University of California

Ernest O. Lawrence Radiation Laboratory

Berkeley, California

DISCLAIMER

UCRL-9728

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

A UNIFIED DYNAMICAL APPROACH TO HIGH- AND LOW-ENERGY STRONG INTERACTIONS

S. C. Frautschi

May 31, 1961

A UNIFIED DYNAMICAL APPROACH TO HIGH- AND LOW-ENERGY STRONG INTERACTIONS

S. C. Frautschi

Lawrence Radiation Laboratory
University of California
Berkeley, California

May 31, 1961

I. INTRODUCTION

In the past, applications of the analyticity properties of the S matrix have usually been confined to limited regions of energy and momentum transfer. A great deal of work has been done on low energy scattering, forward dispersion relations covering all energies at zero momentum transfer have been considered, and very recently the peripheral contributions to high energy, low momentum transfer scattering have received much attention. My report deals with recent work at Berkeley, in which the region of application is enlarged, and the dynamics of high- and low-energy strong interactions are treated in a unified way.

How did we become interested in a unified treatment of all energies? The answer is, we were forced to it by our attempts to understand low energy scattering consistently within the Mandelstam framework. In particular, difficulties were encountered in detailed calculations attempting to incorporate P wave resonances into the π-π [1] and π-N [2] systems, and we believe these difficulties can be resolved only by explicit consideration of higher energies and inelastic effects. The situation is perhaps analogous to the history of the static model, where the very nature of the static approximations made the treatment of certain low-energy phenomena, such as the nucleon magnetic moment, intrinsically unsatisfactory until relativistic methods were introduced. In view of the title of this Conference, however, I shall attempt to concentrate on our approach to high energies, together with the uses high energy considerations may have for understanding low energies.

II. SOME GENERAL CONSEQUENCES OF THE MANDELSTAM REPRESENTATION

I shall begin by describing some general consequences of the Mandelstam representation, before proceeding to approximations and speculations.

Chew and I[3] have found it possible to give a formal definition of a "generalized potential," which leads to suggestive analogies with ordinary potential scattering [at least of the type which obeys a Mandelstam representation], and has strongly influenced our intuition concerning the probable outcome of the detailed dynamical calculations we have proposed. I shall assume that the audience is familiar with the Mandelstam variables and the Mandelstam representation. Spin complications and single spectral functions will be ignored. The variable s serves as barycentric energy squared for channel I, t for channel II, and u for channel III. The amplitude $A(s, t, u)$ satisfies

$$A(s, t, u) = \frac{1}{\pi} \int \frac{dt' \, A_2(t', s)}{t' - t} + \frac{1}{\pi} \int \frac{du' \, A_3(u', s)}{u' - u} . \qquad (1)$$

A_2, the discontinuity in channel II, can be split into

$$A_2(t, s) = V_I^{II}(t, s) + \frac{1}{\pi} \int \frac{ds' \, \rho_{st}^{I(el)} (s', t)}{s' - s} \qquad (2)$$

where V_I^{II} represents scattering into inelastic intermediate states in channel I and $\rho_{st}^{I(e\ell)}$ contains the scattering into elastic intermediate states in channel I. The term V_I^{II} serves as a generalized direct potential for channel I. The generalized exchange potential has a corresponding definition in terms of channel III with u and t interchanged. If the generalized potentials are given, and we diagonalize the S matrix in channel I with respect to all internal quantum numbers (isotopic spin, strangeness, etc.), then the elastic double spectral functions for channel I can be computed by quadrature from

$$\rho_{st}^{I(e\ell)}(s, t) = \frac{1}{\pi q_s \sqrt{s}} \left[\iint \frac{dt'\, dt''\, A_2^*(t', s)\, A_2(t'', s)}{K^{1/2}(q_s^2; t, t', t'')} + \iint \frac{du'\, du''\, A_3^*(u', s)\, A_3(u'', s)}{K^{1/2}(q_s^2; t, u', u'')} \right] \tag{3}$$

and a corresponding equation for $\rho_{su}^{I(e\ell)}$. K and the limits of integration are given by known kinematic expressions. Equation (3) was first derived by Mandelstam.[4] Now comes the important point: the set of Eqs. (1)-(3) has a close analogy in nonrelativistic potential scattering. The generalizations are that the generalized potential is energy-dependent, it becomes complex above the inelastic threshold, and the relativistic kinematic factor $\sqrt{s}$ must be employed in (3).

