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THE GIANT E1 RESONANCE FOR DEFORMED NUCLEI TREATED BY THE RANDOM-PHASE APPROXIMATION

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ABSTRACT

The random-phase approximation has been applied to treat the giant E1 resonance of the deformed nucleus Mg^{24}. Two well separated peaks are predicted on the basis of reasonable force parameters. The backward-going graphs are found not to affect significantly the positions of the 1^- states or the relative distribution of the oscillator strength. On the other hand the Thomas-Kuhn-Reiche sum is diminished for the case of a Ferrel-Visscher force by a magnitude of up to 30 percent by the inclusion of ground state correlations. The violation of the TKR sum rule encountered in the shell-model calculations is thus reduced considerably.

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INTRODUCTION

The giant E1 photo-nuclear resonance has now been studied for a large number of nuclides, and there appears to be experimental indication that the giant E1 resonance exhibits a split peak in strongly deformed nuclei.[1] Danos[2] and Okamoto[3] explained this effect qualitatively in terms of a hydro-dynamic model with different characteristic frequencies along the major and minor axis of the nuclear spheroid. This effect was then also calculated on the basis of the independent-particle picture by Wilkinson[4] and by Mottelson and Nilsson[5] using the single-particle wave functions of an anisotropic harmonic oscillator.

Such a simple description of the E1 giant resonance in terms of non-interacting particles now appears to be contradicted by the empirical fact that the characteristic resonance energy is of the order of a factor of 1.5 to 2 larger than the spacing between two oscillator shells, $\hbar\omega$.[6] Already several years ago Elliott and Flowers[7] were able to explain the photo-excitation spectrum of O^{16} by the perturbation of the simple shell-model states by a residual two-body force of finite range and containing exchange

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operators. As a result, states of mixed configuration are formed with such proportions of the pure states that the 1^- states with T=1 generally appear above $\hbar\omega$ while those with T=0 generally appear below. In particular, the two highest-lying E1 states were found to absorb almost all of the E1 oscillator strength and thus together constitute a true giant state.

Recently, Brown and Bolsterli[8] proposed a very schematic but suggestive picture of the underlying mechanism of shell-model configuration mixing that gives rise to the E1 resonance. In a representation of single-particle states where all E1 single-particle matrix elements are of same sign, and furthermore provided the matrix elements are of roughly equal magnitude, a coherent excited state, collecting the main E1 oscillator strength, is obviously the linear combination of single-particle excitations with the amplitudes being roughly equal and all of the same sign. Such an excited state of one-particle character is indeed also the highest-lying 1^- state provided all matrix elements of the residual interaction are of the same magnitude and sign in this representation. Brown and Bolsterli treated the case of a residual Wigner force of zero range. However, the specific isobaric spin character of the E1 state as well as the antisymmetrisation of the nuclear wave function are neglected in the first of the papers of Ref. 8. A conjectured change of the over-all sign of the interaction matrix elements compensates qualitatively for the mentioned effects.

Recently the approach based on the Random-Phase-Approximation has been applied with success to the study of certain types of collective states of nuclei. The formalism employed was first developed in the early papers by Sawada[9] and his collaborators with application to the electron gas problem. The corresponding methods were introduced into nuclear physics independently by Takagi,[10] Fallieros,[11] Mottelson,[12] and others. This latter type of theory has the distinction relative to the shell-model calculations in that it accounts approximately for the effects of correlations in the ground state. These effects are sometimes discussed in terms of the so-called backward-going graphs which

means that single-particle deexcitations are considered in addition to the excitation, i.e. the lifting of a particle from below the Fermi surface to above it. Only the latter type of processes are included in the usual shell-model calculations.

In the present paper we shall employ the density matrix formulation[13,14] which is especially suitable for the study of higher order nonlinearity effects.

As the specific T=1 character of the giant resonance state appears to be of primary importance, it is of particular interest to study nuclei with such a low Z- value that the isobaric spin is a good constant of the motion. Therefore our first choice has been Mg^{24} which is a prolate nucleus and for which the adiabatic coupling scheme seems well established. In addition, as an example of a possibly oblate nucleus, we have considered C^{12}.

THE RANDOM-PHASE-APPROXIMATION TREATMENT OF THE RESIDUAL INTERACTIONS

We consider a Hamiltonian of the form

$$H = H_o + \Delta V \tag{1}$$

where in the notation of second quantization

$$H_o = \sum_{\mu} \epsilon_{\mu} a_{\mu}^{+} a_{\mu} \tag{2}$$

is the single-particle part of the Hamiltonian, including the shell-model field, and where

$$\Delta V = 1/2 \sum_{\alpha\alpha'\beta\beta'} \langle\alpha\beta|V|\alpha'\beta'\rangle\, a_{\alpha}^{+} a_{\beta}^{+} a_{\beta'} a_{\alpha'} \tag{3}$$

is the residual interaction, i.e. the part of the interaction that is not already included in the field.

The two-body interaction V refers to space, spin and isobaric-spin quantum states of both particles involved in the interaction

$$V = V(\underline{r}^{(1)}, \underline{\sigma}^{(1)}, \underline{\tau}^{(1)}, \underline{r}^{(2)}, \underline{\sigma}^{(2)}, \underline{\tau}^{(2)}).$$

In the absence of a residual interaction, ΔV, the ground state is an eigenvector of H_o corresponding to a sharp Fermi surface in terms of the shell-model wave functions. In the presence of a residual interaction the ground state may contain correlations. As seen from (3), ΔV scatters particles out of the Fermi sea. Obviously such hole-particle pair excitations have to comply with parity and angular-momentum conservation of the ground state.

Our main interest is, however, not the ground state but the excited giant E1 state. Just as the ground state appears to be a complicated linear combination of various one-two- etc. particle excitations, so also the E1 state may have a complicated structure. However, we are concerned only with its relation to the ground state. The simple Random-Phase-Approximation is largely equivalent to the contention that, relative to the correlated ground state, denoted by $|0\rangle$, the "collective state" $|E\rangle$ is of the simple type

$$|E\rangle = (\mathcal{B}^E)^+ \; |0\rangle = \sum_{\nu\nu'} c^E_{\nu\nu'} \; \hat{\rho}_{\nu\nu'} \; |0\rangle \tag{5}$$

where $c^E_{\nu\nu'}$ are coefficients later to be determined and where

$$\hat{\rho}_{\nu\nu'} = a^+_{\nu'} a_\nu \tag{6}$$

is the density matrix operator connected with the promotion of a particle from the single-particle state ν to another state ν'.

The "collective" states thus correspond to linear combinations of one-particle excitations relative to the ground state. The assumption of a state of this particular character to approximate the physical state has strong direct support from what is empirically known about the character of the collective state. As this state (or narrow group of a few states) may be reached from the ground state by an E1 excitation that to a large extent exhausts the total sum rule strength, and as the electromagnetic interaction can excite only one particle at a time to lowest order in $e^2/\hbar c$, the character of the collective

excitation is thus rather well specified empirically as being dominantly of the type expressed by Eq. (5).

The problem of finding the coefficients $c^E_{\nu\nu'}$ is equivalent to the problem of finding the matrix elements of $\hat{\rho}_{\nu\nu'}$ between the ground state $|0\rangle$ and the "collective" state $|E\rangle$.

The operators $\hat{\rho}_{\nu\nu'}$ fulfill the equations of motion

$$i\hbar \frac{\partial \hat{\rho}_{\nu\nu'}}{\partial t} \equiv \left[\hat{\rho}_{\nu\nu'}, H\right] = \left[\hat{\rho}_{\nu\nu'}, H_o\right] + \left[\hat{\rho}_{\nu\nu'}, \Delta V\right] \tag{7}$$

The first commutator on the right hand side is easily evaluated to be

$$\left[\rho_{\nu\nu'}, H_o\right] = \left(\epsilon_\nu - \epsilon_{\nu'}\right) \hat{\rho}_{\nu\nu'} \quad . \tag{8}$$

We omit here the details of the evaluation of the second commutator.

