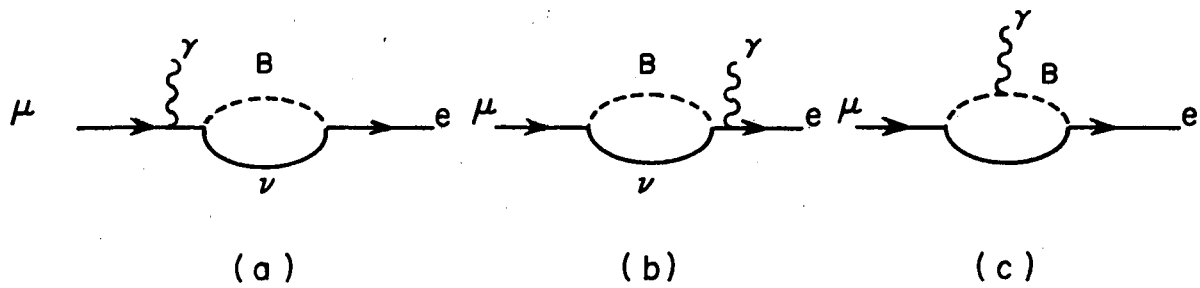

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Fig. 1. Diagrams for decay $\mu \rightarrow e + \gamma$.

B. Branching Ratio $\omega_{\mu\to e+\gamma}/\omega_{\mu\to e+\nu+\bar{\nu}}$

If the $\mu \to e$ conversion proceeds through $\mu \to \nu + B$ and $\nu + B \to e$, then the branching ratio between the unobserved decay $\mu \to e + \gamma$ and the normal decay can be written as

$$\rho = \frac{\omega_{\mu \to e + \gamma}}{\omega_{\mu \to e + \nu + \bar{\nu}}} \cong (3\alpha/8\pi)N^2, \tag{3.3}$$

where α is the fine-structure constant, and N is a number independent of the weak-coupling constant. Amplitude N generally diverges logarithmically with Λ/m, the ratio of cutoff to the B-meson mass. Feinberg[6] and Gell-Mann[13] found (tacitly assuming unit magnetic moment for the vector meson) that for $\Lambda \approx$ nucleon mass, and $m \approx$ K-meson mass, $N \approx 1$. This value for N gives $\rho \approx 10^{-3}$, which is 10^3 times the experimentally measured upper limit for ρ.[15]

Aside from the mild cutoff dependence, there are two reasons in a one-neutrino theory as to why the above-calculated ρ need not be taken as evidence against the B meson. We have already pointed out that there is an infinity of free-particle B-meson Lagrangians which differ in their definition of "normal" magnetic moment. Also, if the B meson exists it must have a large mass (greater than the K-meson mass), and yet the gauge-invariance type of arguments for its presence[16] indicate that it should have a vanishing mass. This implies that the B meson must have a rather complicated structure, so that one should keep an open mind with regard to its electromagnetic properties.

We have recalculated the $\mu e \gamma$ vertex as a function of magnetic moment $(1 + \gamma)\, e\hbar/2mc$ and electric quadrupole moment $Q = -(\gamma+q)(\hbar/mc)^2$, with the interaction Lagrangian given by Eq. (2-28). After a lengthy calculation, the value of N obtained[19] is

$$N = (1-\gamma - q\mu^2/8m^2)\, I'_0 + (1 + 2\gamma + q\mu^2/4m^2)\, I'_1 + (3-\gamma\mu^2/2m^2 + 11\mu^2/6m^2) I'_2 + (22/3 + 4\gamma)\,(\mu^2/m^2) I'_3 + 10\mu^2/m^2 I'_4, \tag{3.4}$$

where

$$I'_n = + \frac{im^{2n}}{\pi^2} \int \frac{d^4q}{(q^2-m^2)^{n+2}} .$$

This result is correct to order μ^2/m^2, terms of order $(\mu/m)^4$ have been dropped, and the electron mass has been set equal to zero. The expression (3.4) for N is consistent with that obtained by Meyer and Salzman[17] and by Ebel and Ernst,[18] who, however, did not calculate terms in μ^2/m^2 or q. Because q was originally defined divided by the square of the boson mass m^2, and the muon mass is the only other quantity of dimensions mass in our calculation, q always appears in N multiplied by μ^2/m^2.

