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Abstract

We study the effect of ellipsoidal nuclear deformation in odd-mass-nuclei rotational band structure, the magnetic moment, and electric-quadrupole reduced transition probabilities. Also, we study the relationship between the rotational bands of an ellipsoidally deformed nucleus and the vibrational and rotational bands of a spheroidally deformed nucleus with γ-vibration-rotation interaction in the limit of γ approaching 0 or $\frac{1}{3}\pi$. Equations for the asymmetric rotor motion are derived. By using T. D. Newton's single-particle eigenvalues and eigenvectors, we then present numerical calculations showing rotational spectra associated with an odd nucleon in an ellipsoidal well. The calculations for the N=4 and N=2 shells were done on an IBM 709 computer. Numerical results are discussed in terms of the β and γ deformation parameters required to give the known spins of the odd-A cesium isotopes Cs^{127} to Cs^{137}. The rotational energy spectrum, magnetic moment of ground state, and various E2 transition probabilities are calculated for Cs^{131} for several deformations, with best energy spectrum fit at β=0.28, γ=38 deg.

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1. Introduction

The Davydov-Filippov model of an ellipsoidally deformed nucleus with three unequal principal axes has been applied extensively to even-even nuclei[1]. For small values of the asymmetry parameter γ the rotational spectra correspond closely to those of the symmetric rotor with γ-vibrational excitations added. Perhaps the greatest utility of the model has been in the regions of nuclei outside the regions of definite spheroidal deformation, where the energy of the second excited 2+ state may be only about twice the energy of the first excited state. We felt that it would be interesting to examine the model for odd-mass nuclei‡‡.

At a late stage in our calculation we learned of similar work by Hecht[2,3], who treats the rotational energies of asymmetric odd-A nuclei with essentially similar results, except that his results are for the spectra of nuclei with A around 190. Filippov[4] has recently made calculations and general examinations of the problem of stability of the asymmetric nuclear shape. From his results it appears that noncylindrical shapes could possibly be of lowest energy in some cases, where one kind of nucleon has nearly completed

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‡‡ Based on a Ph.D. thesis (by LWP), University of California, Berkeley, California (see ref.[13]).

a shell closure and the other kind is just beyond a closed shell. The region of the neutron-deficient cesium isotopes possibly satisfies these requirements.

It is instructive to consider a trivial extension of the Davydov-Filippov model to odd-A; namely, the case where the odd nucleon is in a pure $j = \frac{1}{2}$ state, and hence completely uncoupled from the rotor. In such a case we have just the even-even spectrum but with a ground state spin of $\frac{1}{2}$ and all excited levels doubly degenerate, with spins differing by $\pm \frac{1}{2}$ from the spin in the even-even nucleus. We see that in the rotational band there will be only one state with $I = \frac{1}{2}$, two with $I = \frac{3}{2}$, three with $\frac{5}{2}$, and generally $I + \frac{1}{2}$ states of a given spin I.

The sequence of ground-state spins of the odd-mass cesium isotopes qualitatively suggests a possible explanation in terms of an ellipsoidal deformation which sets in as the neutron number departs sufficiently from 82. The measured spins for neutron number 72, 74, 76, 78, 80, 82 are, respectively, $\frac{1}{2}, \frac{1}{2}, \frac{5}{2}, \frac{7}{2}, \frac{7}{2}, \frac{7}{2}$. The expected $g_{7/2}$ spherical shell model spin appears near the closed shell. Ellipsoidal deformation of the nuclear potential would tend to quench the orbital angular momentum of an odd particle so that for sufficiently large deformation, spin $\frac{1}{2}$ should lie lowest.

The quantitative testing of the model involves considerable mathematical complication. Fortunately, nucleon eigenfunctions and eigenvalues for an ellipsoidal harmonic-oscillator potential had been calculated and tabulated by T. D. Newton[5,6] through the fourth oscillator shell. The Hamiltonian he used was more appropriate for N=4 neutrons than for protons, and we are deeply indebted to him for recalculating, at our request, the fourth oscillator shell by using a proton parameter. With his eigenfunctions we have attempted to calculate rotational spectra of even parity for some odd-proton nuclei in the 50 to 82 shell.

2. Theory of Odd-Mass Nuclei with Fixed Ellipsoidal Deformation

The Hamiltonian of the coupled system of nucleon and asymmetrical rotor core is as follows:

$$\mathcal{H} = \mathcal{H}_p + \mathcal{H}_{int} + T_R, \tag{1}$$

where the terms on the right-hand side are as defined later.

The rotational energy of the general rigid rotor may be expressed as

$$T_R = \frac{1}{2} \sum_{\kappa=1}^{3} R_\kappa^2 \, \mathfrak{I}_\kappa^{-1}, \tag{2}$$

where R_κ denotes the components of angular momentum along the principal axes, and we assume, in accordance with the hydrodynamic model[7)],

$$\mathfrak{I}_\kappa = 4 B\beta^2 \sin^2 (\gamma - \kappa \tfrac{2}{3}\pi), \tag{3}$$

where β and γ are the usual parameters specifying a general ellipsoidal deformation, and B is the inertial parameter for quadrupole surface oscillations.

The single-particle Hamiltonian $\mathcal{H}_p$ is given by

$$\mathcal{H}_p = T_p + V_p(r) + C \, \underset{\sim}{\ell} \cdot \underset{\sim}{s} + D \underset{\sim}{\ell}^2 \tag{4}$$

where

$$T_p = -\frac{1}{2} \frac{\hbar^2}{M} \nabla^2 \tag{5}$$

and

$$V_p = \frac{1}{2} M (\omega_1 X_1^2 + \omega_2 X_2^2 + \omega_3 X_3^2), \tag{6}$$

where M is the single-particle effective mass of a nucleon in the nucleus,

X_1, X_2, X_3 are the Cartesian coordinates of the nucleon in the body fixed system,

$\hbar\omega_1$, $\hbar\omega_2$, $\hbar\omega_3$ are the corresponding quantum energies along the three principal axes, and they satisfy the conservation of nuclear volume conditions

$$\omega_1 \omega_2 \omega_3 = \omega_0^3, \tag{7}$$

$\underset{\sim}{\ell}$ are the infinitesimal pseudo rotation operators,

and C is a spin-orbit potential strength parameter.

The ∇^2 operator in eq. (5) is evaluated in the body fixed coordinate system. The $D\underset{\sim}{\ell}^2$ term in (4) is a correction which depresses high-angular-momentum orbitals, equivalent to the effect of a nuclear potential more square than pure harmonic. The term $\mathcal{H}_{int}$ represents the interaction of the particle with the nuclear deformation. However, in Newton's[6] calculation for ellipsoidal nuclei the energy eigenvalues E_S include both the single-particle energy and the interaction energy.

We shall not consider nuclear shape vibrations but consider the shape fixed. In the case of nonaxial nuclei, if the moments of inertia are sufficiently large, the particle motion will follow nearly adiabatically the rotations of the well, and the wave function will be a linear combination of Nilsson's wave function[8,9] in the following way:

$$\Psi_{IM} = \left(\frac{2I+1}{16\pi^2}\right)^{\frac{1}{2}} \sum_{I_3} b_{I\,I_3} \sum_{J\,J_3} a_{J\,J_3}$$

$$\times\left(D^{I}_{I_3 M}(\theta_i)\,\psi_{N\ell J J_3} + (-)^{I-J} D^{I}_{-I_3 M}(\theta_i)\,\psi_{N\ell J -J_3}\right) \tag{8}$$

where: $\psi_{N\ell J\,J_3}$ is as defined by Newton's[6] eq. 3; the summation over J_3 runs only over alternate half-integral values between J and -J; and I_3 is summed between I and -I on alternate values such that $I_3 - J_3 = 2\nu$. Here ν=0, ±1, ±2, ..., ±N— which is due to the symmetry requirement of invariance with respect to a rotation through π of the 1 and 2 axes about the 3 axis. By convention, $b_{I\,1/2}$ and $a_{J\,1/2}$ are always taken with a positive sign.

The a_{II_3} are the mixing amplitudes which are the eigenvectors of Newton's Table [6], and b_{II_3} are the mixing amplitudes of the rotational states with different I_3 values and the same I. Therefore b_{II_3} are the eigenvectors of the rotational spectra that we calculate. For axial nuclei ($\gamma \to \frac{1}{3}N\pi$), I_3 and J_3 will be approximately constant, and their eigenvalues will be K and Ω. The adiabatic-rotation assumption may not be a good one for the cesium isotopes we later treat, since rotational energy spacings are not much smaller than the single-particle-level spacing. We have neglected the effect of the coriolis interaction in mixing different Newton eigenfunctions.

2.1 ROTATIONAL ENERGY IN THE STRONG-COUPLING APPROXIMATION

The rotational states may have energies much smaller than the single-particle energy and the phonon energy, so in eq. (1) we only have to consider the rotational energy term $T_R = \frac{1}{2}\Sigma_K R_K^2 \mathfrak{J}_K^{-1}$. If we use the representation in which I^2, I_3, J^2, and J_3 are constants of motion, then we write T_R in the following form given by Bohr[7]:

$$T_R = T_{DR} + T_{OR}, \tag{9}$$

where

$$T_{DR} = \left(\frac{\hbar^2}{4\mathfrak{J}_1} + \frac{\hbar^2}{4\mathfrak{J}_2}\right)\left[I(I+1) - I_3^2 + J(J+1) - J_3^2\right] + \frac{\hbar^2}{2\mathfrak{J}_3}(I_3 - J_3)^2. \tag{10}$$

Here T_{DR} is the diagonal term which is the same as the rotational-energy term in the nucleus with axial asymmetry, excluding the case of $J_3 = \frac{1}{2}$:

$$T_{OR} = -\left(\frac{\hbar^2}{\mathfrak{J}_1} I_1 J_1 + \frac{\hbar^2}{\mathfrak{J}_2} I_2 J_2\right) + \left[\left(\frac{\hbar^2}{4\mathfrak{J}_1} - \frac{\hbar^2}{4\mathfrak{J}_2}\right)(J_1^2 - J_2^2)\right]$$

$$+ \left[\left(\frac{\hbar^2}{4\mathfrak{J}_1} - \frac{\hbar^2}{4\mathfrak{J}_2}\right)(I_1^2 - I_2^2)\right]. \tag{11}$$

Here T_{OR} are the off-diagonal terms that vanish in case of axial symmetry, except for $J_3 = \frac{1}{2}$. The nonvanishing matrix elements of eq. (11) will be governed by the following selection rule: The first term will connect states of $\Delta K = \pm 1$ and $\Delta\Omega = \pm 1$, and the second and the third terms will connect states of $\Delta K = 0$ and $\Delta\Omega = \pm 2$, or $\Delta K = \pm 2$ and $\Delta\Omega = 0$.

Substitution of eq. (8) into the rotational-energy equation $T_R \Psi = E_{rot} \Psi$ gives a secular determinant of dimension $I + \frac{1}{2}$ by $I + \frac{1}{2}$ to be solved to determine the eigenvalues and the coefficients b_{II_3}.

All three terms in T_{DR} contribute to the diagonal matrix element. Of the three terms in T_{OR}, the second contributes a constant amount to the diagonal matrix element, and can— for our purposes— be ignored. The first and third terms provide the off-diagonal matrix elements for $\mathcal{H}_{I_3, I_3 - 1}$ and $\mathcal{H}_{I_3, I_3 \pm 2}$. Therefore the secular matrix is separable with only even values of $I_3 - I_3'$ coupled.

