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Kähler Stabilized, Modular Invariant Heterotic String Models

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We review the theory and phenomenology of effective supergravity theories based on orbifold compactifications of the weakly-coupled heterotic string. In particular, we consider theories in which the four-dimensional theory displays target space modular invariance and where the dilatonic mode undergoes Kähler stabilization. A self-contained exposition of effective Lagrangian approaches to gaugino condensation and heterotic string theory is presented, leading to the development of the models of Binétruy, Gaillard and Wu. Various aspects of the phenomenology of this class of models are considered. These include issues of supersymmetry breaking and superpartner spectra, the role of anomalous $U(1)$ factors, issues of flavor and R-parity conservation, collider signatures, axion physics, and early universe cosmology. For the vast majority of phenomenological considerations the theories reviewed here compare quite favorably to other string-derived models in the literature. Theoretical objections to the framework and directions for further research are identified and discussed.

Introduction

Why should we be inclined to believe in string theory? Unless we are remarkably fortunate, and the scale at which string resonances appear is accessible to forthcoming experiments, there will not be – indeed there cannot be – any *direct* evidence that some particular construction of string theory is correct. Wherefore, then, the great interest of the high energy community in this subject? Clearly the answer is in what string theory is (uniquely) capable of *explaining*.

To some the most salient feature of string theory is its great promise as a consistent theory of quantum gravity. But for those whose interest lies in understanding the phenomena relevant at energies closer to the electroweak scale it is rather string theory's capability to explain such manifest properties of particle physics as the presence of three generations, the gauge group of the Standard Model (SM), the representations of the various matter fields, and the Yukawa interactions that do such things as give mass to the fermions but do not do such things as make the proton decay. Consistent string constructions which address these issues generally have supersymmetry present on the string worldsheet (base space), as well as in the spectrum of the effective field theory description (target space). This highly desirable phenomenological property has not been observed in nature. But if supersymmetry is at all relevant in understanding electroweak-scale physics new states and interactions will likely be observed at the CERN Large Hadron Collider (LHC).

There is, therefore, a great opportunity for string theory to make contact with particle physics observations at accessible energies. The elucidation of these points of contact is the purpose of the field of string phenomenology. It is sometimes useful to divide this field into the study of two logically distinct problems. The first we might refer to as the "problem of initial conditions." This is the determination of the massless spectrum of gauge bosons and matter representations – as well as the allowed superpotential couplings amongst these fields – upon compactification of the ten-dimensional (10D) theory to four dimensions (4D) on a manifold which preserves $N = 1$ supersymmetry. The ideal compactification need not necessarily be the one which provides solely the fields of the Minimal Supersymmetric Standard Model (MSSM), but this has often been taken to be the supreme goal of those who study superstring model-building. While some approaches have come close to producing exclusively the MSSM field content, gauge group and renormalizable superpotential it is not unfair to say that none can yet truly claim success in this regard.

Furthermore, different choices of string compactification imply different sets of *moduli* fields in the low-energy effective Lagrangian. Very roughly speaking, these moduli parameterize the geometry of the compact space. Their role in the low-energy Lagrangian is to determine certain dimensionless parameters (such as gauge and Yukawa coupling constants), yet they have no potential at all at the classical level. That is to say, they are truly flat directions in the scalar potential of the theory. There is therefore a second class of problems for the string phenomenologist: the "problem of dynamics." This latter class of problems includes the issues of anomalous $U(1)_X$ cancelation, supersymmetry breaking, dynamical electroweak symmetry breaking, and so forth. The two problems can to a large degree be separated (at least formally), and it is the second class of questions which we wish to address in this review.

Over the years there has been considerable progress in understanding the structure of effective actions describing the low-energy dynamics of massless fields in 4D superstring theory. This success began in the context of (weakly-coupled) heterotic string theory, where the basic program was to extract the relevant terms in the field-theoretical Lagrangian from the S-matrix elements computed within the full-fledged superstring theory.[1] Many important quantities were determined at the classical level, including the Kähler potentials and the gauge and Yukawa couplings for orbifold compactifications of heterotic superstrings. Later, the program of reconstructing effective Lagrangians from string amplitudes was pursued to higher genus in the string loop expansion.[2,3] An important development was the observation that the duality symmetry[4,5] between small and large radius toroidal compactifications extends to a much larger symmetry group of the so-called target space modular transformations acting on the moduli fields.[6] This symmetry can be very helpful when studying the moduli-dependence of the effective actions for orbifold compactifications.[7,8]

In this work we will review the theoretical construction and high-energy motivation for a particular class of weakly-coupled heterotic string models. Much of the machinery, and a great deal of the resulting model-building, will certainly touch on issues of low-energy string phenomenology common to other string theory starting points as well. As the nucleus of this construction was presented in a sequence of papers by Binétruy, Gaillard and Wu we will sometimes refer to the class as the "BGW model," (or models). They are generally characterized by the presence of *Kähler stabilization* to provide a realistic minimum for the dilaton modulus. We will present this mechanism in detail in this review, and study the resulting phenomenology at length. We choose this class not because we necessarily believe it to be the correct model of Nature – though we will see that it has many excellent phenomenological features – but because it is, to date, the most complete model of supersymmetric particle physics arising from string theory in the literature. By this we mean that the phenomenology of this class has been studied in great depth from multiple angles: spectrum and superpotential selection rules, effective Lagrangian construction, gaugino condensation and moduli stabilization, supersymmetry breaking and transmission to the observable sector, anomaly cancelation, superpartner spectra, flavor-changing and rare processes, collider signatures, cosmology, and so on.

In certain phenomenological areas this class of models possesses remarkably favorable characteristics; in other areas less so. There are also certain theoretical objections that can be raised to the treatment presented here and we will give voice to these objections in our concluding chapter. But the goal of this review is not to promote the Kähler stabilized, modular invariant models of the heterotic string; but rather to promote the sometimes arduous act of building complete models of 4D superstring dynamics generally. The most significant property of the class of models considered here, therefore, is that the analysis has been performed in all of these phenomenological areas – for a *single* class of theories. To our knowledge, no other class of models (whether of string-theoretic motivation or otherwise) has been pushed as far in as many different directions as this class. As such, the BGW model provides a paradigm for what a "complete" theory looks like. The importance of this idea of synthesis and completeness has been stressed recently as crucial to any effort in effectively making contact between superstring models and the forthcoming data era we are about to enter.[9,10]

Understanding the phenomenology of any string-derived effective theory begins and ends with the dynamics of the moduli in the theory. We will therefore develop the theory behind the BGW class of heterotic string models by studying the types of moduli present in the theory, their stabilization through nonperturbative dynamics, and their role in transmitting supersymmetry breaking to the Standard Model fields of the observable sector. In Section 1 we consider theories of a single modulus, whose vacuum expectation value (*vev*) determines the gauge coupling of a super Yang-

Mills (YM) theory. This will lead us into a discussion of gaugino condensation from multiple directions. Much of Section 1 reviews material that is more fully treated elsewhere. We conclude the first section with a description of the linear multiplet treatment of the dilaton and the implementation of Kähler stabilization in the effective theory. In Section 2 we introduce the interaction of moduli with the chiral superfields of the MSSM matter sector and discuss the importance of duality symmetries that relate these moduli to themselves. Section 2 concludes with the full effective Lagrangian for the BGW class of theories and examines the sorts of minima that can arise for the effective scalar potential of the moduli. These minima generically break supersymmetry, and Section 3 is devoted to the understanding of how supersymmetry breaking is communicated to the other sectors of the theory – particularly the states of the MSSM. These results are revisited in Section 4 in the presence of anomalous $U(1)$ factors which are generically present in string constructions. In Section 5 we describe a series of phenomenological topics and how they can be addressed in the context of the Kähler stabilized heterotic models. In the final chapter we offer our thoughts on where the model can be improved and how studies such as ours can be used to construct a meaningful string phenomenology.

1. Moduli and their stabilization

The key to understanding the low-energy manifestation of string physics is understanding moduli and their dynamics. The precise definition of a "modulus" sometimes depends on the context, but one property is universal: a modulus is a scalar field for which there is no potential at the classical level. In other words, there is no preference for any particular vacuum expectation value (*vev*) for the scalar field over any other. Moduli are the *sine qua non* of string phenomenology: all string theories, when compactified to four dimensions, possess moduli whose *vevs* determine the size of certain dimensionless constants in the low-energy Lagrangian. An operational definition of a 4D string model is therefore a gauge theory with couplings determined by the scalar components of some set of superfields. It is hard to claim that a particular theory is "stringy" or "string motivated" if it does not contain such fields.

Moduli are also in many ways the "engine" that drive any string-derived 4D effective Lagrangian. The primary focus of any such effective theory is to incorporate various nonperturbative effects – whether of a field theoretic or string theoretic origin – to generate a scalar potential for these fields. A good model would be one in which a nontrivial, finite minimum of this scalar potential exists for all the moduli considered. But this is merely the beginning. The values of the scalar components of these moduli at the minimum of their potential will determine a number of important properties of the theory, such as the gauge and Yukawa couplings. The auxiliary F-term components of these chiral superfields will generally take nonvanishing *vevs* at this minimum, implying a breakdown of supersymmetry (SUSY). The size of these *vevs* will determine the general scale of supersymmetry break-

ing, as well as the potential size of any nonzero vacuum energy at the minimum. Since the moduli couple to matter fields in a well-defined way, the size of the soft supersymmetry breaking in the observable sector can also be calculated. From here a wide array of phenomenological properties of the theory can be considered.

What is more, classes of four-dimensional, effective supergravity theories derived from string constructions can be classified by answering the following questions:

(1) What moduli are present in the low-energy theory?
(2) What symmetries (if any) relate these moduli amongst themselves?
(3) How do these moduli couple to the fields of the observable sector?

A supergravity theory built to describe the low-energy dynamics of weakly-coupled heterotic string theory on an orbifold will have different answers to these questions than one built to describe strongly-coupled heterotic string theory on a Calabi-Yau manifold – and both will be different from the theory describing Type IIA string theory on an orientifold with D-branes at intersections. This is a powerful and under-appreciated fact, suggesting that the phenomenology of these theories (driven as they are by the dynamics of their moduli) will be *different* – perhaps sufficiently different to distinguish them through low-energy observations. This connection is the heart of string phenomenology.

In this work we will be reviewing a class of supergravity models designed to capture the physics of a large class of heterotic string models at weak coupling. Most of the time we will be imagining the compactification of this theory on an orbifold, though much of our discussion is applicable to compactification on more general manifolds. In particular we will *not* be considering theories with nonperturbative structures such as D-branes, the positions and orientations of which would be moduli in the effective low-energy theory. Nor will we be considering moduli associated with the different ways in which one can define a vector bundle V on some compact Calabi-Yau space such that it admits a connection which satisfies the Hermitian Yang-Mills equations. We will rather be concerned with the orbifold limit of such theories, and therefore in a very simple set of moduli: those associated with the fields of the ten-dimensional supergravity Lagrangian.[a] The relevant bosonic 10D fields are the metric g_{MN} $(M, N = 0, \ldots, 9)$, an antisymmetric tensor b_{MN} and the real dilaton scalar ϕ. These fields must be dimensionally reduced to an effective four-dimensional theory. It is common to package these degrees of freedom into chiral superfields. The dimensional reduction of the 10D supergravity Lagrangian and the field redefinitions required to obtain the chiral superfields of the 4D supergravity theory have been reviewed elsewhere.[11,12,13] Here we wish to provide only those facts that are relevant to our subsequent discussion and necessary

[a]Additional twisted sector moduli associated with orbifold or orientifold compactification, such as Wilson line moduli and "blowing-up" moduli, are captured in our treatment to the extent that they can be considered as twisted sector gauge-charged matter in the low-energy effective Lagrangian.

to establish our conventions with respect to other authors.

The most important modulus, and the one that will be of central focus in the class of theories considered here, is the dilaton field s whose real part determines the (universal) gauge coupling. By matching the dimensionally-reduced Yang-Mills action to the canonical form[14] one immediately identifies the gauge coupling as

$$\frac{1}{g_{\rm STR}^2} = e^{3\sigma}\phi^{-3/4} \tag{1}$$

where ϕ is the ten-dimensional dilaton and σ is the real scalar which arises from the dimensional reduction of the graviton. More specifically, σ is often referred to as the "breathing mode" associated to the re-scaling $g_{mn} \to e^{\sigma} g_{mn}$, with $m, n = 4, \ldots, 9$ being the coordinates for the compact space. This allows us to set the scale of the Planck mass. We denote the gauge-coupling as $g_{\rm STR}$ to indicate (a) that it is understood to be the coupling at the string scale and (b) that it is universal to all gauge groups in the low energy theory.[15] Note that the string scale, which is inversely related to the string tension via $M_{\rm STR} = 1/\sqrt{\alpha'}$, is in turn related to the Planck scale via

$$M_{\rm STR} = g_{\rm STR} M_{\rm PL}\,. \tag{2}$$

In the effective supergravity theory the dimensional reduction of the antisymmetric two form $b_{\mu\nu}$ $(\mu, \nu = 0, \ldots, 3)$ appears only through its field strength $H_{\mu\nu\rho} = \partial_{[\rho} b_{\mu\nu]}$. We may identify (1) as the real part of a scalar field s, and $H_{\mu\nu\rho}$ as its pseudoscalar partner a provided we make the duality transformation

$$\epsilon_{\mu\nu\rho\eta}\partial^{\eta} a = \phi^{-3/2} e^{6\sigma} H_{\mu\nu\rho} \tag{3}$$

The field a is the so-called "model-independent" axion field.

In orbifold compactifications in which the six-dimensional compact space is factorizable into three two-torii it is possible to define the Kähler and complex structure moduli in terms of the elements of g_{mn} and b_{mn} (with $m, n = 1, 2$) for each of three internal torii. In particular, for the important case of the Kähler moduli we have in this limit the definition

$$t^I = \phi^{3/4}\sqrt{\det(g^I_{mn})} + \frac{i b^I_{12}}{\sqrt{2}} = (R^I)^2 + \frac{i b^I_{12}}{\sqrt{2}} \tag{4}$$

where $I = 1, 2, 3$ labels the complex plane associated with each torus and R^I is radius of the compactified subspace in string units. The final equality in (4) is strictly true only in the vacuum. We therefore can identify $M_{\rm comp} \equiv 1/R^I = \left\langle \mathrm{Re}\, t^I \right\rangle^{-1/2} M_{\rm STR}$.

1.1. *Basics of gaugino condensation*

Past developments in string phenomenology have by now suggested a number of mechanisms which can be employed to generate potentials for moduli at the non-perturbative level. Yet one source of these effects stands out among the others for its

ubiquity and generality: that of gaugino condensation for some non-Abelian gauge group. This phenomenon is, in principle, a purely field-theoretic effect and can be considered without regard to some underlying string theory construction. Before discussing moduli dynamics it is therefore instructive to consider the physics of gaugino condensation in a general manner. The subject has a long history in the literature and we here intend only to motivate the effective Lagrangian approach to appear in Section 2. More detailed reviews can be found elsewhere.[16,17]

1.1.1. *Strong coupling and confinement*

Non-Abelian gauge groups with weak coupling in some high-energy regime are known to reach strong coupling at low-energies through renormalization group (RG) effects, provided the beta-function coefficient for the gauge coupling takes the correct sign. The sign itself is determined by conventions. We will choose conventions such that the RG equation for the gauge couplings is given by

$$\frac{\partial g_a(\mu)}{\partial t} = -\frac{3b_a}{2} g_a^3(\mu) \ , \tag{5}$$

with

$$b_a = \frac{1}{8\pi^2}\left(C_a - \frac{1}{3}\sum_i C_a^i\right) . \tag{6}$$

In (6) C_a and C_a^i are the quadratic Casimir operators for the gauge group $\mathcal{G}_a$ in the adjoint representation and in the representation of the matter fields Z^i charged under that group, respectively. Note that these conventions imply that a group $\mathcal{G}_a$ with $b_a > 0$ will flow to strong coupling in the infrared.[b] To estimate the energy scale at which strong coupling occurs one can simply solve the RG equation for the gauge coupling. Using the one loop beta-function, the answer is given by

$$\Lambda_a = \mu e^{-1/3b_a g_a^2(\mu)} \ . \tag{7}$$

This expression involves the renormalization scale μ. In our thought experiment we imagine this scale being some high-energy scale where string theory sets the 4D effective Lagrangian; thus it is natural to take $\mu \simeq M_{\rm PL}$ and $g_a^2(\mu) = g_{\rm STR}$.

What happens physically at this scale? Our experience with QCD suggests that confinement can occur. This can be characterized by nonvanishing *vevs* for certain composite objects made up of fermions charged under the strong group. In a pure super-Yang-Mills (SYM) theory (a theory with only gauge supermultiplets and no gauge-charged chiral superfields) the only candidate for such 'condensates' are the fermionic partners of the gauge fields – the gauginos. Thus one might naively expect

[b]The choice of normalization in (5) and (6) was made for future convenience when constructing the effective superspace Lagrangian. To recover the 'standard' conventions of (for example) Martin and Vaughn[18] one must take $b_a \to -(2/3)b_a|_{\rm MV}$.

a nonvanishing $\langle\lambda\lambda\rangle$ to develop at or near the scale Λ. By dimensional analysis we expect

$$\langle\lambda\lambda\rangle \sim \Lambda^3 \tag{8}$$

and using (7) this would seem to suggest

$$\langle\lambda\lambda\rangle \sim M_{\rm PL}^3 e^{-1/b_a g_a^2(M_{\rm PL})} \ . \tag{9}$$

This picture is modified in a number of ways when gauge-charged matter is present. For starters, the beta-function itself changes to reflect the presence of these states in the relevant loop diagrams. More significantly we might expect the chiral matter to confine and form composite operators. We may treat these as dynamical objects or assume them to be very massive 'matter condensates.' Modifications to (9) are known and have been computed by explicit instanton calculations and/or by arguments of anomaly matching and holomorphy.[19,20,21,22] In global superspace a large number of non-Abelian gauge theories, with and without gauge-charged matter, are known to confine and the analogs to (9) are known.[23,24,25,26] We will discuss the presence of matter when we come to the effective Lagrangian approach in Section 1.2 below. For the remainder of this section we return to the pure SYM theory to make some additional remarks.