C. Discussion of N

In our calculation of N, γ and q appear only in the combination

$$\gamma' = \gamma + q\mu^2/8m^2 = (g-1)\,(1-\mu^2/8m^2) - Q\mu^2/8 . \tag{3.5}$$

This means that the rate for $\mu \to e + \gamma$ will depend only on this combination of moments. This result is apparently fortuitous, since, in the monopole form factor f_0 this particular combination does not occur.[19]

1. Finite N

The integral I_0 is logarithmically divergent so that, except for $\gamma' = 1$, N is formally divergent. Since

$$I'_n = - \frac{(-)^n}{n(n+1)} , \tag{3.6}$$

for $\gamma' = 1$, we obtain

$$N = 1 + \frac{2\mu^2}{9m^2} , \tag{3.7}$$

which for any value of the boson mass leads to a branching ratio $\rho > 10^{-3}$. The cutoff independent calculation of N is thus in definite disagreement with experiment.

2. Logarithmically divergent N

N can be made vanishingly small by retaining the integral I_0, making it finite by the formal device of a covariant cutoff Λ. Consistency then requires that all integrals I_n be calculated with the same kind of cutoff. With the Feynman cutoff factor $+\Lambda^2 m^2/(q^2 - \Lambda^2 m^2)$ we obtain the integrals

$$I_n = + \frac{i\, m^{2n}}{\pi^2} \int \frac{d^4 q}{(q^2 - m^2)^{n+2}} \frac{\Lambda^2 m^2}{q^2 - \Lambda^2 m^2} , \tag{3.8}$$

$$I_0 = + \Lambda^2/(1 - \Lambda^2) + \Lambda^4/(1 - \Lambda^2)^2 \log \Lambda^2 ,$$

$$I_1 = - \frac{1}{2} \frac{\Lambda^2}{(1-\Lambda^2)^3} [\, 2 \Lambda^2 \log \Lambda^2 + (1 + \Lambda^2)(1 - \Lambda^2) \,] ,$$

$$I_2 = \frac{1}{3} \frac{\Lambda^2}{(1-\Lambda^2)^4} [\, 3\Lambda^2 \log \Lambda^2 + (1-\Lambda^2)(1 + \frac{1}{2}(5-\Lambda^2) \,] ,$$

$$I_3 = - \frac{1}{4} \frac{\Lambda^2}{(1-\Lambda^2)^5} [\, 4\Lambda^2 \log \Lambda^2 + 1 - \Lambda^2 + \frac{1}{3}(1 - \Lambda^2)(\Lambda^4 - 5\Lambda^2 + 13)/\Lambda^2 \,] ,$$

$$I_4 = \frac{1}{5} \frac{\Lambda^2}{(1-\Lambda^2)^6} [\, 1 - \Lambda^2 + \frac{1}{12} \Lambda^2 (\Lambda^2 - 1)(3\Lambda^6 - 17 \Lambda^4 + 43\Lambda^2 - 77) + 5\Lambda^2 \log \Lambda^2 \,] .$$

By defining $\gamma^1{}_0$ as that value of γ which makes N vanish, we find

$$\gamma^1{}_0 = A + B\epsilon , \tag{3.9}$$

where

$$A = (I_0 + I_1 + 3I_2)/(I_0 - 2I_1) ,$$

$$B = (I_0 - 2I_1)^{-1} (11/6\, I_2 + 22/3\, I_3 + 10\, I_4) - (1/2\, I_2 - 4\, I_3)(I_0 - 2I_1)^{-2} (I_0 + I_1 + 3I_2) ,$$

and $\epsilon = (\mu/m)^2 << 1$; in fact, we expect the upper limit for ϵ to be 1/25, since m must be greater than the K-meson mass. For two representative values of Λ, say $\Lambda = 1$, $\Lambda = 2$, we have

	I_0	I_1	I_2	I_3	I_4	A	B
$\Lambda=1$	0.5000	-0.167	0.084	-0.050	-0.033	0.700	-0.060
$\Lambda=2$	1.13	-0.296	0.125	-0.070	-0.044	0.702	0.058

This table shows that γ_0^1 is insensitive to both the cutoff Λ and the square of the ratio of the masses ϵ (as long as ϵ is small). If $\epsilon = 1/25$, then for $\Lambda = 1$, $\gamma_0^1 = 0.698$ and for $\Lambda = 2$, $\gamma_0^1 = 0.703$. In the expression (3.4) for N, it is evident that we can write

$$N = R\,(1 - \gamma^1/\gamma_0^1)\,,$$

where

$$R = I_0 + I_1 + 3I_2 + \epsilon\,(11/6\, I_2 + 22/3\, I_3 + 10\, I_4)\,.$$

The term proportional to ϵ in R will always be small in comparison with the other terms, so that in R we can neglect ϵ to obtain

$$R = \frac{\Lambda^2}{2(1-\Lambda^2)^4}\left\{2\Lambda^2\,(\Lambda^4 - \Lambda^2 + 3)\log\Lambda^2 + (1 - \Lambda^2)\,(2\Lambda^4+\Lambda^2+3)\right\}.$$

The branching ratio ρ then becomes

$$\rho = 3\alpha/8\pi\, R^2\,(1 - \gamma^1/\gamma_0^1)^2\,.$$

The quantity $3\alpha/8\pi\, R^2$ has been plotted by Ebel and Ernst, and varies from 10^{-4} to 10^{-2} as Λ varies from 1 to 10.

