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ABSTRACT

The spin and parity of the ω meson have been determined to be 1^- by a quantitative comparison of the density of points on the Dalitz plot with the predicted density of points. The simplest matrix element for the $J = 1^-$ meson (which predicts maximum density of points at the center of the Dalitz plot and zero density on the boundary) fits the data remarkably well. On the other hand, the simplest matrix elements for 1^+ and 0^- mesons (both of which predict zero density at the center of the plot) do not fit at all. A quantitative treatment for higher J values has not been attempted. However, the simplest matrix elements for $J = 2^+$ and 2^- also vanish at the center of the Dalitz plot, and are again inconsistent with the data.

SPIN AND PARITY OF THE ω MESON*

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In a previous paper, "Evidence for a T = 0 Three-Pion Resonance",[1,2] we presented data suggesting a spin assignment $J = 1^-$ (vector meson).

The density of points on our Dalitz plot was depopulated near the boundary, and densely populated near the middle. This density was compared with the three possible simple matrix elements for $J \leq 1$. Their qualitative features are summarized in Table I, which is slightly more detailed than the version that we presented earlier. The meaning of the angular momenta $\underline{\ell}$ and $\underline{L}$ is as follows: The, matrix element is analyzed in terms of a single pion plus a dipion. The pions of the dipion are assigned a momentum $\underline{q}$, and an angular momentum $\underline{L}$ (in the dipion rest frame). Then another pair of variables $\underline{p}$ and $\underline{\ell}$, describe the remaining pion in the ω rest frame.[3] A T = 0 state of three pions must be antisymmetric in all pairs, hence all three of the competing matrix elements must vanish where any two pions "touch" in momentum space (i.e., q = 0). This corresponds to regions b, d, and f, on the Dalitz plots in Fig. 1, D and E.

The three competing matrix elements have the following characteristics:

a. The axial matrix element (1- meson) involves cross products of momentum vectors and vanishes whenever all momenta are collinear. This occurs on the boundary of the Dalitz plot.

b. The scalar matrix element (0^- meson) vanishes whenever two pions have the same energy, i.e., along the straight lines that divide the Daltiz plot.

c. The vector matrix element (1^+ meson) vanishes whenever all three pions have the same energy, i.e., at the center of the plot. In contrast to the scalar matrix element, it does not vanish at points a, c, and e, where $|\underline{p}| = 0$ (the finite population here is contributed by the $\ell = 0$ partial wave). Values of $p = 1$ and 2 (units of $m_\pi c$) are indicated in Fig. 1, E.

We concluded that the data suggest that ω is a 1^- meson. This Letter presents quantitative support for that conclusion. Since our earlier letter,[1] we have processed an additional 325 events of the type

$$\bar{p} + p \rightarrow \pi^+ + \pi^+ + \pi^- + \pi^- + \pi^0 . \tag{1}$$

These have been added to our previous 800 events, and the remainder of the discussion is based on all 1125 events.

The square of the matrix element, $|\underline{A}|^2$ has been calculated for the three possible mesons 1^-, 0^-, and 1^+. In terms of $\vec{p}$, $\vec{q}$ and $\mu (= \hat{p} \cdot \hat{q})$, the values of $|\underline{A}|^2$ are:

<u>1^- meson</u>

$$|\underline{A}|^2 = 9\, p^2 q^2 (1 - \mu^2) \tag{2}$$

<u>0^- meson</u>

$$|A|^2 = \frac{\mu^2}{4} \left(\frac{p^2 q^2}{q^2 + 1} \right) \left\{ \left(\sqrt{p^2 + 4(q^2+1)} - 2\sqrt{p^2+1} \right)^2 - \mu^2 \left(\frac{p^2 q^2}{q^2+1} \right) \right\}^2 \tag{3}$$