Why do we expect such an object to break supersymmetry? Consider the equation of motion for the auxiliary field of an arbitrary chiral superfield in supergravity[c]

$$\overline{F}^{\bar{\jmath}} = -e^{K/2}K^{i\bar{\jmath}}(W_i + K_i W) - \frac{1}{4}K^{i\bar{\jmath}}\frac{\partial f_a}{\partial\varphi_i}(\lambda_a\lambda_a) \ , \tag{10}$$

where $W_i = \partial W/\partial\varphi_i$, $K_{i\bar{\jmath}} = \partial^2 K/\partial\varphi_i\partial\overline{\varphi}^{\bar{\jmath}}$ is the Kähler metric and we have set the reduced Planck mass $m_{\rm PL}$ to unity, where $m_{\rm PL} = 1/\sqrt{8\pi G} = 2.44\times 10^{18}$ GeV. The function f_a is the gauge kinetic function which appears in the Yang-Mills kinetic part of the action

$$\mathcal{L}_{\rm YM} = \frac{1}{8}\sum_a \int {\rm d}^4\theta \frac{E}{R} f_a (W^\alpha W_\alpha)_a + \text{ h.c. } \ . \tag{11}$$

In this case we see that *provided the gauge kinetic function is field-dependent* there will generally be a breakdown of supersymmetry via $\langle F\rangle \neq 0$ should some bilinear $\lambda_a\lambda_a$ acquire a vacuum expectation value.[27,28,29,30]

1.1.2. *Gauge coupling as a dynamical field*

In order to proceed further and discuss moduli stabilization we must be somewhat more specific. Therefore, consider a case in which the gauge coupling is determined

[c]Here and throughout we use Kähler $U(1)$ superspace when discussing supergravity as it is the most convenient for string-derived supergravity Lagrangians.[31,32,33] Most intuition from the formalism of Wess & Bagger[34] continues to hold with only a few modifications, particularly in the way expressions are written in superspace notation. For those unfamiliar with Kähler $U(1)$ superspace we have provided some introductory material on the formalism in Appendix A.

by the *vev* for the lowest component of some chiral superfield. In anticipation of weakly coupled heterotic strings we will refer to this chiral superfield S as the dilaton from (1). In the language of superspace effective Lagrangians, this is equivalent to the statement that the gauge kinetic function f_a in the expression (11) is simply given by $f_a = S$ for all gauge groups $\mathcal{G}_a$. Note that the operator $SW^\alpha W_\alpha$ is formally of mass dimension five, so there must be some scale $\Lambda_{\rm UV}$ which enters (11) to restore the canonical mass dimension for $\mathcal{L}_{\rm YM}$. The *vev* of the lowest component $s = S|_{\theta=0}$ determines the (universal) gauge coupling constant g at this scale $\Lambda_{\rm UV}$ via the relation

$$\frac{1}{g_a^2} = k_a \frac{\langle \mathrm{Re}\, s\rangle}{\Lambda_{\rm UV}}. \tag{12}$$

The integer k_a is the affine level of the gauge group $\mathcal{G}_a$, as determined by the conformal field theory of the underlying string construction. For most purposes we will take the simplest case of $k_a = 1$. In what follows we will set $\Lambda_{\rm UV} = m_{\rm PL} = 1$ and consider the *vevs* of all moduli be to be given in units of this scale.

Making the naive replacement suggested by (12) into (9) leads to the hypothesis that gaugino condensation will generically generate a potential for the superfield S (or at least its real part). Let us therefore take the superpotential generated by the gaugino condensation to be (for $k_a = 1$)

$$W_{\rm np}(S) = e^{-\frac{S}{b_a}} \,. \tag{13}$$

Note that in writing (9) in this way we have essentially integrated the gaugino condensates out of the theory, replacing them with a (holomorphic) function of the moduli. To compute the scalar potential for the field S we may use the result familiar from supergravity

$$V = K_{i\bar{\jmath}} F^i \overline{F}^{\bar{\jmath}} - \frac{1}{3} M \overline{M} \,, \tag{14}$$

where F^i is the auxiliary field associated with the chiral superfield φ_i and M is the auxiliary field of supergravity. The auxiliary fields can be identified by their equations of motion

$$F^i = -e^{K/2} K^{i\bar{\jmath}} \left(\overline{W}_{\bar{\jmath}} + K_{\bar{\jmath}} \overline{W}\right) , \quad \overline{M} = -3 e^{K/2} \overline{W} \tag{15}$$

with the gravitino mass given by $m_{3/2} = -\frac{1}{3}\left\langle \overline{M} \right\rangle$. The potential (14) can be written[d]

$$V(s,\bar{s}) = K_{s\bar{s}} |F^S|^2 - 3e^K |W|^2 = e^K K^{s\bar{s}} |W_s + K_s W|^2 - 3e^K |W|^2. \tag{16}$$

At the classical level in string theory, the Kähler potential for the field S has the form

$$K = -\ln(S + \overline{S}) \,, \tag{17}$$

[d]Note that we are here assuming that the superpotential W depends *only* on the chiral dilaton and not on any other modulus. The case of more general $W(S,T)$ will be considered in Section 2.1.2 below.

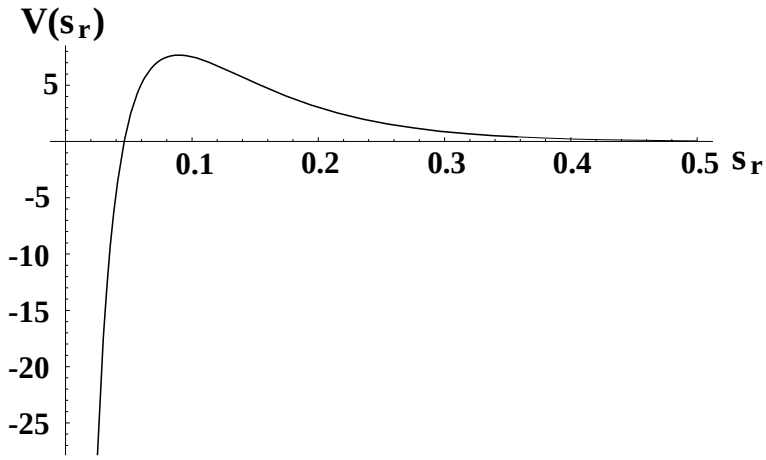

Fig. 1. **Plot of dilaton potential $\mathbf{V(s+\bar{s})}$ for the superpotential $\mathbf{W_{np}(S) = e^{-\frac{S}{b_a}}}$ with $\mathbf{b_a = 5/8\pi^2}$.** The shape of the potential as a function of $s_r = (s+\bar{s}) = 2\,\mathrm{Re}\,s$ is given with the height of the potential in arbitrary units ($m_{\rm PL} = 1$). The asymptotic behavior as $s_r \to \infty$ is the so-called dilaton "runaway" problem.

and the nonperturbatively generated superpotential in (13) gives $W_s = -(1/b_a)W$. The resulting dilaton potential (16) is plotted in Figure 1 for $b_a = 5/8\pi^2$, as would be the case for condensation of pure $SU(5)$ Yang-Mills fields. In the limit $\mathrm{Re}\,s \to 0$ (which corresponds to strong gauge coupling) the potential is unbounded from below. There is an extremum that depends weakly on the precise value of b_a chosen but is generally at a value of Re s where $\alpha = g^2/4\pi \simeq 1$. Finally there is the 'runaway' solution where $\mathrm{Re}\,s \to \infty$ or $g^2 \to 0$. In this limit supersymmetry is restored as both the superpotential and its covariant derivatives vanish.

The potential determined by (13) and (17) is incapable of breaking supersymmetry and providing a realistic minimum for the dilaton. A number of mechanisms were quickly suggested to correct this behavior, and we will address some of them in subsequent sections. For now we continue in the spirit of simplicity (*i.e.* one-modulus models) and here consider the case of multiple condensates.[35,36] A single, simple gauge group $\mathcal{G}$ in the hidden sector was motivated by some of the earliest Calabi-Yau constructions where the hidden sector was an entire factor of E_8.[37,38] In general, however, (and particularly for orbifold compactifications) we expect a hidden sector gauge group that is given by a product of simple groups: $\mathcal{G} = \prod_a \mathcal{G}_a$, or perhaps more generally

$$\mathcal{G}_{\rm hidden} = \prod_{a=1}^{n} \mathcal{G}_a \otimes U(1)^m \ . \tag{18}$$

Some subset of these $\mathcal{G}_a$ will be asymptotically free and can therefore form a gaugino condensate. Taking the simplest case of just two gaugino condensates in the hidden

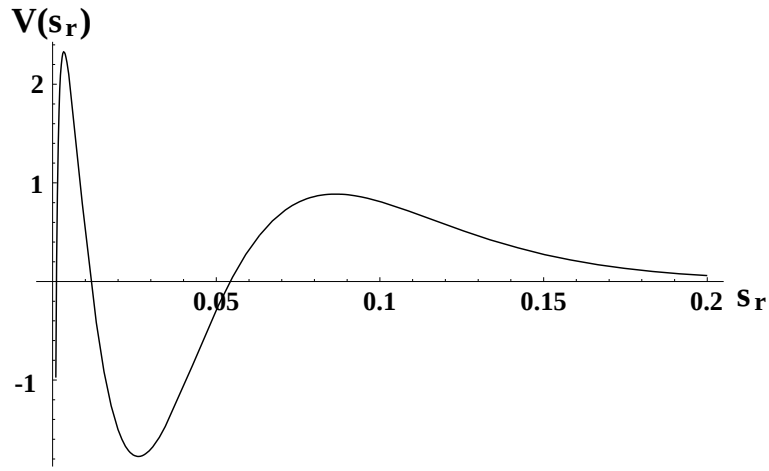

Fig. 2. **Plot of dilaton potential $\mathbf{V(s+\bar{s})}$ for the superpotential of (19)**. The shape of the potential as a function of $s_r = (s+\bar{s}) = 2\,\mathrm{Re}\,s$ is given with the height of the potential in arbitrary units ($m_{\mathrm{PL}} = 1$). Values of the parameter set are given in the text below.

sector we would expect a superpotential of the form[35,39]

$$W(S) = \Lambda^3 \left[d_1 e^{-k_1 S/b_1} + d_2 e^{-k_2 S/b_2} \right]. \tag{19}$$

Here b_1 and b_2 are the beta-function coefficients, defined by (6), for the two condensing groups $\mathcal{G}_1$ and $\mathcal{G}_2$. The quantities d_1 and d_2 parameterize the presence of possible matter in the hidden sector, while the parameters k_1 and k_2 might represent differing affine-level for the gauge groups. For dilaton stabilization with a realistic minimum to occur, we must require that the scalar potential $V(S)$ given in (16) give rise to a minimum such that $\langle s+\bar{s}\rangle/2 = 1/g^2_{\mathrm{STR}} \simeq 2$ while generating a gravitino mass $m_{3/2} = \left\langle e^{K/2} W \right\rangle$ of $\mathcal{O}(1\ \mathrm{TeV})$. Each of the parameters in the set $\{b_1,\, b_2,\, d_1,\, d_2,\, k_1,\, k_2\}$ are not continuously variable, but depend upon particulars of the compactification in a calculable way. Superpotentials of the form (19) are often referred to as "racetrack" models in that two exponential functions must be balanced against one another in a delicate way to achieve the desired minimum.

If we take the simplest possible case[40,41,42] in which $d_a = -b_a/6e$ and take a hidden sector comprising of $\mathcal{G}_1 = SU(7)$ with 8 $\mathbf{7}+\bar{\mathbf{7}}$'s and $\mathcal{G}_2 = SU(8)$ with 15 $\mathbf{8}+\bar{\mathbf{8}}$'s then a solution exists for $k_1 = 2$ and $k_2 = 1$. The balancing of two $SU(N)$ groups with similar beta-function coefficients is a common feature of solutions in the racetrack scenario. The resulting scalar potential is plotted in Figure 2 above. The asymptotic runaway behavior as $s_r \to \infty$ is still present, as is the unbounded from below direction near the origin. But now a nontrivial minimum develops for the real part of the dilaton scalar. This is a major improvement over the single condensate case, though there are two properties that make this minimum unrealistic. First, the value of the dilaton field at this minimum suggests far too strong a gauge coupling

$g^2_{\rm STR} \simeq 30$. Second, the minimum does not occur with vanishing value of $\langle V(\mathrm{Re}\, s)\rangle$ but rather with a large negative value.

The former problem of the dilaton *vev* can be rectified by a number of mechanisms. The relationship between the beta-function parameters b_a and the coefficients d_a can be generalized. For example, one might consider the effect of threshold corrections on the beta-functions for the couplings of groups $\mathcal{G}_1$ and $\mathcal{G}_2$ (as well as on the scales Λ which appear in equation 19). These effects might arise from integrating out heavy vector-like matter charged under those groups, with masses above the condensation scale (such as states charged under some anomalous $U(1)$ factor).[36] If the hidden sector matter is to be integrated out below the scale of gaugino condensation then a nontrivial d_a is generated for each condensing group whose form depends on whether the matter is vector-like in nature[43] or only forms condensates of dimension three or higher.[44] Another solution (often invoked in conjunction with the above) is to allow the nonperturbative superpotential to depend on more than one modulus – in particular, the Kähler modulus of (4).[36,45,46,43] Such a dependence is required in modular invariant treatments of effective Lagrangians from heterotic string theory. We will discuss this further in Section 2.

The success of multi-condensate models in achieving supersymmetry breaking with realistic values of the gauge coupling have made them the preferred starting-point for most phenomenological treatments of string constructions. Yet even in the multi-modulus (and modular invariant) treatments they suffer from a common problem: the minimum of the potential continues to imply negative vacuum energy (*i.e.* the solution is one with anti-de Sitter spacetime). This is a phenomenological disaster that requires the invocation of some additional physics that enters to "rescue" the theory and set the vacuum energy to zero (or very nearly zero).[e] Today the most effective and robust such method is the inclusion of flux – that is, vacuum expectation values for the field strengths of fields from the Ramond-Ramond sector of the string theory.[47] But such mechanisms do not readily present themselves in the weakly coupled heterotic string context. We are thus left to consider other field-theoretic and/or string-theoretic effects which must be included in the effective Lagrangian to allow for a realistic minimum. For the class of models being discussed in this work the mechanism will be that of *Kähler stabilization*, and it will be the focus of Section 1.3 below. But before we can consider the implementation of Kähler stabilization we must discuss the effective Lagrangian approach to gaugino condensation.

[e]It might be hoped that quantum corrections to the vacuum energy of the theory might conspire to "lift" the negative minimum to a positive or vanishing one, without invoking any additional fields or interactions. This is not impossible, but we argue that such corrections are generally insufficient to rectify the problem, and may even make it worse.[48]

1.2. *Condensates as effective operators*

The treatment of condensates in supergravity is greatly simplified by using an effective operator approach.[49,29] The starting point for our analysis is the definition of the composite field operator U_a which represents a $\mathcal{G}_a$-charged gauge condensate chiral superfield

$$U_a \simeq W_a^\alpha W_\alpha^a \ . \tag{20}$$

Note that the lowest component of $u_a = U_a|_{\theta=\bar\theta=0}$ involves the gaugino bilinear $\lambda_a\lambda_a$. We wish to write an effective Lagrangian describing the dynamics of this new chiral superfield such that the behavior of the Lagrangian under symmetry transformation is matched by the behavior of the underlying supersymmetric gauge theory (with unconfined gauginos). For global SUSY the resulting effective Lagrangian (for a single condensate) is that of Veneziano and Yankielowicz (VY)[49]

$$\mathcal{L}_{\rm eff} = \int \mathrm{d}^4\theta (UU^\dagger)^{1/3} - \frac{1}{4}\left[\int \mathrm{d}^2\theta \left(b'\, U \ln\frac{U}{\mu^3} + \frac{1}{g^2}U\right) + \text{ h.c.}\right] \ . \tag{21}$$

Here μ is the renormalization group invariant scale of the condensing group. The logarithmic part of the second term in (21) can be combined with the standard Yang-Mills term in (11) and is represented by the following superpotential expression[50,51,52]

$$W(U,S)_{\rm VY} = \frac{1}{4}U\left[S + b'U\ln(U/\mu^3)\right] \ , \tag{22}$$

where b' is a constant coefficient which we will determine presently. If there are multiple condensates labeled by the index a as in (20), then the final term in (22) will generally involve a different coefficient b'_a for each condensate U_a.

To the Lagrangian in (21) we wish to add the possibility of matter charged under the condensing group to obtain the Veneziano, Yankielowicz and Taylor (VYT) Lagrangian.[49,19,53] From our experience in QCD we generally expect states charged under the strong group to experience confinement and form composites. We will represent these by the composite field operators

$$\Pi_a^\alpha \simeq \prod_i \left(\Phi_i^{(a)}\right)^{n_i^{\alpha,(a)}} \ , \tag{23}$$

where the product involves only those fields $\Phi_i^{(a)}$ charged under the confined group $\mathcal{G}_a$. In (23) the label α is a species index for the matter condensates, each of which may consist of different component fields labeled by the integers $n_i^{\alpha,(a)}$. Note that the canonical mass dimension of this operator Π_a^α is given by

$$\dim\left(\Pi_a^\alpha\right) \equiv d_a^\alpha = \sum_i n_i^{\alpha,(a)} \tag{24}$$

The generalization to supergravity[40,54] of the VYT effective action (that is, the generalization of (21) to include matter in Kähler $U(1)$ superspace) has the following

general form

$$\mathcal{L}_{\rm VYT} = \frac{1}{8}\int {\rm d}^4\theta\, \frac{E}{R}\sum_a U_a \left[b'_a \ln(e^{-K/2}U_a) + \sum_\alpha b^\alpha_a \ln \Pi^\alpha_a \right] + {\rm h.c.}\ , \tag{25}$$

where there are now two separate coefficients b'_a and b^α_a which must be determined for each condensing group $\mathcal{G}_a$. These are obtained by matching the anomalies of the effective theory to those of the underlying theory. The Lagrangian (25) has the correct anomaly structure under Kähler $U(1)$, R-symmetry, conformal transformations, and modular (T-duality) transformations provided the conditions

$$b'_a = \frac{1}{8\pi^2}\left(C_a - \sum_i C^i_a \right), \qquad b^\alpha_a = \sum_{i\in\alpha} \frac{C^i_a}{4\pi^2 d^\alpha_a}\ , \tag{26}$$

are satisfied. The composite chiral superfields Π^α_a are invariant under the nonanomalous symmetries, and may be used to construct an invariant superpotential.[19,55] Provided there are no invariant chiral fields of dimension two, and no additional global symmetries (such as chiral flavor symmetries), the dynamical degrees of freedom associated with the composite fields (20) and (23) acquire masses[56] larger than the condensation scale Λ_a, and may be integrated out. This results in an effective theory constructed as described in (25) with the composite fields taken to be nonpropagating; that is, they do not appear in the Kähler potential.