After multiplication of Eq. (7) by $\langle E|$ from the left and by $|0\rangle$ from the right we obtain

$$\left(\epsilon_{\nu'} - \epsilon_\nu - E\right)\left\langle E|\rho_{\nu\nu'}|0\right\rangle = \sum_{KK's}\left(\left\langle \nu K'|V|sK\right\rangle\left\langle E|\hat{\rho}_{KK'}\hat{\rho}_{s\,\nu'}|0\right\rangle - \left\langle sK'|V|\nu'K\right\rangle\left\langle E|\hat{\rho}_\nu\,\hat{\rho}_{KK'}|0\right\rangle\right) \tag{9}$$

where

$$\langle \nu K'|V|sK\rangle = \int \psi^*_\nu(1)\ \psi^*_{K'}(2)\ V\psi_s(1)\ \psi_K(2)\ d\tau_1 d\tau_2 \tag{9a}$$

This equation thus relates the matrix elements of one ρ to those of a product of two ρ's. By exactly the same procedure an analogous equation can be formed relating two-ρ aggregates to three-ρ aggregates, etc. In the simple Random-Phase-Approximation (RPA) one confines oneself to Eq. (9), which is then linearized according to a certain prescription. In this way Eq. (13) is obtained (see below). Thus quadratic terms of matrix elements of $\rho_{\nu\nu'}$ with

$\nu' \neq \nu$ are neglected as being small of a higher order. Although there are many of these terms, they are assumed to be negligible as they occur incoherently, i.e., with random phases.

For the sake of making a rough estimate of the neglected terms, assume first that all matrix elements involving V are approximately equal.

One may then compare on the one hand the matrix element $\langle E|\hat{\rho}_{KK'}\hat{\rho}_{\nu'\nu}|0\rangle$ and on the other the product of matrix elements $\langle E|\hat{\rho}_{KK'}|E'\rangle\langle E'|\hat{\rho}_{s\nu'}|0\rangle$. The only important intermediate states are other "two-quasi-particle" states $|E'\rangle$ of the same type as $|E\rangle$. Note, however, that $|E'\rangle$ may not necessarily be an E1 state. Under the assumption that $|E\rangle$ and $|E'\rangle$ are linear combinations of on the average Ω elementary particle-hole excitations, it is then easy to see that the following order-of-magnitude estimates are valid

$$\langle E|\hat{\rho}_{KK'}|E'\rangle\langle E'|\hat{\rho}_{s\nu'}|0\rangle \sim \frac{1}{\Omega}\,\frac{1}{\Omega^{1/2}} = \frac{1}{\Omega^{3/2}} \tag{10}$$

and

$$\langle E|\hat{\rho}_{KK'}\,\hat{\rho}_{\nu'\nu'}|0\rangle \sim \frac{1}{\Omega^{1/2}} \tag{11}$$

for ν referring to a single-particle state below the Fermi surface. In Eq. (11) obviously $\hat{\rho}_{\nu'\nu'}$ is the number operator associated with the single-particle level ν'. It is also apparent that only some of the intermediate states $|E'\rangle$ have nonvanishing matrix element with $|E\rangle$ in the approximation that only one-particle-hole excitations ("two-quasi-particle" states) have to be considered. In fact, in this case the sum over s, K, K' and E' together gives just of the order of Ω^3 terms.

If now the Ω^3 off-diagonal terms contributed all coherently, their contribution would be of the order of magnitude $\Omega^{+3/2}$ compared with $\Omega^{1/2}$ for the sum of Ω terms of the type given by Eq. (11). However, the assumption of a random phase relation between these terms instead makes their sum of the

order of magnitude $\Omega^{3/2} \cdot \Omega^{-3/2} = 1$. Thus for large Ω the terms with both annihilation indices different from both creation indices may be neglected.[x]

For the matrix element (11) we make the further approximation

$$\langle E|\, a^+_{\kappa'} a_\kappa a^+_{\nu'} a_s \,|0\rangle_{s=\nu'} \equiv \langle E|\hat{\rho}_{\kappa\kappa'} \hat{\rho}_{\nu'\nu'}|0\rangle = n_{\nu'} \langle E|\hat{\rho}_{\kappa\kappa'}|0\rangle \tag{12}$$

which would be exactly correct for a sharp Fermi surface. In Eq. (12) $n_{\nu'}$ is the occupation number of the single-particle state ν'. (For the second term on the right hand side of Eq. (11) we furthermore assume that also $|E\rangle$ behaves approximately as a sharp Fermi surface when acted upon by the occupation number operator $\hat{\rho}_{\nu\nu}$.)

The fact that an occupation number operator $a^+_{\nu'} a_{\nu'}$ can occur in (12), alternatively if the summation index s equals ν' or if the summation index κ equals ν', gives rise to the usual exchange matrix elements of V, well-known from shell-model calculations. In the usual procedure for linearizing the billinear Eq. (11) to obtain Eq. (13) a factorization of this two-$\hat{\rho}$ product that takes the Pauli principle properly into account is referred to as a Hartree-Fock factorization.

The complete linearized equation can thus be written as follows in terms of the amplitudes $\langle E|\rho_{\nu\nu'}|0\rangle$, which we will denote $\rho_{\nu\nu'}$ for the sake of typographical simplicity

$$(\epsilon_{\nu'} - \epsilon_\nu - E)\, \rho_{\nu\nu'} = (n_{\nu'} - n_\nu) \sum_{\kappa\kappa'} \langle \nu\kappa' | V(1-P^{12}) | \nu'\kappa \rangle \rho_{\kappa\kappa'} +$$

$$+ \sum_\kappa n_\kappa \left[\sum_{\kappa' \neq \nu'} \langle \nu\kappa | V(1-P^{12}) | \kappa'\kappa \rangle \rho_{\kappa'\nu'} - \sum_{\kappa' \neq \nu} \langle \kappa'\kappa | V(1-P^{12}) | \nu'\kappa \rangle \rho_{\nu\kappa'} \right. \tag{13}$$

---

[x] The effect of two-pair excitations neglected in the RPA could be further examined, e.g. by an extension of the Random-Phase Approximation - the "Higher RPA" - consisting of a closed system of equations of motion also connecting the two- and three-operator products where the three-$\hat{\rho}$ aggregates are in turn linearized.

The terms on the second line of Eq. (13) are self-energy terms corresponding to the contribution from the self-energy part of the interaction Hamiltonian

$$\Delta\hat{V}^{self} = 1/2 \sum_{\kappa s} \langle s\kappa | V | s\kappa \rangle \, a_s^+ a_\kappa^+ a_\kappa a_s \tag{14}$$

It appears reasonable to assume that terms of such origin are already effectively included in the shell-model field from which the single-particle energies $\{\epsilon_\nu\}$ have been determined, although we have obviously not attempted to relate the self-energy parts of V to the single-particle energies through a Hartree-Fock procedure or through any other self-consistent method.

After the exclusion of the self-energy terms, Eq. (13) takes the simple form

$$(\epsilon_{\nu'} - \epsilon_\nu - E)\rho_{\nu\nu'} = (n_{\nu'} - n_\nu) \sum_{\kappa\kappa'} \langle \nu\kappa' | V(1-P^{12}) | \nu'\kappa \rangle \rho_{\kappa\kappa'} \tag{15}$$

Another question also related to the arbitrariness in the problem of how the self-consistent field is defined concerns the RPA elements of Eq. (15) that are diagonal in the particle hole-pairs, namely $\langle \nu\nu' | V(1-P^{12}) | \nu'\nu \rangle$. Those obviously correspond to elastic scattering of particle-hole pairs $(\nu\nu')$, and the problem arises whether they also should be thought of as being already included in the single-particle potential. Ordinarily such terms are not included in a Hartree-Fock calculation based on the Brueckner reaction matrix. Unless one employs another type of reaction matrix (see e.g. Sawada[15]) in finding $\epsilon_{\nu'}$ such terms should be retained in the corresponding RPA calculation. However, in the present calculations we have assumed a "phenomenological" shell-model field to supply $\{\epsilon_\nu\}$. Probably the mentioned terms of

the elastic-scattering type are included in this field on the average. However, individual fluctuations relative to this average may be significant and display effects of correlations. For most of our calculations we have chosen to include these diagonal terms, but for some cases they have been excluded. Probably the wave functions and values of the E1 matrix elements corresponding to the former case are to be considered as somewhat more plausible. The energy values are, however, just on account of this ambiguity, uncertain by an order of magnitude of a couple of Mev, and for this reason one might imagine them to be over-estimated in the former case.

It should finally be emphasized that for the case of an uncorrelated ground state (i.e. when deexcitations, or annihilations of hole-particle pairs are excluded) Eq. (15) leads exactly to the eigenvalue problem occurring in the shell-model calculations. The latter type of calculations are obviously limited to including only the very simplest hole-particle pair graphs (one hole-particle pair being exchanged for another hole-particle pair in the interaction process).