1^+ meson

$$|\underline{A}|^2 = \frac{q^2\mu^2}{q^2+1}\left[p^2 - 2(q^2+1) + \sqrt{p^2+1}\sqrt{p^2+4(q^2+1)}\right]^2$$

$$+ q^2(1-\mu^2)\left[2\sqrt{p^2+1} - \sqrt{p^2+4(q^2+1)}\right]^2 . \tag{4}$$

The relationship between p, q, and M_3, the mass of the three pion system is

$$p^2 = \left[(M_3+1)^2 - 4(q^2+1)\right]\left[(M_3-1)^2 - 4(q^2+1)\right] / 4M_3^2 . \tag{5}$$

The boundary of the Dalitz plot corresponds to $\mu^2 = 1$. The straight lines that divide it correspond to $\mu = 0$. From equations (2), (3), and (4), one sees that all the matrix elements vanish for $q = 0$, as required by the $T = 0$ character of three pions. The other qualitative features are also evident.

Figures 1 A, 1 B, and 1 C are isometric drawings of the square of the matrix elements for the 1^+, 0^- and 1^- mesons, respectively. The coordinates in the plane are normalized Dalitz variables. The height above the plane is proportional to $|\underline{A}|^2$. The maximum height above the plane is chosen as unity and contours of equal values of $|\underline{A}|^2$ are drawn at 0.2 intervals.

Before comparing the data with these $|\underline{A}|^2$ distributions, both the data and $|\underline{A}|2$ can be folded six times about the lines of symmetry that divide the Dalitz plot. The points in Fig. 1D show the 241 pion triplets which fell in the control region ($820 < M_3 < 900$ Mev); those in Fig. 1E are the 270 triplets which fall in the ω peak region ($740 < M_3 < 820$ Mev). The

contours on Fig. 1, D and E are formed by projecting the contours of the 1^- meson (Fig. 1C) onto the plane of the Dalitz plot. Since the $|A|^2$ for 1^+ and 0^- mesons show considerable "azimuthal" variation, the folded area has been arbitrarily divided into two sectors A and B. The contours in turn divide each sector into five subsectors, $A_1 \cdots A_5$, $B_1 \cdots B_5$.

The number of peak-region events per unit area within $A_1 \cdots A_5$ and $B_1 \cdots B_5$ are displayed in the upper portions of Fig. 2, A and B, respectively. The normalized density of control-region events are displayed in the lower portions. The normalization accounts for the number of triplets that lie below the background curve in the peak region (169 triplets - see Fig. 3 Thus the density distribution of points in Fig. 1 D is scaled by 169/241. A background subtraction is not meaningful because (a) we are uncertain of the difference in behavior of the $T = 0$ background in the peak region and in the control region and (b) we see no way of predicting what fractions of the $T = 0$ and $T = 1$ plus $T = 2$ states are present in the background.

The smooth curves for the three competing distributions are all normalized to the same number of events (in this case 270 for the peak region). The solid curve is the prediction of the 1^- meson, the dashed curve for the 1^+ meson, and the broken curve for the 0^- meson. In spite of the uncertainty of the background subtraction, the 1^- meson is conclusively favored by the data.

We have tacitly assumed that the ω-meson spin is $J \leq 1$. A quantitative treatment for higher J values has not been attempted. However, the simplest matrix elements for $J = 2^+$ and 2^- also vanish at the center of the Dalitz plot. This is certainly inconsistent with the data. We must conclude that the ω is a vector meson, $J = 1^-$.

Figure 3 shows a portion of the effective-mass distribution. A smooth curve has been drawn through the background and a curve (centered at M_3 = 787 Mev) that represents our experimental resolution function ($\Gamma_{resol.}/2$ = 12 Mev) has been superposed upon it. The area under the resolution function is equal to the number of events that lie above the background curve in the vicinity of the peak. The data reasonably fit the resolution function. We conclude that the true half-width $\Gamma/2$ is consistent with zero, and is less than 12 Mev.