The general formula for diagonal elements of rotational energy is

$$\mathcal{H}_{I_3 I_3} = \left(\frac{\hbar^2}{4\mathcal{J}_1} + \frac{\hbar^2}{4\mathcal{J}_2} \right) \left\{ I(I+1) - I_3^2 + \sum_{J J_3} |a_{J J_3}|^2 \left[J(J+1) - J_3^2 \right] \right\} + \frac{\hbar^2}{2\mathcal{J}_3} \sum_{J J_3} |a_{J J_3}|^2 (I_3 - J_3)^2$$

$$+^{\dagger} - (-)\,\delta_{J_3\,\frac{1}{2}}\,\delta_{I_3\,\frac{1}{2}}\left(I+\frac{1}{2}\right)\left(\frac{\hbar^2}{4\tilde{\mathcal{J}}_1}+\frac{\hbar^2}{4\tilde{\mathcal{J}}_2}\right)$$

$$\times \sum_{J} (-)^{I-J}\, a^2_{J\,\frac{1}{2}}\left(J+\frac{1}{2}\right)$$

$$-\left(\frac{\hbar^2}{4\tilde{\mathcal{J}}_1}+\frac{\hbar^2}{4\tilde{\mathcal{J}}_2}\right)\left(I+\frac{1}{2}\right)\sum_{J\,J_3\,J'\,J_3'} (-)^{I-J}\,\delta_{JJ'}$$

$$\times\, \delta_{J_3'-(J_3-1)}\, a_{J\,J_3'}\, a_{J\,J_3}\left[(J+J_3)(J-J_3+1)\right]^{\frac{1}{2}}$$

$$-\left(\frac{\hbar^2}{4\tilde{\mathcal{J}}_1}-\frac{\hbar^2}{4\tilde{\mathcal{J}}_2}\right)\left(I+\frac{1}{2}\right)\sum_{J\,J_3\,J'\,J_3'} (-)^{I-J}\,\delta_{JJ'}$$

$$\times\, \delta_{J_3'-(J_3+1)}\, a_{J\,J_3'}\, a_{J\,J_3}\left[(J-J_3)(J+J_3+1)\right]^{\frac{1}{2}}. \tag{12}$$

---

† Due to the symmetry of the wave function, the other two matrix elements give the same numerical result, and so are not written out.

The general formula for off-diagonal elements of T_{OR} is

$$\mathcal{H}_{I_3(I_3-1)}{}^{\dagger} = -\delta_{I_3-(I_3'-1)}\left[(I+I_3)(I-I_3+1)\right]^{\frac{1}{2}}$$

$$\times\left\{\left(\frac{\hbar^2}{4\tilde{\mathcal{J}}_1}-\frac{\hbar^2}{4\tilde{\mathcal{J}}_2}\right)\sum_{J\,J_3\,J'J_3'}(-)^{I-J}\,\delta_{JJ'}\,\delta_{J_3'-(J_3+1)}\right.$$

$$\times a_{JJ_3'}\,a_{JJ_3}\left[(J-J_3)(J+J_3+1)\right]^{\frac{1}{2}}$$

$$+\left(\frac{\hbar^2}{4\tilde{\mathcal{J}}_1}+\frac{\hbar^2}{4\tilde{\mathcal{J}}_2}\right)\sum_{J\,J_3\,J'J_3'}(-)^{I-J}\,\delta_{JJ'}\,\delta_{J_3'-(J_3-1)}$$

$$\left.\times a_{JJ_3'}\,a_{JJ_3}\left[(J+J_3)(J-J_3+1)\right]^{\frac{1}{2}}\right\}. \quad (13)$$

and

$$\mathcal{H}_{I_3\,I_3\pm 2} = \left(\frac{\hbar^2}{8\tilde{\mathcal{J}}_1}-\frac{\hbar^2}{8\tilde{\mathcal{J}}_2}\right)$$

$$\times\left[(I\mp I_3)(I\mp I_3-1)(I\pm I_3+1)(I\pm I_3+2)\right]^{\frac{1}{2}}. \quad (14)$$

---

† Due to the symmetry of the wave function, the other two matrix elements give the same numerical result, and so are not written out.

2.2 MAGNETIC MOMENT

The magnetic-moment operator is[10)]

$$\underset{\sim}{\mu}_{op} = \frac{e}{2MC} [\Sigma_i (g_\ell \underset{\sim}{\ell}_i + g_s \underset{\sim}{s}_i) + g_{R_1} R_1 \underset{\sim}{1} + g_{R_2} R_2 \underset{\sim}{2} + g_{R_3} R_3 \underset{\sim}{3}]. \tag{15}$$

If we assume that the three collective g factors are equal to a value g_R, then we may use Nilsson's[8)] expression 20:

$$\mu = \frac{1}{I+1}\Big[(g_s - g_\ell)\langle \underset{\sim}{s} \cdot \underset{\sim}{I}\rangle + (g_\ell - g_R)\langle \underset{\sim}{J} \cdot \underset{\sim}{I}\rangle + g_R\, I(I+1)\Big], \tag{16}$$

where

$$\begin{aligned}\langle \underset{\sim}{J} \cdot \underset{\sim}{I}\rangle = \sum_{I_3 J J_3} & b^2_{I I_3}\, a^2_{J J_3}\, I_3 J_3 + (-)^{I-\frac{1}{2}} \Big\{ b_{I I_3} \\ & \times b_{I -(I_3-1)} (-)^{J-\frac{1}{2}}\, a_{J J_3}\, a_{J -(J_3-1)} \left(\tfrac{1}{2}\right) \\ & \times \left[(I+I_3)(I-I_3+1)(J+J_3)(J-J_3+1)\right]^{\frac{1}{2}} \\ & + b_{I I_3}\, b_{I -(I_3+1)} (-)^{J-\frac{1}{2}}\, a_{J J_3}\, a_{J -(J_3+1)} \left(\tfrac{1}{2}\right) \\ & \times \left[(I-I_3)(I+I_3+1)(J-J_3)(J+J_3+1)\right]^{\frac{1}{2}} \Big\}. \end{aligned} \tag{17}$$

For the calculation of the matrix $\langle \underset{\sim}{s} \cdot I \rangle$ it is more convenient to work in the representation of $\langle \Omega | N \ell \Lambda \Sigma \rangle$, where Nilsson's expansion coefficient $a_{\ell\Lambda\Sigma}$ is related to a_{JJ_3} in the following way[8]:

$$a_{\ell\Lambda\Sigma} = \sum_J \langle \ell \tfrac{1}{2} \Lambda \Sigma | J J_3 \rangle a_{J J_3} ,$$

then

$$\langle \underset{\sim}{s} \cdot \underset{\sim}{I} \rangle = \sum_{I_3 \ell J_3} b^2_{I I_3} \left[a^2_{\ell (J_3 - \frac{1}{2}) \frac{1}{2}} - a^2_{\ell (J_3 + \frac{1}{2}) - \frac{1}{2}} \right]$$

$$\times \left(\frac{I_3}{2}\right) + \sum_{I_3 \ell J_3} (-)^\ell b_{I I_3} \, b_{I -(I_3 - 1)}$$

$$\times a_{\ell (J_3 - \frac{1}{2}) \frac{1}{2}} \, a_{\ell -(J_3 - \frac{1}{2}) \frac{1}{2}} \left(\frac{1}{2}\right) \left[(I + I_3)(I - I_3 + 1) \right]^{\frac{1}{2}}$$

$$+ \sum_{I_3 \ell J_3} (-)^\ell b_{I I_3} \, b_{I -(I_3 + 1)}$$

$$\times a_{\ell (J_3 + \frac{1}{2}) - \frac{1}{2}} \, a_{\ell -(J_3 + \frac{1}{2}) - \frac{1}{2}} \left(\frac{1}{2}\right) \left[(I - I_3)(I + I_3 + 1) \right]^{\frac{1}{2}} . \tag{18}$$

2.3 ELECTRIC-QUADRUPOLE REDUCED TRANSITION PROBABILITIES

The transition probability for electric-quadrupole radiation is given by[11)]

$$T(E2\ I_i \to I_f) = \frac{4\pi}{75\hbar}\left(\frac{E}{\hbar c}\right)^5 B(E2;\ I_i \to I_f), \tag{19}$$

where $B(E2;I_i \to I_f)$ is the reduced transition probability between rotational states I_i and I_f. It can be written as

$$B(E2;I_i \to I_f) = \frac{5}{16\pi(2I_i+1)} \sum_{\mu M_f} \langle f|Q_{2\mu}|\ i\rangle^2. \tag{20}$$

By using the model of a nucleus as an incompressible classical liquid drop with a uniform charge distribution, the nuclear quadrupole-moment operator $Q_{2\mu}$ can be written as[12)]

$$Q_{2\mu} = eQ_0\left[D^2_{\mu 0}\cos\gamma + (D^2_{\mu 2} + D^2_{\mu -2})\ (\sin\gamma)(2)^{-\frac{1}{2}}\right], \tag{21}$$

where Q_0 is the intrinsic quadrupole moment of an axial nucleus and is related to the deformation parameter β by $Q_0 = 3ZR^2\beta(5\pi)^{-\frac{1}{2}}$.

After simple algebraic manipulation, and assuming that there is no change in the internal state of the nucleus in the transition, the reduced transition probability can be expressed in terms of the average value of β and γ as[12)]

$$\begin{aligned} &B(E2;I_i \to I_f) \\ &= \frac{5}{16\pi} e^2 Q_0^2 \Big| \sum_{I_3 I_3'} b_{I_i I_3}\, b_{I_f I_3'} \Big(\cos\gamma \\ &\times \left\langle I_i\ 2\ I_3\ 0 \mid I_f\ I_3' \right\rangle + \left[(\sin\gamma)(2)^{-\frac{1}{2}}\right] \\ &\times \langle I_i\ 2\ I_3 \pm 2 \mid I_f\ I_3' \rangle\Big)\Big|^2. \end{aligned} \tag{22}$$

2.4 ELECTRIC-QUADRUPOLE MOMENT

The formula for the spectroscopic electric quadrupole moment can readily be derived by a specialization of the formulas of subsec. 2.3; that is

$$Q_{spec} = \frac{1}{e} \langle Q_{20} \rangle_{M_I = I}, \tag{23}$$

where

$$Q_{20} = Q_0 \left\{ D^2_{00} \cos\gamma + (D^2_{02} + D^2_{0-2}) \left[(\sin\gamma)(2)^{-\frac{1}{2}} \right] \right\} \tag{24}$$

Substituting eq. (24) into eq. (23), we get

$$\begin{aligned}
Q_{spec} = {} & Q_0 \langle I\ 2\ I\ 0 \mid I\ I \rangle \sum_{I_3 I_3'} b_{I\ I_3}\, b_{I\ I_3'} \\
& \times \Big[\cos\gamma \langle I\ 2\ I_3\ 0 \mid I\ I_3 \rangle \delta_{I_3'\ I_3} \\
& + \left[(\sin\gamma)(2)^{-\frac{1}{2}} \right] \big(\langle I\ 2\ I_3\ 2 \mid I\ I_3 + 2 \rangle \delta_{I_3'\ I_3 + 2} \\
& + \langle I\ 2\ I_3 - 2 \mid I\ I_3 - 2 \rangle \delta_{I_3'\ I_3 - 2} \big) \Big] \\
= {} & \frac{Q_0}{(I+1)(2I+3)} \Bigg\{ \cos\gamma \sum_{I_3} b^2_{II_3} \left[3I_3^2 - I(I+1) \right] \\
& + (\sin\gamma)\left(\tfrac{3}{4}\right)^{\frac{1}{2}} \sum_{I_3 I_3'} b_{II_3}\, b_{II_3'} \Big(\left[(I+I_3+1)(I+I_3+2)(I-I_3-1)(I-I_3) \right]^{\frac{1}{2}} \delta_{I_3'\ I_3+2} \\
& + \left[(I-I_3+1)(I-I_3+2)(I+I_3-1)(I+I_3) \right]^{\frac{1}{2}} \delta_{I_3'\ I_3-2} \Big) \Bigg\} .
\end{aligned} \tag{25}$$

3. Numerical Studies and Comparison with Data

We have used Newton's eigenfunctions and eigenvectors[5,6] for our calculations of rotational spectra; other nuclear properties were calculated by using the formulas of sec. 2.