We note that the conditions (26) are not a unique solution to the set of anomaly constraints. However they are the most straightforward solution, intuitively plausible, and hold in the presence of additional anomalous symmetries, since the weights of the condensates Π are just the products of the weights of their constituents. As an explicit example of a fully constrained model, consider the VYT action for $SU(N_c)$ with chiral flavor symmetry. We have N "quark" and N "anti-quark" chiral supermultiplets Q^A and Q^c_A, respectively. We take the quark condensates to be the matrix-valued "meson" superfield $\mathbf{\Pi}^A_B = Q^A Q^c_B$. We do not assume *a priori* that these are static fields. If $\mathcal{G}_a = SU(N_c) \equiv \mathcal{G}_Q$ we take

$$\mathcal{L}^Q_{\rm VYT} = \frac{1}{8}\int {\rm d}^4\theta \frac{E}{R} U_Q \left[b'_Q \ln(e^{-K/2}U_Q) + b^\alpha_Q \ln(\det \mathbf{\Pi}) \right] +\ {\rm h.c.}\ , \tag{27}$$

where here the form of the matter condensate is dictated by invariance under flavor $SU(N)_L \otimes SU(N)_R$. For the elementary fields we have $C_Q = N_c$, $C^i_Q = \frac{1}{2}$ and $\sum_i C^i_Q = N$. Under Kähler $U(1)$ R-symmetry, anomaly matching requires

$$b'_Q = \frac{1}{8\pi^2}\,(N_c - N)\ , \tag{28}$$

and under the conformal transformation

$$\begin{array}{ll} \lambda_a \to e^{3\sigma/2}\lambda_a & \Phi^i \to e^\sigma \Phi^i \\ U_a \to e^{3\sigma} U_a & \Pi^\alpha_a \to e^{d^\alpha_a \sigma}\Pi^\alpha_a \end{array} \tag{29}$$

with $\mathbf{\Pi} \to e^{2\sigma}\mathbf{\Pi}$, we require

$$3b'_Q + 2N b^\alpha_Q = \frac{1}{8\pi^2}\,(3N_c - N) = 3b_Q, \tag{30}$$

where b_Q is the β-function coefficient defined by (6). Putting these together gives (28) and

$$b_Q^\alpha = \frac{1}{8\pi^2}, \qquad b_Q = b'_Q + \frac{2N}{3} b_Q^\alpha \,, \tag{31}$$

in agreement with the general result (26) for $d_Q^\alpha = 2N$ and $\pi_Q^\alpha = \det \mathbf{\Pi}$. It is easy to see that the anomaly matching condition

$$2N b_Q^\alpha = \sum_i \frac{C_Q^i}{4\pi^2} \tag{32}$$

under chiral $U(1)$ transformations $Q \to e^{i\beta} Q$, $Q^c \to e^{i\beta} Q^c$ is also satisfied by (28) and (31).[f]

It is instructive to compare the effective theory defined by (25) with results[57,58] based on holomorphy of the superpotential by going to the rigid SUSY limit, and neglecting the moduli and the dilaton; $s \to g_0^{-2} =$ constant. Then the superpotential reduces to the standard VYT one:

$$W(U_Q) = \frac{1}{4} U_Q \left[g_0^{-2} + b'_Q \ln(U_Q) + b_Q^\alpha \ln(\det \mathbf{\Pi}) \right] \,, \tag{33}$$

which is the analog of (22). Keeping U_Q static and imposing the equation of motion for the auxiliary field F^Q gives the potential

$$\begin{aligned} -V &= \mathrm{Tr} \left[\bar{\mathbf{F}}^\pi \mathbf{K}'' \mathbf{F}^\pi + \left\{ \mathbf{F}^\pi \left(\mathbf{M} + \frac{1}{4} b_Q^\alpha u_Q \mathbf{\Pi}^{-1} \right) + \text{h.c.} \right\} \right] , \\ u_Q &= e^{-1} \left(\frac{\Lambda_Q^{3N_c - N}}{\det \mathbf{\Pi}} \right)^{1/(N_c - N)} \end{aligned} \tag{34}$$

where $\mathbf{K}''$ is the (tensor-valued) Kähler metric for $\mathbf{\Pi}$. The potential (34) is derivable from the following superpotential for the dynamical superfield $\mathbf{\Pi}$

$$W_{\mathbf{\Pi}} = \mathrm{Tr}(\mathbf{M}\mathbf{\Pi}) - \frac{(N_c - N)}{32\pi^2 e} \left(\frac{\Lambda_Q^{3N_c - N}}{(\det \mathbf{\Pi})} \right)^{1/(N_c - N)} \,, \tag{35}$$

which, up to a factor[g] $-2/e$, is the superpotential found by Davis *et al.*[57]

1.3. *Kähler stabilization and the linear multiplet*

To adopt the form of (22) as an effective superpotential term requires that the gauge kinetic function be written in terms of a linear combination of holomorphic objects – that is, of *chiral* superfields. Most treatments of gaugino condensation

[f]Though we have reserved the issue of modular invariance for the next section, we remark here that it is also very easy to see that should Q, Q^c have nontrivial modular weights under T-duality the modular anomaly matching condition is also satisfied by (28) and (31).

[g]The factor e comes from the fact that we take the derivative of $\int U \ln U$, while Davis *et al.* start with $\int < \lambda\lambda > \ln \Lambda$ and determine $< \lambda\lambda >$ from threshold matching.[57] The minus sign comes from the convention of Ref. 33: $u \sim W^\alpha W_\alpha| = -\lambda\lambda$.

utilize the formalism of the previous section – namely the description of dilaton-like objects as chiral superfields. This has the convenience of familiarity, but almost all extensions of the theory beyond that presented in Sections 1.1 and 1.2 quickly reveal the limitations of the chiral superfield treatment. In this section we will consider an alternative and dual formulation of the Yang-Mills action in terms of *real* dilatonic superfields. This will prove to be convenient in implementing the Kähler stabilization mechanism for solving the vacuum energy problem described in Section 1.1.

1.3.1. *The linear multiplet*

It is worth recalling the properties of the fundamental degrees of freedom obtained from direct dimensional reduction of the massless string spectrum. For each "dilaton" (that is, for each field whose *vev* determines a gauge coupling) the 4D massless modes are a *real* scalar ℓ, an antisymmetric two-index tensor $b_{\mu\nu}$ and a (Majorana) Weyl fermion χ_ℓ. These are precisely the degrees of freedom of the linear multiplet.[59] In particular there is no need to perform the duality transformation (3) in order to generate a pseudoscalar "partner" for the real dilaton. This is particularly important when one considers higher-genus corrections to the effective Lagrangian, where the duality relation (3) is replaced by a much less straightforward field identification.[60] Note also the absence of an auxiliary field in the massless spectrum for this multiplet.

A linear multiplet $\widehat{L}$ is essentially a special case of a real vector superfield defined by the requirement[61,62,63,64]

$$-(\mathcal{D}^\alpha\mathcal{D}_\alpha - 8R^\dagger)\widehat{L} = 0, \quad -(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R)\widehat{L} = 0\ . \tag{36}$$

The requirement that the chiral projection of $\widehat{L}$ vanish already ensures that the vector component v_μ of the multiplet is Hodge-dual to the field strength of a two-index antisymmetric tensor – precisely the field that appears in (3).[64] In other words, the definition (36) automatically enforces the Bianchi identity

$$\partial^\mu v_\mu = 0 \tag{37}$$

for the vector component of $\widehat{L}$, which we identify as

$$v_\mu = \epsilon_{\mu\nu\rho\sigma}\partial^\nu b^{\rho\sigma} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}H^{\nu\rho\sigma}\ . \tag{38}$$

The lowest component ℓ of the superfield $\widehat{L}$ is then the dilaton and the relation (12) is replaced by

$$\frac{g^2_{\rm STR}}{2} = \langle\ell\rangle\ . \tag{39}$$

This quantity represents the string loop expansion parameter. Therefore string-theory information from higher loops is more naturally encoded in terms of this

set of component fields. At the leading order the chiral and linear formalisms are related by the simple superfield identification

$$\widehat{L} = \frac{1}{S + \overline{S}} \tag{40}$$

though this identification fails to be satisfactory at higher loop level.

The antisymmetric tensor field of superstring theories undergoes Yang-Mills gauge transformations, which implies that the superfield $\widehat{L}$ which contains this degree of freedom is not gauge invariant. It is possible to define a *modified linear multiplet* L which recovers gauge invariance by introducing Yang-Mills Chern-Simons forms to the definition in (36).[63] We define the YM Chern-Simons form as

$$\Omega_{\mu\nu\rho} = A^a_{[\mu} F_{\nu\rho]\,a} - \frac{1}{3} f_{abc} A^a_\mu A^b_\nu A^c_\rho \,, \tag{41}$$

where f_{abc} are the structure constants of the group, and then promote this to a real superfield Ω. The new field L will obey the modified linearity conditions

$$\begin{aligned} -(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R)L \;&=\; (\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R)\Omega \;=\; \sum_a (W^\alpha W_\alpha)_a, \\ -(\mathcal{D}^\alpha\mathcal{D}_\alpha - 8R^\dagger)L \;&=\; (\mathcal{D}^\alpha\mathcal{D}_\alpha - 8R^\dagger)\Omega \;=\; \sum_a (\overline{W}_{\dot\alpha}\overline{W}^{\dot\alpha})_a \,. \end{aligned} \tag{42}$$

Neither L nor Ω are gauge-invariant individually, but the combination $\widehat{L} = L + \Omega$ now is.[h] For those subgroups $\mathcal{G}_{a'}$ which experience confinement it is natural to identify the chiral projection of the modified linear multiplet as the chiral superfield $U_{a'}$ in (20). We therefore have

$$\begin{aligned} -(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R)L \;&=\; \sum_a (W^\alpha W_\alpha)_a + \sum_{a'} U_{a'}, \\ -(\mathcal{D}^\alpha\mathcal{D}_\alpha - 8R^\dagger)L \;&=\; \sum_a (\overline{W}_{\dot\alpha}\overline{W}^{\dot\alpha})_a + \sum_{a'} \overline{U}_{a'} \,. \end{aligned} \tag{43}$$

We will see below that this identification is crucial to a correct implementation of certain Bianchi identities in the Yang-Mills sector of the theory.

The generic Lagrangian describing the coupling of the modified linear multiplet to supergravity and chiral superfields, in the presence of Yang-Mills Chern-Simons superforms, is[63,32,65]

$$K = k(L) + K(\Phi, \overline{\Phi}), \quad \mathcal{L} = -3 \int \mathrm{d}^4\theta \, E \, F(\Phi, \overline{\Phi}, L) \tag{44}$$

where the quantity $K(\Phi, \overline{\Phi})$ represents the contribution of chiral superfields (matter and/or additional moduli fields) to the Kähler potential. We will deal with chiral matter more thoroughly in Section 2; in the discussion here they will play

[h]The choice of signs in the relations (42) is one of conventions and several exist in the literature. For example, the conventions used here are those of Gaillard and Taylor.[65] They differ by the presence of the minus sign on the first terms from those of earlier work.[63,32]

only a trivial role. Note that the Kähler potential does not appear explicitly in the superspace Lagrangian, but rather implicitly through the supervierbein E. The component expansion of (44) contains the kinetic terms for the supergravity multiplet as well as the Yang-Mills fields and the linear multiplet itself. The two functions $k(L)$ and $F(L)$ are not entirely arbitrary; they are constrained by the requirement that the Einstein-Hilbert term in the component expansion of (44) have canonical normalization. Under this constraint $k(L)$ and $F(L)$ are related to each other by the following first-order differential equation[63,32]

$$F - L\frac{\partial F}{\partial L} = 1 - \frac{1}{3}L\frac{\partial k}{\partial L}\,. \tag{45}$$

The general solution to (45) reads[32]

$$F(\Phi,\overline{\Phi},L) = 1 + \frac{1}{3}LV(\Phi,\overline{\Phi}) + \frac{1}{3}L\int\frac{\mathrm{d}L}{L}\frac{\partial k(L)}{\partial L}\,. \tag{46}$$

Once the functional form of $k(L)$ is specified the last term in (46) is fixed. This leaves only the second term for a nontrivial interaction between the modified linear multiplet and matter within the function $F(\Phi,\overline{\Phi},L)$. This term is a form of "integration constant" for the differential equation in (45). Such a term will play a very important role in the implementation of the Green-Schwarz mechanism[66] and the inclusion of nonholomorphic threshold corrections to gauge couplings in Section 2 below. Its natural emergence here is one of the benefits of using the linear multiplet in string-derived supergravity models. At tree level we expect from (40) that the Kähler potential for the linear dilaton should be simply $k(L) = \ln L$. This implies the $V = 0$ solution to (46) is a constant $F(L) = 2/3$, and thus

$$\mathcal{L}_{\rm kin} = -2\int \mathrm{d}^4\theta\, E\,. \tag{47}$$

For the general form (44) we must also generalize the duality relationship in (40). Consider the Lagrangian[32,67]

$$\mathcal{L}_{\rm lin} = -3\int \mathrm{d}^4\theta\, E\left[F(\Phi,\overline{\Phi},L) + \frac{1}{3}(L+\Omega)(S+\overline{S})\right] \tag{48}$$

where L is now an unconstrained superfield. After eliminating S by using its classical equation of motion one obtains (44). By varying with respect to L and demanding canonical Einstein term we arrive at (46) and the new duality relation

$$F(\Phi,\overline{\Phi},L) + \frac{1}{3}L(S+\overline{S}) = 1\,. \tag{49}$$

For the simple tree-level case with $F = 2/3 + LV/3$ we have

$$L = \frac{1}{(S+\overline{S}+V)} \tag{50}$$

and we once again recover (40) in the limit of vanishing integration constant.

With the introduction of Yang-Mills Chern-Simons forms, the Bianchi identity satisfied by the vector component of the modified linear multiplet is no longer (37) but is instead[51,68]

$$\partial^\mu v_\mu = \frac{1}{8}\,{}^*\Phi \tag{51}$$

where ${}^*\Phi$ is related to the field strength of a rank-3 antisymmetric tensor field $\Gamma^{\nu\rho\sigma}$

$${}^*\Phi = \frac{1}{3!}\epsilon_{\mu\nu\rho\sigma}\partial^\mu\Gamma^{\nu\rho\sigma}\,. \tag{52}$$

The expression in (52) is the analog to the axionic duality relation in (3).[i] The three-form supermultiplet can be described in flat superspace by a chiral superfield Y and anti-chiral superfield $\overline{Y}$ such that[69]

$$D^2Y - \overline{D}^2\overline{Y} = \frac{8i}{3}\epsilon^{\mu\nu\rho\sigma}\Sigma_{\mu\nu\rho\sigma}\,, \tag{53}$$

where $\Sigma_{\mu\nu\rho\sigma}$ is the gauge-invariant field strength of the rank-three gauge potential superfield $\Gamma_{\nu\rho\sigma}$. Applying (53) to the specific case of our modified linear multiplet in curved superspace we find the constraint

$$\begin{aligned}(\mathcal{D}^\alpha\mathcal{D}_\alpha - 24R^\dagger)U \;-\; (\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 24R)\overline{U} &= \text{total derivative}\\ &= \frac{i}{3}\epsilon_{\mu\nu\rho\sigma}\partial^\mu\Gamma^{\nu\rho\sigma}\\ &= 2i^*\Phi\end{aligned} \tag{54}$$

with a similar constraint for the unconfined YM fields via the replacement $U \to W^\alpha W_\alpha$. Indeed, the composite operator $W^\alpha W_\alpha$ can be interpreted as the degrees of freedom of the three-form field strength.[69,68] The general solution to the constraint equation (54) is a field U which is identified with the chiral projection of a real superfield – precisely as in (43).

The traditional chiral formulation of gaugino condensation is incorrect in that it treats the interpolating field $U = e^{K/2}H^3$ with $H^3 = W^\alpha W_\alpha$ as an ordinary chiral superfield of Kähler chiral weight $w = 2$. But this is inconsistent with (54).[70,71,51,52] In the general formulation of duality transformations, couplings of the dilaton to matter entail duality invariance of the corresponding terms in the Lagrangian, as opposed to their couplings to gauge fields, which are only an invariance of the equations of motion.[52] One must either apply (54) religiously everywhere, or begin by using the modified linear multiplet in the first place.

This is not a purely academic concern: failure to properly incorporate the constrained Yang-Mills geometry inherited from the underlying string dynamics can

[i]The presence of additional terms in the new Bianchi identity (51) implies that the would-be classical shift symmetry (*i.e.* a Peccei-Quinn type symmetry) enjoyed by the model-dependent axion of the dilaton multiplet has been broken to a restricted class of shifts by nonperturbative effects.

have demonstrable effects on the low-energy phenomenology inferred from the component Lagrangian. For example, in the present formalism the condensate superfields U_a of (20) are introduced as static chiral superfields. Their highest components F_{U_a}, defined by

$$\begin{aligned} F_{U_a} &= -\frac{1}{4}\mathcal{D}^\alpha\mathcal{D}_\alpha U_a|_{\theta=\bar\theta=0} \equiv -\frac{1}{4}\mathcal{D}^2 U_a\,, \\ \overline{F}_{\overline{U}_a} &= -\frac{1}{4}\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha}\overline{U}_a|_{\theta=\bar\theta=0} \equiv -\frac{1}{4}\overline{\mathcal{D}}^2\overline{U}_a\,, \end{aligned} \tag{55}$$

therefore appear only linearly in the component Lagrangian. One might believe that they can thus be removed from the theory by solving their equations of motion $F_{U_a} = \overline{F}_{\overline{U}_a} = 0$. However a subtlety arises in deriving the corresponding equations of motion for these auxiliary fields as a result of (54),[72,73] for which we must require[j]

$$-\frac{1}{4}\left(\mathcal{D}^2 U_a - \overline{\mathcal{D}}^2\overline{U}_a\right) = F_{U_a} - \overline{F}_{\overline{U}_a} = 4i\nabla^\mu v^a_\mu + u_a\overline{M} - \overline{u}_a M\,. \tag{57}$$

The first term is the "total derivative" of (54), while the last two terms arise from the lowest components of the superfield terms $24R^\dagger U_a - 24R\overline{U}_a$. At a supersymmetry-breaking minimum of the scalar potential we generally expect nonvanishing *vevs* for both the condensates ($\langle u_a\rangle \sim \langle\lambda_a\lambda_a\rangle$) and the auxiliary field of supergravity ($\langle M\rangle \sim m_{3/2}$, see equation 15). Therefore (57) is a nontrivial constraint that can affect the soft supersymmetry breaking of the observable sector as well as the physics of the axion sector (associated with v^a_μ).[60] When the condensate field U_a is treated as an ordinary chiral superfield these last two terms do not arise automatically (as they do in the linear multiplet treatment) but must be included in the effective theory by hand. These terms also can be shown to vanish in the $M_{\rm PL} \to \infty$ limit of flat superspace, indicating the importance of a proper supergravity treatment for an accurate phenomenology.