Finally, we have studied angular distributions and correlations in ω meson breakup for the 270 triplets in the peak region. Within statistics, the ω meson is produced isotropically. We define the production normal $\underline{n}_1$ as $\underline{p}$ (beam) $\times$ $\underline{p}$ (ω). The breakup normal $\underline{n}_3$ (in the ω rest frame) is the direction normal to its breakup plane. We find that $\underline{n}_3$ is isotropic with respect to $\underline{n}_1$, to $\underline{p}$ (beam), and to $\underline{p}$ (ω). It is not correlated to the direction of breakup of the opposing dipion made in reaction (1).

We wish to point out that although our statistics are sufficiently good to allow a Dalitz-type analysis to be made, they are not such for the analysis of the angular distribution and correlations; a separate study must be made.

We again acknowledge the help that we received from those we mentioned in our earlier Letter. We regret that reference 9 of that Letter was garbled.[4] We also neglected to refer to the theoretical predictions of the ω by Y. Fujii[5] and G. Breit.[6] We are indebted to Professor J. Sakurai for pointing this out to us.

*Work done under the auspices of the U. S. Atomic Energy Commission.

1. B. C. Maglić, L. W. Alvarez, A. H. Rosenfeld, and M. L. Stevenson, Phys. Rev. Letters 7, 178 (1961).

2. The collaborating groups of Johns Hopkins and Duke Universities have informed us of evidence for the ω meson in the reaction $\pi^+ + d \rightarrow p + p + \pi^+ + \pi^- + \pi^0$ at 1.23-Bev/c incident momentum. Further evidence for the ω meson has been found by G. R. Lynch and N. H. Xuong in the reaction $\bar{p} + p \rightarrow 3\pi^+ + 3\pi^- + \pi^0$. In their paper, UCRL-9857, they report $M_\omega = 780$ and $\Gamma/2 < 18$.

3. C. N. Yang (Brookhaven National Laboratory, private communication) points out that the matrix elements of Table I must be formed of linear combinations of different values of L and ℓ Furthermore, L and ℓ are meaningful only in the limit of nonrelativistic pions. In this limit, the following linear combinations are required:

1^+ meson, $\ell = 2$ and 0, $L = 1$

0^- meson, $\ell = 3$ and 1, $L = 3$ and 1,

where $\ell = 1$, $L = 1$ is the minimum complexity necessary for the 1^- meson. These additions are included in Table I.

4. The last part of reference 9 should have read:

For previous evidence see: I. Derado, Nuovo cimento 15, 853 (1960) E. Pickup, F. Ayer, and E. O. Salant, Phys. Rev. Letters 5, 161 (1960); J. G. Rushbrooke and D. Radojcic, Phys. Rev. Letters 5, 567 (1960); J. Anderson, V. Bang, P. Burke, D. Carmony, and N. Schmitz, Phys. Rev. Letters 6, 365 (1961); A. R. Erwin, R. March, W. D. Walker, and E. West Phys. Rev. Letters 6, 628 (1961); D. Stonehill, C. Baltay, H. Courant, W. Fickinger, E. C. Fowler, H. Kraybill, J. Sandweiss, J. Sanford, and H. Taft, Phys. Rev. Letters 6, 624 (1961).

5. Y. Fujii, Progr. Theoret. Physics (Kyoto) 21, 232 (1959).

6. G. Breit, Proc. Nat'l Acad Sci 46, 746 (1960). Phys. Rev. 120, 287 (1960).

Table I. Possible three-pion resonances with $T = 0$, $J \leq 1$.

Meson		Matrix element			
Type,	J	ℓ	L	Simple example	Vanishes at (See Fig. 1, D and E)
V	1^-	1	1	$(\vec{p}_0 \times \vec{p}_+) + (\vec{p}_+ \times \vec{p}_-) + (\vec{p}_- \times \vec{p}_0)$	Whole boundary
PS	0^-	1 and 3	1 and 3	$(E_- - E_0)(E_0 - E_+)(E_+ - E_-)$	Straight lines
A	1^+	0 and 2	1	$E_-(\vec{p}_0 - \vec{p}_+) + E_0(\vec{p}_+ - \vec{p}_-) + E_+(\vec{p}_- - \vec{p}_0)$	Center b, d, f.