The branching ratio ρ, when it does not vanish (i.e., for $\gamma^1 \neq \gamma_0^1$), is sensitive to the value of Λ. The combination of γ and q necessary to forbid the $\mu \to e + \gamma$ decay is thus certainly ad hoc. On the other hand, we know of no criterion for fixing on a choice of γ and q a priori.

Now only one combination of the two parameters γ and q is involved in choosing γ^1 to forbid the process $\mu \to e + \gamma$. Another different combination of γ and q will determine the rate of the coherent μ+nucleus $\to$ e + nucleus process. In other words, we expect to be able to choose γ and q so that $f_1(0)^2$ and $\left\{f_1(\mu^2) + f_0(\mu^2)\right\}^2$ are both small enough not to exclude the vector meson hypothesis.

D. Two-Neutrino Hypothesis

Another explanation for the absence of $\mu \to e$ conversion consists in the assumption[20] that two different neutrinos ν and ν' are involved in μ decay, ν being coupled to the electron, and ν' to the muon. Since these neutrinos are different, they are not capable of annihilating each other, and thus any $\mu \to e$ processes are strictly forbidden. The implications of this alternative are not pursued here.

IV. PAIR CREATION OF VECTOR MESONS BY PHOTONS

If the B mesons do exist as real intermediaries in the weak interactions, then it should be possible to produce them in some way amenable to experiment. Lee and Yang[2] have proposed using high-energy neutrinos to search for the onset of the semiweak process $\nu \to e + B$ or $\nu \to \mu + B$ in the Coulomb field of a nucleus. We here consider the possibility of their pair production in the electromagnetic process $\gamma \to B^+ + B^-$ in the field of a nucleus. This latter process has a cross section which instead of being semiweak ($\sigma \approx 10^{-37} cm^2$) is typically electromagnetic ($\sigma \approx 10^{-31}$ cm^2). In this section we compare qualitatively the advantages and disadvantages of these two methods of production.

A. Discussion of Energy Dependence of the Cross Sections

The cross sections for vector-meson processes have a much stronger energy dependence than those for Dirac particles and, in Born approximation, increase indefinitely with energy. We now investigate the Coulomb scattering of vector mesons to illustrate that this singular behavior is associated with the extra longitudinal spin degree of freedom that spin-1 particles possess.

1. Plane-Wave Expansion

We begin with the plane-wave expansion of the vector meson field $U_\mu(X)$,

$$U_\mu(X) = \frac{1}{(2\pi)^{3/2}} \sum_{r=1}^{3} \int \frac{d^3p}{\sqrt{2E}} \left[\epsilon_\mu^{\ r} a^r e^{-iP\cdot X} + \epsilon_\mu^{\ +r} b^{+r} e^{iP\cdot X} \right] , \tag{4.1}$$

where E is the meson energy, and a^r, b^r are respectively the destruction and creation operators for particle and antiparticle of spin polarization $\epsilon_\mu^{\ r}$. We choose the z axis in the direction of propagation $\vec{P}$ so that r = 3 denotes longitudinal and r = 1, 2 transverse polarizations. Then since $\epsilon_\mu^{\ 2} = 1$ and $\partial_\mu U_\mu = 0$, $P\cdot\epsilon^r = 0$, for the transverse polarization vectors $\epsilon_\mu^{\ 1,2}$,

$$\epsilon_\mu^{1,2} = (\vec{\epsilon}^{\,1,2}, 0) , \tag{4.2}$$

where $\vec{\epsilon}^{\,1,2}$ are unit vectors perpendicular to $\vec{P}$; and for the longitudinal polarization vector

$$\epsilon_\mu^{3} = (E\vec{P}/mp,\ iP/m) . \tag{4.3}$$

The covariant polarization sum is given by

$$\sum_{r=1}^{3} \epsilon_\mu^{r} \epsilon_\nu^{r} = (\delta_{\mu\nu} + P_\mu P_\nu/m^2) , \tag{4.4}$$

while for the transverse polarizations only we obtain

$$\sum_{r=1}^{2} \epsilon_i^{r} \epsilon_j^{r} = (\delta_{ij} - P_i P_j/P^2),\ (i,\ j = 1, 2, 3) . \tag{4.5}$$

2. Vertex Operator

For the matrix element of the vector meson current operator between free-particle states of initial and final momenta P, and P' , corresponding to the emission or absorption of a photon of momentum K(P = P' + K), we have

$$\langle P' | J_\mu | P \rangle = - (e/2)\ (2\pi)^{-3} (EE')^{-1/2} \left[(P_\mu + P'_\mu)\ \epsilon \cdot \epsilon' + K_\nu \cdot (\epsilon'_\mu\ \epsilon_\nu - \epsilon'_\nu \epsilon_\mu) \right] . \tag{4.6}$$

(We are here specializing to vector mesons of unit magnetic moment and no quadrupole moment $g = 1,\ q = 0$).