We now turn to work by Froissart,[5] who has shown that the cross section for the reaction $a + b \to c + d$ is limited by $\sigma(s) \leqslant \ln^2 s$ as $s \to \infty$. Although his proof was carried out only for scalar particles, it is believed that the extension to particles with spin will be straightforward. Froissart's treatment is quite rigorous, but we must content ourselves here with an heuristic argument based on the generalized Yukawa potential

$$g(s) \; \frac{e^{-\mu r}}{r} \quad . \tag{4}$$

A particle with impact parameter a sees a total interaction of order $g(s)e^{-\mu a}$. The scattering is strong when the total interaction is of order one or greater, so the radius within which strong scattering occurs is $a \simeq \mu^{-1} \ln g(s)$, corresponding to a cross section

$$\sigma \simeq \frac{\pi}{\mu^2} \ln^2 g(s) \quad . \tag{5}$$

Since the Mandelstam representation contains at most a finite number of subtraction terms, the strength g of the generalized potential can increase no faster than some power of s. Therefore the cross section increases at most as $\ln^2 s$.

In Froissart's rigorous proof he develops a method powerful enough to give further bounds on scattering at nonforward angles. For the covariant amplitude A he obtains

$$A \leqslant s \ln^2 s \tag{6}$$

(which corresponds to (5)) for forward (where $\mathrm{Im}\, A \propto s\, \sigma$) and backward angles, and

$$A \leq s^{3/4} \ln^{3/2} s \qquad (7)$$

at any other fixed physical angle. It is not known whether the logarithmic increases will be consistent with more detailed dynamical considerations.

III. THE STRIP APPROXIMATION

Although the generalized potential theory is formally satisfactory, it gains practical significance only to the extent that the generalized potential can be calculated. At this point we are forced to introduce some approximation. Chew and I,[6] as well as Ter-Martirosyan,[7] Gribov,[8] Wilson,[9] and other workers in this field, have chosen the "strip approximation," in which one concentrates on representing the long range part of the potential adequately. To be precise, the one-pion exchange potential is calculated exactly if it is present, the two-pion exchange potential is calculated by formally correct equations, and the potential due to exchanges of heavier systems is neglected. The approximation can be visualized with the aid of Fig. 1, which represents the equal-mass case [e.g. pion-pion scattering]. The double spectral functions are nonzero in the shaded regions. In the <u>heavily</u> shaded strips the complete unitarity condition involves only two-pion intermediate states in one of the channels; for example the strip $\rho_2(t, s)$ is associated with purely two-pion intermediate states for $4 < t < 16$ in the t physical region. The strip approximation consists of calculating only those portions of the double spectral function which correspond to two-pion intermediate states in some channel. Since this restriction is valid only in the "strips," the approximation is best for the singularities nearest the physical regions.

In Fig. 2 we see Cutkosky diagrams which are generated by the strip approximation: (a) corresponds to two-pion intermediate states in the s channel ($\rho_1(s, t)$ of Fig. 1), while (b) is calculated as scattering into two-pion intermediate states in the t channel ($\rho_2(t, s)$ of Fig. 1), but also has the significance of a two-pion exchange potential in the s channel. In the latter capacity, Fig. 2(b) represents scattering into a sum over inelastic intermediate states containing definite numbers of particles in the s channel. A practical calculation would begin by iterating relations such as Eq. (3) to find the scattering into two-pion intermediate states in a given channel. The result provides a two-pion exchange potential for one of the other channels, which can be used in turn to calculate scattering into two-pion intermediate states in that channel, and one continues in this way until consistency is achieved. At each step one uses relations such as Eq. (3) which involve only two-particle absorptive amplitudes, and thus the strip approximation avoids the full complexity of the many-body problem.