FIGURE LEGENDS

Fig. 1. (A), (B), and (C). Isometric drawings of the square of matrix elements for the 1^+, 0^-, and 1^- mesons expressed in terms of the normalized Dalitz variables $x = \left(T_- - T_2\right)/\sqrt{3\,Q}$, and $y = T_0/Q$, $(Q = T_+ + T_- + T_0 = M_3 - 3)$.

(D) and (E). Dalitz plots, folded six times. (D) displays 241 triplets in the control region and (E) displays 270 triplets in the peak region. The contour lines represent the projection of the contours of (C) (1^- meson).

Fig. 2. (a) Density of points on the Dalitz plot for sectors A_1 through A_5. (b) Density of points for sectors B_1 through B_5. The sectors are defined by the contours for the 1^- meson. The smooth curves are the probability distributions $|A|^2$ for the three competing matrix elements.

Fig. 3. Number of $\pi^+ \pi^- \pi^0$ triplets versus effective mass (M_3) of the triplets for the reaction $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + \pi^0$. The smooth curve drawn through the background has superposed upon it the experimental resolution function ($\Gamma_{resol.}/2 = 12$ Mev). The area under the resolution function is equal to the number of events above the background curve.

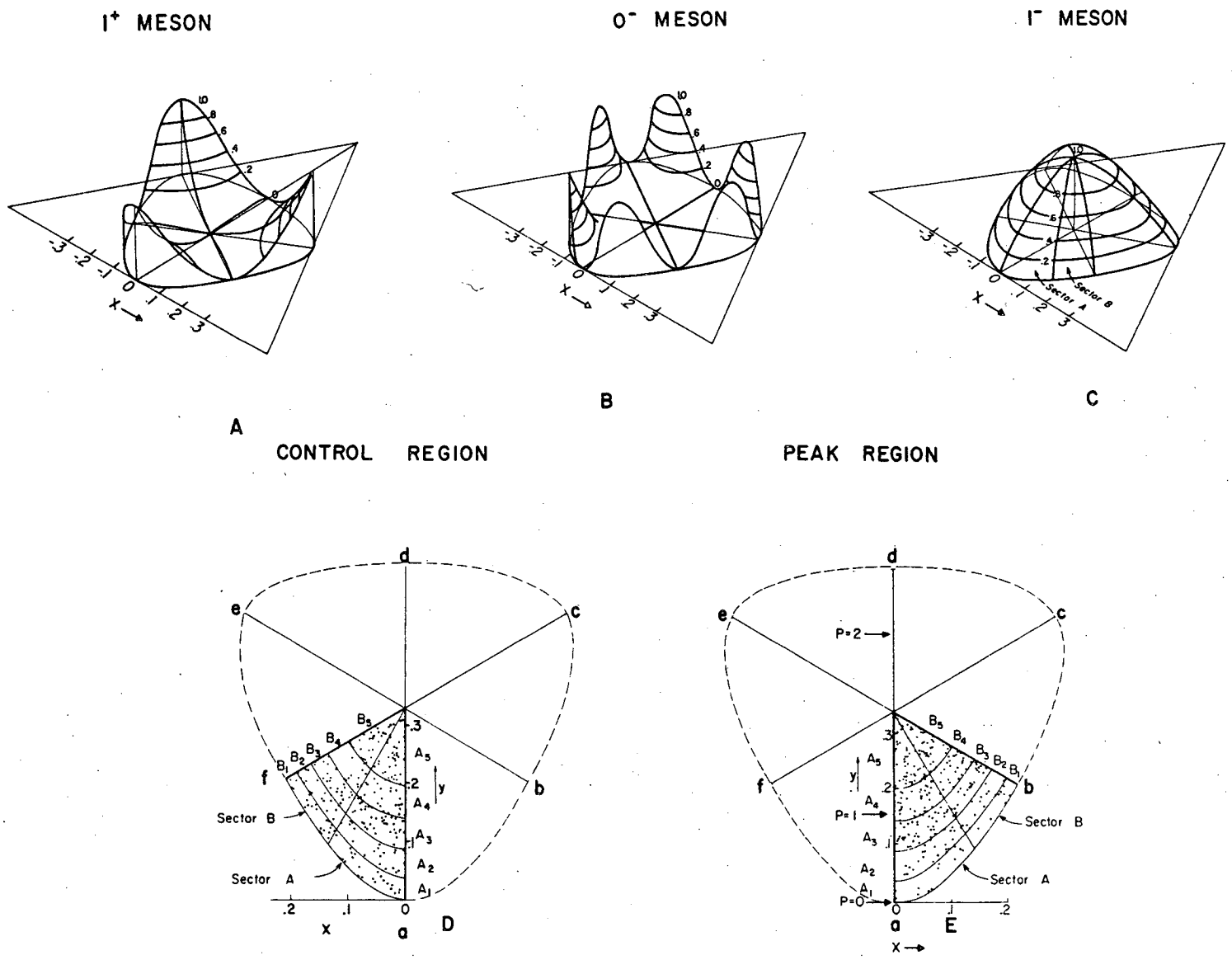
1+ MESON
0- MESON
1- MESON
A
B
C
Sector A
Sector B
CONTROL REGION
PEAK REGION
Sector B
Sector A
P=2
P=1
P=0
D
E
MUB-783