3. Coulomb Scattering

For the Coulomb scattering differential cross section, corresponding to the diagram in Fig. 2, we find

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z^2 a^2}{4P^2V^2} \frac{1}{\sin^4\theta/2}\right)\frac{1}{4E^2} \Big[4(P\cdot n)^2 (\epsilon\cdot\epsilon')^2 + (P\cdot\epsilon')^2(n\cdot\epsilon)^2 + (P'\cdot\epsilon)^2(n\cdot\epsilon')^2$$

$$+ 2(P\cdot\epsilon')(P'\cdot\epsilon)(n\cdot\epsilon)(n\cdot\epsilon') - 4(P\cdot n)(P\cdot\epsilon')(n\cdot\epsilon)(\epsilon\cdot\epsilon')$$

$$- 4(P'\cdot\epsilon)(\epsilon'\cdot n)(P\cdot n)(\epsilon\cdot\epsilon')\Big], \tag{4.7}$$

where primed and unprimed quantities refer to initial and final meson respectively, $n = (0, 0, 0, i)$ is the virtual photon polarization, and θ is the scattering angle. If we sum over final polarizations and average over initial polarizations we obtain, for the total differential cross section[21] $d\sigma_T/d\Omega$,

$$\frac{d\sigma_T}{d\Omega} = \frac{Z^2 a^2}{4P^2V^2} \frac{1}{\sin^4\theta/2}\left[1 + \frac{(\gamma^2-1)^2}{6\gamma^2}\sin^2\theta\right]. \tag{4.8}$$

Here $\gamma^2 = (1 - V^2/c^2)^{-1}$. In contrast to the case of spin-1/2 and spin-0 particles $d\sigma_T/d\Omega$ does not approach zero with increasing incident meson energy. We now separate out of the total cross section (4.8) the separate contributions of longitudinal and transverse polarized mesons.

(a) Transverse-Transverse Spin Transitions. When both the initial and final meson are transverse-polarized, Eqs. (4.2) and (4.7) give

$$\frac{d\sigma}{d\Omega} = \frac{Z^2 a^2}{4P^2V^2} \frac{1}{\sin^4\theta/2} (\vec{\epsilon}\cdot\vec{\epsilon}')^2 . \tag{4.9}$$

Let $\sigma_R = Z^2 a^2 \sin^{-4}(\theta/2)(4P^2V^2)^{-1}$, the relativistic Rutherford cross section. If we sum Eq. (4.7) over the transverse polarizations by using Eq. (4.5), we find

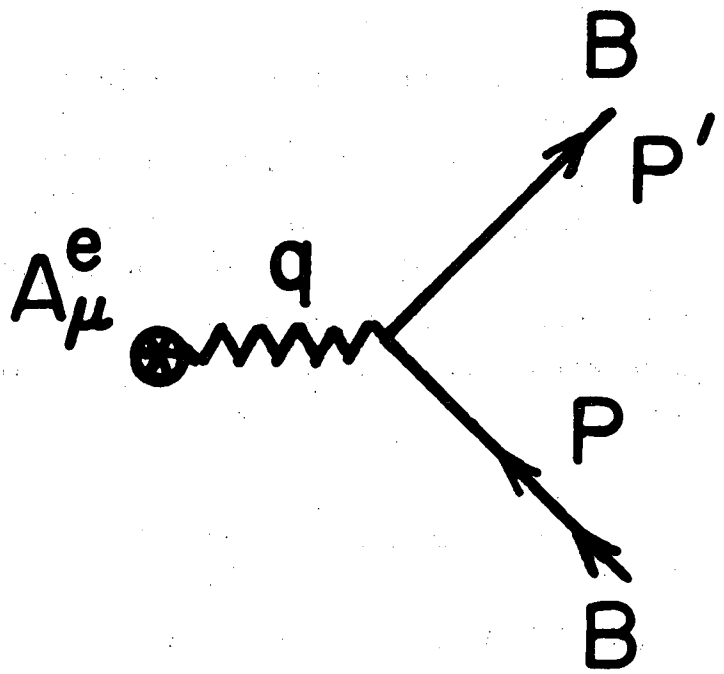

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Fig. 2. Diagram for Coulomb scattering of B mesons.