In the approximation employed, our occupation factors n_ν and $n_{\nu'}$ have the values 1 and 0, corresponding to a sharp Fermi surface. We then also confine ourselves to excited states that, relative to the ground state, represent transitions of a particle <u>across</u> the Fermi surface. Thereby obviously a transition of a particle originally being above the Fermi surface in the correlated ground state to another state above the Fermi surface is excluded. The same holds true for a hole transition below the Fermi surface. However, as already emphasized, the formalism allows for both creation and destruction of hole-particle pairs relative to the correlated ground state.

Equation (15) is an eigenvalue equation in terms of the matrix elements of the density matrix $\rho_{\nu\nu'}$. The corresponding transposed matrix equation holds for the coefficients $c^{E}_{\nu\nu'}$, which are the components of the "collective" eigenvector $|E\rangle$ defined in Eq. (5). This equation may be written in the form[16]

$$\begin{pmatrix} \underset{\sim}{A} & -\underset{\sim}{B} \\ \underset{\sim}{B} & -\underset{\sim}{A} \end{pmatrix} \begin{pmatrix} \underset{\sim}{u} \\ \underset{\sim}{v} \end{pmatrix} = E \begin{pmatrix} \underset{\sim}{u} \\ \underset{\sim}{v} \end{pmatrix} \tag{16}$$

where $\underset{\sim}{u}$ corresponds to creation of a hole-particle pair relative to the ground state while $\underset{\sim}{v}$ corresponds to the annihilation of such a pair relative to the ground state. Thus in a pure shell model calculation $\underset{\sim}{v} = 0$. The matrix $\underset{\sim}{B}$ is obviously associated with the coupling between the excitations and the de-excitations. While both of the matrices $\underset{\sim}{A}$ and $\underset{\sim}{B}$ are Hermitian, the total matrix is non-Hermitian.

The reality of E depends on the actual strength of interaction and is for the present case always very well ensured. It is easy to verify that Eq. (16) has the simple property that to each positive eigenvalue E there corresponds a negative and unphysical eigenvalue - E, which may be ruled out by imposing the extra condition on the correlated ground state

$$\mathcal{B}^{E}\|0\rangle = 0. \tag{17}$$

Also, as the typical matrix elements of $\underset{\sim}{B}$ are of the order of, or less than, 1 Mev for the light nuclei considered compared to a separation of the roots of $\underset{\sim}{A}$ and $-\underset{\sim}{A}$ by a magnitude of the order of $2\hbar\omega$, the amount of mixture of de-excitations in the eigenfunctions of Eq. (16) is expected to be rather small in the treated cases. This is also born out in the explicit calculation.

As a result of the non-hermiticity of the secular problem the corresponding eigenvectors satisfy orthonormality and completeness relations characteristic of an indefinite metric:

$$\sum_{\nu\nu'} \left[\left(c^{E_n}_{\nu\nu'}\right)^* c^{E_m}_{\nu\nu'} - \left(c^{E_n}_{\nu'\nu}\right)^* c^{E_m}_{\nu'\nu} \right] = \frac{E_n}{|E_n|} \delta_{mn} \tag{18a}$$

$$\sum_{E_n, E_m \neq 0} \left(c^{E_n}_{\nu\nu'}\right)^* c^{E_m}_{\kappa\kappa'} \frac{E_n}{|E_n|} = \delta_{\nu\kappa}\delta_{\nu'\kappa'} \tag{18b}$$

where the sum over ν in (18a) runs over all single-particle states below the Fermi surface and the sum over ν' includes all such states above it. The E1 matrix element between the 1^- states and the ground state is most easily expressed in terms of the $\langle E|\hat{\rho}_{\nu\nu'}|0\rangle$ matrix elements

$$\langle E| \sum_i e_i r_i Y_1(i) |0\rangle = \sum_{\nu\nu'} \langle \nu'|rY_1|\nu\rangle \left(e_p \langle E|\hat{\rho}^p_{\nu\nu'}|0\rangle + e_n \langle E|\hat{\rho}^n_{\nu\nu'}|0\rangle\right) \tag{19}$$

DETAILED CALCULATIONS

In our calculations we have considered the self-conjugate nuclei Mg^{24} and C^{12} which have ground states with T = 0. Excitations of E1 character relative to the ground state have thus the isobaric spin T = 1 and are generated by the operators[x]

$$\hat{\Phi}_{\nu\nu'} = \frac{1}{\sqrt{2}} \left(\hat{\rho}^p_{\nu\nu'} - \hat{\rho}^n_{\nu\nu'}\right) \tag{20}$$

where, from here on, ν and ν' label only the space and spin coordinates but not the charge of the state. If we rewrite Eq. (15) in terms of the matrix elements of $\hat{\Phi}_{\nu\nu'}$ denoted $\Phi_{\nu\nu'}$, it takes the form:

$$(\epsilon_{\nu'} - \epsilon_\nu - E)\, \Phi_{\nu\nu'} = (n_{\nu'} - n_\nu) \sum_{\kappa\kappa'} \langle \nu\kappa'|U|\nu'\kappa\rangle\, \Phi_{\kappa\kappa'} \tag{21}$$

where $\langle \nu\kappa'|U|\nu'\kappa\rangle$ is the matrix element of $V(1-P^{12})$ in terms of the T = 1 wave functions (see Eq. (20)).

* That only T=1 states are excited can be seen from Eq. (19) and the fact that for such self-conjugate nuclei considered here the effective charges e_p and e_n of the proton and neutron are e/2 and -e/2, respectively. Physically the T=1 state, as seen from Eq. (20), corresponds to neutrons and protons moving 180° out of phase relative to each other, a picture which retains an essential feature of the original Goldhaber and Teller[17] two-fluid model.

Restricting ourselves to an interaction of the form

$$V = J(r_{12})(w + bP^{\sigma} + mP^{r} + hP^{r}P^{\sigma}) \tag{22}$$

and introducing the integrals

$$W = \int \psi_{\nu}^{*}(1)\ \psi_{\kappa'}^{*}(2)\ J\ \psi_{\nu'}(1)\ \psi_{\kappa}(2)\ d\tau_1 d\tau_2 \tag{23a}$$

$$B = \int \psi_{\nu}^{*}(1)\ \psi_{\kappa'}^{*}(2)\ JP^{\sigma}\ \psi_{\nu'}(1)\ \psi_{\kappa}(2)\ d\tau_1 d\tau_2 \tag{23b}$$

$$M = \int \psi_{\nu}^{*}(1)\ \psi_{\kappa'}^{*}(2)\ JP^{r}\ \psi_{\nu'}(1)\ \psi_{\kappa}(2)\ d\tau_1 d\tau_2 \tag{23c}$$

$$H = \int \psi_{\nu}^{*}(1)\ \psi_{\kappa'}^{*}(2)\ JP^{r}P^{\sigma}\ \psi_{\nu'}(1)\ \psi_{\kappa}(2)\ d\tau_1 d\tau_2 \tag{23d}$$

we may write

$$\langle \nu\kappa' | U | \nu'\kappa \rangle = -wH - bM - mB - hW. \tag{24}$$

The calculation of the interaction matrix elements as well as the E1 transition matrix elements is particularly simple if one uses the "asymptotic" wave functions,[5] valid in the limit that the quadrupole part of the shell model field is strong relative to the spin-orbit coupling. In this limit appropriate at large deformations only the diagonal parts of the spin-orbit interaction is retained. In the Appendix we present the results of such calculations applied to the C^{12} nucleus. Unfortunately the applicability of not only the adiabatic coupling scheme but in particular the asymptotic wave functions to the nucleus is very uncertain, and the results obtained are only of interest as illucidating the general mechanism behind the calculations that employ the more detailed wave functions of Ref. 18. They should thus not be directly compared with the empirical data.

In the coupling scheme appropriate to deformed nuclei, there are two separate modes of excitations corresponding to K=0 and K=1. These are basically characterized by the two different oscillator frequencies ω_z and $\omega_\perp$

where $\omega_z < \omega_\perp$ for prolate nuclei as Mg^{24} and $\omega_z > \omega_\perp$ for oblate nuclei, of which we have assumed C^{12} to be an example (see Table 1). Within each of the K=0 and K=1 groups of states a giant E1 excitation is then formed.