What quantities can one hope to calculate in the strip approximation? The price paid for treating only nearby singularities well is that at best only the edges of the physical region lying near such singularities can be treated reliably. From Fig. 1 we see that these edges correspond to small momentum transfers at arbitrary energy; in the s channel for example the physical regions $t \sim 0$ or $u \sim 0$ are treated fairly well at all s. Fortunately both low-energy scattering and the diffraction peaks which dominate high-energy elastic scattering fall within these "edge regions." Furthermore, we recall that the potential for each channel represents a sum over inelastic intermediate states in that channel; the contribution from an individual intermediate state (e.g. $n\pi$) to the imaginary part of the forward elastic amplitude can be calculated, and the optical theorem then gives the cross section $\sigma(2\pi \to n\pi)$. One can see from Fig. 2(b) that this estimate for $\sigma(2\pi \to n\pi)$ is closely related to the one-pion exchange calculations which will be reported at this meeting by Selleri and Ferrari, Salzman, Baker, and others. There are significant differences, however; our approach has the advantage of not requiring any extension of physical cross sections off the mass shell, the disadvantage of not providing the angular dependence of inelastic cross sections (and of much greater mathematical complexity!).

IV. ASYMPTOTIC BEHAVIOR AND OSCILLATIONS IN THE STRIP DIRECTIONS

It is indicated experimentally and very plausible from classical arguments that (to within logarithmic factors) total cross sections approach constant high-energy limits and that the widths (in momentum transfer) of the corresponding forward-diffraction peaks also approach constants. Now a diffraction peak of constant width $\Delta t \sim 20\ \mu^2$ can arise quite naturally from an imaginary amplitude such as

$$\int_4^\infty \frac{dt'\ \rho_{st}(s,\ t')}{t' - t} \tag{8}$$

if the strip region $4 < t' < 16$ dominates the integral. Therefore the experimental occurrence of such peaks increases our faith in the strip approximation. It is furthermore possible that diffraction-like peaks may occur in the physical regions near each strip, even though some of these regions correspond to backward or inelastic (e.g. $\pi^+ + \pi^- \to \pi^0 + \pi^0$) scattering. But to the extent that the Pomeranchuk theorems[10] hold, large peaks will occur only in forward elastic amplitudes.

The assumption of constant high-energy limits for total cross sections implies, for the elastic amplitude in the t channel, that $A_t \sim t$ as $t \to \infty$ at zero momentum transfer. This asymptotic behavior allows us to make some very remarkable statements about low energy behavior. For a class of nonrelativistic potentials which satisfy the Mandelstam representation, Regge[11] has used an independent method to show rigorously that $A_t(t, s) \sim t^{\alpha(s)}$ as $t \to \infty$ at fixed s. He has also shown that partial waves with $\ell \leq [\mathrm{Re}\,\alpha]_{max}$ may have bound states or resonances, while those with $\ell > [\mathrm{Re}\,\alpha]_{max}$ will necessarily have small phase shifts. On the basis of the potential analogy we believe that Regge's results are relevant to the relativistic S matrix. Thus constant total cross sections imply strong S and P wave scattering. In cases such as π-π or π-N scattering where all the strips are close to physical regions, continuity arguments give us cause to hope that α will not be much greater in the elastic scattering region [e.g. $16 > s > 4$ for π-π scattering] than it was at $s = 0$. In these cases Regge's criterion predicts small D and higher waves in the elastic scattering region.

Even more information about low energies can be gained from asymptotic behavior. Froissart[5] has used the asymptotic limits he derived for scalar particles in the physical region (Eqs. (6), (7)) to give a rigorous discussion of how many independent parameters are permitted when the double spectral function is given. He finds that at most the S and P waves in each channel, and one overall subtraction constant, can be independent; if higher partial waves were independent the asymptotic limits would be violated. If either of the Pomeranchuk theorems[10] is valid then the P wave is also determined by the double spectral function [of course the S wave must satisfy a unitarity condition, so it too is determined except for possible

Castillejo-Dalitz-Dyson[12] poles]. Detailed dynamical considerations will be required if any further reductions in the number of parameters are to be made. Chew and I suspect that such reductions will in fact occur.[6] From the experimental observation of P wave resonances at low energies and constant cross sections at high energies, we hypothesize that strong interactions saturate the unitarity condition,[6] and that this condition determines the coupling parameters.