3.1 ROTATIONAL SPECTRA

In the low-lying rotational-energy calculation, we assumed that the odd-nucleon state of motion in the nuclear well is not changed for different states of rotational motion. We also neglected possible vibrational effects and the collective rotation-vibration interaction. The rotational energy E_R and its associated eigenvector b_{II_3} — which are obtained by diagonalization of a $(I+\frac{1}{2})$ by $(I+\frac{1}{2})$ matrix for a state with nuclear spin I — were calculated by means of an IBM 709 digital computer[13].

Both the calculated values of the rotational energy E_R and the mixing coefficient b_{II_3} for each value of the single-particle energy ES[$ES=E_S-(N+\frac{3}{2})$] are tabulated in Appendix B of ref.[13] from 0 to $\frac{1}{3}\pi$ in steps of $\frac{1}{24}\pi$. By symmetry, the states with deformation parameters β and γ, and the states with deformation parameters $-\beta$ and $\frac{1}{3}\pi-\gamma$ are equivalent. Curves fitted through the calculated values of β and γ were used for interpolation.

Curves showing the total energy $E = E_S+E_R$ for the lowest single-particle state of the N=2 shell, $\beta = 0.2\,\frac{B}{\hbar^2} = 77.25\ \mathrm{MeV}^{-1}$, are plotted as a function of γ in fig. 1 for the first rotational band, in fig. 2 for the second rotational band, and in fig. 3 for the third rotational band. For comparison, the rotational energy of even-even nuclei[14] with the same values of β and B are plotted as a function of γ in fig. 4. From these four figures we can clearly see the analogy between the rotational spectra of an odd nucleon in an ellipsoidally deformed nucleus and the Davydov-Filippov even-even nuclear rotational-energy spectra[1,15]. This is expressed schematically in the correlation diagram in fig. 5.

As discussed in the Introduction, in the limiting case of a pure $J = \frac{1}{2}$ odd nucleon, the nucleon motion is completely decoupled from the collective surface motion. Also, one gets a system of levels with the same spacing as in even-even but with a doubly degenerate state of spin $I \pm \frac{1}{2}$ in place of each level of spin I in the corresponding even-even nucleus, except that a spin 0 goes to a spin $\frac{1}{2}$ level. The existence of the nearly degenerate states is clearly seen in fig. 6, where the energy is plotted against γ for a single-particle state of almost pure $s_{1/2}$.

3.2 NUCLEAR PROPERTIES OF Cs^{131}

Of the light cesium isotopes with ground-state spin less than $\frac{7}{2}$, only Cs^{131} has enough known even-parity excited states for significant comparison. The rotational energies are calculated for $B\beta^2/\hbar^2 = 13.85\ \mathrm{MeV}^{-1}$ and $\hbar\omega_o = 8.07$ MeV. The results are tabulated in table 1 together with theoretical spins and the experimental energy values[16,17]. Interpolated energy values for optimum parameters $\beta = 0.28$ and $\gamma = 38$ deg are given, along with the energy levels calculated for neighboring values of deformation parameters. The γ value used in table 1 is rather close to the value required to fit levels of the neighboring even-even nucleus Xe^{130}. The ratios of rotational energy of Xe^{130} are [18]

$$\frac{E_1(4+)}{E_1(2+)} = 2.25, \quad \frac{E_1(5+)}{E_1(2+)} = 4.43, \text{ and } \frac{E_1(6+)}{E_1(2+)} = 3.66,$$

which corresponds to a deformation $\gamma = 35$ or 25 deg in Day and Mallmann's table[19]. The decay scheme of Ba^{131} in fig. 7 is taken from the recent study by Bodenstedt et al.[20]. They propose and justify spins of $\frac{1}{2}$ and $\frac{7}{2}$ for excited states at 124 and 133 keV, respectively. The locations and spins of these states— together with the spin of the ground state— essentially fix

the parameters β and γ in our calculation. The energy match with four additional higher levels test the model, and the agreement is good. The theoretical spins are not inconsistent with the observation of all the gamma transitions in the experimental decay scheme shown in fig. 7, except for one transition; namely, there cannot be a transition JC because of a spin change of three by our assignments. We suggest that there is enough uncertainty in the experimental energies so that the reported 917 keV transition could fit as transition JD. The experimental literature is somewhat contradictory with respect to multipolarity assignments. It is clear that our spin assignments could be tested by multipolarity assignments. Transitions CA, GC, ID, and JE must be pure E2, since $|\Delta I| = 2$. A search for the unobserved higher spin levels predicted here could be valuable. The experimental level at 703 keV does not seem to have a counterpart among the theoretical values. We are also somewhat concerned about the difficulty of populating a $\frac{7}{2}+$ state at 1039 keV, presuming Ba^{131} to have a $\frac{1}{2}+$ or $\frac{3}{2}+$ ground state.

The reason for not seeing the two $\frac{9}{2}+$ states and higher spin states could be that the high spin itself and the predominance of high-K components in these states inhibit population by β decay from Ba^{131} or by γ transition from higher energy states.

Before presenting the calculations of the ground-state magnetic moment and some relative transition probabilities, we list the eigenfunctions of the states involved, the fifteen a_{JJ_3} Newton coefficients used, the three $b_{5/2\ K}$ values, and the four $b_{7/2\ K}$ values.

For N=4, β= -0.3, and γ = 22.5 deg

ℓ	J	J_3	a_{JJ_3}
4	$\frac{9}{2}$	$\frac{9}{2}$	0.04129
4	$\frac{9}{2}$	$-\frac{7}{2}$	0.14200
4	$\frac{7}{2}$	$-\frac{7}{2}$	0.01109
4	$\frac{9}{2}$	$\frac{5}{2}$	-0.28351
4	$\frac{7}{2}$	$\frac{5}{2}$	0.00604
2	$\frac{5}{2}$	$\frac{5}{2}$	-0.14104
4	$\frac{9}{2}$	$-\frac{3}{2}$	-0.51944
4	$\frac{7}{2}$	$-\frac{3}{2}$	-0.01793
2	$\frac{5}{2}$	$-\frac{3}{2}$	-0.24832
2	$\frac{3}{2}$	$-\frac{3}{2}$	-0.05451
4	$\frac{9}{2}$	$\frac{1}{2}$	0.61698
4	$\frac{7}{2}$	$\frac{1}{2}$	0.02807
2	$\frac{5}{2}$	$\frac{1}{2}$	0.35165
2	$\frac{3}{2}$	$\frac{1}{2}$	0.04707
0	$\frac{1}{2}$	$\frac{1}{2}$	0.18867

For I= $\frac{5}{2}$

K	$b_{5/2\,K}$
$\frac{1}{2}$	-0.87287
$-\frac{3}{2}$	0.47423
$\frac{5}{2}$	0.11491

For I= $\frac{7}{2}$

K	$b_{7/2\,K}$
$\frac{1}{2}$	-0.44320
$-\frac{3}{2}$	-0.78613
$\frac{5}{2}$	0.42070
$-\frac{7}{2}$	0.09263

We calculated the magnetic moment of the ground state of Cs^{131} by setting collective g_R values equal to Z/A, g_ℓ=1 and first setting g_s for the odd proton equal to the "quenched" value 4.0 found generally applicable by Chiao and Rasmussen[21]. A second calculation sometimes follows in parentheses, using free-space g_s=5.585. We feel that the quenched values are more appropriate, since they take account of a general tendency by the odd nucleon toward polarizing the spins of neighboring nucleons in such a way as to reduce the intrinsic-spin contribution to the magnetic moment.

The experimental magnetic moment of Cs^{131} is +3.48 nm[22]. Table 2 gives our theoretical moment at several values of β and γ near the optimum (0.28,38 deg) in units of the nuclear magneton. The interpolated moment for β=0.28, γ =38 deg is 2.82 (unquenched value is 3.14). For purpose of comparison, the spherical shell model with quenched g factor gives μ=4.0 for

Table 2

Theoretical magnetic moment

	γ = 30 deg	γ = 37.5 deg	γ = 45 deg	γ = 38 deg
β = 0.2	2.35	2.66	2.79	2.70
β = 0.3	2.88	2.86	2.74	2.85

$d_{5/2}$ (unquenched, 4.793) and μ=1.68 for $(g_{7/2+})^{-3}_{J=5/2}$ (unquenched, 1.225). Nilsson's model[8,9] for a pure $g_{7/2}$ proton with projection $\Omega = \frac{5}{2}$ gives μ=1.49 (unquenched, 1.18).

The agreement is not good for all the model and the experimental magnetic moment lies in between that of asymmetric-rotor and single-particle models. The magnetic moment is probably not a good test of the model, since the moment will be quite sensitive to the precise amount of $g_{7/2}$ vs $d_{5/2}$ configuration mixing.

We have used our formulas of subsec. 2.3 to calculate reduced E2 transition probabilities for a number of transitions. In most cases the experimental data are at present not sufficient to test the calculations carefully, since M1-E2 mixing ratios are not well determined. Evidence seems good that the prominent 122-keV transition is E2, and of course it may not have admixture of M1 if the spin assignments $\frac{1}{2}+ \rightarrow \frac{5}{2}+$ are correct. Its measured half-life is 3.77×10^{-9} sec (ref.[20]). We estimate the total conversion coefficient to be about 0.9 from Sliv's K- and L-shell calculations[23]. The mean life for photon emission should thus be $\approx 7.16\times10^{-9}$ sec. This corresponds to a reduced transition probability B(E2) of 31.34×10^{-50} e^2cm^4. The single-particle B(E2) value as used by Kerman[11] for a ($0\rightarrow 2$ transition) is $2.0\times10^{-50}e^2cm^4$ for A=131. Hence, the transition rate is 15.7 times the single-particle reference value.

Note from fig. 8 that our theoretical estimate near $\gamma = 38$ deg overestimates the experimental value by about a factor of 1.5. In retrospect it would have been more realistic to use an inertial parameter B considerably larger than the irrotational value. Since we used such a small B, we were forced to a deformation value of β=0.28 in order to fit the energy spectrum. Certainly the experience from the spheroidal nuclear region would lead us to believe that a β value about half of this is more realistic for Cs^{131}. Such a modification of our calculations away from the hydrodynamic moments of inertia would lower the predicted B(E2) value, since they vary roughly as β^2.

We find a serious discrepancy in trying to compare experimental and theoretical transition rates for the 133-keV transition presumed to be $\frac{7}{2}+$ to $\frac{5}{2}+$ and predominantly M1. Its lifetime is 13.3×10^{-9} sec (ref.[20]) and, allowing for a conversion coefficient of ≈ 0.5, this given an upper limit B(E2) $< 7.0\times10^{-50}$ e^2cm^4. Our theoretical calculation (fig. 9) for B(E2) is

3.4 times this limit. If future experimental work firmly establishes the position and spin associated with the 13.3×10^{-9}-sec state according to the proposal of Bodenstedt et al.[20], then it will be clear that we should not have fitted that state as the $\frac{7}{2}$ + number of the ground rotational family. It may be that the 13.3-nsec state belongs to a different intrinsic Newton proton state than the ground state.

We have also calculated the relative B(E2) values for three transitions each from the first two excited states of spin $\frac{3}{2}$ (i.e., at 216 keV and 620 keV). The ratios to the ground transition are shown in figs. 10 and 11. Points at 0 deg and 60 deg are calculated in the spheroidal limits through squares of Clebsch-Gordan coefficients with $K=\frac{1}{2}$ for I equal to $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$ on the prolate side; and $K=\frac{1}{2}$ for $I_{3/2}$; $K=\frac{5}{2}$ for $I_{5/2}$, $I_{7/2}$ on the oblate side. Experimental data on M1-E2 mixing ratios are presently insufficient to test any of these calculations. They are presented here in part to illustrate the drastic differences between $\frac{3}{2}$ + and $\frac{3}{2}$ +' states, whereby there appears clearly an approximate selection rule from the limiting model of γ vibrations of a spheroid (see Appendix). In this latter model the $\frac{5}{2}$ + and $\frac{7}{2}$ + states have no phonons of gamma vibration, the $\frac{1}{2}$ + and $\frac{3}{2}$ + have one phonon, and the $\frac{3}{2}$ +' has two phonons. This can be seen from the top curve of fig. 11.