Furthermore in the case of deformed nuclei, an E1 transition from the ground state populates only the lowest member of a K=1 rotational band and only the I=1 member of a K=0 rotational band. In this case there is thus no sharing of the intrinsic E1 strength on different rotational states. The transition probability is therefore given exclusively in terms of the intrinsic wave functions.[18]

Most of the degeneracy of the spherical problem is removed by the distortion of the nuclear field. The remaining degeneracy associated with the time reversal degeneracy of the single-particle orbitals may be exploited to reduce the secular matrix in the K=0 case by the introduction of the new state vectors generated by

$$\Phi_I = \frac{1}{\sqrt{2}} (\Phi_{ab} + \Phi_{-a-b}) \tag{25}$$

$$\Phi_{II} = \frac{1}{\sqrt{2}} (\Phi_{ab} - \Phi_{-a-b}) \tag{26}$$

It is easily seen that the matrix elements of V vanish between states of those two different types. Furthermore it is also easy to verify that the second group of states are associated with a vanishing E1 matrix element with the ground state and therefore can be left out of the discussion.

In calculating the strength of the K=1 transition we will have to remember to double their relative strength corresponding to the additional degeneracy of the K=1 states (the angular momentum component may be +1 or -1).

RESIDUAL INTERACTION EMPLOYED

In these calculations we have limited ourselves to considering a phenomenological interaction V simulating the "actual" nuclear force. We have chosen the "empirical" force employed by Ferrell and Visscher, having a Gaussian radial shape and a particular exchange mixture. To isolate the effects to exchange mixtures, we have also considered the case of a pure Wigner force.

In addition we have also in some cases considered a force of the same radial dependence as the interactions above but with the exchange parameters of a "Rosenfeld mixture". (We will somewhat inadequately label this potential as "Rosenfeld" in the tables.) The parameters characterizing these forces may be found in Table 2.

RESULTS OF CALCULATIONS

a. Excitation spectra

The effects of the inclusion of the backward-going graphs are, as pointed out, rather small as far as the positions of the roots and the relative distribution of E1 strength is concerned. Generally the resonances are slightly lowered by some tens or hundreds of kev.

Of more interest is the way in which the sum rules are affected. This will, however, be discussed later in this section.

Complete calculations have been performed for Mg^{24}, and the results can be studied in Table 4. For C^{12} the calculations are complete only for B = 0, i.e. with ground state correlations neglected. The results may be studied in Table 5.

The matrix elements, computed numerically on the IBM 709 of the Lawrence Radiation Laboratory, exhibit some of the features conjectured earlier (e.g., Ref. 8). Thus, for instance, the effective interaction for T=1 is repulsive in most cases, thereby generally pushing the roots above their single-particle values. The matrix elements are in no way constant, however, and fluctuations in their size is of decisive importance. Instead the single-particle excitations that originally carry most of the E1 oscillator strength (asymptotically unhindered) have also the large interaction matrix elements in between themselves. It is effectively on one or two of the states in this smaller group of states that most of the E1 strength is collected as they are being pushed upwards due to the interaction.

Thus, e.g., for Mg^{24} the single-particle excitations carrying most of the E1 oscillator strength scatter in energy around $\hbar\omega_z$ or say 11-13 Mev for K=0 and around $\hbar\omega_z$ or about 15-17 Mev for K=1 (with the well parameters assumed as η=4 and κ=0.08).

The one or two states collecting the giant strength now appear at energies higher by about 5 Mev for K=0 and about 7 Mev for K=1, as is shown in Table 6. These figures refer specifically to the Ferrell and Visscher case with diagonal elements retained. Note that, in the total spectrum of states reached by E1 excitations the giant states are by no means the highest-lying as in the spherical case. Thus for the mentioned case of Mg^{24} with K=1 (as exhibited in Table 4b) the highest-lying root is more than 5-Mev above the K=1 giant resonance peak, and for K=0 about 12 Mev above the corresponding giant peak. Indeed in the latter case the giant peak is found in the lower part of the energy spectrum.

One may also note that the interaction mechanism described tries to enlarge the splitting between the two peaks, relative to their splitting in the independent particle description (see Table 6). This should be compared with the energy splitting existing after the interaction has been turned on, which is about 5.7 Mev. It may be pointed out that the ratio between the energy difference between two peaks and the average E1 resonance energy is roughly preserved relative to the independent-particle case and is approximately equal to the distortion parameter ϵ.

A calculation where a pure Wigner force was assumed, gives a concentrated K=0 peak, while for K=1 the strength is distributed over states several Mev apart. On the whole the difference between this and the Ferrell and Visscher case is not too marked, however, apart from differences in the TKR sum rule (see below).

Turning attention to the case of C^{12} (Table 5), where the whole coupling scheme may be very much less appropriate, we note that the clear separation encountered in Mg^{24} into two peaks of K=0 and K=1, respectively, does not occur here. In analogy with the prolate Mg^{24}, the oblate C^{12} would be expected to exhibit a low-lying K=1 peak and a higher-lying K=0 peak with a splitting of order $\epsilon\hbar\omega$. In C^{12} there is indeed for K=1 a strong low-lying peak. However, 7 Mev higher there is still another peak for K=1 which may be a remnant of the spherical coupling scheme, from which we are much less distant in C^{12} (with $\eta = -2$) than in Mg^{24} (with $\eta = 4$). For K=0 there is essentially one, high-lying peak in C^{12}.

The gross features of the empirical gamma-ray absorption cross section for C^{12} and Mg^{24} may be obtained from the (γ,n) and (γ,p) reactions and the reactions $B^{11}(p,\gamma)C^{12}$ and $Na^{23}(p,\gamma)Mg^{24}$. The experimental results[19-25] indicate a sharp peak at 22 Mev for C^{12} and a somewhat wider peak at 19 or 20 Mev for Mg^{24}. At higher energies the details of the cross section are still unknown.

In the proton capture experiment it was found that up to energies of 24 Mev the excited 1^- states of C^{12} decay primarily by ground-state gamma transitions whereas, above that energy, transitions to the rotational 2^+ state at 4.43 Mev are favoured. Although the data on Mg^{24} are more fragmentary, there is evidence that the opposite situation exists there; i.e. decay through the 1.37 Mev rotational 2^+ state in Mg^{24} is strongly preferred at least for excitations up to 23 Mev.

It is therefore tempting to identify the observed giant resonances in C^{12} with the theoretically calculated K=1 group of "giant" states and to identify the peak observed in Mg^{24} with the narrow group of K=0 states predicted theoretically around 18 Mev.[x] The reduced E1 transition rates from an $I=1^-$, K=1 state to the 0^+ and 2^+, K=0 states should be in the ratio 2:1 theoretically. The same ratio from the $I=1^-$, K=0 state should be 1:2. Those two ratios correspond qualitatively to those inferred from the experimental data. The K=0 state in C^{12} and the K=1 state in Mg^{24} would then presumably be located at a higher energy.

The experimental information from the mentioned gamma-ray branching in The $B^{11}(p,\gamma)$ reaction might on this basis be taken to indicate a K=0 character of the 1^- spectrum above an excitation energy of 24 Mev in C^{12}. However, any conclusions are obscured by the occurrence of rotational 2^- states which in C^{12} should lie about 3 Mev above the giant $I=1^-$, K=1 states. These are expected

[x] The rotational energy correction is of the order of 0.5 Mev in Mg^{24} I=1, K=0 0.8 Mev in C^{12} I=1, K=1.

to be populated in the $B^{11}(p,\gamma)$ reaction, and will probably decay mainly to the low-lying 2^+ rotational state. In Mg^{24} the $I = 3^-$, $K = 0$ rotational states are expected to occur about 2 Mev above giant 1^- states. They may possibly be responsible for a part of the peak observed in the $Na^{23}(p,\gamma)$ reaction just above 20 Mev.

b. Discussion of sum rules

It is obvious that a pure shell-model calculation (ground state correlations not considered, cf. Tables 4-5) retains the sum over squared E1 matrix elements unchanged. In the shell model case the Thomas-Kuhn-Reiche sum proportional to $\sum_n E_n |M_n|^2$ will exceed the sum rule value $\frac{NZ}{A}$, derived for velocity-independent and non-exchange interactions, since the energy values E_n are pushed upwards relative to the independent-particle case. The effect is usually larger with the force mixtures considered that contain exchange components. However, also the Wigner interaction treated in the approximation where backward-going graphs are neglected violates the sum rule, due to the fact that only a particular set of graphs are included in the conventional shell-model treatment of this interaction. This is borne out in Table 4.