1.3.2. *Kähler stabilization*

In the previous subsection we have argued that it is the linear multiplet formulation which hews most closely to the underlying string theory. The fundamental degrees of freedom of the dilaton multiplet, particularly in the presence of Yang-Mills Chern-Simons forms, are easily incorporated into its structure. Kähler $U(1)$ superspace then provides a framework for naturally including this constrained Yang-Mills geometry into four-dimensional supergravity. In Section 2.2 we will also see how the integration constant associated with (46) allows a beautiful implementation of the

[j]One way to ensure this constraint is to first rewrite F_{U_a} as

$$F_{U_a} = \frac{1}{2}\left(F_{U_a} + \overline{F}_{\overline{U}_a}\right) + 2i\nabla^\mu v^a_\mu + \frac{1}{2}\left(u_a\overline{M} - \bar u_a M\right)\,, \tag{56}$$

(and the conjugate expression for $\overline{F}_{\overline{U}_a}$), and then vary the Lagrangian with respect to the *unconstrained* combination $F_{U_a} + \overline{F}_{\overline{U}_a}$.

Green-Schwarz mechanism of anomaly cancelation. Before introducing that complication, however, we wish to recast the familiar language of gaugino condensation from Sections 1.1 and 1.2 in terms of our less familiar linear multiplet. Along the way we will consider possible nonperturbative corrections to the effective action which will allow us to achieve moduli stabilization.

On general grounds we expect nonperturbative effects to alter the form of any and all functions which determine the low-energy effective supergravity Lagrangian. These may be of field-theoretic or string-theoretic origin. An example is the gaugino condensation of the previous sections, which we imagine to be a nonperturbative effect from field theory which generates corrections to the holomorphic superpotential for the moduli fields. It is certainly not unreasonable to believe that instanton effects – whether of the world-sheet or of target space – will correct the Kähler potential as well. Indeed, such stringy effects were conjectured some time ago by Shenker[74] and have since been explicitly demonstrated to exist in certain string contexts.[75,76,77]

We expect corrections to the Kähler potential to involve the confinement scale Λ_a of (7), and on dimensional grounds we expect corrections of field-theoretic origin which scale like

$$L^{-m} e^{-n/6b_a L} / M_{\rm PL}^{n-2} \tag{58}$$

where $n \geq 2$ and $m \geq 0$.[78,71,60] The simplest example of such an effect would be to consider the leading-order nonperturbative contribution ($n = 2$ and $m = 0$) to the Kähler potential

$$f(L) \;=\; A\, e^{-\,1/3bL}, \tag{59}$$

where A is a constant to be determined by the nonperturbative dynamics and b is some effective beta-function coefficient. Another possibility is to consider instanton contributions from the string world-sheet, in which the function $f\,(L)$ derived from (58) is slightly modified to[74]

$$f(L) = \sum_n A_n (\sqrt{L})^{-n} e^{-B/\sqrt{L}}\ . \tag{60}$$

It is an important feature of (60) that these string instanton effects scale like $e^{-1/g}$ (when we use $\ell \sim g^2$) and are thus stronger than analogous nonperturbative effects in field theory which have the form e^{-1/g^2}. Thus they can be of significance even in cases where the effective four-dimensional gauge coupling at the string scale is weak.[78]

The corrections described above are, strictly speaking, not corrections to the Kähler potential of the dilaton but to its *action*, which is best investigated in component form. While these corrections can be written in terms of modifications to the effective four-dimensional Kähler potential, it is simpler to implement the changes directly at the superfield level by modifying the kinetic energy part of the superspace Lagrangian. We thus follow the form of (44) and introduce two functions

of the effective superfield L which parameterize these nonperturbative corrections arising from instanton effects as follows

$$\mathcal{L}_{\rm KE} = \int {\rm d}^4\theta\, E\left[-2 + f(L)\right], \quad k(L) = \ln\, L + g(L), \tag{61}$$

where (45) implies

$$L\frac{{\rm d}g(L)}{{\rm d}L} \;=\; -L\frac{{\rm d}f(L)}{{\rm d}L} \;+\; f(L). \tag{62}$$

In addition, we wish to obtain the classical limit $f = g = 0$ at weak coupling, so we must demand a further boundary condition at vanishing coupling

$$g(L=0) \;=\; 0 \quad \text{and} \quad f(L=0) \;=\; 0. \tag{63}$$

In the presence of these nonperturbative effects the relationship between the dilaton and the effective field theory gauge coupling is modified from the relation in (39) in a manner dictated by the duality relation in (49)

$$\frac{g^2_{\rm STR}}{2} = \left\langle \frac{\ell}{1+f\,(\ell)} \right\rangle . \tag{64}$$

We continue to use the form of (25) to describe gaugino condensation. For simplicity we will work in this section only with a single gaugino condensate. We will thus take the hidden sector to be a pure Yang-Mills theory with no chiral matter. Then we have

$$\mathcal{L}_{\rm VY} = \frac{b}{8}\int {\rm d}^4\theta\, \frac{E}{R}\, U \ln(e^{-K/2}U/\mu^3) + \text{ h.c. }, \tag{65}$$

and from (26) we have $b = C/8\pi^2$ with C being the quadratic Casimir operator for the condensing group in the adjoint representation. Using superspace integration by parts, and the property (43), it is possible to write the complete Lagrangian for this simple system as

$$\mathcal{L}_{\rm eff} = \int {\rm d}^4\theta\, E\,\{\, -2 \,+\, f(L) \,+\, bL\ln(e^{-K}\overline{U}U/\mu^6)\,\}\,. \tag{66}$$

The method for obtaining the component field Lagrangian from D-density expressions in Kähler $U(1)$ superspace is outlined in the Appendix. Recalling the discussion surrounding the auxiliary fields F_U and $\overline{F}_{\overline{U}}$ in (57), we are careful to solve for the equations of motion for the combination $(F_U + \overline{F}_{\overline{U}})$ from which we obtain

$$f \,+\, 1 \,+\, b\ell\ln(e^{-k}\bar{u}u/\mu^6) \,+\, 2b\ell \,=\, 0\,, \tag{67}$$

where $k = k(\ell)$ is the dilaton Kähler potential in (61). This equation is easily solved to provide an expression for the magnitude of the gaugino condensate

$$\bar{u}u \,=\, \frac{1}{e^2}\ell\mu^6 e^{g\,-\,(f+1)/b\ell}\,, \tag{68}$$

where the basic form of (9) is recovered in the limit $f(\ell) = g(\ell) = 0$, as it should. The equation of motion for the supergravity auxiliary field gives

$$M = \frac{3}{4} b u \,; \qquad \overline{M} = \frac{3}{4} b \overline{u} \,, \tag{69}$$

from which the gravitino mass can be computed. Eliminating all auxiliary fields via their equations of motion generates the scalar potential for the dilaton

$$\begin{aligned} V(\ell) &= \frac{1}{16\ell^2} \left[\left(1 + \ell \frac{\mathrm{d}g}{\mathrm{d}\ell} \right) (1 + b\ell)^2 \, - \, 3b^2 \ell^2 \right] u\overline{u}, \\ &= \frac{1}{16 e^2 \ell} \left[\left(1 + \ell \frac{\mathrm{d}g}{\mathrm{d}\ell} \right) (1 + b\ell)^2 \, - \, 3b^2 \ell^2 \right] \mu^6 e^{g \, - \, (f+1)/b\ell} \end{aligned} \tag{70}$$

which depends only on the dilaton ℓ. In the case of multiple condensates each combination $(F_{U_a} + \overline{F}_{\overline{U}_a})$ gives rise to an equation (67) for the individual condensates u_a, and the solutions (68), (69) and (70) are generalized to

$$\bar{u}_a u_a \, = \, \frac{1}{e^2} \ell \mu^6 e^{g \, - \, (f+1)/b_a \ell} \,, \tag{71}$$

$$m_{3/2} = \frac{1}{3} \left\langle |M| \right\rangle = \frac{1}{4} \left\langle \left| \sum_a b_a u_a \right| \right\rangle \,, \tag{72}$$

$$V\left(\ell\right) = \frac{1}{16\ell^2} \left(1 + \ell \frac{\mathrm{d}g}{\mathrm{d}\ell} \right) \left| \sum_a \left(1 + b_a \ell\right) u_a \right|^2 - \frac{3}{16} \left| \sum_a b_a u_a \right|^2 . \tag{73}$$

It is instructive to return to the case of Section 1 in which all nonperturbative corrections are vanishing and see how the run-away behavior is manifest in the linear multiplet formulation. Taking (70) for $f = g = 0$ and $b = b_{E_8} = 30/8\pi^2$ we plot the behavior of the dilaton scalar potential in Planck units in Figure 3. Note that the general behavior is that of Figure 1, but now the asymptotic approach to vanishing coupling occurs for $\ell \to 0$ while the unbounded-from-below direction is for $\ell \to \infty$, which implies strong coupling. From the identity (62) we can derive the necessary and sufficient condition on the function $f(\ell)$ such that $V(\ell)$ in (70) or (73) is bounded from below:

$$f - \ell \, \frac{\mathrm{d}f(\ell)}{\mathrm{d}\ell} \geq -\mathcal{O}(\ell e^{1/b_a \ell}) \quad \text{for} \quad \ell \to 0, \tag{74}$$

$$f - \ell \, \frac{\mathrm{d}f(\ell)}{\mathrm{d}\ell} \geq \quad 2 \qquad\qquad \text{for} \qquad \ell \to \infty \,. \tag{75}$$

It is clear that condition (74) is not at all restrictive, and therefore has no nontrivial implication. On the other hand, condition (75) is quite restrictive; in particular the simple tree-level model with $f = g = 0$ violates this condition – hence the runaway solution. According to our assumption of boundedness for $g(\ell)$ and $f(\ell)$ it must be that ℓ=0 is the only pole of $g \, - \, (f+1)/b\ell$. We therefore recognize a relation between $\langle \bar{u} u \rangle$ and $\langle \ell \rangle$: gauginos condense (i.e., $\langle \bar{u} u \rangle \neq 0$) if and only if the dilaton is stabilized (i.e., $\langle \ell \rangle \neq 0$).

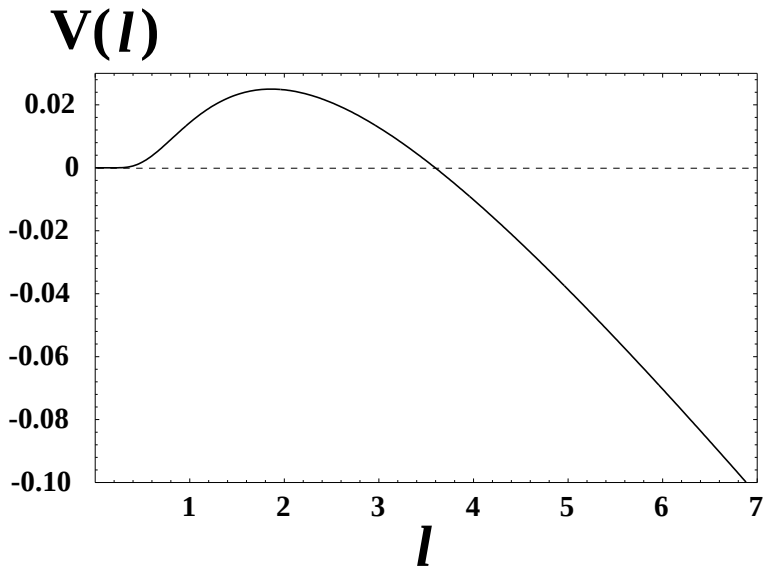

Fig. 3. **Plot of dilaton potential $\mathbf{V}(\ell)$ for the potential of (70) without nonperturbative effects.** The shape of the potential as a function of $\ell = L|_{\theta=\bar{\theta}=0}$ is given with the height of the potential in arbitrary units ($m_{\rm PL} = 1$). For this case we have taken the condensing group to be E_8.

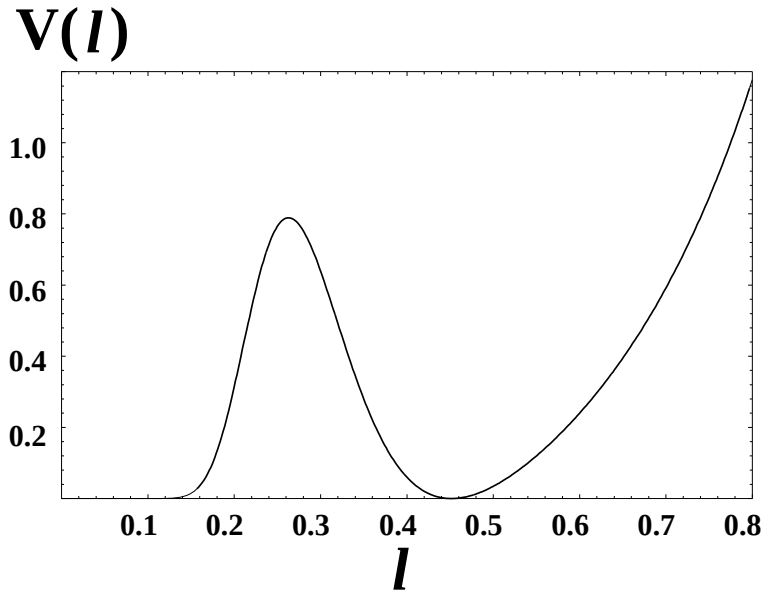

Fig. 4. **Plot of dilaton potential $\mathbf{V}(\ell)$ for the potential of (70) with nonperturbative effects parameterized by (59).** The shape of the potential as a function of $\ell = L|_{\theta=\bar{\theta}=0}$ is given with the height of the potential in arbitrary units ($m_{\rm PL} = 1$). For this case we have taken the condensing group to be E_8 and chosen the parameter $A = 6.92$ in (59).

Let us demonstrate the Kähler stabilization effect by using (59) as our ansatz for $f(\ell)$. Note that the boundedness condition (75) requires $A \geq 2$. We will therefore take $b = b_{E_8}$ and choose $A = 6.92$. The resulting scalar potential for the dilaton is plotted in Figure 4. Again, the plots give the value of the dilaton potential in arbitrary units; the absolute size of the potential values have been rescaled to more easily exhibit the shape of the potential. For large values of ℓ the potential is now bounded. A minimum occurs for $\langle \ell \rangle \simeq 0.45$ for which $f(\ell = 0.45) \simeq 1$ and thus $g^2_{\rm STR} \simeq 1/2$ from (64). The choice $A = 6.92$ was made so as to ensure $\langle V(\ell) \rangle = 0$ at the minimum – the other properties of the low-energy phenomenology are generally insensitive to the precise choice.

It has occasionally been argued that utilizing such nonperturbative corrections in the Kähler potential should be avoided on the grounds that the absence of holomorphy arguments implies that we have (as yet) no real theoretical control over the magnitude of these effects. We would like to emphasize that no matter what mechanism generates the corrections to the Kähler potential, conditions (74) and (75) tell us precisely how to modify the theory so as to have a stabilized dilaton (and therefore broken supersymmetry by the argument of above). These arguments are quite general. Provided (75) holds the potential of the modified model in the strong-coupling regime is always bounded from below, and in most cases rises as ℓ increases. Joining the weak-coupling behavior of the modified model to its strong-coupling behavior therefore strongly suggests that its potential has a nontrivial minimum (at $\ell \neq 0$). Furthermore, if this nontrivial minimum is global, then the dilaton is stabilized. Though we are unable to study the exact Kähler potential at present, it is nevertheless interesting to study models with reasonable Kähler potentials for the purpose of illustrating the significance of conditions (74) and (75) as well as displaying explicit examples with supersymmetry breaking. Furthermore, as we will see in Sections 3 - 5, these corrections give rise to a distinctive phenomenology that is in many respects beneficial.

2. The interaction of moduli with matter

In the previous section we considered a very simple system of moduli, focusing on those closed-string moduli (or geometric moduli) which determine the gauge couplings of the low-energy effective theory. We saw that the driving mechanism for moduli stabilization and supersymmetry breaking is gaugino condensation. We briefly considered some of the classes of mechanisms available to the effective field theorist for achieving a realistic supersymmetry-breaking minimum. Looking at the theory from the four-dimensional effective field theory point of view allows us to treat many possible constructions at once. However, not all the mechanisms discussed in Section 1 can be operative in the same theory at the same time. These issues depend on the nature of the underlying string construction chosen. Given such a specific construction, however, it is usually possible to do better than the crude picture in Section 1 would indicate. This is because much is known about the

moduli that appear in the four-dimensional effective field theory for most specific constructions – including such important quantities as the moduli dependence of Yukawa couplings and kinetic terms, threshold corrections to gauge couplings, and the existence of certain field theory counterterms descended from the string theory. In this section we will focus on the case of orbifold compactification of weakly-coupled heterotic (WCH) string theory as this is an area in which some of the most concrete examples can be constructed.

2.1. *Introducing the Kähler moduli*

2.1.1. *Modular symmetries*

Orbifold compactifications of heterotic string theory are essentially toroidal backgrounds. Early constructions generally took the compact six-dimensional space to factorize into three two-torii $\mathcal{M} = T^2 \times T^2 \times T^2$, though the current renaissance in orbifold model building has often exploited nonfactorizable compactifications.[79,80,81,82] As mentioned in Section 1, the presence of toroidal geometry implies moduli which dictate the sizes and relative orientations of the torii. These moduli will enjoy various symmetry transformations that leave the spectrum and equations of motion for the low-energy effective theory unchanged. We will refer to these symmetries collectively as *modular symmetries*, though this term is more precisely used in the context of string theory to refer to symmetries of the string worldsheet which relate equivalent surfaces. The symmetry group of target space modular transformations depends on the particular orbifold (or orientifold) background.