3.3 GROUND STATE SPIN[22,24,25]

We have determined the lowest state for a variety of β and γ values for nucleon numbers of 51, 53, 55, and 57; and have plotted the deformation regions where a given spin is lowest. This map is in polar coordinates, where β is the radial coordinate and γ is the angular coordinate. All shapes of quadrupole deformation are of course represented within the 60-deg sector. Fig. 12 represents the 51st level of the proton in the N=4 shell, fig. 13 the 53rd level, fig. 14 the 55th level, and fig. 15 the 57th level.

The lowest spin state along γ=0 in these four figures should correspond with the lowest Nilsson state for that proton number from the diagram of fig. 3 of Mottelson and Nilsson[9]. One sees from all figures except fig. 12 that our intuition about the stabilization of spin $\frac{1}{2}$ states in asymmetric wells is borne out by the calculations.

Note that at the Cs^{131} reference point (β=0.28, γ=38 deg) on fig. 13 the $\frac{5}{2}$ + spin is lowest; for the Cs^{127} and Cs^{129} only a shift of γ of 2 to 5 deg toward a smaller γ value brings spin $\frac{1}{2}$ lowest. The spin $\frac{1}{2}$ area for the 55-proton nuclei does encompass the γ=0 line out to β=0.2 and is there designated by the Nilsson model as the $\frac{1}{2}$ + (420) state at prolate deformation. Some experimental knowledge of excited states of Cs^{127} and Cs^{129} would be most desirable, to test between predictions of the spheroidal and the asymmetric nuclear models.

3.4. MAGNETIC MOMENTS OF SPIN $\frac{1}{2}$ NUCLEI

There is one region in which the approximate calculation of magnetic moments for spin $\frac{1}{2}$ nuclei is especially simple. This is the region of nucleon number 65 to 81, where the $g_{7/2}$ and $d_{5/2}$ orbitals are presumably filled, and even-parity states will generally consist of configurations involving the close-lying $d_{3/2}$ and $d_{5/2}$ orbitals. An examination of the Newton eigenfunctions of the top two states in the fourth oscillator shell for $\gamma < 30$ deg shows that these states consist mainly of $s_{1/2}$-$d_{3/2}$ admixture in comparable amounts.

Likewise, Nilsson's highest $\Omega = \frac{1}{2}$ state number 51 in his two sets of calculations shows the same admixture[8]. The relative mixing ratios do not depend strongly on deformation. If a Nilsson-type calculation is made with only degenerate $s_{1/2}$ and $d_{3/2}$ orbitals considered, and if the radial matrix

elements of r^2 between 3s and 2d and between 2d and itself are considered equal (Nilsson's eqs. 11a and 11b have them differing by only 4%)[8], then the eigenfunction of the top $\Omega = \frac{1}{2}$ state for prolate (next to the top for oblate) has

$$a_s = \left(\frac{2}{3}\right)^{\frac{1}{2}} \text{ and } a_{d_{3/2}} = \left(\frac{1}{3}\right)^{\frac{1}{2}},$$

independent of deformation.

The predicted moment is two-thirds of the simple $s_{1/2}$ nucleon value plus one-third of the moment resulting from a $d_{3/2}$ nucleon coupled to a core angular momentum of 2 to a resultant $\frac{1}{2}$. The magnetic moment of the $d_{3/2}$ plus phonon coupled to $\frac{1}{2}$ is $\mu = g_R - g_J$. If we use the quenched g_s factor of -2.4 for an odd neutron and a g_R factor for collective motion of about 0.4, the magnetic moment of a pure $s_{1/2}$ neutron state is about -1.2, and the moment of the $d_{3/2}$ plus phonon is about zero. Thus, the nucleus with an odd neutron in this state should have a magnetic moment of about -0.8 nm.

Likewise, the second from the top $\Omega = \frac{1}{2}$ state in the prolate spheroid (top on oblate) is one-third of $s_{1/2}$ character and two-thirds of $d_{3/2}$ plus phonon. An odd neutron in this state should give rise to a magnetic moment of -0.4 nm.

Intuitively, we would expect that models with a stable nonaxial deformation or with quadrupole oscillations about a spherical equilibrium shape might give rise to intermediate theoretical values for the odd neutron in either state.

Table 3 lists nuclear moments of those spin $\frac{1}{2}$ nuclei to which the model might possibly apply. The tin isotope is in obvious disagreement, but the other isotopes are consistent with some sort of quadrupole coupling—mixed s-d model, as discussed above. A reduction in the collective g_R factor

Table 3

Magnetic moment values of certain spin $\frac{1}{2}$ nuclei

Isotope	Experimental μ (nm)
${}_{48}Cd^{111}_{63}$	-0.59
${}_{48}Cd^{113}_{65}$	-0.62
${}_{50}Sn^{119}_{69}$	-1.04
${}_{52}Te^{123}_{71}$	-0.73
${}_{52}Te^{125}_{73}$	-0.88
${}_{54}Xe^{129}_{75}$	-0.77

for tin, corresponding to the proton closed shell, could explain its deviation in terms of the coupling scheme here proposed. The magnetic moment of spin $\frac{1}{2}$ nuclei does not give much help in deciding between the nonaxial model and other quadrupole deformation models.

We shall examine Newton's[6] eigenfunctions for the three highest states in the fourth oscillator shell in order to be able to understand more quantitatively the predictions of his model. If much $d_{3/2}$ — $d_{5/2}$ admixture is involved in the wave function, there will be serious shifts in the calculated moment away from the simple formula. At β=0.1 and γ=30 deg, for example, the Newton wave function with the most $s_{1/2}$ character is actually 28% $s_{1/2}$, 55% $d_{3/2}$, 8.3% $d_{5/2}$, and 9% higher. It has the second from the highest eigenvalue.

We calculate a magnetic moment of about -0.8 nm, but here the $d_{5/2}-d_{3/2}$ cross-term (calculated with eq. Ap II.5 of ref. [26]) contributes almost half the value. Therefore, the basis of the previously derived simple approximate formula (μ= -1.2× fraction $s_{1/2}$), based only on $s_{1/2}$- $d_{3/2}$ admixture, is clearly violated, and may be trusted only at deformation parameters well below β=0.1 for asymmetric shapes. For prolate (γ=0) symmetric shapes the validity goes to somewhat larger β. That is, for γ=0, β=0.1 we have 62% $s_{1/2}$, 31% $d_{3/2}$, and 3.8% $d_{5/2}$; and the $d_{3/2}-d_{5/2}$ cross term contributes only -0.17 nm shift to the magnetic moment.

4. Conclusions

We have attempted serious and detailed testing of the fixed asymmetric-rotor model for odd-A nuclei. Certainly the large E2 transition probability in odd-A nuclei somewhat removed from closed shells yet not within the regions of spheroidal nuclei are strong indications of collective motion. The availability of nuclear eigenfunctions for an asymmetric well as calculated by Newton[6] make various calculations feasible. Our attempt to fit all the well-established levels of Cs^{131} as a single rotational band gives encouragement but is inconclusive. Better experimental information is much needed. Also, our difficulties with the E2 transition probability of the 133-keV transition suggests that the theoretical calculations need to be repeated to include perhaps two or more Newton intrinsic states with rotation-particle coupling—a coupling we have completely ignored for the sake of simplifying calculations.

The magnetic moment of spin $\frac{1}{2}$ nuclei ranging from 65 to 81 neutrons agrees with predictions of a model involving coupling to a core angular momentum, but do not distinguish to any degree the core shape or magnitude of deformation.

Successful results with this model do not necessarily prove that the nucleus literally has fixed asymmetric deformation. We show in the Appendix a spheroid with γ vibrations gives quite similar results at small γ values. The asymmetric rotor model does provide a prescription for calculations and a point of attack on nuclei in regions not yet very amenable to theoretical interpretation.

Appendix

RELATIONSHIP OF A SYMMETRIC ROTOR PLUS γ VIBRATION WITH A SLIGHTLY ASYMMETRIC ROTOR

It is evident that at $\gamma=0$ and $\gamma=\frac{1}{3}\pi$, the first rotational band has the same structure as that of a spheroid; and the energy values of the higher rotational band become infinite, with hydrodynamic moments of inertia. However, in the cases of $\gamma=\frac{1}{24}\pi$ or $\frac{7}{24}\pi$ the computed spectra of the ellipsoidally deformed nuclei (see Appendix B of ref. [13]) show the general features of the rotational - γ-vibrational spectra of a spheroid. In this section we shall see that in these limits it is possible to reproduce the properties of the ground, second, and third rotational bands of an ellipsoidal nucleus by considering the γ-vibration-rotation interaction between rotational bands, and the one- and two-phonon γ-vibrational bands of a spheroidal nucleus.

Since it is impossible to write a simple expression for ellipsoidal nuclei in the general case, one specific example will be explained in detail; the other cases are quite analogous. The illustration is for an intrinsic state with projection $\Omega=\frac{5}{2}$.

	I	π	K	n_γ
———	$\frac{5}{2}$	+	$\frac{3}{2}$	2
———	$\frac{3}{2}$	+	$\frac{3}{2}$	2
———	$\frac{5}{2}$	+	$\frac{1}{2}$	1
———	$\frac{3}{2}$	+	$\frac{1}{2}$	1
———	$\frac{1}{2}$	+	$\frac{1}{2}$	1
———	$\frac{7}{2}$	+	$\frac{5}{2}$	0
———	$\frac{5}{2}$	+	$\frac{5}{2}$	0

The Hamiltonian $\mathcal{H}'$ for γ-vibration - rotation interaction may be derived from the last term of eq. (11) when γ is small[27)]

$$\mathcal{H}' = -\frac{\hbar^2(I_1^{\,2} - I_2^{\,2})\gamma}{\sqrt{3}\,\mathfrak{J}} \tag{A.1}$$

In the γ-asymmetrical model the energies of the bands are determined by the β and γ values. In axially symmetric nuclei the energies of the rotational and the vibrational bands depend upon the phonon energy and the moment of inertia. If the adiabatic approximation holds and if the γ-vibration phonon energy $\left[\hbar\omega = (C_\gamma/B_\gamma)^{\frac{1}{2}}\right]$ and the moment of inertia ($\mathfrak{J} = 3B_\gamma\beta^2$) are the same for all the bands, the energy of the nth highest energy state with spin I is expressed as

$$E_n(I) = \frac{\hbar^2}{2\mathfrak{J}}\left\{\left[I(I+1)-K^2\right] - (-)^{I-J}(J+\tfrac{1}{2})(I+\tfrac{1}{2})\delta_{K\,1/2}\delta_{\Omega\,1/2}\right\} + Q\hbar\omega, \tag{A.2}$$

where

$$Q=n - 1 \text{ for } I \geq \frac{5}{2}$$

and

$$Q=n \qquad \text{for } I = \frac{1}{2},\frac{3}{2} \text{ for the first three bands.}$$

Since the mixing amplitudes and the B(E2) values for different bands are proportional to the term $(B_\gamma C_\gamma \beta^4)^{\frac{1}{2}}$, which is approximately equal to $(\frac{1}{3}\hbar\omega\mathfrak{J})$, we define an energy-difference ratio

$$p = \frac{E_2(\frac{5}{2}) - E_1(\frac{5}{2})}{E_1(\frac{7}{2}) - E_1(\frac{5}{2})}, \tag{A.3}$$

which is related to the $(B_\gamma C_\gamma \beta^4)^{\frac{1}{2}}$ in a linear way, and we will express the mixing amplitudes and the B(E2) ratio in terms of p by second-order perturbation theory. For comparison, the values of β are tabulated in column a

of table A-1 and the values of the mixing amplitudes

$$a(\tfrac{5}{2}, \tfrac{5}{2}; \tfrac{5}{2}, \tfrac{1}{2}),$$

$$a(\tfrac{3}{2}, \tfrac{3}{2}; \tfrac{3}{2}, \tfrac{1}{2}),$$

and

$$\frac{B[E2\ \frac{3}{2}\ (2) \rightarrow \frac{1}{2}(1)]}{B[E2\ \frac{3}{2}\ (2) \rightarrow \frac{5}{2}(1)]} \quad \text{ratio}$$

are in columns b, d, and f for the spheroidal model, in c, e, and g for the ellipsoidal model. The notation a(I, K; I, K') signifies the mixing amplitude of the angular momentum projection K' in the state of spin I and predominant component K. The p values for the spheroidal model are calculated using the results in Appendix B of ref.[13] for $\gamma = 52.5$ deg.