$$\frac{d\sigma}{d\Omega} = \sigma_R \, (1 + \cos^2 \theta) \, . \tag{4.10}$$

[transverse-transverse transitions]

(b) Longitudinal-Longitudinal Spin Transitions. When both the initial and final meson are longitudinally polarized, Eqs. (4.3) and (4.7) give

$$d\sigma/d\Omega = \sigma_R \cos^2 \theta \, . \tag{4.11}$$

[longitudinal-longitudinal transitions]

(c) Transverse-Longitudinal Spin Transitions. Finally, when the initial meson is transverse-polarized and the final meson longitudinally polarized (or vice versa), we have

$$d\sigma/d\Omega = \sigma_R \, (E^2 + m^2)^2/(2mE)^2 \, (\vec{P}' \cdot \vec{\epsilon})^2/P^2 \, . \tag{4.12}$$

By summing over the transverse polarizations we obtain

$$d\sigma/d\Omega = \sigma_R \, [\, (E^2 + m^2)^2/4m^2E^2] \, \sin^2 \theta \tag{4.13}$$

[transverse-longitudinal transitions] .

After adding Eqs. (4.10), (4.12) and twice (4.13) (to account for both transverse-longitudinal and longitudinal-transverse transitions), and then dividing the sum by 3 (the statistical weight of the initial meson), we obtain Eq. (4.8). This calculation illustrates how, in Coulomb scattering at least, the high-energy behavior of the vector meson cross sections is due to the transverse-longitudinal and the longitudinal-transverse spin transitions, the matrix element of which increases indefinitely with energy.

In the spin-1/2 case, the second term in the bracket of Eq. (4.8), which is just the spin-correction term to the relativistic Rutherford cross section σ_R, is replaced by $(V/c)^2 \sin^2 \theta/2$, while for spin 0 there is no such term. This comparison of the Coulomb scattering cross sections illustrates how the electromagnetic interaction of vector mesons increases with increasing energy, contrary to the behavior we are used to in the spin-1/2 and spin-0 cases. This energy dependence will appear again in our discussion of the Compton scattering and pair-production formulae.

The above considerations can be applied to the decay $\mu \rightarrow e + \gamma$. If one calculates N by assuming a scalar intermediate meson[6] a finite value for N is obtained, while for a vector meson the value for N is logarithmically divergent. The reason for this is the energy dependence of the longitudinal-transverse spin transitions, which can occur when the vector meson emits magnetic dipole radiation in spin-flip transitions.

B. Photo-Pair Production of Vector Mesons

In this section we employ the Weizsacker-Williams approximation[22] to calculate the cross section for forward production of vector meson pairs in the limit of high incident-photon energies.[23] An exact Born approximation calculation would be considerably more tedious and, because of the unphysical quadratic increase of the cross section with energy, would be, in any case, no more reasonable physically. These cross sections are only suggestive, since at high energies the neglect of vector-meson form-factor structure is unjustified.

1. Weizsacker-Williams Approximation

We present here a brief description of the Weizsacker-Williams approximation. Consider a fast meson ($V \approx c$) moving by a fixed nucleus of charge Ze, and use the reference frame in which the incoming meson is at rest and the nucleus moves by with a velocity V_0. In this moving frame the field of the nucleus is contracted along the direction of motion of the nucleus and can be considered to consist of

virtual quanta directed along this direction. At a distance r from the nucleus the number I_ν of quanta per unit frequency interval per unit area is given by

$$I_\nu = 2(Ze)^2/\pi r^2 \qquad \text{for} \quad \nu < E_0/2\pi mr ,$$

$$I_\nu = 0 \qquad \text{for} \quad \nu > E_0/2\pi mr ,$$

where E_0 is initial meson energy in original frame. These quanta are scattered by the meson, and we find those that have energy ϵE_0 $(0 < \epsilon \leqslant 1)$ after the scattering in the original frame. In the moving frame let K_0 and K be the momenta of the incident and scattered quanta respectively, and θ the angle between $\vec{K}_0$ and $\vec{K}$. The energy of the scattered quantum in the original frame is

$$\epsilon E_0 = K (1 - V_0 \cos \theta) (1 - V_0^2)^{-1/2} \sim K(1 - \cos \theta) E_0/m. \tag{4.14}$$

By the Compton relation we have

$$K_0 = K/(1 - (K/m) (1 - \cos \theta)) ,$$

and thus, by Eq. (4.14),

$$K = m \epsilon (1 - \cos \theta)^{-1}, \; K_0 = m \epsilon (1 - \epsilon)^{-1} (1 - \cos \theta)^{-1} . \tag{4.15}$$