For the Ferrell and Visscher force the inclusion of the backward-going graphs decreases the sum of squared E1 matrix elements by as much as 25 percent or more for Mg^{24}. Also the energies are somewhat lowered when the backward-going graphs are included. The TKR-sum is therefore considerably reduced, in fact almost down to the sum-rule estimate. This is in line with the result[26,27] that the random-phase approximation with inclusion of only "exchange" graphs leads to this sum rule being exactly obeyed.

On the whole, it may be considered one of the gratifying results of these calculations that for a force of finite range and realistic exchange

mixture the TKR sum is very much improved and brought into line with general experimental findings that the TKR sum is generally exceeded for light elements[28] by no more than about 10 percent below an energy of 2 $\hbar\omega$, which excess is then attributed to exchange forces.

However, for the unrealistic but interesting case of a pure Wigner force the effect of the inclusion of the "backward-going graphs" is rather small. Indeed the contribution from the correlation terms appears to be almost completely incoherent with the main shell-model contribution. The TKR sum rule is thus still violated by a considerable amount, which may reflect on the insufficiency of the RPA approximation for the treatment of this particular force.

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TABLE LEGENDS

Table 1. Parameters of the shell-model potential appropriate to the description of deformed nuclei. For the meaning of the parameters see Ref. 18.

Table 2. The exchange mixture parameters of the force $V_o\ (w+bP^{\sigma}+mP^{r}+hP^{r}\ P^{\sigma})$ $\exp - \frac{r}{\alpha}^{2}$, where $V_o = -51.9$ Mev and $\alpha = 1.732$ fm.

Table 3. Elementary single-particle excitations for Mg^{24}. Table 3a refers to K=0 and Table 3b to K=1. The energy values are taken from Ref. 18. In this reference the N=0, 1, 2 shells are calculated with an assumed $\mu=0$ (coefficient of ℓ^2-part of potential); on the other hand μ is assumed =0.35 for N=3. The energy eigenvalues of the N=3 shell are now corrected a posteriori to correspond to $\mu=0$ while the wave functions calculated for $\mu=0.35$ are left as before. This inconsistency as well as the slightly inaccurate energy adjustment does not affect the sum of squared E1 matrix elements. (The table lists the pure single-particle matrix elements of z and $\frac{x+iy}{\sqrt{2}}$ in units of $\sqrt{\frac{\hbar}{M\omega_z}}$ and $\sqrt{\frac{\hbar}{M\omega_\perp}}$ respectively.) However, it affects somewhat the distribution on the different states and is responsible for the fact that the Thomas-Kuhn-Reiche sum rule $\sum_n f_{on} = \frac{NZ}{A}$ (for K=1, $\frac{2NZ}{A}$, as both K=1 and -1 are included) is somewhat exceeded in the independent-particle case. The matrix elements are defined for negative angular-momentum states with a phase that differs from the time-reversal convention by a factor $(-)^{\Omega-1/2}$. In accordance with this convention the following relation holds $|-(-\Omega)\rangle = |\Omega\rangle$.

Table 4. Shell-model calculations for Mg^{24}, K=0(a) and K=1(b). The energies of the calculated 1^- states are given in column one. The squared matrix elements $|M|^2$ of z and $\frac{1}{\sqrt{2}}$ (x+iy) in terms of the basic linear combinations of Eq. (20) and (25) are listed in column two in units of $\hbar/M\omega_z$ and $\hbar/M\omega_\perp$, respectively. The sum of matrix elements is compared with the single-particle sums of the included single-particle excitations. Column three lists the oscillator strength (the quantity occurring in the Thomas-Kuhn-Reiche sum), f_{on}, which for K=0 equals $(2M/h^2)(E_n|M_n|^2) \cdot (1/4)$, where the factor 1/4 comes from the effective charge. The f_{on} sums may be compared with the estimate NZ/A. For K=1, we have added the contribution from $\frac{1}{\sqrt{2}}$ (x+iy) and $\frac{1}{\sqrt{2}}$ (x-iy). The comparison in this case should be made with the estimate 2 NZ/A. Column four lists the gamma widths $\Gamma_\gamma = 2(e^2/\hbar c) \cdot (E_n{}^2/Mc^2) \cdot f_{on}$. The diagonal hole-pair interaction matrix elements were retained in these calculations. This accounts for the shift upward of the sum of the eigenvalues over the sum of the single-particle energies listed in Table 3.

Table 5. Results of calculations for C^{12} K=0 and K=1. For a detailed explanation see the caption to Table 4. Only the Ferrell-Visscher force is considered in the case of C^{12}. The number of elementary excitations is not quite complete. Three transitions associated with very weak E1 matrix elements are left out for K=1 and two for K=0. This is the reason why we fall short of the TKR sum rule by a few percent in the independent-particle case.

For a discussion of the listed cases of the diagonal hole-pair interaction matrix elements being alternatively excluded and retained, see the main text.

Table 6. The average peak position (in Mev) for the K=0 and K=1 states, found by weighting the energy levels according to their absorption cross section is shown for the independent-particle system based on the pure shell model wave functions (see Table 3) and the random-phase approximation calculation with the Ferrell-Visscher force (Table 4).

Table A-1. The K=0 matrix elements of J(r) with the C^{12} "asymptotic" wave functions expressed in terms of the Talmi integrals I_p.

Table A-2. The T=1, S=0, odd-parity states of the K=0 and K=1 groups and their respective E1 strengths for the C^{12} nucleus computed with the "asymptotic" wave functions. Results for two different interaction potentials and two values of the deformation parameter, $\eta = -2$ and $\eta = -4$, are compared. Ground state correlations are included in the calculations.

Table 1.

	η	κ	$\hbar\omega_z$ (Mev)	$\hbar\omega$ (Mev)
C^{12}	-2 (-4)	0.10	20.3	16.8
Mg^{24}	4	0.08	11.4	15.9

Table 2.

Exchange mixture	w	b	m	h	Triplet even	Singlet even	Triplet odd	Singlet odd
Ferrell-Visscher	0.317	0.0	0.5	0.183	1.0	0.634	-0.366	0.0
Rosenfeld	-0.13	0.46	0.93	-0.26	1.0	0.6	-0.34	-1.78

Table 3a.

Hole-particle excitation		$\epsilon_{\nu'}-\epsilon_{\nu}$ (Mev)	Single-particle matrix elements $\langle\|z\|\rangle$	Classif. according to asympt. rule	$\langle\|z\|\rangle^2$	f_{on}
Hole state $[Nn_z\Lambda\Omega]$	Particle state $[Nn_z\Lambda\Omega]$					
[110 1/2]	[211 1/2]	17.59	0.1693	h	0.029	0.09
[110 1/2]	[200 1/2]	21.61	0.0181	h	0.000	0.00
[101 3/2]	[202 3/2]	19.94	0.1815	h	0.033	0.12
[101 1/2]	[211 1/2]	11.06	0.6645	u	0.442	0.86
[101 1/2]	[200 1/2]	15.08	0.3017	h	0.091	0.24
[220 1/2]	[330 1/2]	12.34	1.1278	u	1.272	2.76
[211 3/2]	[321 3/2]	11.21	0.9528	u	0.908	1.79
[211 3/2]	[312 3/2]	18.44	0.2215	h	0.049	0.17
[211 3/2]	[301 3/2]	22.43	-0.1008	h	0.010	0.00
[220 1/2]	[321 1/2]	16.92	0.3669	h	0.135	0.40
[220 1/2]	[310 1/2]	21.25	-0.1774	h	0.032	0.12
[220 1/2]	[301 1/2]	28.54	-0.0125	h	0.000	0.00
					Sum =4x3.000	6.55

Table 3b.