Table A-1

Comparison of mixing amplitude and B(E2) ratio for ellipsoidal and spheroidal models

a	b	c	d	e	f	g
	$a(\frac{5}{2}, \frac{5}{2}; \frac{5}{2}, \frac{1}{2})$		$a(\frac{3}{2}, \frac{3}{2}; \frac{3}{2}, \frac{1}{2})$		$\frac{B[E2\ \frac{3}{2}\ (2)\rightarrow \frac{1}{2}\ (1)]}{B[E2\ \frac{3}{2}\ (2)\rightarrow \frac{5}{2}\ (1)]}$	
	spheroidal	ellipsoidal	spheroidal	ellipsoidal	spheroidal	ellipsoidal
0.1	0.00278	0.00276	-0.00238	-0.00049	2.93×10^{9}	1.59×10^{7}
0.2	0.00285	0.00226	-0.00250	-0.00046	2.50×10^{9}	4.82×10^{7}
0.3	0.00318	0.00304	-0.00273	-0.00049	1.87×10^{9}	1.75×10^{7}

From table A-1 we see that both models yield similar results, and that the B(E2) ratio shows the effect of the forbidden transition of $\Delta n=2$, although the magnitudes are not equal. We believe that as γ goes to smaller values the two models will approach identical predictions. Note that in the B(E2) ratio calculation the denominator contains two terms of opposite sign and nearly equal values, which will cause the B(E2) ratio to be very sensitive to the a values.

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Table 1

Energy levels, experimental and theoretical energy values for Cs^{131}

Level label	Spin$_{(theo)}$ [a]	E_{exp} [b] (keV)	E_{theo} [c] ($\frac{B\beta^2}{\hbar^2}$=13.85)(Newton state no. 27) (keV)				
			0.28,38°	0.3,30°	0.3,37.5°	0.3,45°	0.2,37.5°
J	$\frac{7}{2}+'$	1039	1022	1682.7	1106.7	1873.7	1224.0
I	$\frac{3}{2}+'$	620	644	1404.4	650.7	1714.7	1023.1
H	$\frac{9}{2}+'$	----	448	722.9	462.6	684.9	822.4
G	$\frac{5}{2}+'$	373	380	899.7	452.1	458.8	575.2
F	$\frac{11}{2}+$	----	264	531.7	260.2	419.1	731.2
E	$\frac{3}{2}+$	216	224	780.6	276.3	377.3	466.9
D	$\underline{\frac{7}{2}+}$	133	136	529.2	196.7	183.9	478.9
C	$\underline{\frac{1}{2}+}$	124	122	717.8	183.8	341.9	432.6
B	$\frac{9}{2}+$	----	40	330.8	0	304.1	606.4
A	$\underline{\frac{5}{2}+}$	0	0	383.5	2.61	97.4	373.9

[a] Underlined spins have been experimentally determined or inferred.

[b] See ref. [16] for compilation of various determinations of energies.

[c] Interpolated energy values of this column are mostly subject to an uncertainty of about 8 keV.

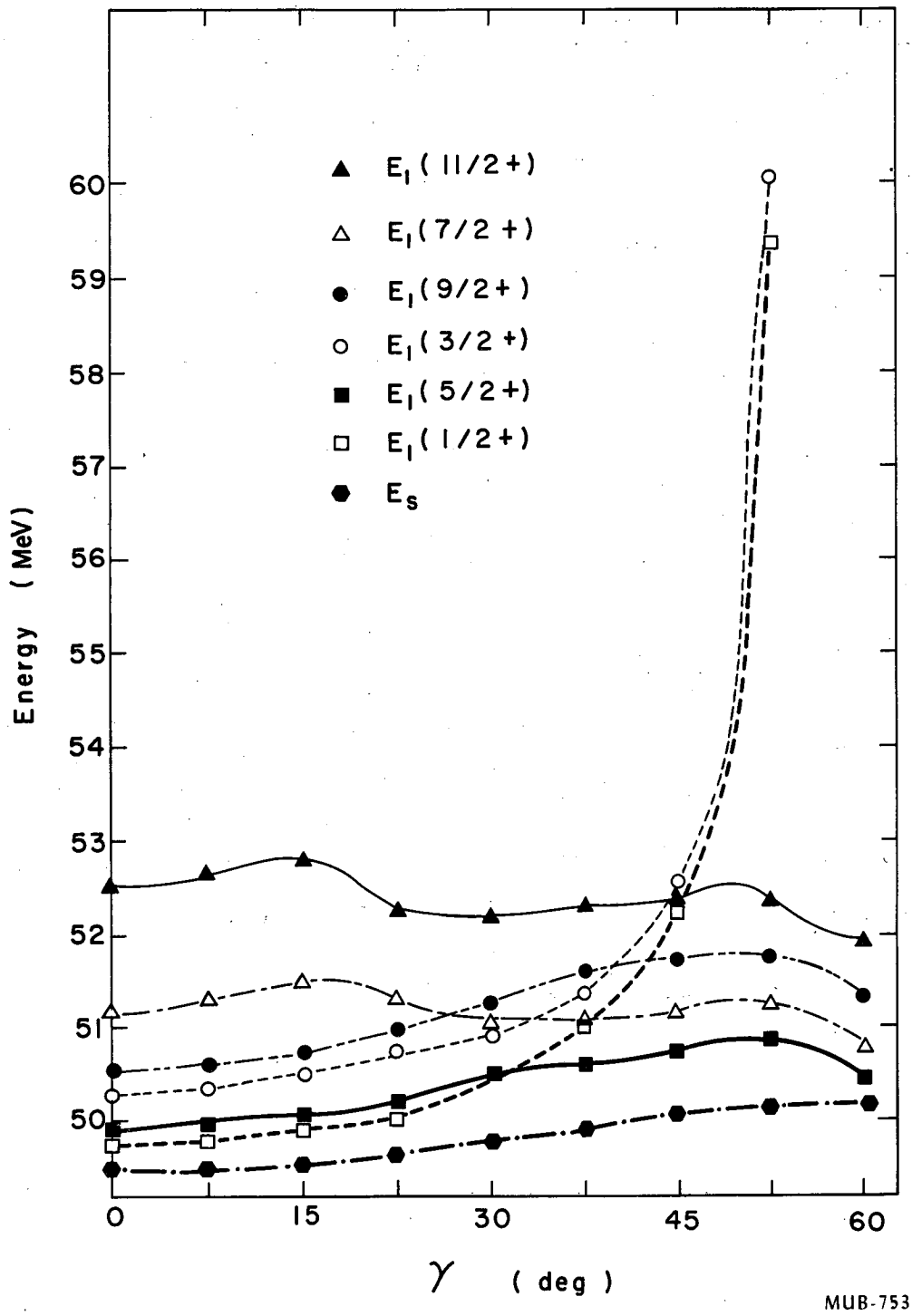


Fig. 1. Energy diagram of the first rotational band for the lowest single-particle state of the N=2 shell as a function of γ.

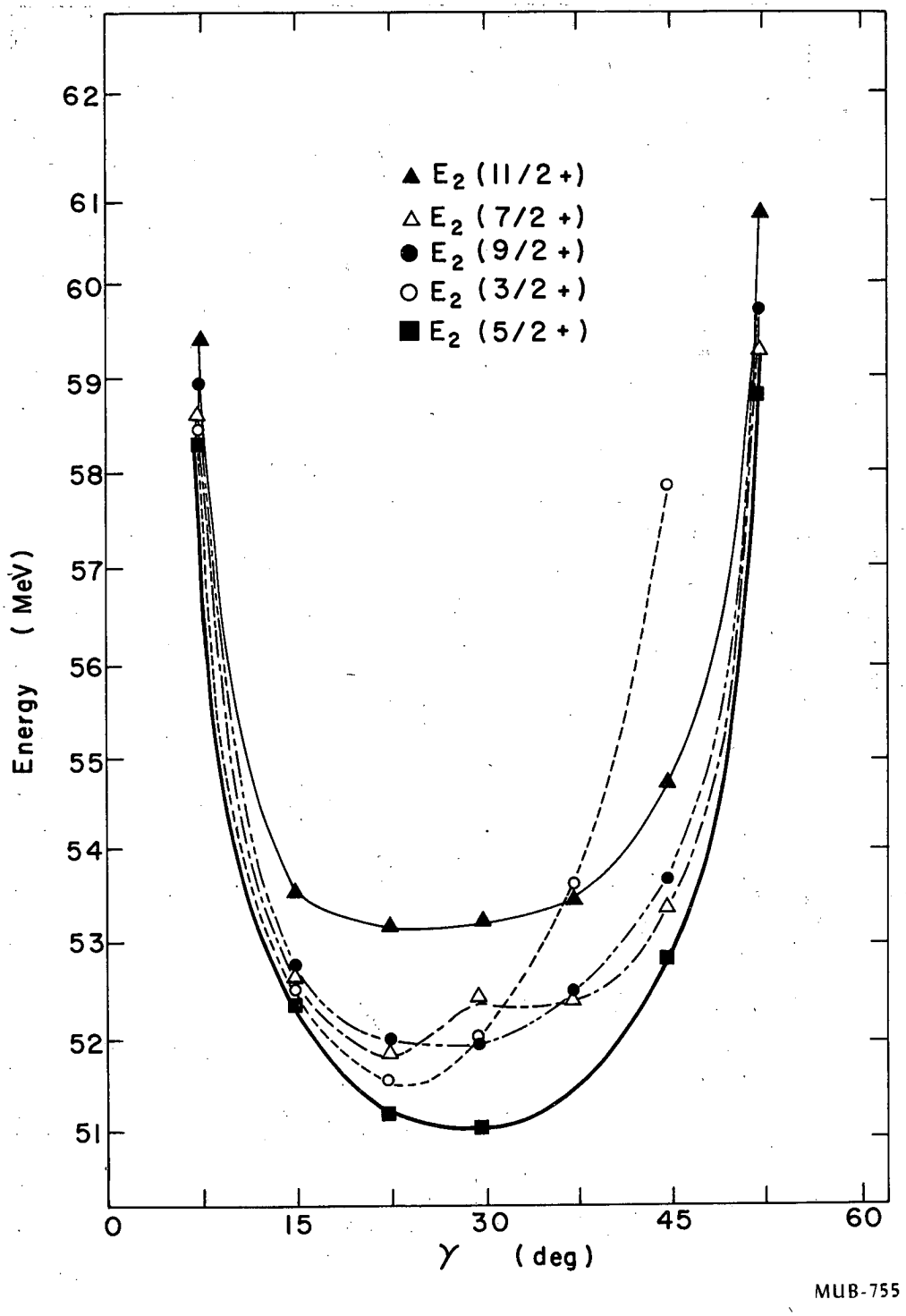


Fig. 2. Energy diagram of the second rotational band for the lowest single-particle state of the N=2 shell as a function of γ.