Consequently the bremsstrahlung cross section (from which we can obtain the pair production cross section) for emission of a quantum of energy between ϵE_0 and $(\epsilon + d\epsilon) E_0$ is

$$\phi(\epsilon)d\epsilon = \frac{1}{\epsilon} \int_d^{\tilde{R}} 2\pi r dr \int_{-1}^{1-\eta} d(\cos \theta) \frac{K}{K_0} \frac{d\sigma}{d\Omega} \frac{2\pi}{(1-\epsilon)^2} \frac{Z^2 e^2}{\pi^2 r^2} , \tag{4.16}$$

where $d\sigma/d\Omega$ is the Compton cross section, d is the smallest impact parameter, i.e., the nuclear radius, K and K_0 are given by Eq. (4.15), and

$$R = 2\,E_0\,(1-\epsilon)/m^2\epsilon, \qquad \eta = rm^2(\epsilon/E_0)\,(1-\epsilon)^{-1}\,.$$

2. Compton Effect for $g = 1$ Vector Mesons

In order to find the pair-production cross section from Eq. (4.16) it is necessary to evaluate the Compton cross section for the particle in question. The Compton cross section for vector mesons of unit magnetic moment ($g = 1$) was calculated by Booth and Wilson.[26] In the rest frame of the initial meson their result, corresponding to the diagrams of Fig. 3, is

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\,r_0^{\,2}\left(\frac{K}{K_0}\right)^2\left[1 + \cos^2\theta + \frac{KK_0}{12m^2}\,(7 - 16\cos\theta + 3\cos^2\theta)\right.$$
$$\left. + (29 - 16\cos\theta + \cos^2\theta)\,(K^2 + K_0^{\,2})/48\,m^2\right], \qquad (4.17)$$

where K, K_0 are the final and initial photon momenta respectively, θ is the scattering angle, and $r_0 = \alpha/m$ is the classical "meson" radius.

3. Compton Effect for $g = 0$ Vector Mesons

It would be interesting to see the dependence of the vector meson Compton cross section, and hence pair-production cross section, on arbitrary magnetic moment and quadrupole moment, but the inclusion of such terms would in general make the calculation very difficult. By way of illustration, however, we are able to do the calculation in the special case $\gamma = -1$ ($q = 0$), which corresponds to $g = 0$. The matrix element for Compton scattering corresponding to the diagrams of Fig. 3, is obtained from the Lagrangian (2.28) with $\gamma = -1$, $Q = 0$. We find

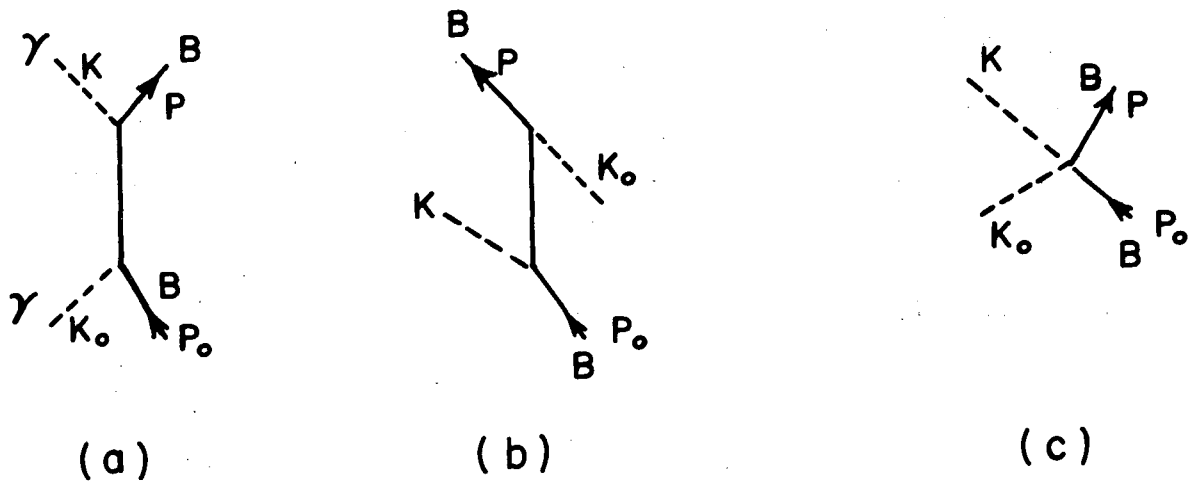

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Fig. 3. Diagrams for Compton scattering of B mesons.

