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ABSTRACT

It is shown that under very simple and general assumptions the existence of an antiunitary reflection transformation and the charge gauge group implies the existence of an antiparticle corresponding to a given charged particle. Similar consequences follow on replacing the charge gauge group by the baryon gauge group. No assumptions as to specific wave equations, or indeed the existence of local fields, are made.

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INTRODUCTION

In the conventional discussion of charge conjugation,[1] one starts from specific wave equations for free-particle operators (usually the Dirac equation), and shows that for a consistent interpretation one must have antiparticles corresponding to particles of a given charge. The question arises as to whether there are other possible wave equations which do not have this property. More generally, we can ask if the antiparticles must occur independently of whether there is indeed a consistent local field theory. Here we try to answer the question purely group theoretically. The mathematical tool used is the theory of corepresentations developed by Wigner.[2] Our main result is the theorem that the existence of an antiunitary reflection transformation for free-particle states together with the unitary charge gauge transformations implies the existence of antiparticle states corresponding to given charge states. Replacing the charge gauge transformations by baryon gauge transformations yields the result that neutral baryons must also have antiparticles associated with them.

PROOF OF THE THEOREM

We postulate that free-particle states form the basis for an irreducible corepresentation of our group of quantum mechanical operators. It is sufficient here to assume this group to be a direct product of:

(a) A group of unitary operators corresponding to proper inhomogeneous transformations, and an antiunitary operator corresponding to the combined space and time reflection.

and

(b) The unitary group of operators generated by the charge operator. Since only integer values of the charge occur in nature, we will assume this group to be isomorphic with the two-dimensional rotation group (rotation about the charge axis).

Theorem:

If the representation $e^{im\phi}$ of the charge gauge subgroup occurs in an irreducible corepresentation of our quantum mechanical group, then so does the representation $e^{-im\phi}$.

Proof:

To avoid unnecessary complications, we restrict the proper Lorentz transformations to pure translations. (Carrying along the homogeneous Lorentz transformations merely complicates the computation without shedding any new light.) Our group of operators is then generated by the unitary operators

$$T(a) \, , \tag{1}$$

which correspond to the translation

$$x'_\mu = x_\mu + a_\mu \, ; \tag{2}$$

the unitary operators

$$e^{iQ\phi} \, , \tag{3}$$

which describe the charge gauge group; and the antiunitary operator Θ, corresponding to the transformation

$$x'_\mu = -x_\mu \ . \tag{4}$$

Multiplication rules for these operators are

$$\Theta \, T(a) \, \Theta^{-1} = T(-a) \ , \tag{5}$$

$$\Theta \, e^{iQ\phi} \, \Theta^{-1} = e^{iQ\phi} \ , \tag{6}$$

and

$$T(a) \, e^{iQ\phi} = e^{iQ\phi} \, T(a) \ . \tag{7}$$

The proof of the theorem merely consists of following Wigner's general discussion.[2]

Consider first the subgroup of unitary operators T, $e^{iQ\phi}$. Because this is an Abelian group it has only one-dimensional irreducible representations. Suppose the irreducible representation p,m occurs in an irreducible corepresentation of our full group. Then, there is a state of four-momentum p, charge m such that

$$T(a) \, | \, pm \rangle = e^{i(p,a)} \, | \, pm \rangle$$

and

$$e^{iQ\phi} \, | \, pm \rangle = e^{im\phi} \, | \, pm \rangle \ . \tag{8}$$

Let $| \, \lambda \rangle$ denote an arbitrary state of the corepresentation. We then have

$$\Theta \, | \, pm \rangle = \sum_\lambda | \, \lambda \rangle \langle \lambda \, | \, \Theta \, | \, pm \rangle \ . \tag{9}$$

If U denotes any element of our unitary subgroup,

$$U \, \Theta \, | \, pm \rangle = \sum_\lambda | \, \lambda \rangle \langle \lambda \, | \, U \, \Theta \, | \, pm \rangle \ . \tag{10}$$

However,

$$U\,\Theta = \Theta\,\Theta^{-1}\,U\,\Theta\ , \tag{11}$$

and for a corepresentation,

$$\langle \lambda \mid \Theta\,\Theta^{-1}\,U\,\Theta \mid pm \rangle$$

$$= \sum_{\lambda'} \langle \lambda \mid \Theta \mid \lambda' \rangle \langle \lambda' \mid \Theta^{-1}\,U\,\Theta \mid pm \rangle^{*}\ , \tag{12}$$

therefore

$$U\,\Theta \mid pm \rangle = \sum_{\lambda\lambda'} \mid \lambda \rangle \langle \lambda \mid \Theta \mid \lambda' \rangle \langle \lambda' \mid \Theta^{-1}\,U\,\Theta \mid pm \rangle^{*}\ . \tag{13}$$

By inserting

$$U = T(a)\,e^{iQ\phi}\ , \tag{14}$$

and by using our multiplication rules (5) to (7), we get

$$T(a)\,e^{iQ\phi}\,\Theta \mid pm \rangle = \sum_{\lambda\lambda'} \mid \lambda \rangle \langle \lambda \mid \Theta \mid \lambda' \rangle \langle \lambda' \mid T(-a)\,e^{iQ\phi} \mid pm \rangle^{*}\ , \tag{15}$$

but

$$\langle \lambda' \mid T(-a)\,e^{iQ\phi} \mid pm \rangle = e^{-i(p,a)}\,e^{im\phi}\,\delta\,(\lambda',\,pm)\ . \tag{16}$$

Hence

$$T(a)\,e^{iQ\phi}\,\Theta \mid pm \rangle = \sum_{\lambda} \mid \lambda \rangle \langle \lambda \mid \Theta \mid pm \rangle\,e^{+i(p,a)}\,e^{-im\phi}$$

$$= e^{i(p,a)}\,e^{-im\phi}\,\Theta \mid pm \rangle\ . \tag{17}$$

Thus $\Theta \mid pm \rangle$ is a state of four-momentum p, and charge $-m$; i.e.,

$$\Theta \mid pm \rangle = \mid p, -m \rangle \quad . \tag{18}$$

We see that the antiunitary reflection transformation requires that for a state with nonvanishing charge we have necessarily a degeneracy with a state of opposite charge.

DISCUSSION

The connection of this result with the TCP theorem for free particles is quite obvious. Since we have the two degenerate states we can clearly introduce an operator (C) which interchanges the two. However, we have seen that Θ (which is just PT) already does this. Hence, the combined operator is merely one which carries our state vectors into themselves.

It may be noted that the above theorem says nothing about the neutral particles--as, for example, a Λ_0 . However, if there is another gauge group which can be substituted for the charge group, we can draw exactly similar conclusions. Thus, postulating baryon conservation, we can conclude that the Λ_0 must have an antiparticle. Similarly, even without assuming charge independence, we can conclude that the Σ_0 and Ξ_0 must have antiparticles. For the K_0 we can only draw this conclusion subject to the usually assumed doublet structure <u>and</u> charge independence, unless one wants to postulate the strangeness gauge group.

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REFERENCES

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1. For example, G. C. Wick, Ann. Rev. Nuclear Sci. 8, 1 (1958).

2. E. P. Wigner, Group Theory (Academic Press, Inc., New York, 1959) Chap. 26.