Hole-particle excitation: Hole state $[Nn_z\Lambda\Omega]$	Particle state $[Nn_z\Lambda\Omega]$	$\epsilon_{\nu'}-\epsilon_{\nu}$ (Mev)	Single-particle matrix elements $\left\langle \left\lvert \frac{x+iy}{\sqrt{2}} \right\rvert \right\rangle$	Classified according to asympt. rule	$\left\langle \left\lvert \frac{x+iy}{\sqrt{2}} \right\rvert \right\rangle^2$	f_{on}
[110-1/2]	[211 1/2]	17.59	-0.6540	u	0.428	0.95
[110-1/2]	[200 1/2]	21.61	-0.1143	h	0.013	0.04
[101 3/2]	[202 5/2]	14.78	1.0000	u	1.000	1.86
[10-1-3/2]	[21-1-1/2]	13.76	-0.2919	h	0.085	0.15
[10-1-3/2]	[200-1/2]	17.78	0.6427	u	0.413	0.92
[10-1-1/2]	[211 1/2]	11.06	0.1297	h	0.017	0.02
[10-1-1/2]	[200 1/2]	15.08	-0.6947	u	0.483	0.92
[101 1/2]	[202 3/2]	17.24	0.9809	u	0.962	2.09
[220 -1/2]	[330 1/2]	12.34	0.3898	h	0.152	0.23
[220 1/2]	[321 3/2]	14.86	0.7423	u	0.551	1.03
[21-1-3/2]	[330-1/2]	8.70	0.2012	h	0.041	0.04
[211 3/2]	[312 5/2]	17.15	1.0142	u	1.029	2.22
[220-1/2]	[321 1/2]	16.92	-0.5839	u	0.341	0.73
[220-1/2]	[310 1/2]	21.25	-0.0667	h	0.004	0.01
[220-1/2]	[301 1/2]	28.54	0.0671	h	0.005	0.02
[220 1/2]	[312 3/2]	22.08	-0.0534	h	0.003	0.01
[220 1/2]	[301 3/2]	26.07	-0.0772	h	0.006	0.02
[21-1-3/2]	[32-1-1/2]	13.42	-0.2381	h	0.057	0.10
[21-1-3/2]	[310-1/2]	17.60	0.6347	u	0.403	0.89
[21-1-3/2]	[30-1-1/2]	24.90	0.0019	h	0.000	0
[211 3/2]	[303 5/2]	23.94	-0.0667	h	0.004	0.01
					Sum=2 x 5.997	12.26

Table 4a

	Ferrell and Visscher								Wigner							
	Shell-model				Correlations included				Shell-model				Correlations included			
	E (Mev)	$\lvert M\rvert^2$	f_{on}	Γ_γ (kev)	E (Mev)	$\lvert M\rvert^2$	f_{on}	Γ_γ (kev)	E (Mev)	$\lvert M\rvert^2$	f_{on}	Γ_γ (kev)	E (Mev)	$\lvert M\rvert^2$	f_{on}	Γ_γ (kev)
	29.41	0.01	0.01	0.1	29.41	0.00	0.00	0.0	30.98	0.00	0.00	0.0	30.98	0.00	0.00	0.0
	25.96	0.38	0.44	4.6	25.57	0.15	0.17	1.7	27.46	0.00	0.00	0.0	27.37	0.00	0.00	0.0
	24.13	0.22	0.23	2.1	23.95	0.32	0.33	3.0	26.50	0.77	0.90	9.9	26.35	0.72	0.84	9.0
Mg^{24}	22.94	0.02	0.02	0.1	22.93	0.01	0.01	0.1	24.44	0.02	0.02	0.2	24.42	0.03	0.03	0.3
K = 0	22.17	0.21	0.20	1.6	22.17	0.18	0.17	1.3	23.69	0.21	0.22	2.0	23.65	0.19	0.20	1.8
	20.73	0.06	0.06	0.4	20.68	0.08	0.08	0.5	22.87	0.03	0.03	0.3	22.80	0.03	0.03	0.2
	18.82	1.00	0.83	4.6	18.79	0.22	0.18	1.0	20.36	0.56	0.50	3.2	20.33	0.44	0.39	2.5
	18.44	4.84	3.93	20.8	18.24	1.87	1.50	7.8	19.46	0.53	0.45	2.7	19.44	0.44	0.38	2.2
	17.68	3.40	2.65	12.9	17.35	3.50	2.67	12.5	18.92	0.11	0.09	0.5	18.92	0.09	0.08	0.4
	16.57	1.63	1.19	5.1	16.40	2.64	1.90	8.0	17.10	8.46	6.37	29.0	16.96	7.67	5.73	25.6
	13.46	0.15	0.09	0.3	13.36	0.15	0.09	0.2	15.04	0.77	0.51	1.8	14.90	0.61	0.40	1.4
	12.23	0.07	0.04	0.1	12.21	0.07	0.04	0.1	13.81	0.52	0.31	0.9	13.71	0.47	0.29	0.8
Sums		12.00	9.68			9.18	7.15			12.00	9.43			10.70	8.35	
Single particle-sums		12.00	6.55			12.00	6.55			12.00	6.55			12.00	6.55	

Table 4b

	Ferrell and Visscher								Wigner							
	Shell-model				Correlations included				Shell-model				Correlations included			
	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)
	29.70	0.04	0.07	1.0	29.43	0.02	0.04	0.5	31.08	0.00	0.00	0.0	31.07	0.00	0.00	0.0
	27.28	0.86	1.48	17.1	27.18	0.29	0.48	5.6	29.97	0.06	0.12	1.5	28.95	0.05	0.09	1.2
	26.29	0.55	0.91	8.8	26.20	0.15	0.24	2.6	27.81	0.01	0.02	0.2	26.79	0.31	0.52	5.8
	25.61	1.10	1.77	18.1	25.47	0.38	0.61	6.1	26.87	0.41	0.69	7.7	26.06	0.42	0.69	7.3
	24.97	4.50	7.07	68.6	24.61	2.34	3.63	34.2	26.11	0.53	0.87	9.3	25.11	3.47	2.74	26.9
	23.69	3.79	5.65	49.3	23.22	3.65	5.33	44.7	25.19	2.32	3.67	36.2	24.29	5.42	4.14	38.0
Mg^{24}	23.20	0.15	0.22	1.8	23.10	1.08	1.57	13.0	24.36	2.69	4.12	38.0	24.15	1.36	2.07	18.8
	22.14	0.02	0.02	1.7	22.12	0.02	0.03	0.2	23.52	0.03	0.05	0.4	23.51	0.02	0.03	0.3
K=1	21.09	0.40	0.54	3.7	20.82	0.56	0.74	5.0	22.49	0.31	0.44	3.5	22.38	0.09	0.13	1.0
	19.76	0.02	0.02	0.1	19.79	0.03	0.04	0.3	22.09	4.88	6.78	51.4	21.92	4.05	6.27	46.9
	18.95	0.12	0.14	0.8	18.93	0.12	0.14	0.8	20.73	0.01	0.01	0.1	20.70	0.01	0.01	0.1
	18.32	0.02	0.03	0.1	18.30	0.03	0.03	0.2	19.82	0.01	0.01	0.1	19.79	0.01	0.01	0.1
	18.01	0.02	0.03	0.1	18.09	0.02	0.02	0.1	19.45	0.13	0.16	0.9	19.39	0.14	0.17	1.0
	17.53	0.02	0.03	0.1	17.48	0.01	0.01	0.1	19.35	0.03	0.03	0.2	19.30	0.00	0.00	0.0
	16.64	0.14	0.14	0.6	16.89	0.27	0.29	1.3	18.78	0.27	0.32	1.8	18.74	0.24	0.28	1.5
	16.41	0.00	0.00	0.0	16.26	0.02	0.02	0.1	18.03	0.08	0.09	0.4	17.92	0.08	0.09	0.4
	16.12	0.05	0.05	0.2	16.03	0.00	0.00	0.0	17.70	0.01	0.01	0.1	17.68	0.01	0.01	0.1
	15.49	0.13	0.13	0.5	14.66	0.02	0.02	0.1	16.50	0.15	0.15	0.6	16.32	0.13	0.14	0.6
	13.60	0.01	0.02	0.0	14.06	0.00	0.00	0.0	15.49	0.01	0.01	0.0	15.45	0.01	0.01	0.0
	12.69	0.02	0.01	0.0	12.64	0.02	0.01	0.0	14.99	0.00	0.00	0.0	14.85	0.00	0.00	0.0
	10.04	0.00	0.00	0.0	10.00	0.01	0.00	0.0	11.63	0.06	0.04	0.1	11.58	0.05	0.04	0.1
Sums		11.95	18.33			9.03	13.26			11.99	17.59			11.94	17.45	
Indep. sums		11.99	12.26			11.99	12.26			11.99	12.26			11.99	12.26	