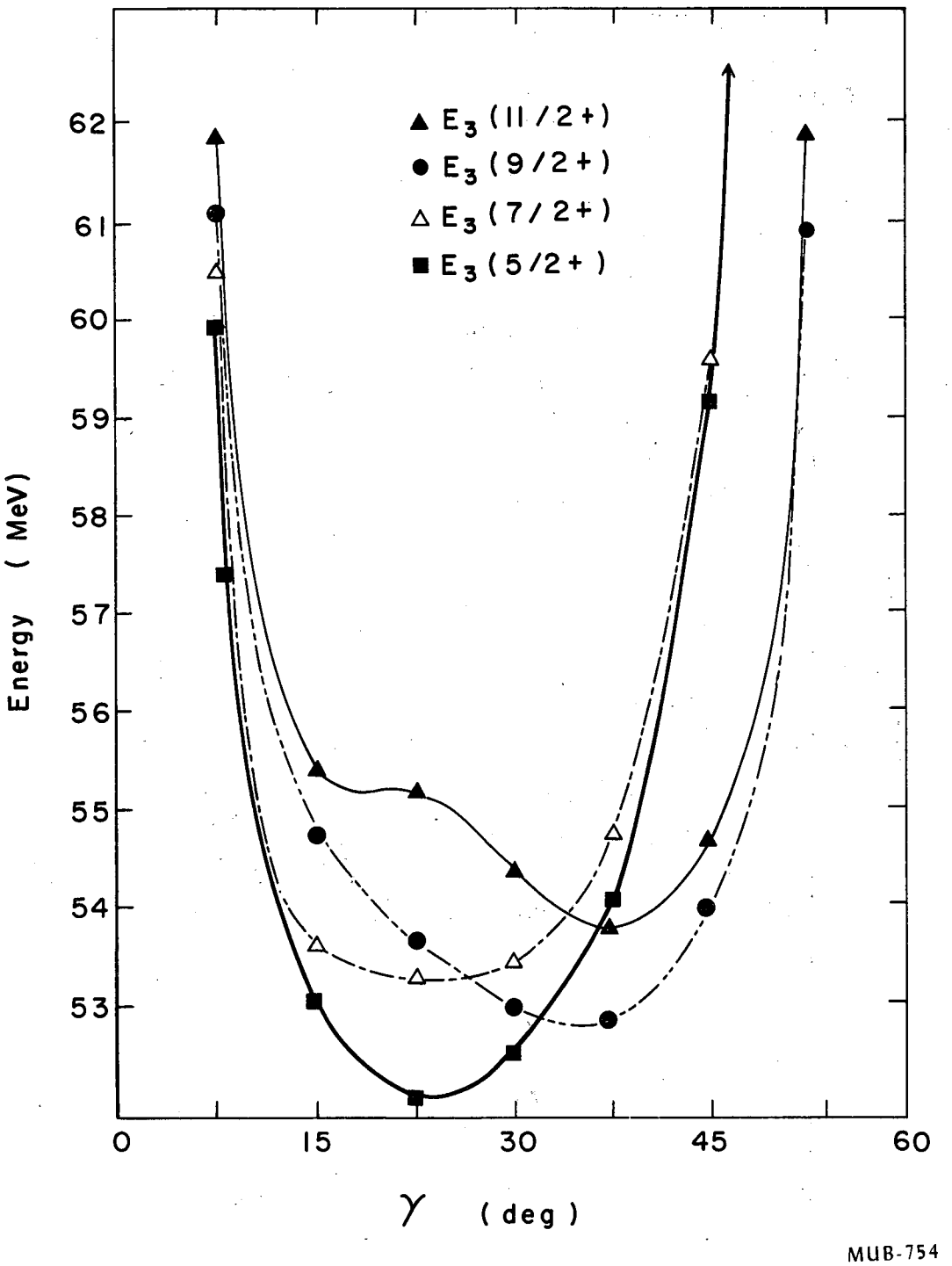


Fig. 3. Energy diagram of the third rotational band for the lowest single-particle state of the N=2 shell as a function of γ.

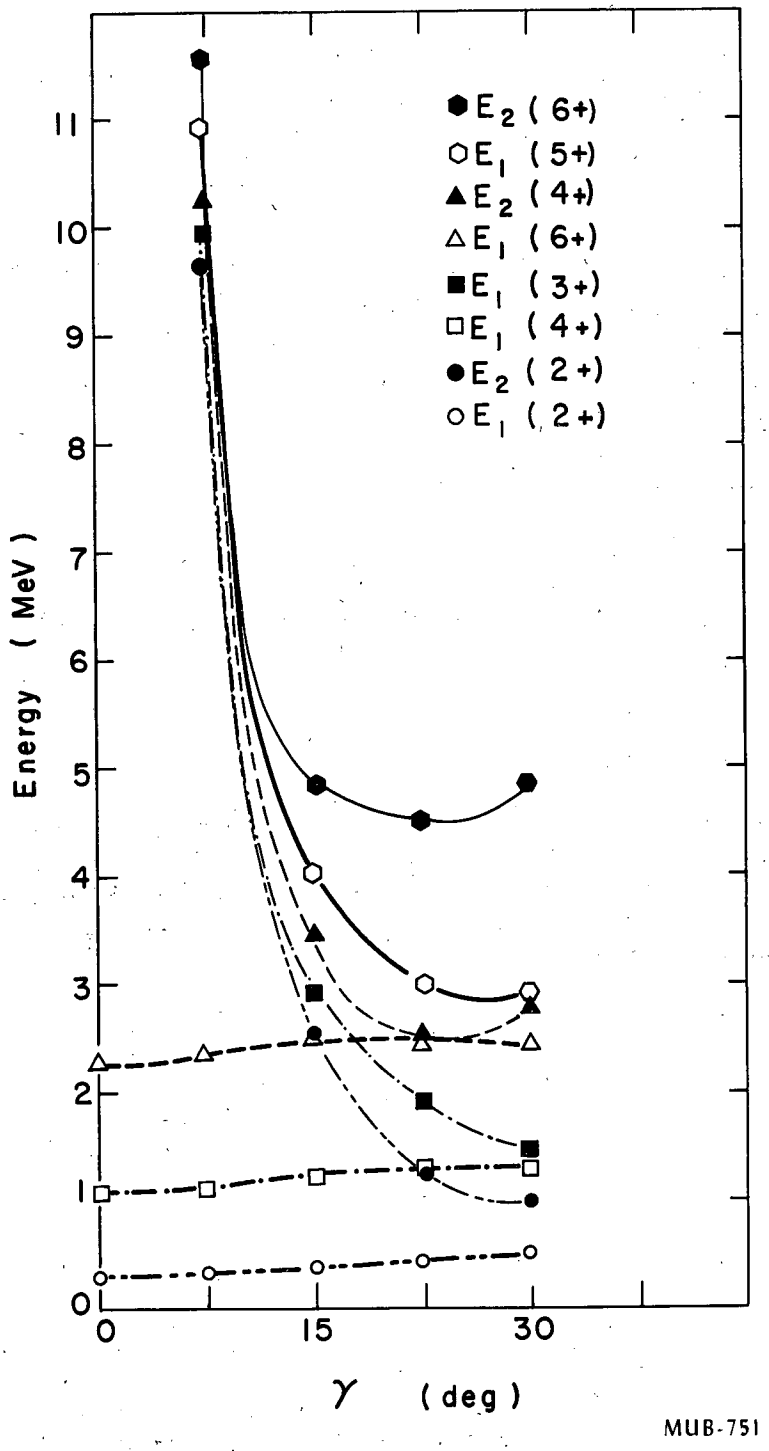


Fig. 4. Rotational-energy diagram of an even-even nucleus E_R (in MeV) as a function of γ.

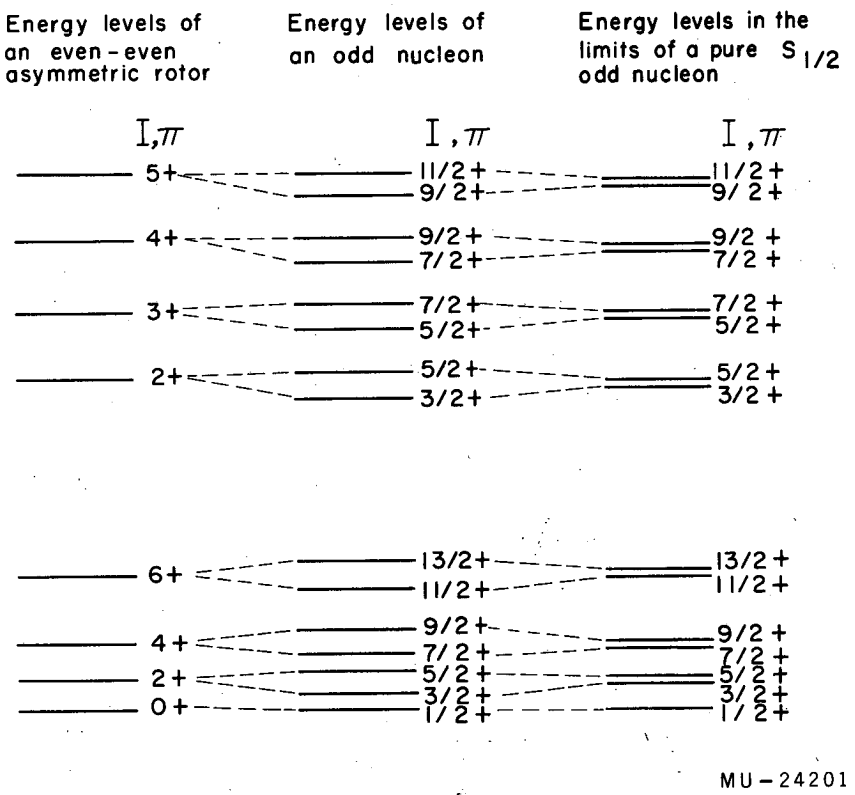


Fig. 5. Relation of spectrum between an even-even asymmetric rotor and an odd mass nucleus of ellipsoidal shape.

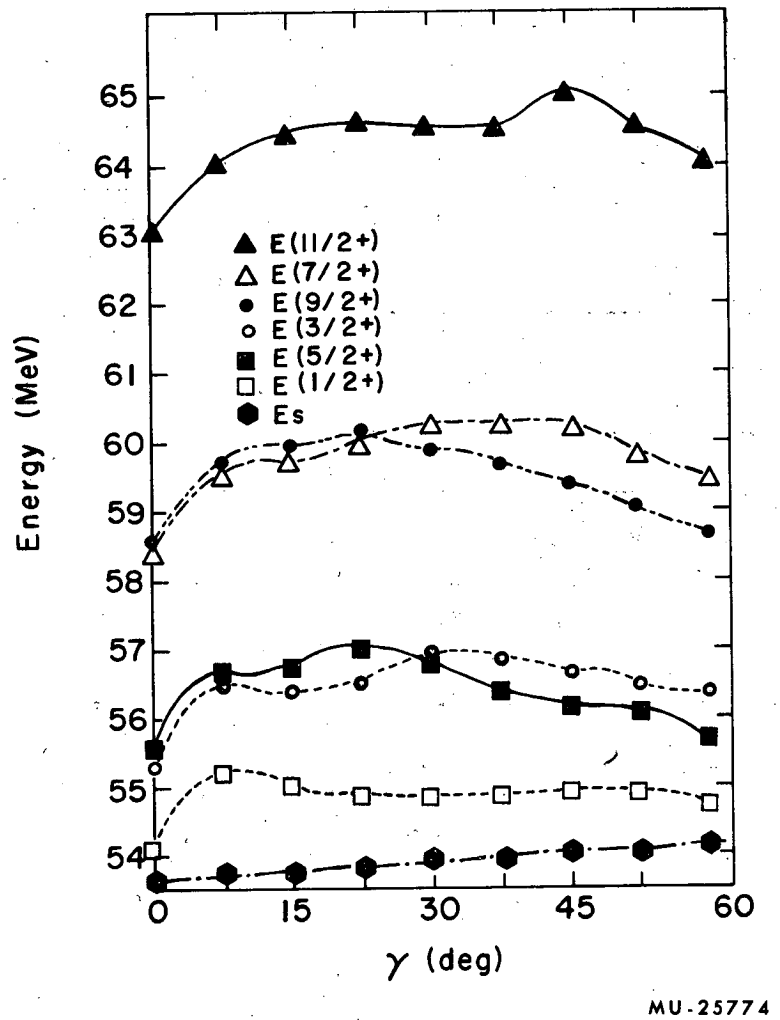


Fig. 6. Energy diagram of a nearly pure $s_{1/2}$ state as a function of γ, for the N=2 shell, with β=0.1 and $B/\hbar^2$ = 77.25 MeV^{-1}.