We turn now to the matter of reconciling asymptotic behavior with the detailed dynamical equations of the strip approximation. Some further features discovered by Regge[11] are relevant here: $\alpha(s)$ becomes complex in the double spectral region $s > 4$, and the resulting factor $t^{i\ \mathrm{Im}\ \alpha(s)} = \exp[i\ \mathrm{Im}\ \alpha(s)\ \ln t]$ oscillates an infinite number of times in the double spectral region (but not in the physical region!). Guided by these results we have assumed the generalized behavior

$$A_t(t, s) \sim t^{\alpha(s)} (\ln t)^{\beta} \qquad (9)$$

as $t \to \infty$, and substituted this behavior into Eqs. (2), (3) to see if the result is consistent with the initial assumption. We find consistency for all β if α is complex, whereas (as shown earlier by Gribov[8]), consistency requires $\beta < -1$ if α is real. Russian workers have studied the case of real α and have concluded that both elastic[8] and inelastic[13] cross sections must fall off faster than $\ln^{-1} s$ with increasing s [the inelastic result was derived[13] from a study of one-pion exchange, but can also be obtained from the behavior of the elastic forward amplitude combined with the optical theorem]. On the other hand we at Berkeley, after vigorous prodding from Mandelstam, have come to believe that complex, variable α will be required to describe strong interactions. We are greatly influenced by the potential analogy: we believe that Eqs. (1)-(3) are sufficiently similar in the nonrelativistic and relativistic cases for Regge's result to provide a guide. Additional terms with logarithms, and perhaps with constant α , may be present, but terms of the Regge type are needed in the double spectral regions so that the nonrelativistic behavior can emerge in a suitable limit (low energy scattering, with the mass of the exchanged system negligible relative to the mass of the scattering system).

The complicated, oscillating asymptotic behavior which we now anticipate for the double spectral function makes the prospects of any simple extrapolation from the physical region of high energies and low momentum transfers to the strips of the double spectral function appear quite dim. It is at this point that our "on the mass shell" method may give very different results from the one-pion exchange calculations where one extrapolates physical cross sections to nearby unphysical regions.

V. CALCULATIONS IN PROGRESS

Active investigation of the new approach is being carried out along several lines at Berkeley. Charap is attempting to calculate by machine the spectral functions generated by iteration of a nonrelativistic Yukawa potential. His object is to study the oscillations and ascertain how rapidly the asymptotic behavior sets in, in a problem which is relatively simple and known to be soluble.

Chew and I intend to calculate pion-pion scattering adiabatically, starting with a small value for the usual interaction parameter λ , and then building it up. At small λ we should obtain an S-dominant solution;[15] the question is whether a large P wave can be built up by increasing λ. In their investigation of the S-dominant solution, Chew, Mandelstam, and Noyes[15] obtained small P waves even at large λ, but their complete neglect of the double spectral function on the left cut is suspect. If large P waves cannot be obtained by the adiabatic approach we may need recourse to the method of *assuming* a P wave resonance[1] and then establishing consistency.

Analogous methods are also being applied to pion-nucleon scattering by Singh and Udgaonkar,[16] and to nucleon-nucleon scattering by Charap, Jones, Lubkin, Muzinich, and Scotti.

To summarize, the unified approach gives a framework for discussing high-energy phenomena, and indicates that high energy considerations can help in understanding low energies and vice versa. We are working on it hopefully. But towering mathematical complexities, especially in the form of oscillations, seem to stand between us and the quantitative results we seek.