Recall from Section 1 that for orbifolds we expect at minimum three untwisted Kähler moduli T^I which describe the size of the three complex planes. Depending on the orbifold action imposed there may be additional Kähler moduli as well as some number of degrees of freedom U^I related to deformations of the complex structure. For example, one of the most studied orbifolds is the $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifold[83,84,85] which has three Kähler moduli $T^I,\ I = 1, 2, 3$ and three complex structure moduli $U^I,\ I = 1, 2, 3$. Another commonly studied example is the Z_3 orbifold,[86,87,88] where there are nine Kähler moduli fields labeled by T^{IJ}, with $I,\ J = 1, 2, 3$. It is common to make the assumption that the off-diagonal components of this matrix are fixed at the string scale and do not correspond to dynamical degrees of freedom in the low energy theory. One thus works with only the diagonal entries $T^I \equiv T^{II}$. Finally, there exist cases such as the (2,2) Abelian orbifolds[1] which gives rise to the low-energy gauge group $E_8 \otimes E_6 \otimes U(1)^2$ in which there are precisely the minimal number of three untwisted Kähler moduli $T^I,\ I = 1, 2, 3$.

Both complex structure and Kähler moduli will generally transform under some set of $SL(2, \mathbf{Z})$ symmetry groups (or their subgroups).[4,5] The $SL(2, \mathbf{Z})$ group acts as follows on a generic single-index modulus M^I as

$$M^I \to \frac{a^I M^I - ib^I}{ic^I M^I + d^I} \ , \qquad a^I d^I - b^I c^I = 1 \ , \qquad a^I, b^I, c^I, d^I \in \mathbf{Z} \ , \tag{76}$$

with an analogous matrix equation for moduli which carry two complex-plane indices. This set of transformations can be generated from the two transformations $M^I \to 1/M^I$ and $M^I \to M^I + i$. From the standard field definition of (4) we note that for Kähler moduli these transformations are generated by the well-known duality transformation $R^I \to 1/R^I$ as well as by discrete shifts of the axionic field b^I_{mn} associated with each of the three complex planes. The Kähler potentials describing these untwisted moduli can be inferred from the dimensional truncation of the ten-dimensional supergravity Lagrangian. At leading order this Kähler potential is simply $K = -\ln V$ where V is the volume of the compact space. Thus

$$K = \sum_I g^I \; ; \qquad g^I = -\ln(T^I + \overline{T}^I) \;\text{ or }\; g^I = -\ln(U^I + \overline{U}^I) \tag{77}$$

for single-index fields, and

$$K = -\ln\det(T^{IJ} + \overline{T}^{IJ}) \tag{78}$$

for the more general case.[89] For more details and examples, the reader is referred to the review of Bailin and Love,[13] and references therein. For the remainder of this section we will choose the simple case of three diagonal Kähler moduli as this is sufficient to illustrate the types of structures one expects in this class of theories.

Note that under an $SL(2,\mathbf{Z})$ transformation (76) we have

$$g^I \to g^I + F^I + \overline{F}^I \; ; \qquad F^I = \ln(icT^I + d) \tag{79}$$

and since the fields T^I have no (classical) superpotential we immediately recognize (79) as a Kähler transformation. The classical effective supergravity action is therefore invariant under (76). This is welcome, since modular invariance is known to be (perturbatively) preserved in string theory;[90,91] that is, the set of transformations (76) on the various T^I and U^I should be symmetries of the low-energy effective Lagrangian to all-loop order in string perturbation theory.[a]

Let us see how this picture is modified by the inclusion of matter fields. We denote chiral superfields of gauge-charged matter by Z^i, with lower-case Latin indices. In orbifold models the Kähler metric for the matter fields arising from untwisted sectors is precisely known. It continues to be given by the volume of the compact space, but in this case the relation in (4) is modified[14,92,93] to

$$2(R^I)^2 = T^I + \overline{T}^I - \sum_i |(Z^i)^I|^2 \; , \tag{80}$$

where the fields $(Z^i)^I$ are identified (upon dimensional reduction) with the components of the 10D gauge fields that project into the compact directions associated

[a]When nonperturbative objects, such as D-branes, are present in the construction these $SL(2,\mathbf{Z})$ symmetries can be absent in the low-energy theory. See, for example, Reference 94.

with each of the T^I. Therefore the Kähler potential for this sector can be immediately identified as

$$K = -\sum_I \ln\left(T^I + \overline{T}^I - \sum_i |(Z^i)^I|^2\right) . \tag{81}$$

For twisted sector matter, the corresponding expressions are not known exactly but only to leading order in the matter fields,[1] though in some cases additional information on higher-order terms can be inferred. Continuing to work in the approximation of only three Kähler moduli (and no complex structure moduli), the various matter Kähler metrics can be summarized in one form

$$K_{i\bar{j}} = \kappa_i(T,\overline{T})\delta_{i\bar{j}} + \mathcal{O}(|Z|^2) \quad \text{with} \quad \kappa_i(T,\overline{T}) = \prod_I (T^I + \overline{T}^I)^{-q_i^I} . \tag{82}$$

The parameter q_i^I is the *modular weight* associated with the field Z^i. For untwisted matter with Kähler potential (81), for example, we have $q_i^I = 1$ for one of the three values for I. It is common in phenomenological studies to treat the three diagonal Kähler moduli as one common (overall) size modulus T. Under this simplification the combined matter/modulus Kähler potential is given by

$$K(T,\overline{T};\, Z,\overline{Z}) = -3\ln(T+\overline{T}) + \sum_i \frac{|Z^i|^2}{(T+\overline{T})^{q_i}} , \tag{83}$$

where $q_i = \sum_I q_i^I$. The modular weights q_i^I of the twisted sector fields can readily be computed in Abelian orbifold theories.[89] They are generally fractional numbers, but the quantity q_i (when q_i^I are summed over all three complex planes) will yield an $\mathcal{O}(1)$ integer.

A matter field Z^i of modular weight q_i^I transforms under (76) as

$$Z^i \to \prod_I (ic^I T^I + d^I)^{-q_i^I} Z^i = \exp(-\sum_I q_i^I F^I) Z^i , \tag{84}$$

or simply $Z^i \to (icT+d)^{-q_i} Z^i$ for one overall Kähler modulus.[b] Writing the Kähler potential (83) as

$$K = \sum_I g^I + \sum_i \exp(\sum_I q_i^I g^I)|Z^i|^2 + \mathcal{O}(Z^4) \tag{85}$$

it is easy to see that the total Kähler potential continues to transform as $K \to K + \sum_I (F^I + \overline{F}^I)$ under (76), or $K \to K + 3(F+\overline{F})$ with $F = \ln(icT+d)$ in the overall modulus case. Therefore the classical symmetry will be preserved provided the superpotential for gauge-charged matter transforms as [33]

$$W \to W\,(icT+d)^{-3} . \tag{86}$$

[b]While (84) is strictly true for untwisted fields, it is possible for fields in twisted sectors with the same modular weight to mix amongst themselves under $SL(2,\mathbf{Z})$ transformations.[97]

To ensure this transformation property the superpotential of string-derived models has a moduli dependence of the form[95,8,96]

$$W_{ijk} = w_{ijk}\,[\eta(T)]^{-2(3-q_i-q_j-q_k)}\,. \tag{87}$$

where $W_{ijk} = \partial^3 W(Z^N)/\partial Z^i \partial Z^j \partial Z^k$ and w_{ijk} is a constant independent of the moduli. The function $\eta(T)$ is the classical Dedekind eta function

$$\eta(T) = e^{-\pi T/12} \prod_{n=1}^{\infty} (1 - e^{-2\pi n T}) \tag{88}$$

and it has a well-defined transformation under (76) given by

$$\eta(T) \to (icT + d)^{1/2}\,\eta(T)\,. \tag{89}$$

The form of (87) can be readily inferred from the requirement of modular invariance under (76) for the classical supergravity Lagrangian.[c] But this requirement does not prevent the multiplication of the effective Yukawa couplings by any function that is truly invariant under modular transformations.[d] Such functions have often been considered, in particular in conjunction with gaugino condensation in a hidden sector.[45] In our study we will not consider the presence of such functions of the Kähler moduli; since they are modular invariant we do not expect them to shift the eventual minimum of the potential away from the self-dual points. We refer the interested reader to the existing literature.[98,99]

Note that the leading (large T or large radius) behavior of (88) is $\exp(-\pi T/12)$. For three untwisted fields, the Yukawa interaction (if allowed by string selection rules) has no modulus-dependence, as can be seen from (87) taking $q_i = 1$ for all fields. Yukawa interactions involving twisted fields where $q_i > 1$, however, will instead come with an exponential *suppression* involving the *vev* of some modulus. The origin of this suppression factor is readily understood: the Yukawa coupling in the twisted field case is the result of stringy nonperturbative effects, specifically world-sheet instantons, which depend on the size of the compact space. Such behavior is common to all string theory models in which chiral fermions are localized at certain fixed points in the compact space and have been useful in efforts toward building a string-theoretic understanding of flavor.[102,103,104] In the BGW model, however, the Kähler moduli are stabilized at self-dual points where $\mathrm{Re}, t \sim 1$ and the exponential factors are therefore not important in the weakly-coupled vacuum.

We have chosen to write the effect of a modular transformation (76) in terms of a rotation at the chiral superfield level in (84). In fact, however, the Kähler transformation of (79) is a continuous *R-transformation* which affects scalars and

[c]To actually see this dependence of the superpotential on the Kähler moduli and Dedekind function emerge from the underlying string theory is not trivial. It involves factoring the level-one Euler characters for $SU(3)$ from the three-point vertex amplitude calculation in string theory.[100,95,101]

[d]Similarly, in the Yang-Mills sector, there may be modular invariant holomorphic functions of the Kähler moduli that appear as a universal threshold correction to the Yang-Mills kinetic function in models with $N = 2$ sectors.[105]

fermions differently. This can be accounted for by assigning the transformation property

$$\theta \to e^{-\frac{i}{2}\sum_I \mathrm{Im}F^I}\theta \tag{90}$$

to the Grassmanian theta-parameter. All fermionic modes (modes with unit chiral weight) in the supersymmetric Lagrangian receive such a chiral rotation, including the gaugino fields of the gauge supermultiplet. These transformations can be written as

$$\lambda_a \to e^{-\frac{i}{2}\sum_I \mathrm{Im}\,F^I}\lambda_a, \quad \chi_A \to e^{\frac{1}{2}\sum_I (i\mathrm{Im}\,F^I - 2q^I_A F^I)}\chi_A\ . \tag{91}$$

Note also that the composite operator $U_a = \mathrm{Tr}(W^\alpha W_\alpha)_a$ introduced in (20) for the confined gauginos will also have a transformation deriving from (91) given by

$$U_a \to e^{-i\sum_I \mathrm{Im}F^I}U_a\ . \tag{92}$$

That (90), (91) and (92) involve phase rotations suggests its interpretation as a type of $U(1)_R$ gauge transformation. This is the guiding principle behind Kähler $U(1)$ supergravity, which was briefly discussed in Section 1 and is outlined in Appendix A.

2.1.2. *General considerations of* $W(S,T)$

Having introduced the classical symmetries of Kähler moduli in weakly-coupled heterotic string theory, we are in position to describe the nature of multi-modulus condensation models. We review them here in part for the sake of historical completeness, but also to provide a foil for the Kähler stabilization case which concludes Section 2. For the treatment of this subsection we will use the chiral formulation of the dilaton.

Prior to the recognition that the set of transformations (76) were perturbatively good symmetries of the low-energy effective Lagrangian, it was common to assume that both the tree-level superpotential of the observable sector W_O and the effective scalar potential generated by gaugino condensation (13) would continue to be independent of the Kähler moduli. In such a situation the effective supergravity model is determined by the functions

$$\begin{aligned} K &= -\ln(S+\overline{S}) - 3\ln(T+\overline{T})\,, \\ W &= W_0 + W(S) = W_0 + e^{-S/b_a}\,, \end{aligned} \tag{93}$$

where W_O is independent of all moduli and we are taking the case of one overall Kähler modulus $T^1 = T^2 = T^3 = T$. The factor of three in the Kähler potential (or equivalently, the presence of three diagonal Kähler moduli) is significant, in that it generates a *no-scale* model for the T-moduli. In particular

$$K_{T\overline{T}}F^T\overline{F}^{\overline{T}} = 3e^K|W|^2 \tag{94}$$

and the final term in (16) is canceled. The potential continues to have a run-away solution to weak-coupling ($\langle s\rangle \to \infty$), but in this case the potential is positive-definite and monotonically decreasing.

A Lagrangian described by (93) is not invariant under (76). Classically the invariance of the Lagrangian can be restored simply by the replacement $W \to W\eta^{-6}(T)$, and this was the approach taken in the earliest treatments of modular-invariant gaugino condensation.[41,106,107] Consider a gaugino condensate-induced effective superpotential of the general form

$$W(S,T) = \Omega(S)\eta^{-6}(T)\,, \quad \text{with} \quad \langle\Omega\rangle \neq 0\,. \tag{95}$$

Minimization of the potential with respect to the field s gives rise to two possible solutions[45]

$$\text{Solution (1)}: \qquad (\Omega_s + K_s\Omega) = 0 \tag{96}$$

$$\text{Solution (2)}: \qquad (s+\bar{s})^2\Omega_{ss} = e^{2i\gamma}\Omega^*\left[2 - 3|(t+\bar{t})G_2\left(t,\bar{t}\right)|^2\right] \tag{97}$$

where $\gamma = \arg(\Omega_s + K_s\Omega)$ and we have introduced the modified Eisenstein function

$$G_2\left(t,\bar{t}\right) = \left(2\zeta(t) + \frac{1}{t+\bar{t}}\right)\,; \quad \zeta(T) = \frac{1}{\eta(T)}\frac{d\eta(T)}{dT}\,. \tag{98}$$

These two solutions imply very different properties for the resulting ground state – and hence for how supersymmetry breaking will ultimately be transmitted to the observable sector. In the first solution it is clear that $\left\langle F^S\right\rangle = 0$, and it can be demonstrated that $\left\langle F^T\right\rangle \neq 0$. Such an outcome is typically called *moduli domination* (or more specifically Kähler moduli domination). The value of the (overall) Kähler modulus *vev* can be computed in these two regimes for an arbitrary function $\Omega(S)$. In the first case the minimum can easily be shown[108] to be $\langle \mathrm{Re}\, t\rangle \simeq 1.23$ and the self-dual points are local maxima. For the second solution the zeroes of the Eisenstein function are indeed minima and $\langle \mathrm{Re}\, t\rangle = 1$.[e]

The situation determined by (96) with $\langle \mathrm{Re}\, t\rangle \simeq 1.23$ has been studied frequently in the context of weakly-coupled heterotic string models – so much so, in fact, that this minimum is sometimes stated to be the *only* outcome possible in modular-invariant treatments of gaugino condensation. This statement, though erroneous, is understandable: achieving solution (97) requires a deviation of the Kähler potential for the dilaton from its tree-level value. If this is achieved, the potential for the Kähler moduli must be re-examined. Consider the scalar potential for (95)[108]

$$V = \frac{1}{(s+\bar{s})(t+\bar{t})^3|\eta(t)|^{12}}\left\{|(s+\bar{s})\Omega_s - \Omega|^2 + 3|\Omega|^2\left[(t+\bar{t})^2|G_2\left(t,\bar{t}\right)|^2 - 1\right]\right\}\,. \tag{99}$$

When (96) holds the first term in braces vanishes. The potential in the Re t direction then has a minimum at $\langle \mathrm{Re}\, t\rangle \simeq 1.23$, as can be seen in the top curve in Figure 5. But let us now imagine that $|(s+\bar{s})\Omega_s - \Omega|^2 = K^{-1}_{s\bar{s}}F^s\overline{F}^{\bar{s}}$ is not vanishing, but

[e]More generally, the Kähler moduli will be stabilized at one of two self-dual points $\langle t_I\rangle = 1$ or $\langle t_I\rangle = \exp\left(i\pi/6\right)$ where the Eisenstein function vanishes.

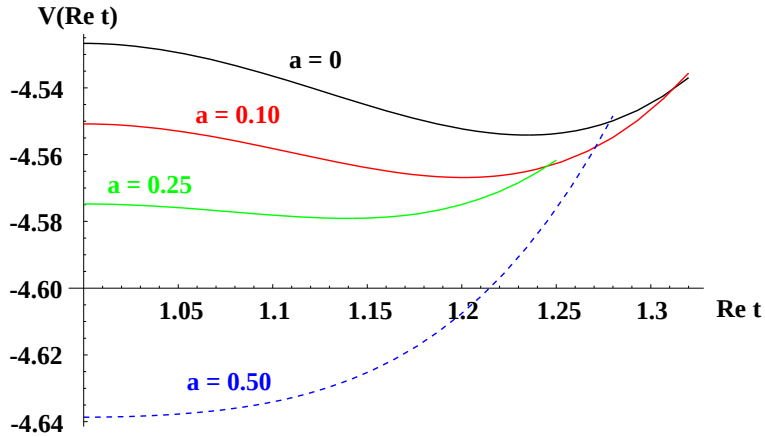

Fig. 5. **Shape of $\mathbf{V(t,\bar{t})}$ in the** Re **t direction for fixed $\langle \mathbf{s}+\bar{\mathbf{s}}\rangle$.** The value of the potential in (99) is given for Re t in the neighborhood of the fixed point at $t=1$. The dilatonic F-term is taken to have a value parameterized by $|(s+\bar{s})\Omega_s-\Omega|^2 = a|\Omega|^2$ with four different values of the parameter a. For the case $a=0$ we recover the result $\langle \mathrm{Re}\, t\rangle \simeq 1.23$. As in earlier plots we have taken $m_{\rm PL}=1$. The value of $V(t,\bar{t})$ for each value of a has been re-scaled independently in order to place each plot on the same figure.

instead is given by some coefficient a times $|\Omega|^2$. The case of (96) is then simply the case of $a=0$. As the size of the (nonvanishing) dilaton F-term increases, the potential for Re t becomes increasingly shallow, as is demonstrated in Figure 5. More importantly, the minimum occurs for values of Re t approaching the self-dual point of $\langle \mathrm{Re}\, t\rangle = 1$. For some critical value (here at $a=0.5$) the Kähler moduli are fixed at their self-dual points. This is the outcome of (97) and, as we will see in Section 2.3, it is precisely the outcome that the BGW model with Kähler stabilization is designed to achieve. Before considering this point, we must look at the transformations (76) beyond the classical level.