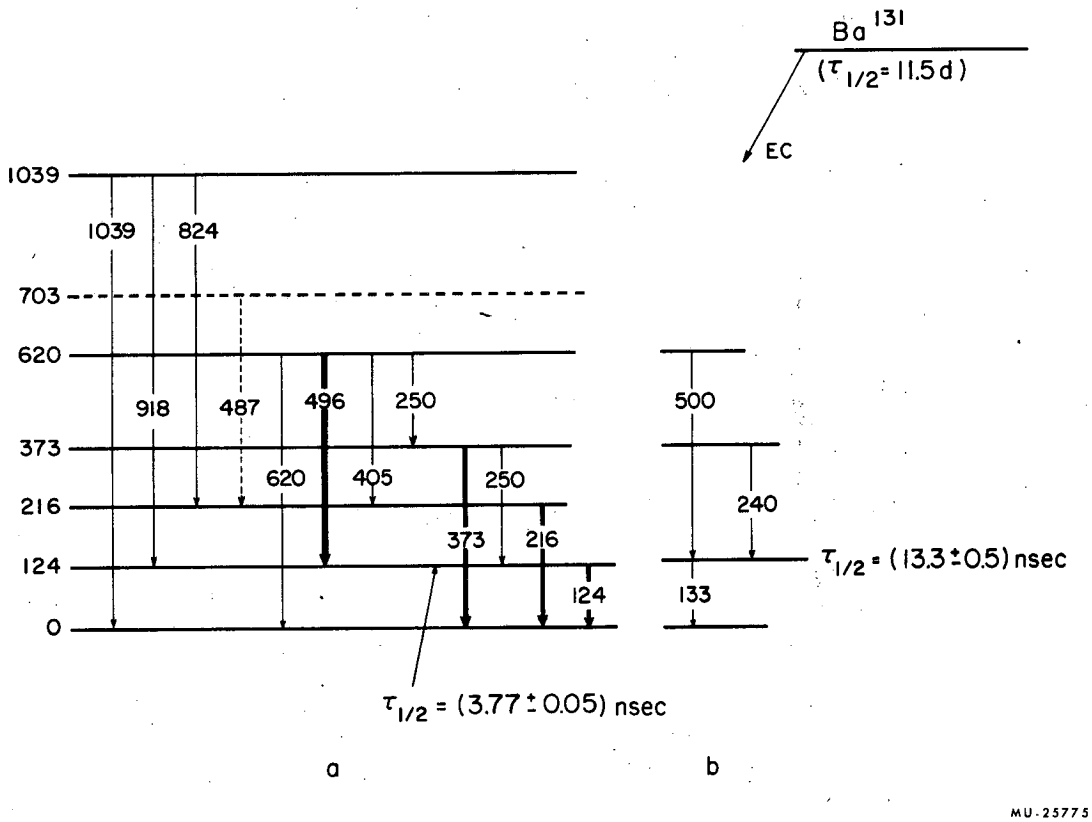


Fig. 7. Decay scheme of Ba^{131}.

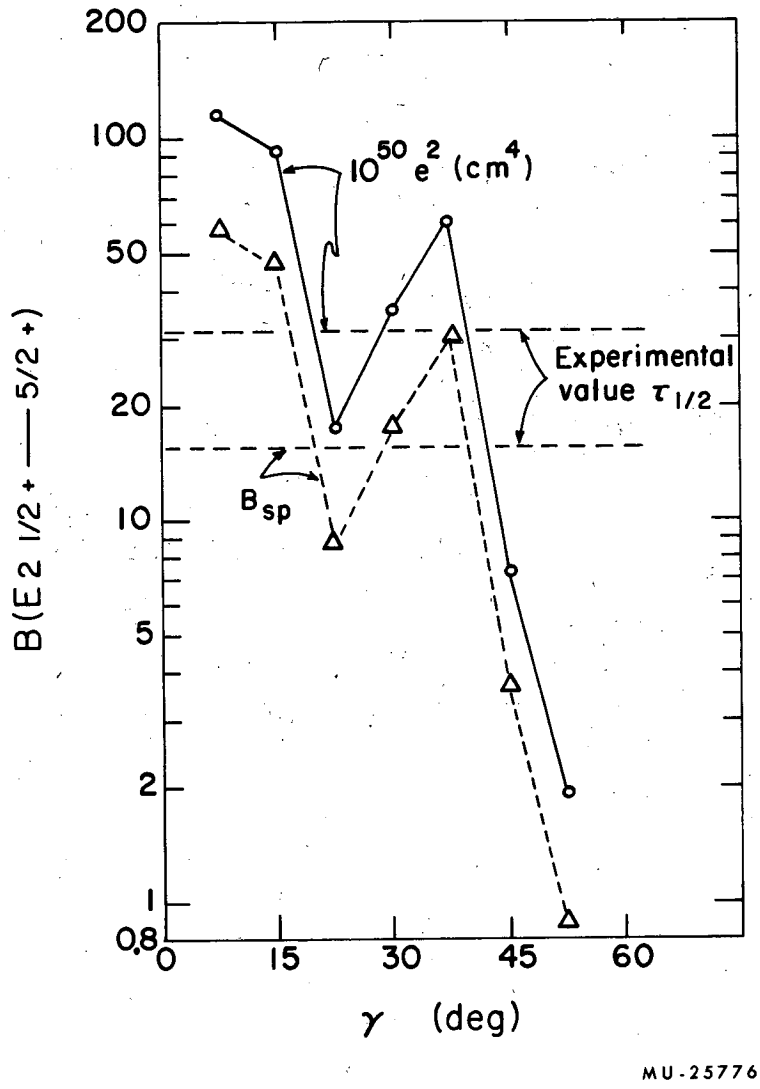
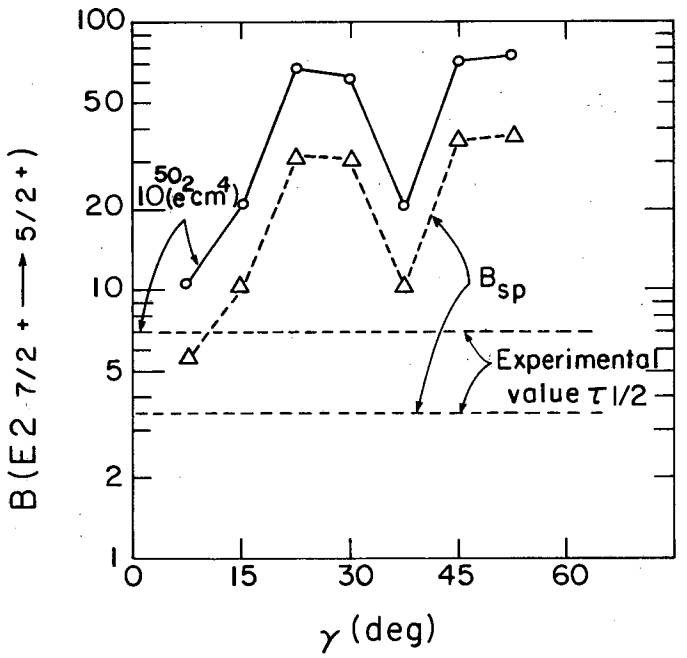


Fig. 8. Comparison of the experimental $B(E2\ \frac{1}{2}+\rightarrow\frac{5}{2}+)$ value with the theoretical values as a function of γ, with β=0.3.



MU-25777

Fig. 9. Comparison of the experimental $B(E2\ \frac{7}{2}+ \rightarrow \frac{5}{2}+)$ values with the theoretical values as a function of γ, with β=0.3.

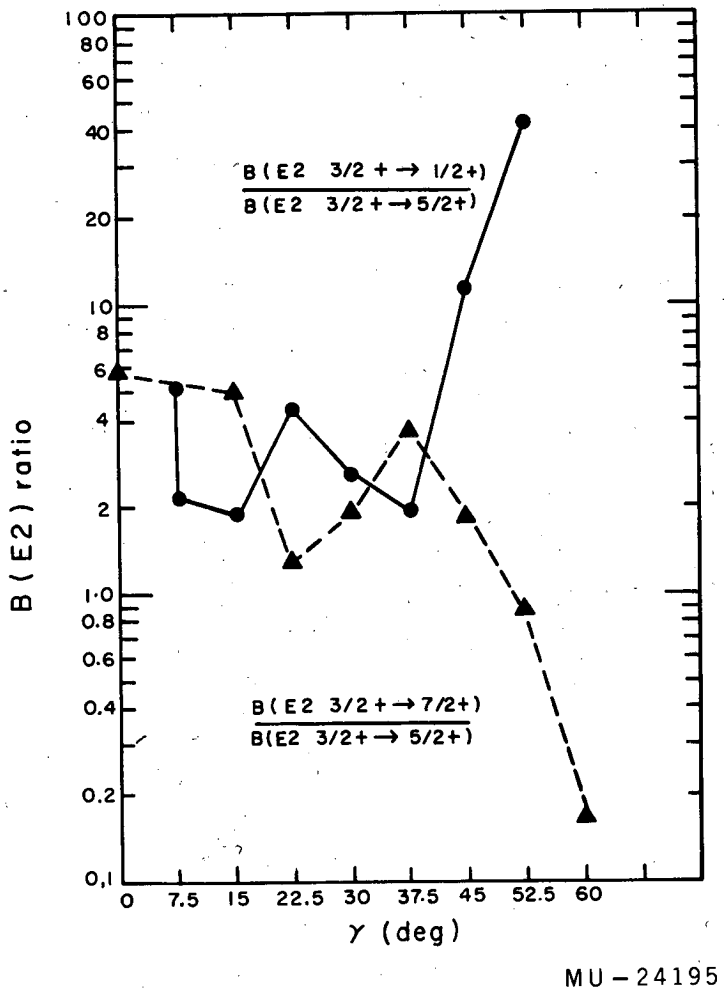


Fig. 10. Theoretical ratios

$$\frac{B(E2\ \frac{3}{2}+\ \rightarrow \frac{1}{2}+)}{B(E2\ \frac{3}{2}+\ \rightarrow \frac{5}{2}+)} \quad \text{and} \quad \frac{B(E2\ \frac{3}{2}+\ \rightarrow \frac{7}{2}+)}{B(E2\ \frac{3}{2}+\ \rightarrow \frac{5}{2}+)}$$

as a function of γ, with $\beta = 0.3$.