Fig. 1.

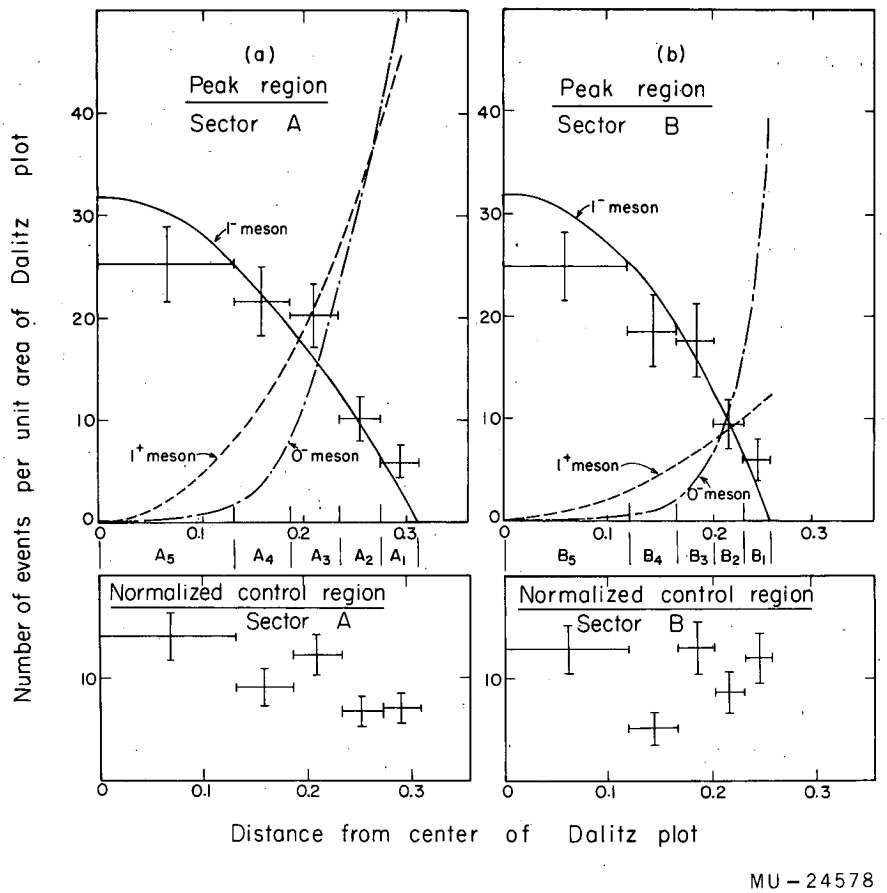
(a)
Peak region
Sector A
(b)
Peak region
Sector B
1⁻ meson
1⁺ meson
0⁻ meson
A5
A4
A3
A2
A1
B5
B4
B3
B2
B1
Normalized control region
Sector A
Normalized control region
Sector B
Number of events per unit area of Dalitz plot
Distance from center of Dalitz plot
MU-24578

Fig. 2.