2.2. *Anomalies and counterterms*

While the diagonal modular transformations of (76) leave the classical effective supergravity Lagrangian invariant, the form of (91) leads us to expect the symmetry to be anomalous at the quantum level. This is indeed the case. The variation of the one loop Lagrangian under a modular transformation is[109,110]

$$\delta\mathcal{L}_{1-\rm loop} = \frac{1}{64\pi^2}\sum_I \int \mathrm{d}^2\theta \frac{E}{R}\sum_a \alpha_a^I (W^\alpha W_\alpha)_a F^I + \text{ h.c. } , \tag{100}$$

where $F^I = \ln(icT^I + d)$ and the coefficient α_a^I can be computed from the effective field theory[109,65]

$$\alpha_a^I = -C_a + \sum_i \left(1-2q_i^I\right) C_a^i \, . \tag{101}$$

Here C_a and C_a^i are quadratic Casimir operators in the adjoint and matter representations, respectively, and q_i^I are the modular weights of the matter superfields Z^i charged under the group $\mathcal{G}_a$.

Since the set of $SL(2,\mathbf{Z})$ transformations should remain good symmetries to all loop order, we expect some mechanism should exist to cancel (100) and leave the resulting effective Lagrangian invariant once again. Indeed, counterterms must be added to the effective theory which correspond to massive modes of the string theory which have been integrated out. From the point of view of the effective supergravity Lagrangian, as well as the underlying string theory, it is convenient to separate these contributions into two types of counterterms: (1) operators which we may interpret as threshold corrections to gauge kinetic functions and (2) Green-Schwarz (GS) type counterterms which are analogous to the GS anomaly cancelation mechanism in ten-dimensional string theory. While the former are easy to interpret in the chiral formulation of the dilaton, the latter find their natural interpretation in terms of a linear multiplet formalism. We will stress the linear multiplet in what follows.

The GS counterterm is universal in weakly-coupled heterotic string models.[110,111,112] In the simplest case we have

$$\mathcal{L}_{\rm GS} = b_{\rm GS} \int \mathrm{d}^4\theta \, EL \sum_I g^I \tag{102}$$

where the GS coefficient $b_{\rm GS} = C_{\rm GS}/8\pi^2$ can be computed from knowledge of the string spectrum. Note that this Green-Schwarz coefficient $b_{\rm GS}$ is truly universal; it does not depend on the gauge group a or complex plane I which labels (101). This is remarkable, since the anomaly coefficients in (101) seem to depend very strongly on the matter content of each sector of the theory. Nevertheless, the anomaly cancelation condition $b_{\rm GS} = \alpha_a^I \; \forall a, I$ holds for a wide variety of orbifold constructions, including the $\mathbf{Z}_3$ and $\mathbf{Z}_7$ orbifolds.[3,113] In some sense this universality was to be expected, in that the object involved in anomaly cancelation for the Kähler $U(1)$ symmetry is the antisymmetric two-index $b_{\mu\nu}$ of the universal dilaton field. This is not the case in the open string models where multiple closed-string moduli play this role.[114,115]

Even in cases where the universal term is not sufficient to cancel the entire anomaly (that is, where $b_{\rm GS} \neq \alpha_a^I$ for some set of $\{a, I\}$), there is still always some part which is universal to all sectors. We therefore will write

$$\alpha_a^I = -C_{\rm GS} + b_a^I \tag{103}$$

with

$$b_a^I = C_{\rm GS} - C_a + \sum_i \left(1 - 2q_i^I\right) C_a^i \; , \tag{104}$$

thereby separating out the universal contribution. The model-dependent contribution is given by the string threshold corrections.[116] These corrections vanish

completely unless the field T^I corresponds to an internal plane which is left invariant under some orbifold group transformations. This only occurs if an $N = 2$ supersymmetric twisted sector is present. Specifically,

$$\mathcal{L}_{\rm th} = -\sum_I \frac{1}{64\pi^2} \int {\rm d}^4\theta \, \frac{E}{R} \left[\sum_a b_a^I (W^\alpha W_\alpha)_a + \sum_{a'} b_{a'}^I U_{a'} \right] \ln \eta^2(T^I) + {\rm h.c.} \quad (105)$$

where the two gauge sums run over unconfined and confined gauge groups, respectively. The parameters b_a^I vanish for orbifold compactifications with no $N = 2$ supersymmetry sector,[3] such as in the $\mathbf{Z}_3$ and $\mathbf{Z}_7$ orbifolds mentioned above..

The form of (105) is suggestive of a correction to the gauge kinetic functions of the chiral formulation. This is how such terms are normally presented.[117,118,119,43,120,71] For example, modifying the universal gauge kinetic function $f_a = S$ of (11) by the addition of the group-dependent term

$$\delta f_a = -\sum_I \frac{b_a^I}{8\pi^2} \ln \eta^2(T^I) \quad (106)$$

produces the necessary effect to cancel the anomalies, *provided* the Kähler potential for the chiral dilaton is suitably modified. In particular, the universal contribution associated with (102) is incorporated via

$$K(S, \overline{S}) \to -\ln \left[(S + \overline{S}) + b_{\rm GS} G \right] \ , \quad (107)$$

where $G = -\sum_I \ln(T^I + \overline{T}^I)$. Finally, for overall modular invariance to hold, the formerly invariant dilaton must now be made to transform under (76) as

$$S \to S - b_{\rm GS} \sum_I F^I . \quad (108)$$

This manner of implementing the modular invariance requirements in the effective supergravity Lagrangian will produce the same results (to leading order in $g_{\rm STR}$) as the linear multiplet treatment we are about to use. Yet it has only one virtue: retaining the familiar chiral formulation for the dilaton and the gauge kinetic terms. But we feel that the downside of forcing an unphysical Kähler transformation upon the dilaton – and the resulting kinetic mixing between dilaton and Kähler moduli – make the linear multiplet formulation superior.

Furthermore, the actual string calculation prefers the linear multiplet treatment. To see this, consider the actual correction to the gauge coupling (not the gauge kinetic function) as computed from the underlying conformal field theory.[2] The resulting correction is given by

$$\delta \left(\frac{1}{g_a^2} \right) = -\sum_I \frac{b_a^I}{8\pi^2} \ln \left[(T^I + \overline{T}^I) |\eta^2(T^I)|^2 \right] \ ; \quad (109)$$

which is not the real part of a holomorphic quantity. Attempting to identify a holomorphic quantity in (109) to then factor out and place in the gauge kinetic function leads to the complications of (107) and (108), as well as additional confusion that

requires considering the meaning of the Wilsonian versus 2PI effective action for the dilaton.[71]

Finally, the virtue of using Kähler $U(1)$ superspace in this context is also apparent. Recall the general treatment of the coupling of linear multiplets to supergravity in Section 1.3. There it was pointed out that consistency restricts the form of the kinetic Lagrangian in (44) only up to an integration constant in (46) once the Kähler potential $k(L)$ is specified. The Green-Schwarz mechanism utilizes this integration constant with $V = \sum_I g^I$ in (102).[f] The presence of this constant implies an additional invariance of the theory under which the function V transforms as $V(\Phi, \overline{\Phi}) \to V(\Phi, \overline{\Phi}) + H(\Phi) + \overline{H}(\overline{\Phi})$. The presence of a term $\int \mathrm{d}^4\theta L\, V$ in the Lagrangian implies, upon integration by parts in superspace, that terms of the form $H(\Phi) W^\alpha W_\alpha +$ h.c. are produced when this transformation symmetry is applied. In the specific case of (102) we see how the anomalous terms in (100) are thereby naturally canceled with this mechanism.

Since the linear multiplet is itself real the correction in (109) is simple to implement. Taking (102) together with the (tree-level) kinetic Lagrangian and gaugino condensate terms of (66) we arrive at the following combined Lagrangian for models with $b_a^I = 0$

$$\mathcal{L}_{\text{eff}} = \int \mathrm{d}^4\theta\, E\, \{ -2 + f(L) + b_{\text{GS}} L G + b' L \ln(e^{-K} \overline{U} U / \mu^6) \}\,. \tag{110}$$

In the above form the modular anomaly cancelation by the Green-Schwarz counterterm is transparent. The second and third terms in (110) are *not* modular invariant separately, but their sum is modular invariant, which ensures the modular invariance of the full theory. Note that if we consider a situation in which the various threshold corrections b_a^I vanish then anomaly cancelation implies $b' = b_{\text{GS}}$. If we now consider nonvanishing b_a^I then using (43) it is clear that (105) can also be written, upon integration by parts in superspace, as a D-term superspace integral involving the modified linear multiplet. Combining the expression in (110) with (105) we obtain

$$\begin{aligned}\mathcal{L}_{\text{eff}} = \int \mathrm{d}^4\theta\, E\, \{ & -2 + f(L) \\ & + L \left(b' \ln(e^{-K} \overline{U} U / \mu^6) - \sum_I \frac{b_a^I}{8\pi^2} \ln \left[(T^I + \overline{T}^I) |\eta^2(T^I)|^2 \right] \right) \Bigg\}\,,\end{aligned} \tag{111}$$

and the correction of (109) is naturally implemented in the effective theory.

2.3. *The BGW stabilization model*

We now have the basic pieces which make up the BGW model of moduli stabilization. In this section we will gather all the parts to the effective Lagrangian and

[f] In Section 4 we will use this integration constant to implement a Green-Schwarz term for anomalous $U(1)$ gauge factors as well.

exhibit the stabilization of the moduli. We do so for the multi-condensate case, and we will allow for the presence of matter fields in both the hidden and observable sectors. For notational convenience we will introduce for each gaugino condensate a vector superfield L_a such that the gaugino condensate superfields $U_a \simeq (W^\alpha W_\alpha)_a$ are then identified as the (anti-)chiral projections of the vector superfields:

$$U_a = -\left(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R\right) L_a\,, \quad \overline{U}_a = -\left(\mathcal{D}^\alpha\mathcal{D}_\alpha - 8\overline{R}\right) L_a\,. \tag{112}$$

This is a trivial notational generalization of the fundamental definition in (43). Its justification lies in the fact that the individual $L_a|_{\theta=\bar\theta=0}$ turn out to be nonpropagating degrees of freedom and none will appear in the effective theory component Lagrangian. In other words, the dilaton field itself is the lowest component of the field $L = \sum_a L_a$. Similarly only one antisymmetric tensor field (also associated with $L = \sum_a L_a$) is dynamical.

2.3.1. *Elements of the effective Lagrangian*

We continue to allow for only three untwisted $(1,1)$ Kähler moduli T^I as in the previous sections. We also continue to make the assumption that the Kähler potential for the charged-matter/moduli system can be written as

$$K = k\,(L) + \sum_I g^I + \sum_i e^{\sum_I q_i^I g^I}\left|Z^i\right|^2 + \mathcal{O}\left(Z^4\right), \quad g^I = -\ln(T^I + \overline{T}^I), \tag{113}$$

where $k(L)$ is assumed to be of the form given in (61). The relevant part of the complete effective Lagrangian is then

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm KE} + \mathcal{L}_{\rm GS} + \mathcal{L}_{\rm th} + \sum_a \mathcal{L}_{\rm a} + \mathcal{L}_{\rm VYT} + \mathcal{L}_{\rm pot}\,, \tag{114}$$

with the kinetic terms given by (61). The minimal form of the Green-Schwarz counterterm in (102) is sufficient for canceling anomalies associated with modular transformations (76). Any modular-invariant function of the matter chiral superfields may also appear in the integration constant of (46), however. As the exact form of the Green-Schwarz counterterm has not been computed from the underlying string theory, we will allow the possibility that the constant V in (46) involves the entire Kähler potential for chiral fields. We therefore take $\mathcal{L}_{\rm GS}$ to be of the form

$$\mathcal{L}_{\rm GS} = \int {\rm d}^4\theta\, E\, L\, V_{\rm GS}, \tag{115}$$

$$V_{\rm GS} = b_{\rm GS}\sum_I g^I + \sum_i p_i e^{\sum_I q_i^I g^I}\left|Z^i\right|^2 + \mathcal{O}\left(Z^4\right)\,, \tag{116}$$

with the coefficients p_i being as yet undetermined. A string computation of axionic vertices in the presence of nonzero backgrounds for twisted moduli and matter fields is needed to impose further restrictions on $V_{\rm GS}$ (such as the values of the constants p_i).[g] The contribution from possible threshold effects in $\mathcal{L}_{\rm th}$ is given in (105).

[g]The computation which found the "minimal form" of (102) set such background fields to zero.[65]

The terms $\mathcal{L}_{\rm GS}$ and $\mathcal{L}_{\rm th}$ are contributions to the effective supergravity theory that are inherited from the underlying string theory. Their role is to cancel the modular anomaly which arises at the one-loop level (100). To ensure a manifestly modular-invariant theory, however, we must also include operators which *generate* the terms in (100) in the first place. For the confined gauge sector this is already present in the form of the VY superpotential of (25), but we still lack the equivalent expression for the unconfined/light degrees of freedom. This is the role of the $\mathcal{L}_a$ terms. Their specific form in curved superspace is given by[121]

$$\mathcal{L}_a = -\frac{1}{64\pi^2}\int d^4\theta \frac{E}{R}\left(W_\alpha\left[P_\chi B_a\right]W^\alpha\right)_a + \text{h.c.}\,, \tag{117}$$

where P_χ is the chiral projection operator $P_\chi W^\alpha = W^\alpha$, that reduces in the flat space limit to $(16\Box)^{-1}\overline{\mathcal{D}}^2\mathcal{D}^2$. Here we understand a to label *unconfined* gauge groups. The functions B_a are[65]

$$B_a = -\sum_I \alpha_a^I g^I + \left(C_a - \sum_i C_a^i\right)k(L) + 2\sum_i C_a^i \ln\left(1+p_i L\right)\,. \tag{118}$$

Due to the assumed invariance of L under modular transformations it is only the first term in (117) which contributes to the modular anomaly and thus (100) is recovered. The other terms, together with the threshold corrections (105) and Green-Schwarz term, determine the renormalization of the coupling constants g_a at the string scale

$$\begin{aligned} g_a^{-2}(\mu_{\rm STR}) = \Bigg\langle \frac{1+f}{2\ell} - b_a' k(\ell) + \sum_i \frac{C_a^i}{8\pi^2}\ln(1+p_i\ell) \\ - \sum_I \frac{b_a^I}{16\pi^2}\ln\left[|\eta(t^I)|^2(t^I+\overline{t}^I)\right]\Bigg\rangle\,, \end{aligned} \tag{119}$$

where $t^I \equiv T^I|_{\theta=\bar\theta=0}$.

The Lagrangian describing the condensates is of the VYT-type, as described in Section 1.2 above and explicitly written in (25):

$$\mathcal{L}_{\rm VYT} = \frac{1}{8}\int {\rm d}^4\theta\,\frac{E}{R}\sum_a U_a\left[b_a'\ln(e^{-K/2}U_a) + \sum_\alpha b_a^\alpha \ln\Pi_a^\alpha\right] + \text{h.c.}\,. \tag{120}$$

Recall the anomaly-matching conditions (26) that the coefficients b_a' and b_a^α must satisfy:

$$b_a' = \frac{1}{8\pi^2}\left(C_a - \sum_i C_a^i\right), \qquad b_a^\alpha = \sum_{i\in\alpha}\frac{C_a^i}{4\pi^2 d_a^\alpha}\,, \tag{121}$$

where d_a^α is the mass dimension of the corresponding matter condensate superfield Π_a^α. Note the important property that when $d_a^\alpha = 3$ for all Π's charged with respect to the condensing group $\mathcal{G}_a$ we have the identity

$$b_a' + \sum_\alpha b_a^\alpha = b_a \tag{122}$$

with b_a being the beta-function coefficient (6) associated with the coupling for the group $\mathcal{G}_a$. We will make the assumption that $d_a^\alpha = 3$ in what follows below, but the more general case is easily analyzed by a slight modification of the effective Lagrangian.[122]

The final term in (114) is a superpotential term for the matter condensates consistent with the symmetries of the underlying theory

$$\mathcal{L}_{\rm pot} = \frac{1}{2}\int \mathrm{d}^4\theta \frac{E}{R} e^{K/2} W\left(\Pi^\alpha,\, T^I\right) + \text{ h.c. } . \tag{123}$$

We will adopt the simplifying assumption[44] that for fixed α, $b_a^\alpha \neq 0$ for only one value of a. In other words, we assume that each matter condensate is made up fields charged under only one of the confining groups. This is not a necessary requirement, but it will make the phenomenological analysis of the model much easier to perform. We next assume that there are no unconfined matter fields charged under the confined hidden sector gauge groups. This allows a simple factorization of the superpotential of the form

$$W\left(\Pi^\alpha,\, T^I\right) = \sum_\alpha W_\alpha\left(T\right)\Pi^\alpha, \tag{124}$$

where the functions W_α are given by

$$W_\alpha\left(T\right) = c_\alpha \prod_I \left[\eta\left(T^I\right)\right]^{2(q_I^\alpha - 1)} . \tag{125}$$

Here $q_I^\alpha = \sum_i n_i^\alpha q_i^I$ is the effective modular weight for the matter condensate and the Yukawa coefficients c_α, while *a priori* unknown variables, are taken to be $\mathcal{O}\left(1\right)$.