Table 5

Shell-model calculation (C^{12})								
Diagonal m.e.	included				excluded			
K=0								
	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)
	31.91	3.52	2.77	48.8	27.43	3.46	2.33	27.3
	26.31	1.26	0.83	8.9	22.32	2.16	1.20	9.3
	23.74	1.08	0.64	5.6	19.31	0.22	0.11	0.6
	20.85	0.08	0.04	0.3	17.27	0.12	0.05	0.2
	17.82	0.00	0.00	0.0	14.29	0.00	0.00	0.0
sums		5.94	4.28			5.96	3.69	
indep. p sums		5.98	3.00			5.98	3.00	
$\int\sigma_\gamma dE$		257 (Mev-mb)				222 (Mev-mb)		
K=1								
	29.50	1.04	1.82	24.7	25.58	1.10	1.67	17.0
	24.77	0.04	0.06	0.6	21.90	0.02	0.03	0.2
	24.08	0.00	0.01	0.0	20.50	0.16	0.19	1.3
	22.97	1.08	1.48	12.2	20.27	0.06	0.06	0.4
	22.21	3.66	4.85	37.2	18.84	4.10	4.60	25.4
	19.74	0.08	0.10	0.6	16.48	0.40	0.40	1.7
	17.82	0.00	0.00	0.0	14.62	0.02	0.01	0.0
	17.12	0.00	0.00	0.0	13.36	0.08	0.01	0.0
sums		5.92	8.32			5.92	6.96	
indep. p sums		5.92	5.92			5.92	5.92	
$\int\sigma_\gamma dE$		499 (Mev-mb)				418 (Mev-mb)		

Table 6

	K=1	K=0	Difference
Interacting-particle system	23.5	17.8	5.7
Independent-particle system	16.4	12.8	3.6
Difference	7.1	5.0	2.1

Table A-1

$\langle 000\ 110 \mid J(r) \mid 110\ 000 \rangle$	$= 1/2\ (I_0 - I_1)$
$\langle 000\ 110 \mid J(r) \mid 000\ 110 \rangle$	$= 1/2\ (I_0 + I_1)$
$\langle 000\ 211 \mid J(r) \mid 110\ 101 \rangle$	$= 1/4\ (I_0 - I_2)$
$\langle 000\ 211 \mid J(r) \mid 101\ 110 \rangle$	$= 1/4\ (I_0 - I_2)$
$\langle 000\ 101 \mid J(r) \mid 211\ 110 \rangle$	$= 1/4\ (I_0 - 2I_1 + I_2)$
$\langle 101\ 211 \mid J(r) \mid 211\ 101 \rangle$	$= 1/4\ (I_0 - I_1 + I_2 - I_3)$
$\langle 101\ 211 \mid J(r) \mid 101\ 211 \rangle$	$= 1/4\ (I_0 + I_1 + I_2 + I_3)$
$\langle 101\ 21\text{-}1 \mid J(r) \mid 10\text{-}1\ 211 \rangle$	$= 1/4\ (I_0 - I_1 - I_2 + I_3)$
$\langle 101\ 10\text{-}1 \mid J(r) \mid 21\text{-}1\ 211 \rangle$	$= 1/4\ (I_0 - 3I_1 + 3I_2 - I_3)$

Table A-2

NUCLIDE	η	FORCE	DIAGONAL MATRIX ELEMENTS	K	E (Mev)	OSCILLATOR STRENGTH f_{on}	$\epsilon_{\nu'} - \epsilon_{\nu}$ (Mev)
$^{12}_{6}C_{6}$	-2	FERRELL and VISSCHER	INCLUDED	0	28.5 / 21.6	3.13 / 0.09	20.3 = $\hbar\omega_z$
				1	22.1 / 18.1	6.38 / 0.02	16.8 = $\hbar\omega_\perp$
			EXCLUDED	0	24.3 / 17.2	2.67 / 0.00	20.3
				1	18.2 / 14.8	4.84 / 0.35	16.8
		ELLIOTT and FLOWERS	INCLUDED	0	28.0 / 22.4	3.23 / 0.15	20.3
				1	21.5 / 18.8	6.43 / 0.13	16.8
			EXCLUDED	0	23.6 / 17.9	2.79 / 0.00	20.3
				1	18.0 / 15.3	5.17 / 0.15	16.8
	-4	FERRELL and VISSCHER	INCLUDED	0	31.0 / 24.0	3.10 / 0.10	22.7
				1	21.1 / 17.1	6.42 / 0.02	15.8

APPENDIX

Calculations with the Asymptotic Wave Functions

In the limit when the deformation energy is large compared to the off-diagonal elements of the spin-orbit coupling we can use the approximation in which the spin motion is decoupled from the orbital motion.[5,14]

In this approximation we shall use the harmonic oscillator wave functions $\psi_{Nn_z\Lambda}(\rho, z, \phi)$ in the representation of the (dimensionless) cylindrical coordinates $\left(\rho = \sqrt{x^2+y^2},\ z,\ \phi;\ x,\ y = \sqrt{\frac{\hbar}{M\omega_\perp}}\, x', y';\ z = \sqrt{\frac{\hbar}{M\omega_z}}\, z'\right)$: where Λ has the meaning of the projection of the orbital angular momentum $\vec{\ell}$ on the body axis z', n_z and $n_\perp = N - n_z$ are the oscillator quantum numbers referring to oscillations parallel to the z'-axis and perpendicular to it, respectively.

Using the above wave functions it is easy to eliminate the degenerate spin substates from the equation of motion (Eq.(15)). For a general collective state of isotopic spin T and spin S and with the two-body interaction V of Eq. (22) we obtain the RPA "equation of motion" of the $\rho_{\alpha\alpha'}$'s in the form:

$$(\varepsilon_{\nu'} - \varepsilon_\nu - E)\, \mathcal{X}^{ST}_{\nu\nu'} = (n_{\nu'} - n_\nu) \sum_{\varkappa \neq \varkappa'} \Big\{ \langle \nu \varkappa' | \mathcal{J}(r) | \nu' \varkappa \rangle \big[(w + b - m - h) + (-)^S (w - h) + (-)^T (w + b + (-)^S w) \big] - \langle \nu \varkappa' | \mathcal{J}(r) | \varkappa \nu' \rangle \big[(w + b - m - h) + (-)^S (b - m) + - (-)^T (m + h + (-)^S m) \big] \Big\} \mathcal{X}^{ST}_{\varkappa\varkappa'} \tag{A1}$$

$$\mathcal{X}^{ST}_{\alpha\alpha'} \equiv \frac{1}{2} \Big(\rho_{\alpha\alpha'}(\tau\tau, \Sigma\Sigma) + (-)^S \rho_{\alpha\alpha'}(\tau\tau, -\Sigma-\Sigma) + (-)^T \rho_{\alpha\alpha'}(-\tau-\tau, \Sigma\Sigma) + (-)^{S+T} \rho_{\alpha\alpha'}(-\tau-\tau, -\Sigma-\Sigma) \Big) \tag{A2}$$

Here $\rho_{\alpha\alpha'}$ $(\tau\tau, \Sigma\Sigma)$ means a $\rho_{\alpha\alpha'}$-component with the (α,α') pair characterized by the charge state τ and spin state Σ. In the giant E1 problem, i.e., for S = 0, T = 1 we have:

$$(\varepsilon_{\nu'}-\varepsilon_{\nu}-E)\,\chi^{01}_{\nu\nu'} = -(n_{\nu'}-n_{\nu})\sum_{\varkappa\neq\varkappa'}\left[\langle\nu\varkappa'|\mathcal{J}(r)|\nu'\varkappa\rangle(m+2h)+\langle\nu\varkappa'|\mathcal{J}(r)|\varkappa\nu'\rangle \times(w+2b)\right]\chi^{01}_{\varkappa\varkappa'} \tag{A3}$$

Here the matrix elements are expressed in terms of spacial wave functions only, i.e. the subscripts refer specifically to spacial states.