REFERENCES

1. G. F. Chew and S. Mandelstam, Nuovo cimento 19, 752 (1961). See also G. F. Chew, Proc. 1960 Annual Conf. on High Energy Physics, Rochester, 1960, p.273.

2. S. Frautschi and D. Walecka, Phys. Rev. 120, 1486 (1960); W. Frazer and J. Fulco, Phys. Rev. 119, 1420 (1960); W. Frazer, Proc. 1960 Annual Conf. on High Energy Physics, Rochester, 1960, p. 282.

3. G. F. Chew and S. Frautschi, "The Role of the Potential and Unstable Elementary Particles in the S-Matrix Theory of Strong Interactions," Lawrence Radiation Laboratory Report UCRL-9685, 1961.

4. S. Mandelstam, Phys. Rev. 112, 1344 (1958).

5. M. Froissart, "Asymptotic Behavior and Subtractions in the Mandelstam Representation," University of California (Berkeley) Physics Department preprint (1961).

6. G. F. Chew and S. Frautschi, Phys. Rev. Letters 5, 580 (1960); also "A Dynamical Theory for Strong Interactions at Low Momentum Transfers but Arbitrary Energies," Lawrence Radiation Laboratory Report UCRL-9510, 1960.

7. K. A. Ter-Martirosyan, Soviet Physics-JETP 12, 575 (1961).

8. V. N. Gribov, Nucl. Phys. 22, 249 (1961).

9. K. Wilson, Harvard University Physics Department preprint, 1960.

10. I. Pomeranchuk, Soviet Physics-JETP 3, 306 (1956) and 7, 499 (1958).

11. T. Regge, Nuovo cimento 14, 951 (1959) and 18, 947 (1960).

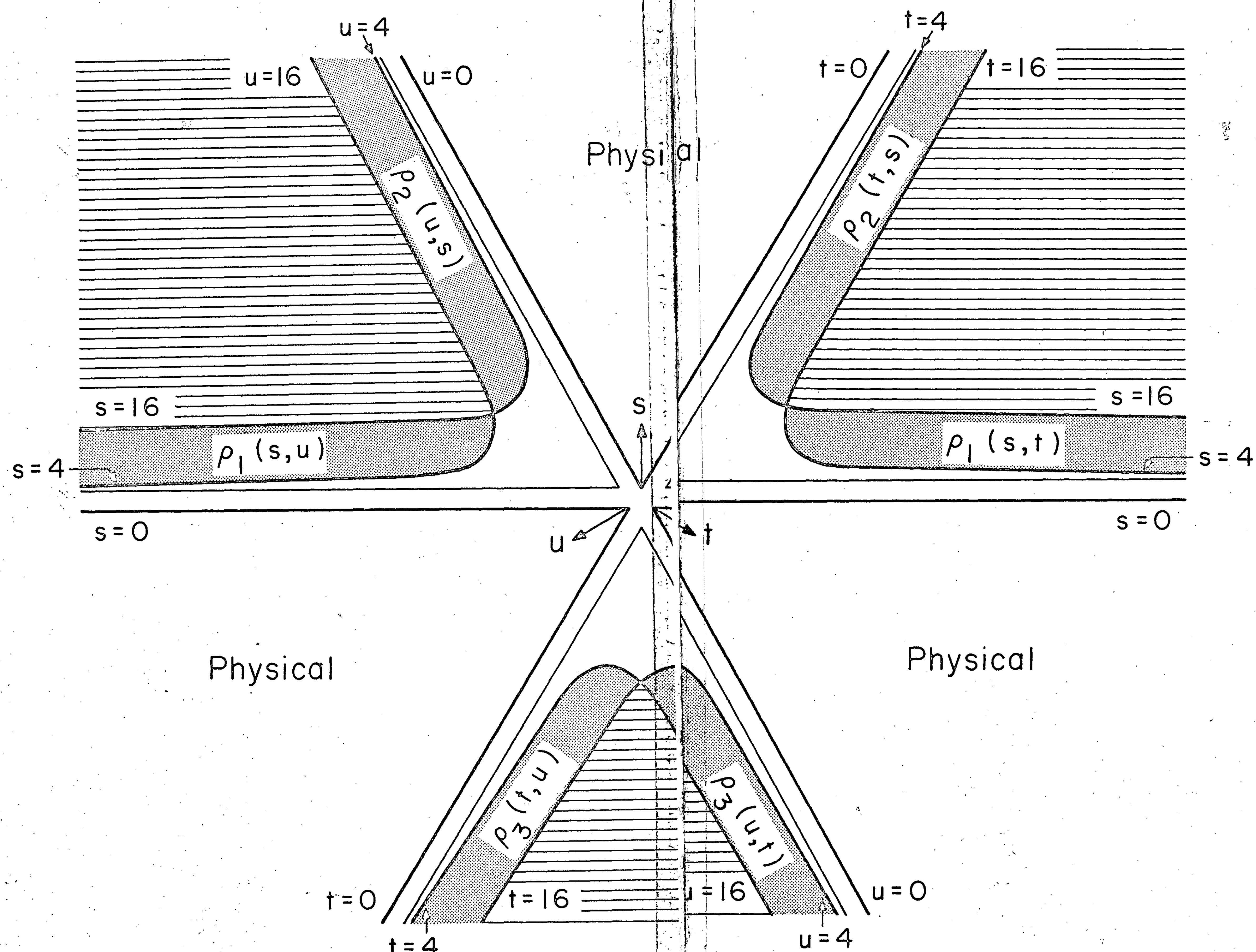
12. Castillejo, Dalitz, and Dyson, Phys. Rev. 101, 453 (1956).