Just as in Section 1.3, we can solve the equations of motion for the auxiliary fields in the theory to eliminate them from the Lagrangian. The equation of motion for $F_{U_a} + \overline{F}_{\overline{U}_a}$ gives a formula for the real part of the gaugino condensates analogous to the simple case of (71):

$$\rho_a^2 = e^{-2\frac{b_a'}{b_a}} e^K e^{-\frac{(1+f)}{b_a \ell}} e^{-\frac{b_{\rm GS}}{b_a}\sum_I g^I} \prod_I \left|\eta\left(t^I\right)\right|^{\frac{4(b_{\rm GS}-b_a)}{b_a}} \prod_\alpha \left|b_a^\alpha/4c_\alpha\right|^{-2\frac{b_a^\alpha}{b_a}} , \tag{126}$$

where we have introduced the notation $u_a = U_a|_{\theta=\bar\theta=0} \equiv \rho_a e^{i\omega_a}$. It is not hard to see that by using (64) and taking $b_a' = b_a = b_{\rm GS}$ we obtain the simple expression in (71) – *i.e.* the expected one-instanton form for gaugino condensation. Expression (126) encodes more information, however, than simply the one-loop running of the gauge coupling which led us to (9) at the beginning of this review. Consider the renormalization group invariant quantity[123]

$$\delta_a = \frac{1}{g_a^2\left(\mu\right)} - \frac{3b_a}{2}\ln\mu^2 + \frac{2C_a}{16\pi^2}\ln g_a^2\left(\mu\right) + \frac{2}{16\pi^2}\sum_i C_a^i \ln Z_a^i\left(\mu\right). \tag{127}$$

Using the above expression it is possible to solve for the scale at which the $1/g^2(\mu)$ term becomes negligible relative to the $\ln g^2(\mu)$ term – effectively looking for the

"all loop" Landau pole for the coupling constant. This scale is related to the string scale by the relation

$$\mu_L{}^2 \sim \mu_{\rm STR}{}^2 e^{-\frac{2}{3b_a g_a^2(\mu)}} \prod_i \left[Z_a^i\left(\mu_{\rm STR}\right) / Z_a^i\left(\mu_L\right)\right]^{\frac{C_a^i}{12\pi^2 b_a}} . \tag{128}$$

Comparing the expression in (128) with that in (126) shows that the two agree provided we identify the wave-function renormalization coefficients Z_a^i with the quantity $|4W_\alpha/b_a^\alpha|^2$. This is precisely what is needed to provide agreement between (127) and (119), indicating that the condensation scale represents the scale at which the coupling becomes strong as would be computed using the so-called "exact" beta-function.

The equation of motion for the auxiliary field F^α of the chiral supermultiplets Π^α gives

$$0 \,=\, \sum_a b_a^\alpha u_a \,+\, 4\pi^\alpha e^{K/2} W_\alpha \quad \forall \;\; \alpha \,, \tag{129}$$

while that for the auxiliary field of supergravity gives

$$M = \frac{3}{4}\left(\sum_a b'_a u_a - 4W e^{K/2}\right) . \tag{130}$$

Using these rules the potential for the fields ℓ and t^I is

$$\begin{aligned} V &= \frac{1}{16\ell^2}\left(v_1 - v_2 + v_3\right), \\ v_1 &= \left(1 + \ell\frac{{\rm d}g}{{\rm d}\ell}\right)\left|\sum_a \left(1 + b_a\ell\right) u_a\right|^2 , \quad v_2 = 3\ell^2\left|\sum_a b_a u_a\right|^2 , \\ v_3 &= \frac{\ell^2}{(1+b\ell)}\sum_I \left|\sum_a d_a(t^I) u_a\right|^2 , \end{aligned} \tag{131}$$

where the quantity $d_a(t^I)$ is

$$d_a(t^I) = \left(b_{\rm GS} - b_a\right)\left(1 + 4\zeta(t^I){\rm Re}\, t^I\right) \tag{132}$$

and the Riemann zeta-function is defined in (98). Note that $d_a(t^I) \propto F^I$ vanishes at the self-dual point $t^I = 1$, for which $\zeta(t^I) = -1/4$ and $\eta(t^I) \approx .77$. Therefore we can immediately conclude that the F-terms for the Kähler moduli vanish at the minimum of the potential. We will return to this important phenomenological property in subsequent sections. After eliminating v_3 in (131) we are left with the same potential as (73).

2.3.2. *Minimizing the scalar potential*

In order to go further and make quantitative statements about the scale of gaugino condensation (and hence supersymmetry breaking) it is necessary to choose a specific form for the nonperturbative effects characterized by $f(L)$ and $g(L)$. For this

example we will choose the form (60), suggested originally by Shenker, and include the first two terms in the summation

$$f\left(L\right)=\left[A_0+A_1/\sqrt{L}\right]e^{-B/\sqrt{L}}\,. \tag{133}$$

With this function it is possible to minimize (131) with vanishing cosmological constant and $\alpha_{\rm str}=0.04$ for $A_0=3.25, A_1=-1.70$ and $B=0.4$ in (133). Other combinations of these parameters can also be employed to stabilize the dilaton at weak coupling with vanishing vacuum energy. In general we would not expect such a truncation as (133) to be valid for all values of L – it need only be a valid parameterization in the neighborhood of $\langle\ell\rangle$ consistent with the weak-coupling limits of (74) and (75).

With the choice of (133) the scale of gaugino condensation can be obtained once the following are specified: (**1**) the condensing subgroup(s) of the original hidden sector gauge group E_8, (**2**) the representations of the matter fields charged under the condensing subgroup(s), (**3**) the Yukawa coefficients in the superpotential for the hidden sector matter fields and (**4**) the value of the string coupling constant at the compactification scale, which in turn constrains the coefficients in (133) necessary to minimize the scalar potential (131).

The above parameter space can be simplified greatly by assuming that all of the matter in the hidden sector which transforms under a given subgroup $\mathcal{G}_a$ is of the same representation, such as the fundamental representation. This is not unreasonable given known heterotic string constructions. In this case the sum of the coefficients b_a^α over the number of condensates can be replaced by one effective variable

$$\sum_\alpha b_a^\alpha \longrightarrow \left(b_a^\alpha\right)_{\rm eff}\;; \qquad \left(b_a^\alpha\right)_{\rm eff}=N_c b_a^{\rm rep}\,. \tag{134}$$

In the above equation $b_a^{\rm rep}$ is proportional to the quadratic Casimir operator for the matter fields in the common representation and the number of condensates, N_c, can range from zero to a maximum value determined by the condition that the gauge group presumed to be condensing must remain asymptotically free. The variable b_a^α can then be eliminated in (126) in favor of $\left(b_a^\alpha\right)_{\rm eff}$ provided the simultaneous redefinition $c_\alpha\longrightarrow\left(c_\alpha\right)_{\rm eff}$ is made so as to keep the final product in (126) invariant. Combined with the assumption of universal representations for the matter fields, this implies

$$\left(c_\alpha\right)_{\rm eff}\equiv N_c\left(\prod_{\alpha=1}^{N_c}c_\alpha\right)^{\frac{1}{N_c}} \tag{135}$$

which we assume to be an $\mathcal{O}\left(1\right)$ number, if not slightly smaller.

From a determination of the condensate value ρ the supersymmetry-breaking scale can be found by solving for the gravitino mass, given by

$$m_{3/2}=\frac{1}{3}\left\langle|M|\right\rangle=\frac{1}{4}\left\langle\left|\sum_a b_a u_a\right|\right\rangle. \tag{136}$$

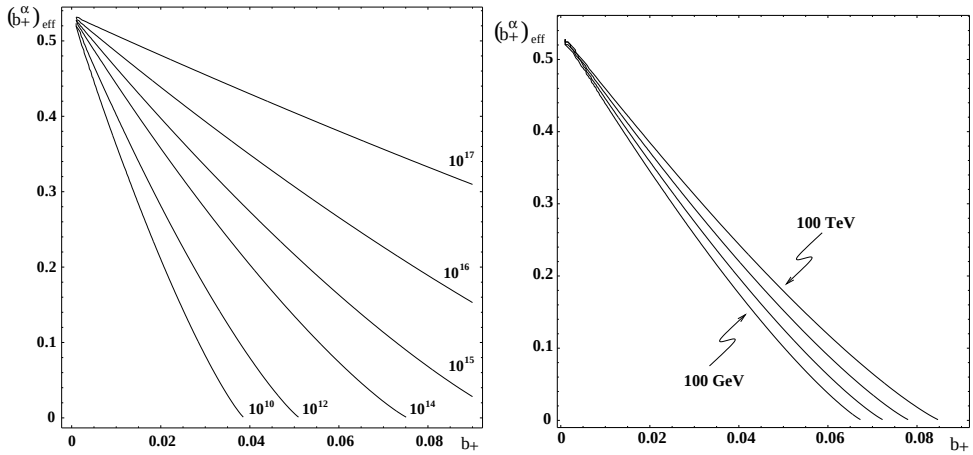

Fig. 6. **Condensation scale and gravitino mass**. Contours give the scale of gaugino condensation in GeV in the left panel and gravitino masses of 10^2 through 10^5 GeV in the right panel for $(c_\alpha)_{\rm eff} = 3$.

In the presence of multiple gaugino condensates the low-energy phenomenology is dominated by the condensate with the largest one-loop beta-function coefficient. For example, the gravitino mass for the case of pure supersymmetric Yang-Mills $SU(5)$ condensation (no hidden sector matter fields) would be 4 TeV. The addition of an additional condensation of pure supersymmetric Yang-Mills $SU(4)$ gauginos would only add an additional 0.004 GeV to the mass. Therefore let us consider the case with just one condensate with beta-function coefficient denoted b_+:

$$m_{3/2} = \frac{1}{4} b_+ \left\langle |u_+| \right\rangle . \tag{137}$$

Now for a given choice of the effective Yukawa coupling $(c_\alpha)_{\rm eff}$ and unification-scale gauge coupling $g_{\rm STR}$ the condensation scale

$$\Lambda_{\rm cond} = (M_{\rm PL}) \left\langle \rho_+^2 \right\rangle^{1/6} \tag{138}$$

and gravitino mass can be plotted in the $\left\{ b_+, \left(b_+^\alpha\right)_{\rm eff} \right\}$ plane. Both quantities are shown in the two panels of Figure 6 for $(c_\alpha)_{\rm eff} = 3$. The importance of including the possible effects of matter condensates is clear from the left panel in Figure 6, in that two different hidden sectors involving condensing groups with the same beta-function coefficient can give rise to different scales of gaugino condensation. This scale is a much weaker function of the relevant parameters than that of the gravitino mass in the right panel of Figure 6.

For fixed values of the unknown Yukawa coefficients c_α, the region of parameter space for which a phenomenologically preferred value of the supersymmetry-breaking scale occurs is a rather limited slice of the entire space available. But this is somewhat deceptive. The variation of the gravitino mass as a function of the Yukawa parameters c_α is shown in Figure 7. On the horizontal axis there are no

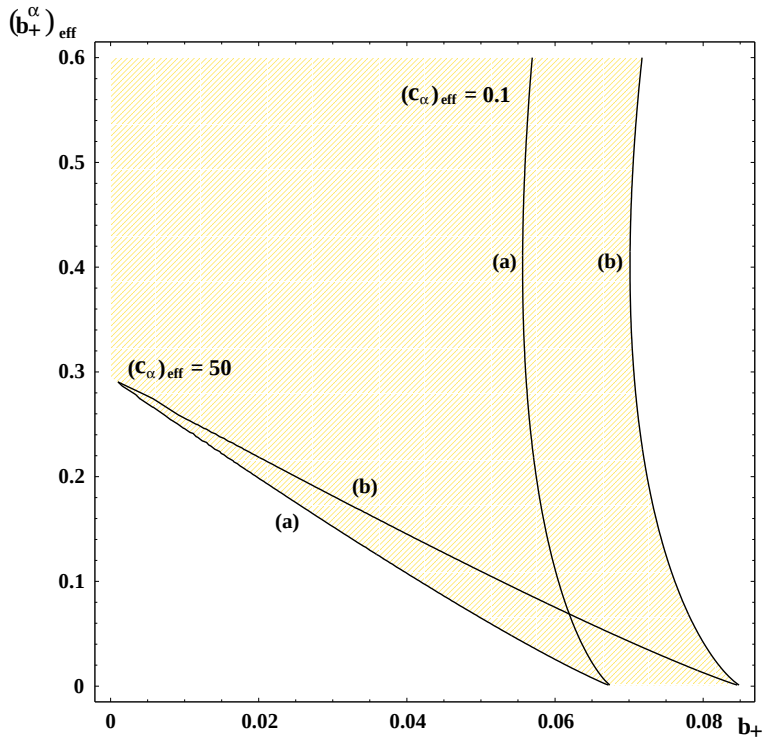

Fig. 7. **Gravitino mass regions as a function of effective Yukawa parameter.** Gravitino mass contours for (a) 100 GeV and (b) 10 TeV are shown for $(c_\alpha)_{\rm eff} = 50$ and $(c_\alpha)_{\rm eff} = 0.1$ with $\alpha_{\rm STR} = 0.04$. The region between the two sets of curves can be considered roughly the region of phenomenological viability.

matter condensates ($b_a^\alpha = 0,\ \forall\alpha$) so there is no dependence on the variable $(c_\alpha)_{\rm eff}$. For very large values of the effective Yukawa parameter the gravitino mass contours approach an asymptotic value very close to the case shown here for $(c_\alpha)_{\rm eff} = 50$. We might therefore consider the shaded region between the two sets of contours as roughly the maximal region of viable parameter space for a given value of the unified coupling at the string scale.

In Figure 6 and 7 we have chosen to show a range in the beta-function parameter b_+ for which $b_+ \lesssim 0.09$. In principle the condensing gauge group can be as large as E_8 in the weakly-coupled heterotic string, for which $b_{E_8} = 30/8\pi^2 = 0.38$. In general we expect the hidden sector gauge group to be a product of subgroups of E_8. The set of all such possible breakings has been computed for Abelian orbifolds:[124,125]

$$\begin{cases} E_7,\ E_6 \\ SO(16),\ SO(14),\ SO(12),\ SO(10),\ SO(8) \\ SU(9),\ SU(8),\ SU(7),\ SU(6),\ SU(5),\ SU(4),\ SU(3) \end{cases} . \tag{139}$$

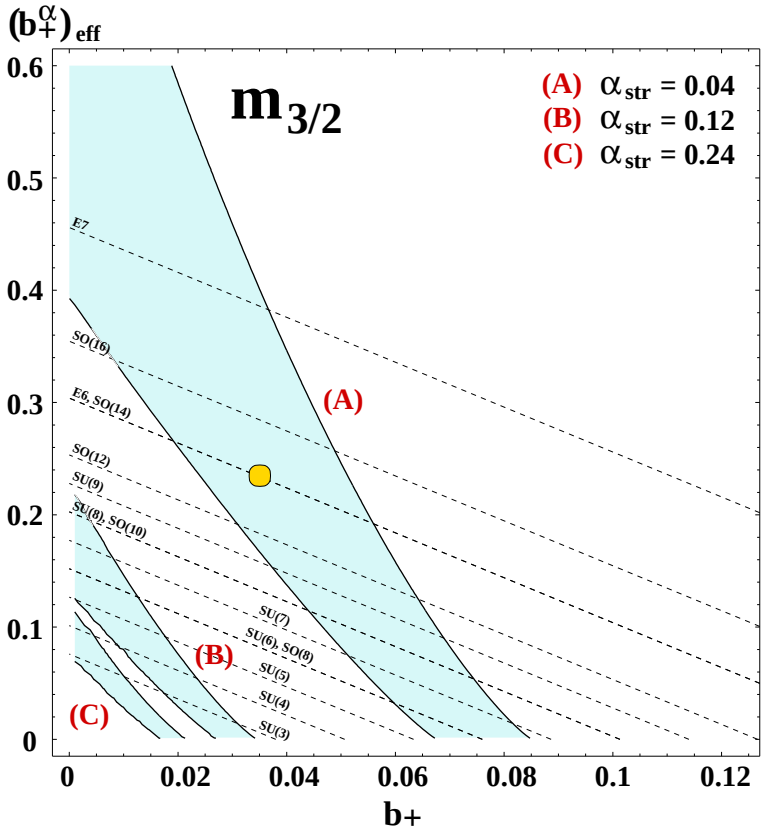

Fig. 8. **Constraints on the hidden sector.** The shaded regions give three different "viable" regions depending on the value of the unified coupling strength at the string scale. The upper limit in each case represents a 10 TeV gravitino mass contour with $(c_\alpha)_{\rm eff} = 1$, while the lower bound represents a 100 GeV gravitino mass contour with $(c_\alpha)_{\rm eff} = 10$. The dot indicates an example point with a condensing E_6 gauge group and 9 **27**'s of matter forming baryonic condensates.

For each of these groups, one can define a line in the $\left\{b_+, \left(b_+^\alpha\right)_{\rm eff}\right\}$ plane via the relations (6), (121) and (122). These lines will all be parallel to one another with horizontal intercepts at the beta-function coefficient for a pure Yang-Mills theory. The vertical intercept will then indicate the amount of matter required to prevent the group from being asymptotically free, thereby eliminating it as a candidate source for the supersymmetry breaking.

In Figure 8 we have overlaid these gauge lines on a plot similar to Figure 7. We restrict the Yukawa couplings of the hidden sector to the more reasonable range of $1 \leq (c_\alpha)_{\rm eff} \leq 10$ and give three different values of the string coupling at the string scale. A typical matter configuration would be represented in Figure 8 by a point on one of the gauge group lines. As each field adds a discrete amount to $(b_a^\alpha)_{\rm eff}$ and the fields must come in gauge-invariant multiples, the set of all such possible hidden

sector configurations is necessarily a finite one. For example, the point in Figure 8 with $\mathcal{G}_c = E_6$ and 9 **27**'s of matter forming baryonic condensates has $b_+ = 3/8\pi^2 \simeq 0.038$ and is indicated in the plot by the circle. Note that one cannot obtain values of b_+ arbitrarily close to zero in practical model building. The number of possible configurations consistent with a given choice of $\{\alpha_{\rm STR}, (c_\alpha)_{\rm eff}\}$ and supersymmetry-breaking scale $m_{3/2}$ is quite restricted. Furthermore, even moderately larger values of the string coupling at unification become increasingly difficult to obtain as it is necessary to postulate a hidden sector with very small gauge group and particular combinations of matter to force the beta-function coefficient to small values.

3. Soft supersymmetry breaking

In this section we will look in greater detail at how supersymmetry is broken by F-terms from various chiral superfields in the theory. Ultimately we wish to make contact with the formalism of Brignole et al.[120] which describes soft supersymmetry breaking in the observable sector in terms of chiral superfields S, T^I and their auxiliary fields. After translating the explicit results of the BGW model from Section 2.3 into this notation, we will generalize to the case of arbitrary supersymmetry breaking in models arising from weakly-coupled heterotic strings. This will allow us to compare the Kähler stabilized case to those of other moduli stabilization regimes in the literature. It will also allow us to exhibit the one-loop corrections to the tree-level soft Lagrangian in all generality. Finally we will return to the specific case of Kähler stabilization in the BGW model to discuss the superpartner spectrum. Further phenomenological consequences of these results will follow in Section 5.