Our wave functions given in cylindrical representation can be expanded in the spherical functions as

$$\Psi_{N n_z \Lambda}(\rho, z, \varphi) = \sum_{\ell} a_{\ell\Lambda}\Psi_{N\ell\Lambda}(\vec{r}) \tag{A4}$$

The matrix element $\langle\beta\delta|\mathcal{J}|\alpha\gamma\rangle$ are now obviously

$$\langle\beta\delta|\mathcal{J}(r)|\alpha\gamma\rangle = \sum_{\ell_\alpha\ell_\beta\ell_\gamma\ell_\delta} a_{\ell_\alpha\Lambda_\alpha}a_{\ell_\beta\Lambda_\beta}a_{\ell_\gamma\Lambda_\gamma}a_{\ell_\delta\Lambda_\delta}\langle N_\beta\ell_\beta\Lambda_\beta, N_\delta\ell_\delta\Lambda_\delta| \times|\mathcal{J}(r_{12})|N_\alpha\ell_\alpha\Lambda_\alpha, N_\gamma\ell_\gamma\Lambda_\gamma\rangle \tag{A5}$$

Sometimes it is convenient to use the Talmi method of separation of the center of mass and the relative coordinates of the particles "1" and "2". This method is especially useful when the radial shape of $\mathcal{J}(r_{12})$ is complicated and it is difficult to employ an electronic computer for the evaulation of $\langle\beta\delta|\mathcal{J}|\alpha\gamma\rangle$ of Eq. (A5). We can expand the product $\psi_{(\alpha)}(\vec{r}_1)\psi_{(\gamma)}(\vec{r}_2)$ as:

$$\Psi_{N_\alpha\ell_\alpha\Lambda_\alpha}(\vec{r}_1)\Psi_{N_\gamma\ell_\gamma\Lambda_\gamma}(\vec{r}_2) = \sum_{\lambda\mu}(\ell_\alpha\ell_\gamma\Lambda_\alpha\Lambda_\gamma|\lambda\mu)\sum_{n\ell N L}\langle n\ell, NL, \lambda|N_\alpha\ell_\alpha, N_\gamma\ell_\gamma, \lambda\rangle \times|n\ell, NL, \lambda\mu\rangle \tag{A6}$$

where the "transformation brackets" $\langle | \rangle$ are defined and extensively tabulated by Moshinsky and Brody.[30] The relative motion (nl)- and center of mass motion ($\mathcal{N}L$) wave functions and the "ket" $|n\ell, \mathcal{N}L, \lambda\mu\rangle$ are as defined in Ref. 30.

Using Eq. (A6) we can now express $\langle \beta\delta | \mathcal{J} | \alpha\gamma \rangle$ as:

$$\langle \beta\delta | \mathcal{J}(r) | \alpha\gamma \rangle = \sum_{\ell_\alpha \ell_\beta \ell_\gamma \ell_\delta} a_{\ell_\alpha \Lambda_\alpha} \cdots a_{\ell_\delta \Lambda_\delta} \sum_{\lambda} \sum_{nn'\ell\mathcal{N}L} (\ell_\alpha \ell_\gamma \Lambda_\alpha \Lambda_\gamma | \lambda\mu) \times$$
$$\times (\ell_\beta \ell_\delta \Lambda_\beta \Lambda_\delta | \lambda\mu) \langle n\ell, \mathcal{N}L, \lambda | N_\alpha \ell_\alpha, N_\gamma \ell_\gamma, \lambda \rangle \langle n'\ell, \mathcal{N}L, \lambda | N_\beta \ell_\beta, N_\delta \ell_\delta, \lambda \rangle \times$$
$$\times \langle n'\ell \| \mathcal{J}(r) \| n\ell \rangle \tag{A7}$$

where the reduced radial integral $\langle n'\ell \| J(r) \| n\ell \rangle$ can be expressed in terms of the Talmi integrals I_p as:

$$\langle n'\ell \| \mathcal{J}(r) \| n\ell \rangle = \sum_p B(n\ell, n'\ell, p) I_p \tag{A8}$$

where the coefficients $B(n\ell, n'\ell, p)$ are tabulated in Ref. 30.

We have performed explicit computations for the problem of the 1^-, T=1, states in C^{12}. We have employed two different interactions:

$$\text{I)} \quad \mathcal{J}(r) = -V_0 e^{-r^2/\alpha^2} \quad ; \quad V_0 = 51.9 \text{ MeV}, \alpha = 1.732 \times 10^{-13} \text{ cm} \tag{A9a}$$

w = 0.317, m = 0.5, b = 0, h = 0.183

—identical with the interaction listed in Table 2

$$\text{II)} \quad \mathcal{J}(r) = -V_0 \left[e^{-r/r_0} / r/r_0 \right] ; \; V_0 = 43 \text{ MeV}, r_0 = 1.4 \times 10^{-13} \text{ cm} \tag{A9b}$$

w = -0.13, m = 0.93, b = 0.46, h = -0.26

i.e. the Rosenfeld mixture used by Elliott and Flowers.[7]

In the C^{12} problem we have the "occupied" ($n_\nu = 1$) states $[Nn_z\Lambda] = [000]$, $[10\pm1]$ and the "unfilled" ($n_\nu = 0$) states [110], $[20\pm2]$, $[21\pm1]$, and [200], where the $\pm\Lambda$ states are degenerate in energy. The above states, denoted $|\rangle$,

are expressible in terms of spherical states denoted $|\rangle\rangle$ (where the phase convention of the radial wave function is that of Ref. 18, i.e., such that the sign of the term of highest power in the radial polynomial is always positive) as follows:

$$|000\rangle = |N = 0;\ \ell = 0, \Lambda = 0\rangle\rangle,$$

$$|10\pm1\rangle = N = 1,\ \ell = 1, \Lambda = \pm1\rangle\rangle,$$

$$|110\rangle = N = 1,\ \ell = 1, \Lambda = 0\rangle\rangle,$$

$$|20\pm2\rangle = N = 2,\ \ell = 2, \Lambda = \pm2\rangle\rangle,$$

$$|21\pm1\rangle = N = 2,\ \ell = 2, \Lambda = \pm1\rangle\rangle,$$

$$|200\rangle = -\sqrt{\frac{1}{3}}\ |N = 2,\ \ell = 2,\ \Lambda = 0\rangle\rangle + \sqrt{\frac{2}{3}}\ |N = 2,\ \ell = 0,\ \Lambda = 0\rangle\rangle$$

In Table A1 we give the explicit expressions for the relevant matrix elements for the case K = 0 in terms of the Talmi[29] integrals I_p.* The basic single particle E1 matrix elements $(\alpha'|z|\alpha)$ are in units of $\sqrt{\frac{\hbar}{M\omega_z}}$: $\langle 110|z|000\rangle = \langle 211|z|101\rangle = \langle 21\text{-}1|z|10\text{-}1\rangle = 1/\sqrt{2}$; the $\left\langle \alpha' \left| \frac{x+iy}{\sqrt{2}} \right| \alpha \right\rangle$ elements in units of $\sqrt{\frac{\hbar}{M\omega_\perp}}$: $\left\langle 202 \left| \frac{x+iy}{2} \right| 101 \right\rangle = \left\langle 10\text{-}1 \left| \frac{x+iy}{2} \right| 20\text{-}2 \right\rangle = 1$; $\left\langle 200 \left| \frac{x+iy}{2} \right| 10\text{-}1 \right\rangle = \left\langle 101 \left| \frac{x+iy}{2} \right| 200 \right\rangle = 1/\sqrt{2}$. The computed oscillator strengths for K = 1 have to be doubled to take account of the two nuclear spin projections = ± 1. Our final numerical results are displayed in Table A2. The computations were performed for two values of the oblate deformation $\eta = -2$ and $\eta = -4$. The splitting of the giant E1 states corresponding to K = 0 and K = 1 is roughly in agreement with the more detailed calculations. The second K = 0 giant peak occurring in the latter calculations does not appear in the asymptotic

* For the range of force considered here I_0 is the dominant term, which together with our choice of parameters w, m, b, and h in (A3) ensures that all the occurring matrix elements are of about equal magnitude and have one and the same sign. The condition for I_0 to be dominant is that $\alpha \ll \hbar/M\omega_0$. In the limit of an extremely long range interaction we have instead $I_0 = I_1 = I_2$ etc. The interaction matrix elements then all vanish, except the diagonal ones, as seen from Table A1. Thus in this limit the coherence is again destroyed.

limit and is obviously associated with the fact that the wave functions of Ref. 17 are intermediate between the spherical shell-model wave functions and the asymptotic ones.

Our results for the Rosenfeld mixture of Elliott and Flowers[7] with a Yukawa well (Eq. (A9b)) are quite similar to those for the rather different interaction of Ferrell and Visscher (Eq. (A9a)).

We have also computed the self-energy terms originating from the self-energy hamiltonian Eq. (13). Some of them are very large (of the order of 40 Mev for the interaction of Eq. (A9a)). This indicates that a self-consistency calculation for the single particle energy spectrum, or conversely, a self-consistent determination of the residual interaction for a given single-particle model is generally important.

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