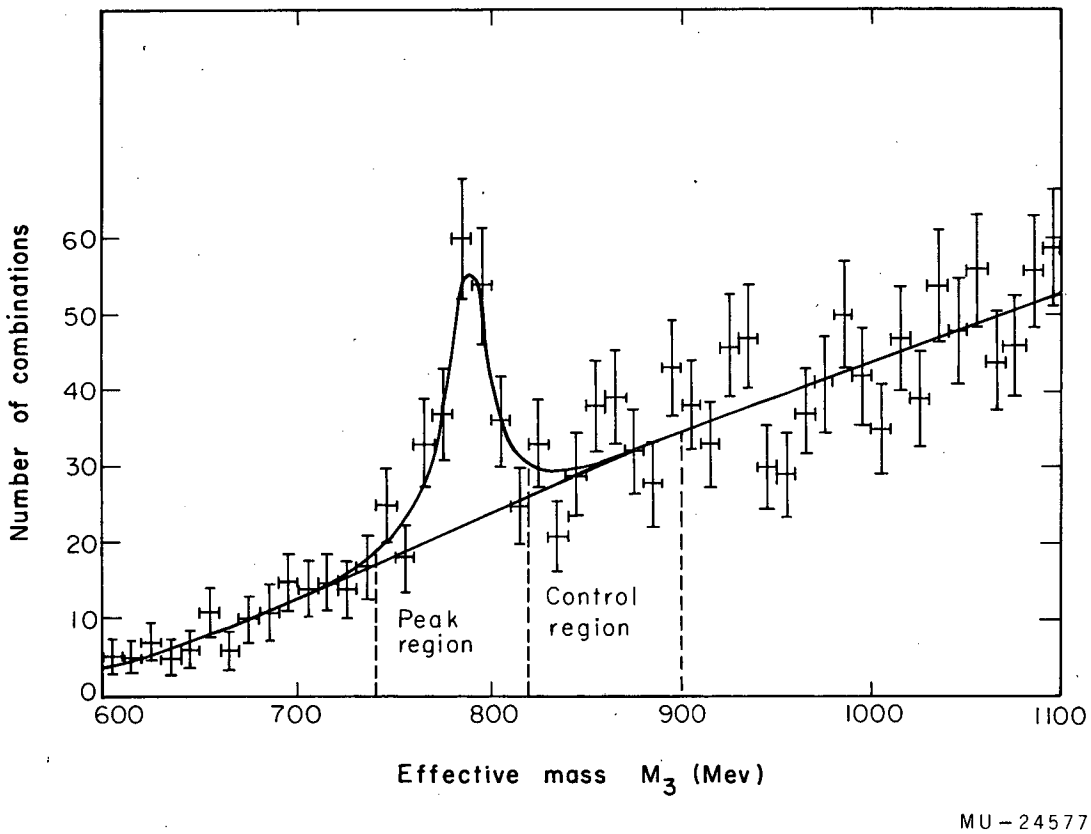
Number of combinations
0
10
20
30
40
50
60
600
700
800
900
1000
1100
Effective mass M_3 (Mev)
Peak region
Control region
MU-24577