$$\mathcal{M} = \frac{ie^2}{2(2\pi)^2} \frac{\delta^4(P + K - P_0 - K_0)}{\sqrt{KK_0\, EE_0}} \Bigg[\epsilon \cdot \epsilon_0 \cdot \left(\frac{P \cdot \epsilon^P P_0 \cdot \epsilon_0^P}{P_0 \cdot K_0} - \frac{P \cdot \epsilon_0^P P_0 \cdot \epsilon^P}{P_0 \cdot K} - \epsilon_0^P \cdot \epsilon^P \right)$$

$$+ \frac{P_0 \cdot \epsilon_0^P K_0 \cdot \epsilon_0}{m^2} \left(\frac{P \cdot \epsilon^P K \cdot \epsilon}{P \cdot K_0} - \epsilon \cdot \epsilon^P \right)$$

$$+ \frac{P_0 \cdot \epsilon^P K \cdot \epsilon_0}{m^2} \left(\epsilon \cdot \epsilon_0^P - \frac{P \cdot \epsilon_0^P K_0 \cdot \epsilon}{P \cdot K_0} \right),$$

$$+ \frac{\epsilon_0 \cdot \epsilon_0^P}{m^2} (P_0 \cdot K_0\, \epsilon \cdot \epsilon^P - K \cdot \epsilon\, P \cdot \epsilon^P) + \frac{\epsilon_0 \cdot \epsilon^P}{m^2} \left(P \cdot \epsilon_0^P K_0 \cdot \epsilon - P_0 \cdot K\, \epsilon \cdot \epsilon_0^P \right) \Bigg],$$

where ϵ_0^P, ϵ^P are respectively the initial and final photon polarization four-vectors and the notation is evident from Fig. 3. If we now choose the initial meson to be at rest, and the photon gauge so that $P_0 \cdot \epsilon_0^P = P_0 \cdot \epsilon^P = 0$, then $\mathcal{M}$ simplifies considerably, a simplification that occurs only for $g = 0$. In this frame (which is just the frame required for the Weiszacker-Williams approximation) the differential Compton cross section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_0^2 \left(\frac{K}{K_0} \right)^2 \left[1 + \cos^2\theta + \frac{K^2 + K_0^2}{3m^2} (5 + \cos^2\theta) - \frac{4}{3} \frac{KK_0}{m^2} \cos\theta\, (1 - \cos\theta) \right]. \tag{4.18}$$

4. Compton Cross Sections For Spin 1/2 and Spin 0

By way of comparison we quote here the Compton cross sections for both spin 1/2[27] and spin 0.[26] They are

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_0^2 \left(\frac{K}{K_0}\right)^2 (1 + \cos^2\theta) \quad \text{for spin 0 ,}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_0^2 \left(\frac{K}{K_0}\right)^2 \left[1 + \cos^2\theta + \frac{K}{K_0} + \frac{K_0}{K} - 2\right]$$

for spin 1/2 .

By comparing these two results with Eqs. (4.17) and (4.18), we see that the cross section for vector mesons has a much stronger energy dependence than the cross sections for spin 0 and spin 1/2, and also that the energy dependence for both $g = 0$ and $g = 1$ in the vector-meson case is the same. This increased energy dependence for spin-1 particles is caused by the longitudinal-transverse spin transitions as in Coulomb scattering.

5. High-Energy Limit of Pair Production of Vector Mesons.

By using Eq. (4.17) in Eq. (4.18), we are able to calculate the bremsstrahlung cross section and hence, by the substitution rule, the pair-production cross section. The bremsstrahlung cross section was obtained by Christy and Kusaka[28] for this case $g = 1$, $q = 0$.

For the total pair production cross section for mesons with $g = 1$, we find

$$\sigma_T = \frac{a^2(eZ)^2}{m^2} \left\{ \frac{7K_0}{12\, m^2 d} - \frac{1}{36} \log^3\left(\frac{2K_0}{m^2 d}\right) + 9 \log^2\left(\frac{2K_0}{m^2 d}\right) + 16 \log\left(\frac{2K_0}{m^2 d}\right) - 4/3 \right\} . \tag{4.19}$$

This result is valid only for high incident-photon energies, $K_0/m >> 1$; terms of order m/K_0 were dropped in the expression (4.19). In σ_T, d is taken to be the "root-mean-square" nuclear radius in the equivalent uniform-charge model of the nucleus.[29] This quantity is related to $r_0' A^{1/3}$, the range ($\approx 1.3 \times 10^{-13}$ cm) in the exponential model by

$$d = \sqrt{3/5}\, r_0' A^{1/3} ,$$

where A is the nuclear mass number.