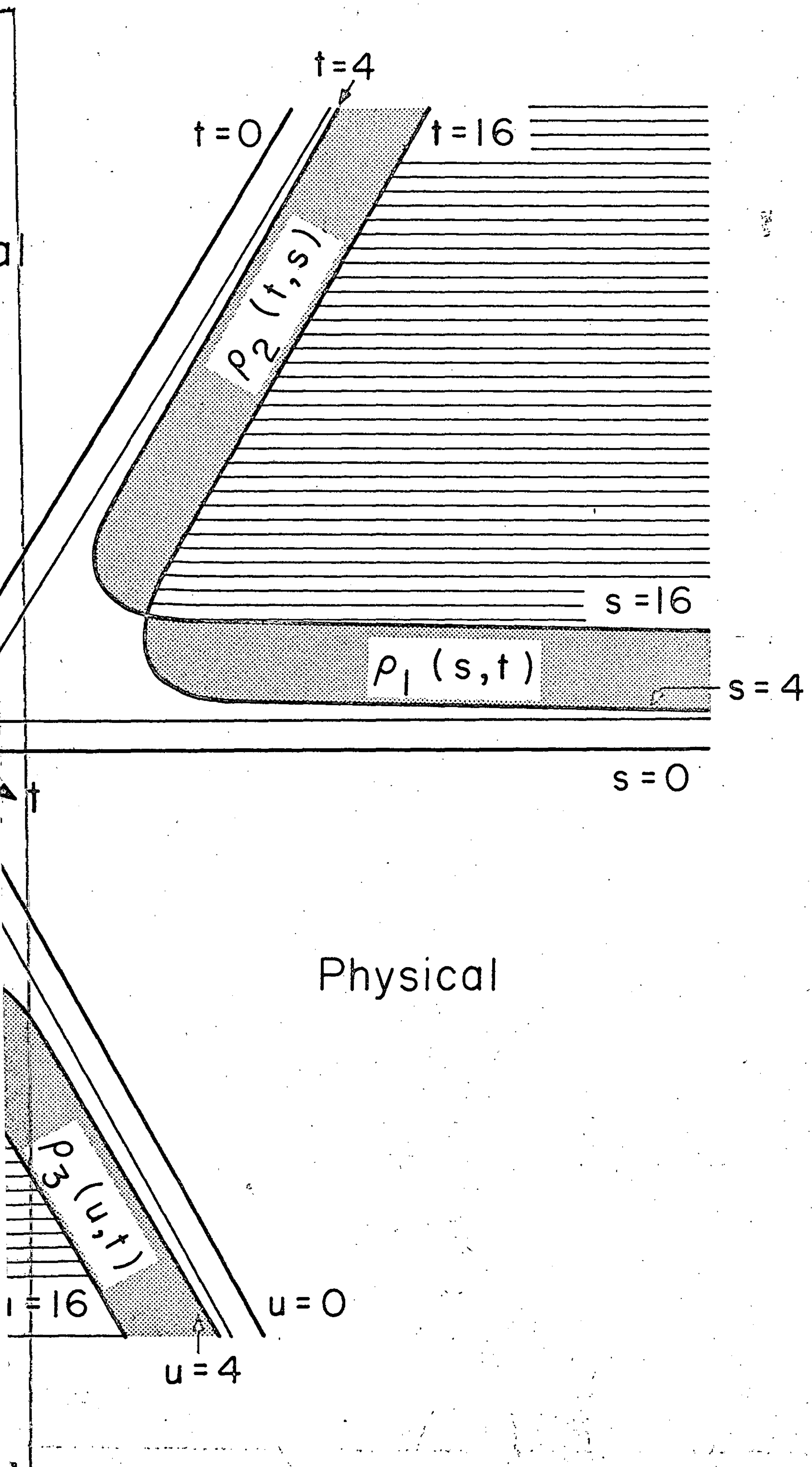
13. V. Berestetsky and I. Pomeranchuk, Nucl. Phys. 22, 629 (1961).