Fig. 3

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NOTES ADDED IN PROOF

I. Is the G Parity of the ω Meson -1?

A. Dalitz Plot Evidence for G = -1.

If the true width of the 3-pion resonance were consistent with zero, one might question whether the observed $\pi^+ \pi^- \pi^0$ decay mode is an "allowed" transition. Duerr and Heisenberg have suggested that our data are also consistent with $J = 1^{++}, 0^{-+}$ (the first superscript refers to the parity and the second to the G parity).[7] Both of these can give a uniformly populated Dalitz plot. They argue that our Dalitz plot might really be uniformly populated, but that measurement errors can depopulate the area near the boundary, making an originally flat distribution resemble that for a vector meson. Our counter argument is as follows: A flat Dalitz plot corresponds to an isotropic distribution of the vector $\underline{q}$ representing the breakup of the dipion. If we start with isotropy it is difficult for us to see how random errors can introduce anisotropy. This argument is supported by two experimentally flat Dalitz plots: (a) Our control region, as well as that of Xuong and Lynch,[2] shows no significant depopulation near the boundary. (b) An analysis of 1347 τ^- decays in flight shows that experimental errors do not depopulate the boundary of the Dalitz plot.[8]

For our data to fit a flat distribution, χ^2 is 93 where nine is expected. It would take a fluctuation of more than ten standard deviations to produce such a distribution, and so we conclude that it is definitely inconsistent with being flat.

B. Further Evidence Against 0^{-+} and 1^{++}

1. For the neutral-to-charged branching ratio of 0^{-+}, Duerr and Heisenberg predict $(\omega \to 3\pi^0)/(\omega \to \pi^+ + \pi^- + \pi^0) = 3/2$. If this hypothesis were correct, we should be able to see the $3\pi^0$ decay mode in the reaction $\bar{p} + p \to 2\pi^+ + 2\pi^- + 3\pi^0$ as a peak of 45-Mev (experimental) half-width in the missing-mass distribution. We estimate that we would notice such a peak if it contained $\gtrsim$50 to 100 events. The same experiment that yielded over 1125 reactions (1) also yielded 76 ± 18 6-prong annihilations identified as $\bar{p} + p \to 2\pi^+ + 2\pi^- + (\omega \to \pi^+ + \pi^- + \pi^0)$.[2] We should see (3/2)(76 ± 18) = 114 ± 25 neutral decays in our 4-prong missing-mass peak. We find no evidence for such a large peak.

2. Additional evidence against 1^{++} comes from the absence of the 4-pion decay mode of the ω both in the 6-prong data of Xuong and Lynch[9] and in our 4-prong data. According to Duerr and Heisenberg this is an allowed decay mode and could compete with the electromagnetic 3-pion decay mode. We have also looked in vain for the $\pi^+ + \pi^- + \gamma$ decay mode of the ω.

The Dalitz plot, the absence of the $3\pi^0$ decay mode, and the absence of a 4-pion decay mode are strong evidence that the ω meson is 1^{--}.

II. Effect of the T = 1, J = 1 Pion-Pion Resonance on the Final States of the ω Decay

At the suggestion of M. Gell-Mann, we have included the effect of the T = 1, J = 1 pion-pion interaction on the final state of ω decay. The square of the matrix element for the 1^- meson is multiplied by the product of three resonant denominators of the form $(M_{ij}^2 - M_\rho^2)^2 + M_\rho^2 \Gamma_\rho^2 (M_{ij})$; where $\Gamma_\rho(M_{ij}) = \Gamma[(M_{ij}^2 - 4)/(M_\rho^2 - 4)]^{3/2}/(M_\rho/M_{ij})$, M_ρ is the mass of the resonance and Γ its width, ij is first 0+ then +- then -0, M_{0+} is the invariant mass of the $\pi^+\pi^0$ system, etc. For M_ρ = 5.37 (=750 Mev) and Γ = 1 (=140 Mev), this

factor is 1.20 times larger at the center of the Dalitz plot than it is at point "a" in Fig. 1E. Therefore the $T = 1$, $J = 1^-$ pion-pion resonance does not seriously distort the matrix elements.

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8. M. Marro-Luna, D. H. Miller, J. J. Murray, A. H. Rosenfeld, and R. D. Tripp, Lawrence Radiation Laboratory Report UCRL-9798, August 1961 (submitted to Nuovo cimento).

9. N. Muong and J. Lynch, Lawrence Radiation Laboratory, private communication.