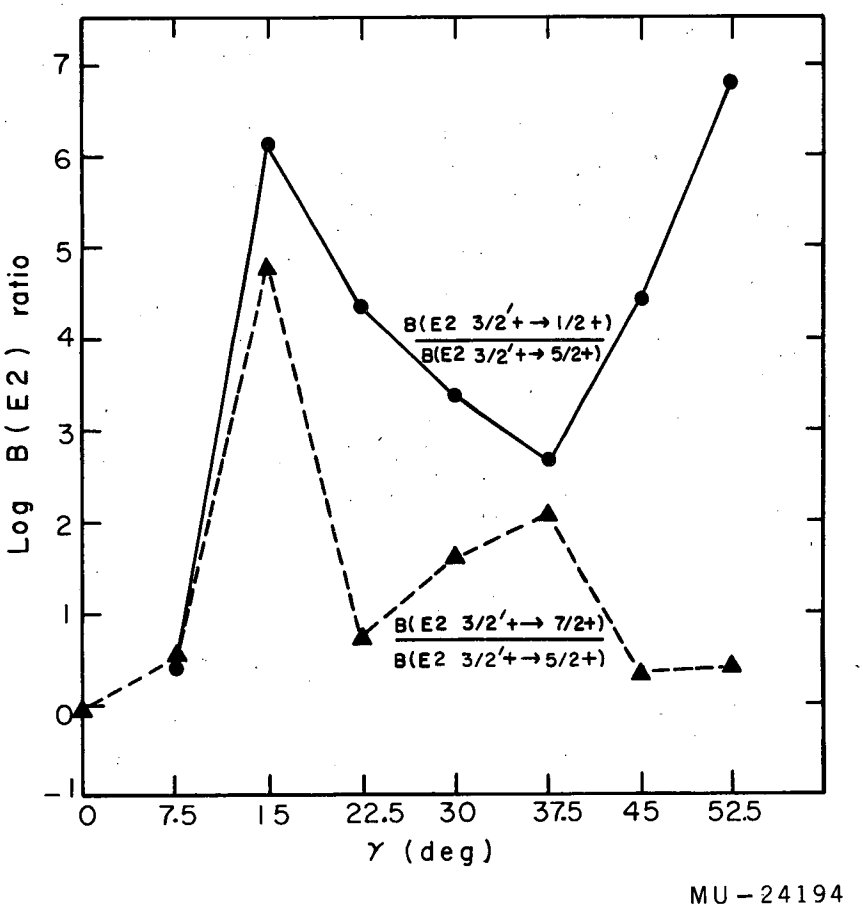
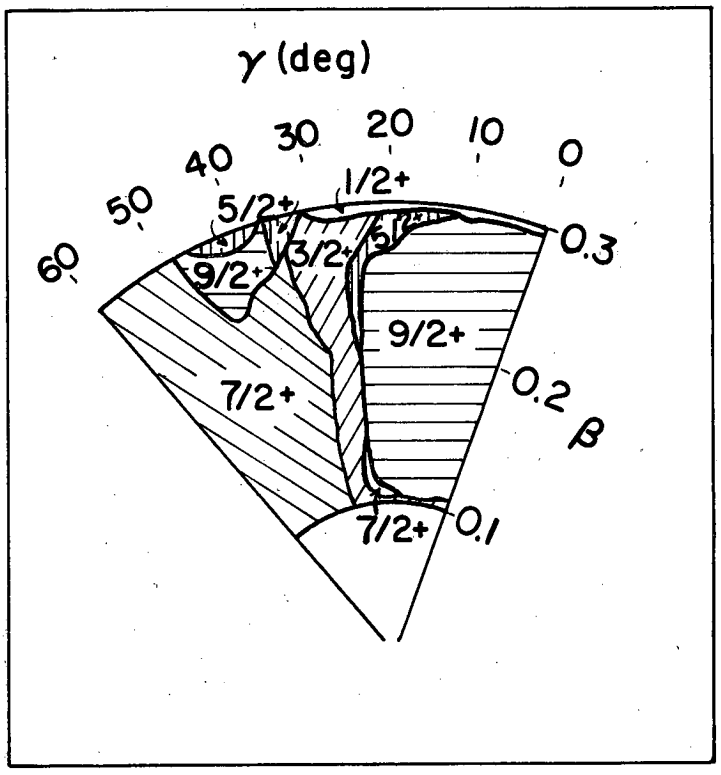


Fig. 11. The logarithmic theoretical ratios

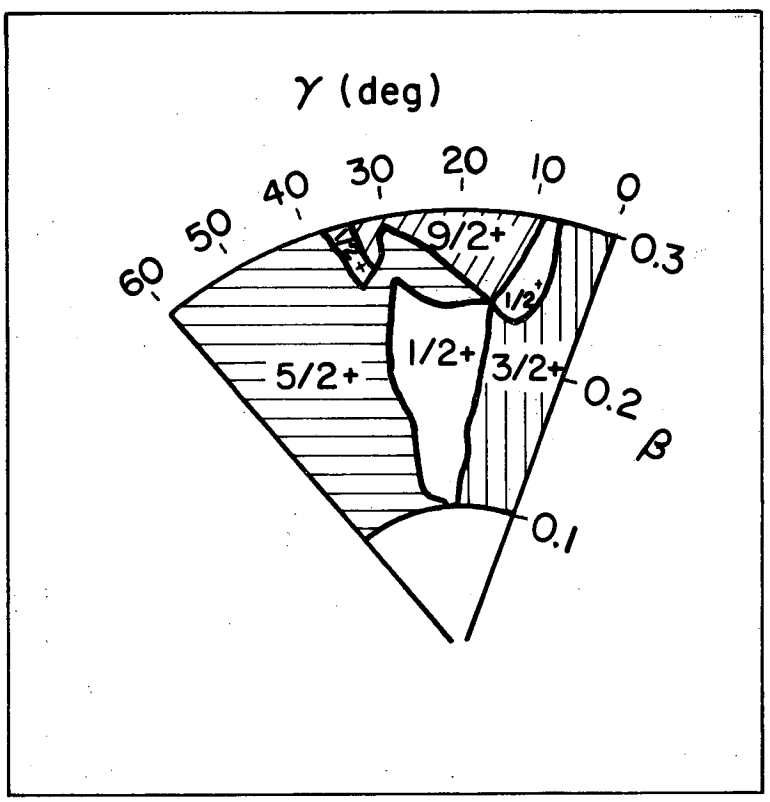
$$\frac{B(E2\ \frac{3}{2}'+ \rightarrow \frac{1}{2}+)}{B(E2\ \frac{3}{2}'+ \rightarrow \frac{5}{2}+)} \text{ and } \frac{B(E2\ \frac{3}{2}'+ \rightarrow \frac{7}{2}+)}{B(E2\ \frac{3}{2}'+ \rightarrow \frac{5}{2}+)}$$

as a function of γ, with β=0.3.



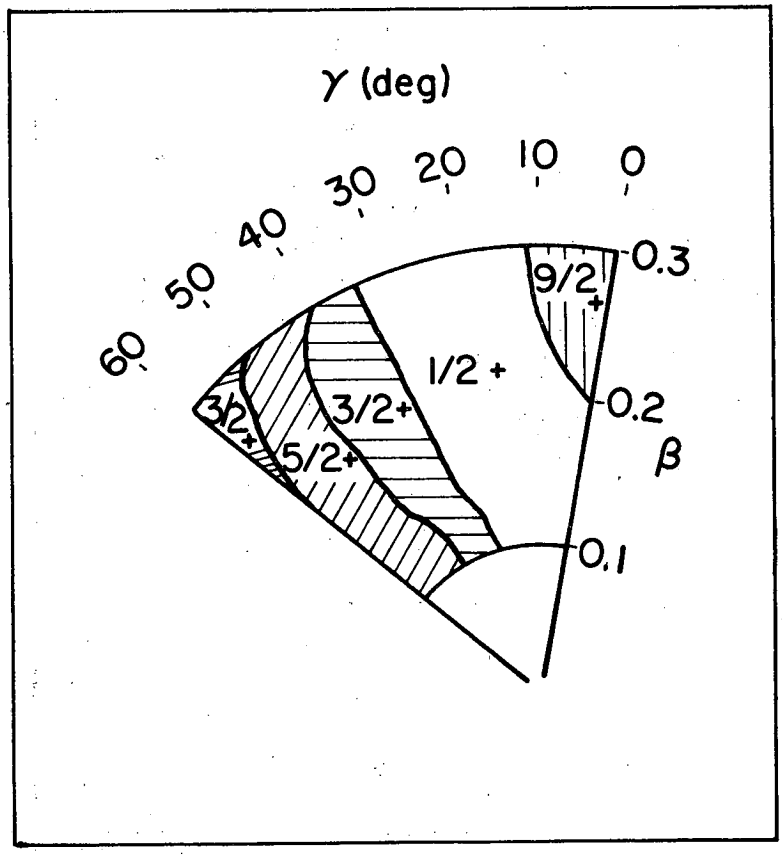
MU-25778

Fig. 12. Plot of the ground-state spin of the 51st level of the proton as a function of β, γ; with $B\beta^2/\hbar^2$ = 9.41 MeV^{-1} and $\hbar\omega_o$=8.16 MeV.



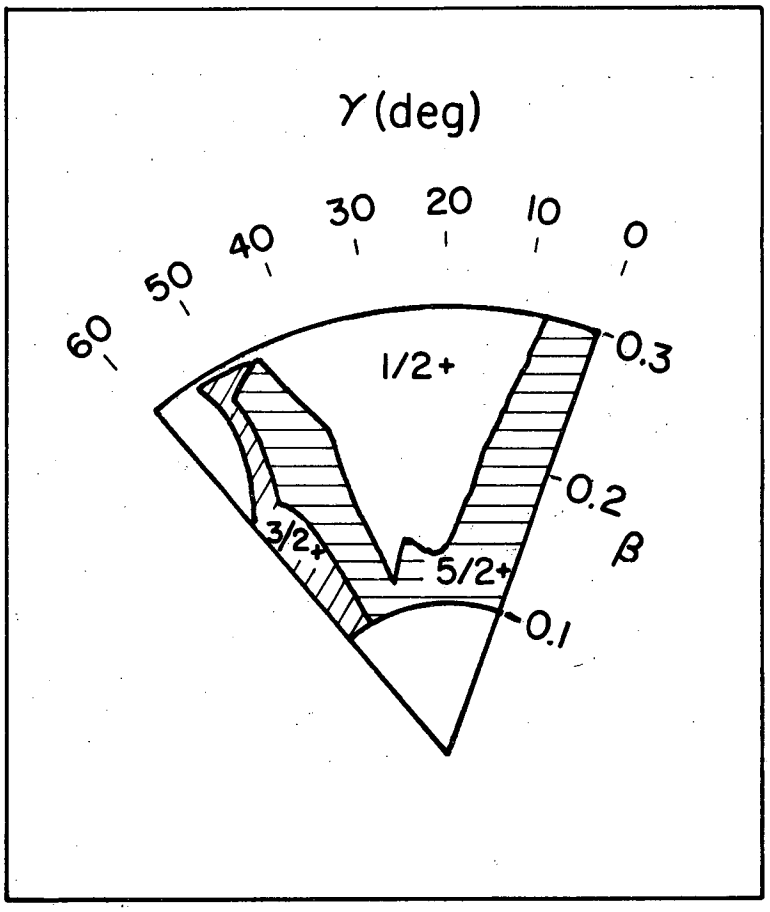
MU-25779

Fig. 13. Plot of the ground-state spin of the 53rd level of the proton as a function of β, γ; with $B\beta^2/\hbar^2 = 9.41\ \mathrm{MeV}^{-1}$ and $\hbar\omega_o = 8.16$ MeV.



MU-25780

Fig. 14. Plot of the ground-state spin of the 55th level of the proton as a function of β, γ; with $B\beta^2/\hbar^2$ = 9.41 MeV^{-1} and $\hbar\omega_o$=8.16 MeV.



MU-25781

Fig. 15. Plot of the ground-state spin of the 57th level of proton as a function of β, γ; with $B\beta^2/\hbar^2 = 9.41\ \mathrm{MeV}^{-1}$ and $\hbar\omega_o = 8.16$ MeV.

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