14. S. Mandelstam, Discussion at the Conference on Strong Interactions, Berkeley, 1960, to be published in Revs. Modern Phys.

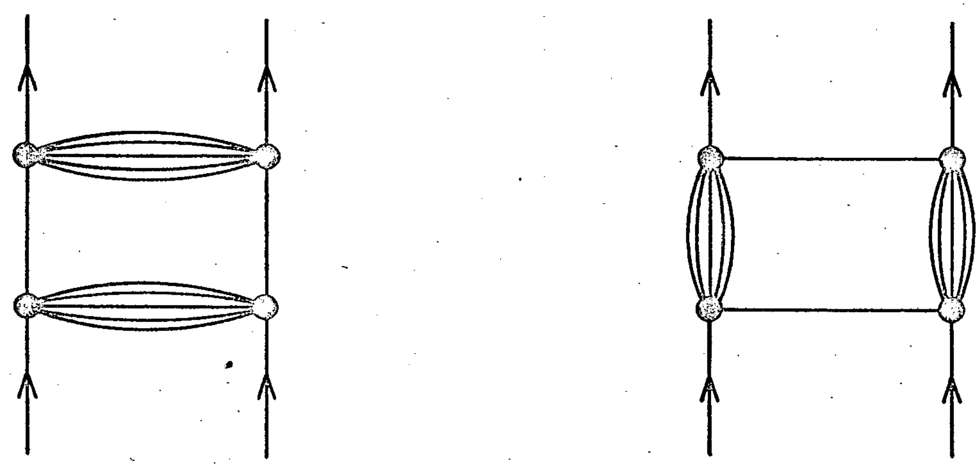
15. G. F. Chew, S. Mandelstam, and H. P. Noyes, Phys. Rev. 119, 478 (1960).

16. V. Singh and B. M. Udgaonkar, "Theory of π-N Scattering in the Strip Approximation to the Mandelstam Representation," Lawrence Radiation Laboratory Report UCRL-9561 (1961).


u=4
u=16
u=0
t=4
t=0
t=16
Physi al
$\rho_2(u,s)$
$\rho_2(t,s)$
s=16
s=16
$\rho_1(s,u)$
$\rho_1(s,t)$
s=4
s=4
s
s=0
s=0
u
t
Physical
Physical
$\rho_3(t,u)$
$\rho_3(u,t)$
t=0
t=16
u=16
u=0
t=4
u=4
58499-2



58499-2



(a) (b)

58500-1