If we now use Eq. (4.18) in Eq. (4.16), the leading term in the pair-production cross section obtained is

$$\sigma_T = \frac{4}{3} \alpha^2 \frac{(Ze)^2}{m^2} \frac{K_0}{m^2 d} .$$

We thus find that the energy dependence of the leading term is the same for $g = 0$ as for $g = 1$. Although this result may be fortuitous, it suggests that the leading term in the energy dependence of the cross section is not very g-dependent.

C. Discussion of the Proposed Experiment

The photoproduction of B-meson pairs is in many experimental ways complementary to the proposed high-energy neutrino production. The neutrino reaction $\nu \rightarrow \mu + B$ has a small cross section (about one count per hour per ton of detector is expected), but enjoys the advantage of freedom from competing semiweak processes. For pair production, on the other hand, the cross section is very much larger, but a background of even more likely electromagnetic processes must be contended with. It is this problem of discrimination against background that we wish to discuss qualitatively in this section.

Although the neutrino production of B mesons requires high-energy particle accelerators, the electromagnetic pair production is feasible at any synchrotron facility possessing the requisite threshold energy $2m_B$. The neutrino production experiments constitute part of a large program of high-energy neutrino experiments, while the photoproduction experiments constitute part of a rather different program of high-energy electromagnetic experiments.

1. Electron Background

The background in the pair-production experiment consists mainly of directly produced lighter-mass pairs: e^+e^-, $\mu^+\mu^-$, $\pi^+\pi^-$, K^+K^-. Because of the dimensional factor $\alpha Z^2(e^2/m^2)$ in all the pair-production cross sections the electrons will, because of their small mass, provide by far the greatest background contamination. At high energies the ratio of total cross sections for B and e pair production $(\sigma_T)_B$ is given by Eq. (4-16)), and $(\sigma_T)_e = \alpha Z^2(e^2/m)^2 \frac{28}{9} \log\left(\frac{2K_0}{m_e}\right)$ [27] will be

$$\frac{(\sigma_T)_B}{(\sigma_T)_e} = \frac{3}{16} \left(\frac{m_e}{m_B}\right)^2 \frac{K_0}{m_B^2 d} \Big/ \ln\left(\frac{2K_0}{m_e}\right)$$

At $K_0 = 2$ Bev this gives $(\sigma_T)_B / (\sigma_T)_e = 10^{-5}$ for B mesons of mass 500 Mev.

The probability that an electron will be produced in pair production with energy E_e and at an angle greater than θ is approximately[30] $(m_e/E_e)^2(3/8)(1 - \cos\theta)^{-1}$, while the heavier B mesons, and more importantly their decay products, will be produced rather isotropically. Unfortunately, without knowing the mass of the assumed B meson, we cannot be more quantitative about these kinematic factors.

2. B-Meson Signatures

The pair-produced B mesons decay promptly into $\mu + \nu$, $e + \nu$, and other products. The B leptonic decays will therefore lead to $e^+ + e^- + \nu + \bar{\nu}$, $\mu^+ + \mu^- + \nu + \bar{\nu}$, $e^+ + \mu^- + \nu + \bar{\nu}$, and $e^- + \mu^+ + \nu + \bar{\nu}$ with about equal frequency. This $\mu^\pm - e^\mp$ signature in half the decays is unique to B-pair production.

The muons and electrons from $B^\pm$ production are polarized because of the semi weak decay, while the electromagnetically produced electrons and muons are not. The products of B decay will also be energetically distinguished from directly produced muons and electrons.

V. CONCLUSION

The decay $\mu \rightarrow e + \gamma$ is certainly very important for the intermediate-vector meson hypothesis in the weak interactions if we assume the two neutrinos in ordinary μ decay are the same. As we have shown, absence of the $\mu \rightarrow e + \gamma$ decay mode does not provide conclusive evidence that the B-meson hypothesis is incorrect, since by a judicious combination of magnetic dipole and electric quadrupole moments, the decay amplitude can be made to vanish for both $\mu \rightarrow e + \gamma$ and the coherent conversion $\mu + n \rightarrow e + n$. The rather ad hoc nature of this solution is unsatisfactory, but lack of any criteria for fixing the electromagnetic properties of the B meson prevents us from definitely excluding the intermediary meson solely because of the failure to observe $\mu \rightarrow e$ conversion processes.

A definitive test for the existence of the B meson is, of course, its production either by high-energy neutrinos or by photons in the Coulomb field of a nucleus. Although there are serious background difficulties in the detection of pair production of B mesons, the cross section is large enough that, if B mesons exist, this process should ultimately be observed. If neither of the proposed production experiments is successful, then the attractive Yukawa mechanism for the Fermi interactions will have to be rejected.

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