3.1. *Bosonic sector of the BGW model*

Returning to the superspace Lagrangian defined by (114), we can derive the lowest component expression through the means outlined in Appendix A. The result for the bosonic part of the component Lagrangian is given by

$$\begin{aligned}
\frac{1}{e}\mathcal{L}_B \;=\; & -\frac{1}{2}\mathcal{R} \;-\; (1+b\ell)\sum_I \frac{1}{(t^I+\bar{t}^I)^2}\left(\partial^\mu \bar{t}^I\,\partial_\mu t^I - \overline{F}^I F^I\right) \\
& -\frac{1}{16\ell^2}\,(\ell g'+1)\left[4\left(\partial^\mu\ell\,\partial_\mu\ell - B^\mu B_\mu\right) + \bar{u}u - 4e^{K/2}\ell\left(W\bar{u}+u\bar{W}\right)\right] \\
& +\frac{1}{9}\,(\ell g'-2)\left[\overline{M}M - b^\mu b_\mu - \frac{3}{4}\left\{\overline{M}\left(\sum_b b'_b u_b - 4We^{K/2}\right) + \text{ h.c. }\right\}\right] \\
& +\frac{1}{8}\sum_a\left\{\frac{f+1}{\ell} + b'_a\ln(e^{2-K}\bar{u}_a u_a) + \sum_\alpha b^\alpha_a \ln(\pi^\alpha\bar{\pi}^\alpha)\right. \\
& \qquad \left. +\sum_I\left[bg^I - \frac{b^I_a}{4\pi^2}\ln|\eta(t^I)|^2\right]\right\}\left(F_a - u_a\overline{M} + \text{ h.c. }\right)
\end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{16\ell}\sum_a\left[b'_a\left(\ell g'+1\right)\bar{u}u_a-4\ell u_a\left(\sum_\alpha b^\alpha_a\frac{F^\alpha}{\pi^\alpha}+(b'_a-b)\frac{F^I}{2\mathrm{Re}t^I}\right)+\ \mathrm{h.c.}\right]\\
&+\frac{i}{2}\sum_a\left[b'_a\ln(\frac{u_a}{\bar{u}_a})+\sum_\alpha b^\alpha_a\ln(\frac{\pi^\alpha}{\bar{\pi}^\alpha})\right]\nabla^\mu B^a_\mu\ -\ \frac{b}{2}\sum_I\frac{\partial^\mu\mathrm{Im}\,t^I}{\mathrm{Re}\,t^I}B_\mu,\\
&+\sum_{I,a}\frac{b^I_a}{16\pi^2}\left[\zeta(t^I)\left(2iB^\mu_a\nabla_\mu t^I-u_aF^I\right)+\ \mathrm{h.c.}\,\right]\\
&+e^{K/2}\left[\sum_I F^I\left(W_I+K_IW\right)+\sum_\alpha F^\alpha W_\alpha+\ \mathrm{h.c.}\,\right],
\end{aligned}
\tag{140}
$$

where $\ell=L|_{\theta=\bar\theta=0}$, $g'=g'(\ell)=\mathrm{d}g(L)/\mathrm{d}L|_{\theta=\bar\theta=0}$, b_μ and $M=(\overline{M})^\dagger=-6R|_{\theta=\bar\theta=0}$ are auxiliary components of the supergravity multiplet and we have employed the following definitions

$$
\begin{aligned}
\sigma^\mu_{\alpha\dot\alpha}B^a_\mu\ &=\ \frac{1}{2}[\mathcal{D}_\alpha,\mathcal{D}_{\dot\alpha}\,]L_a|_{\theta=\bar\theta=0}\ +\ \frac{2}{3}\ell_a\sigma^\mu_{\alpha\dot\alpha}b_\mu,\quad B^\mu=\sum_a B^\mu_a,\\
u_a\ &=\ U_a|_{\theta=\bar\theta=0}\ =\ -(\overline{\mathcal{D}}^2-8R)L_a|_{\theta=\bar\theta=0},\quad u=\sum_a u_a,\\
\bar{u}_a\ &=\ \overline{U}_a|_{\theta=\bar\theta=0}\ =\ -(\mathcal{D}^2-8R^\dagger)L_a|_{\theta=\bar\theta=0},\quad \bar{u}=\sum_a\bar{u}_a,\\
-4F^a\ &=\ \mathcal{D}^2U_a|_{\theta=\bar\theta=0},\quad -4\overline{F}^a\ =\ \overline{\mathcal{D}}^2\overline{U}_a|_{\theta=\bar\theta=0},\quad F_U=\sum_a F^a,\\
\pi^\alpha\ &=\ \Pi^\alpha|_{\theta=\bar\theta=0}\quad \bar\pi^\alpha\ =\ \overline{\Pi}^\alpha|_{\theta=\bar\theta=0}\\
-4F^\alpha\ &=\ \mathcal{D}^2\Pi^\alpha|_{\theta=\bar\theta=0},\quad -4\overline{F}^\alpha\ =\ \overline{\mathcal{D}}^2\overline{\Pi}^\alpha|_{\theta=\bar\theta=0},\\
t^I\ &=\ T^I|_{\theta=\bar\theta=0},\quad -4F^I\ =\ \mathcal{D}^2T^I|_{\theta=\bar\theta=0},\\
\bar{t}^I\ &=\ \overline{T}^I|_{\theta=\bar\theta=0},\quad -4\overline{F}^I\ =\ \overline{\mathcal{D}}^2\bar{T}^I|_{\theta=\bar\theta=0}\,.
\end{aligned}
\tag{141}
$$

The auxiliary fields in the bosonic Lagrangian can be eliminated via their equations of motion. For the auxiliary fields of the supergravity multiplet these are simply

$$
b_\mu\ =\ 0,\qquad M\ =\ \frac{3}{4}\left(\sum_a b'_au_a-4We^{K/2}\right),
\tag{142}
$$

while those associated with the chiral superfields give

$$
\begin{aligned}
F^I\ &=\ \frac{\mathrm{Re}\,t^I}{2(1+b\ell)}\left\{\sum_a\bar{u}_a\left[(b-b'_a)+\frac{b^I_a}{2\pi^2}\zeta(\bar{t}^I)\mathrm{Re}\,t^I\right]-4e^{K/2}\left(2\mathrm{Re}\,t^I\overline{W}_I-\bar{W}\right)\right\},\\
0\ &=\ \sum_a b^\alpha_au_a\ +\ 4\pi^\alpha e^{K/2}W_\alpha\quad\forall\ \alpha,\\
\bar{u}_au_a\ &=\ \frac{\ell}{e^2}e^{g-(f+1)/b'_a\ell-\sum_I b^I_ag^I/8\pi^2b'_a}\prod_I|\eta(t^I)|^{b^I_a/2\pi^2b'_a}\prod_\alpha(\pi^\alpha_r\bar\pi^\alpha_r)^{-b^\alpha_a/b'_a},
\end{aligned}
\tag{143}
$$

for F^I, F^α and $F^a + \overline{F}^a$, respectively. In the above we have defined the modular invariant quantity

$$\pi_r^\alpha = \Pi_r^\alpha|_{\theta=\bar\theta=0} = e^{\sum_I q_I^\alpha g^I/2}\Pi^\alpha|_{\theta=\bar\theta=0}\,. \tag{144}$$

With these (140) becomes simply

$$\begin{aligned}\frac{1}{e}\mathcal{L}_B \;=\; &-\frac{1}{2}\mathcal{R} \;-\; (1+b\ell)\sum_I \frac{\partial^\mu \bar t^I\,\partial_\mu t^I}{(t^I+\bar t^I)^2} \;-\; \frac{1}{4\ell^2}\,(\ell g'+1)\,(\partial^\mu\ell\,\partial_\mu\ell - B^\mu B_\mu)\\ &-\sum_a\left(b'_a\omega_a + \sum_\alpha b_a^\alpha\phi^\alpha\right)\nabla^\mu B_\mu^a \;-\; \frac{b}{2}\sum_I \frac{\partial^\mu \mathrm{Im}\,t^I}{\mathrm{Re}\,t^I}B_\mu\\ &+i\sum_{I,a}\frac{b_a^I}{8\pi^2}\left[\zeta(t^I)B_a^\mu\nabla_\mu t^I - \text{ h.c. }\right] - V\,,\end{aligned} \tag{145}$$

with the field definitions

$$u_a = \rho_a e^{i\omega_a}, \quad \pi^\alpha = \eta^\alpha e^{i\phi^\alpha}, \quad 2\phi^\alpha = -i\ln\left(\frac{\sum_a b_a^\alpha u_a \bar W_\alpha}{\sum_a b_a^\alpha \bar u_a W_\alpha}\right) \quad \text{if } W_\alpha \neq 0\,. \tag{146}$$

The final form of the scalar potential is then

$$\begin{aligned}V \;=\; &\frac{(\ell g'+1)}{16\ell^2}\left\{\bar u u + \ell\left[\bar u\left(\sum_a b'_a u_a - 4e^{K/2}W\right) + \text{ h.c. }\right]\right\}\\ &+\frac{1}{16(1+b\ell)}\sum_I\left|\sum_a u_a\left(b - b'_a + \frac{b_a^I}{2\pi^2}\zeta(t^I)\mathrm{Re}\,t^I\right) - 4e^{K/2}\left(2\mathrm{Re}\,t^I W_I - W\right)\right|^2\\ &+\frac{1}{16}\,(\ell g'-2)\left|\sum_b b'_b u_b - 4We^{K/2}\right|^2\,,\end{aligned} \tag{147}$$

which, upon substitution of relations (62) and (122), reduces to (131) when the form (125) is used for the matter condensate superpotential. The expression for the condensate itself, as a function of model parameters, is precisely the expression in (126) in this case.

As indicated in the last chapter, supersymmetry is broken at the minimum of the scalar potential in (147). The result should be the appearance of new soft supersymmetry-breaking terms in the component Lagrangian for the observable sector. These can be identified directly by looking at the scalar potential for the observable sector as determined from the component expansion of (114). This direct approach was, in fact, the method employed in the initial studies of this class of theories.[126,122] Using the one-condensate approximation these tree level soft terms are given by

$$M_a^0 = -\frac{g_a^2(\mu)}{2}\left[\frac{3b_+}{1+b_+\ell} + \sum_i \frac{p_i C_a^i}{4\pi^2 b_+\,(1+p_i\ell)}\right]m_{3/2}\,, \tag{148}$$

and

$$A_{ijk} = \frac{\bar{u}_+}{4}\left[\frac{b_+}{1+b_+\ell}\frac{(p_i-b_+)}{(1+p_i\ell)}\right] + (i\to j) + (i\to k)\,, \tag{149}$$

$$(M_i^0)^2 = \frac{|u_+|^2}{16}\left(\frac{p_i-b_+}{1+p_i\ell}\right)^2\,, \tag{150}$$

where p_i is the possibly nonvanishing coupling of the chiral matter to the Green-Schwarz term in (116). To obtain these it was necessary to use (62) and the vacuum condition $\langle V\rangle = 0$ in (131).

A great deal of simplification is possible, particularly in finding the one-loop corrected values of these expressions, in circumstances in which the supersymmetry breaking is associated with nonvanishing F-terms for chiral superfields.[127] This simplification was exploited by Brignole et al. to systematize possible patterns of soft supersymmetry breaking in a wide class of string-inspired models.[120,128] In this parameterization supersymmetry-breaking effects are computed in a context in which closed-string moduli (such as the dilaton, Kähler moduli and complex structure moduli) are represented by chiral superfields, and each is allowed to participate in supersymmetry breaking via nonvanishing auxiliary field *vevs*. Attempting to use this parameterization in the present case immediately gives rise to an apparent contradiction. We are here working in a context in which we imagine no complex structure moduli. The Kähler moduli have vanishing F^I at the minimum of the scalar potential. This leaves only the dilaton to do the job of communicating the supersymmetry breaking of the hidden sector to the fields of the observable sector – a situation commonly referred to as *dilaton domination*. However, the dilaton multiplet in the linear formulation has no auxiliary field! What plays the role of nonvanishing $\langle F^S\rangle$ in our case?

3.2. *Moduli as messengers of supersymmetry breaking*

The key to understand the apparent paradox can be found in the modified linearity conditions of (43) and the resulting F-term expressions in (55). The equation of motion for $F^{U_a} + \overline{F}^{\overline{U}_a}$ generates the expression for the condensate in (143). In a sense, the lowest component of the condensate (which is acquiring a vacuum expectation value at the minimum of the scalar potential) plays the role of "auxiliary field" for the dilaton in the modified linear multiplet. Let us see more explicitly how this comes about.

In the presence of a (nonperturbatively induced) potential for the dilaton, the tree level scalar Lagrangian for the condensate/moduli sector takes the form

$$\mathcal{L}_{\rm scalar} = -\sum_\alpha \frac{\partial_\mu t^I \partial^\mu \bar{t}^I}{(t^I+\bar{t}^I)^2} - \frac{k'(\ell)}{4\ell}\partial_\mu\ell\partial^\mu\ell - \frac{\ell}{k'(\ell)}\partial_\mu a\partial^\mu a - V\,, \tag{151}$$

where the axion a is related to the two-form $b_{\mu\nu}$ of the linear multiplet by a duality

transformation which follows from (1), (3) and (38):

$$\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\partial_\nu b_{\rho\sigma} = -\frac{2\ell}{k'(\ell)}\partial^\mu a\,. \tag{152}$$

The scalar potential V can be cast in the following suggestive form

$$V = \sum_\alpha \frac{1}{(t^I+\bar{t}^I)^2}F^\alpha\overline{F}^\alpha + \frac{\ell}{k'(\ell)}F^2 - \frac{1}{3}M\overline{M} \tag{153}$$

provided we make the identification

$$F = \frac{k'(\ell)}{4\ell}f(\ell,t^I,z^i) \tag{154}$$

where $f(\ell,t^I,z^i)$ is a complex but *nonholomorphic* function of the scalar fields. For example in the class of models being considered here

$$f(\ell,t^I,z^i) = -\sum_a(1+\ell b_a)\bar{u}_a \approx -(1+\ell b_+)\bar{u}_+, \tag{155}$$

where $\bar{u}_a(\ell,t^I,z^i)$ is the value of the gaugino condensate for hidden gauge group $\mathcal{G}_a$. It is the function F, defined via (154), that will play the role of F^S in this model.

To exhibit this connection, consider the variable $x(\ell) = 2g_{\rm STR}^{-2}(M_{\rm STR})$. This variable can be related to the function $k(\ell)$ via the differential equations

$$k'(\ell) = -\ell x'(\ell), \quad \partial\ell = -\frac{\ell}{k'(\ell)}\partial x\,, \tag{156}$$

from which we derive the following relations

$$\begin{aligned}
&\frac{\partial k(x)}{\partial x} = k'(\ell)\frac{\partial\ell}{\partial x} = -\ell, \quad \frac{\partial^2 k(x)}{\partial x^2} = -\frac{\partial\ell}{\partial x} = \frac{\ell}{k'(\ell)}\,,\\
&\frac{k'(\ell)}{4\ell}\partial_\mu\ell\partial^\mu\ell = \frac{\ell}{4k'(\ell)}\partial_\mu x\partial^\mu x = \frac{1}{4}\frac{\partial^2 k(x)}{\partial x^2}\partial_\mu x\partial^\mu x\,.
\end{aligned} \tag{157}$$

We would like to recast (151) into the standard form we expect for a theory of only chiral superfields:

$$\begin{aligned}
\mathcal{L}_{\rm scalar} &= -\sum_N K_{N\overline{N}}\left(\partial_\mu z^N\partial^\mu\bar{z}^{\overline{N}} + F^N\overline{F}^{\overline{N}}\right) + \frac{1}{3}M\overline{M}\,,\\
K &= k\,(s+\bar{s}) + K(t^I,\bar{t}^I) + \sum_i \kappa_i|z^i|^2\,,
\end{aligned} \tag{158}$$

where we now let the index N run over the chiral dilaton, Kähler moduli and gauge-charged matter. This can be accomplished by setting $x = s+\bar{s}$ and $a = \mathrm{Im}\,s$ in (157) provided we identify $F = F^S$ and $k_{s\bar{s}} = \ell/k'(\ell)$. These relations change slightly when we include a Green-Schwarz counterterm,[129] but the basic correspondence remains the same.

We therefore conclude that it is the dilaton that acts as the messenger of supersymmetry breaking in this model, albeit in an indirect way. Since the operators which connect this field to those of the observable sector involve one inverse power

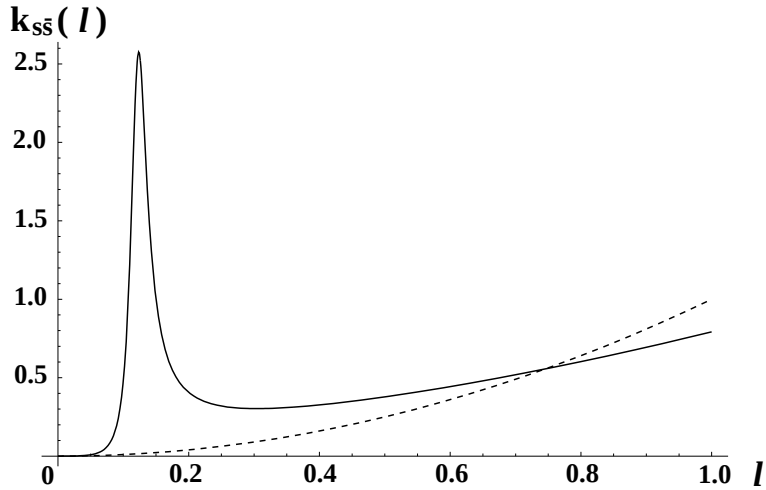

Fig. 9. **Effective metric for the chiral dilaton field.** The effective metric $k_{s\bar{s}}$ of (159) is plotted as a function of ℓ for the tree-level case (dashed line) and the case with nonperturbative corrections (solid line). The nonperturbative corrections produce a large maximum in the effective metric near the minimum of $V(\ell)$.

of the Planck scale (or, strictly speaking, the string scale) this is rightly called an instance of "gravity mediation" as in the general parlance. From (157) we have the desired translation from the linear multiplet to the chiral multiplet notation

$$\langle k_s \rangle = -\ell \qquad\qquad \langle k_{s\bar{s}} \rangle = \frac{\ell^2}{1 + \ell g'(\ell)} \tag{159}$$

which is a property of the vacuum state of the theory. The key feature of (159) is the deviation of the dilaton Kähler metric from its tree level (perturbative) value.

The importance of this fact can be appreciated from another direction. Taking the expression of (126), let us assume the nonperturbative superpotential generated by gaugino condensation for the *chiral* dilaton would be of the form of (13): $W(S) \propto e^{-S/2b_+}$ with b_+ being the largest beta-function coefficient among the condensing gauge groups of the hidden sector. From the equations of motion (15) one immediately sees that requiring the potential in (16) to vanish at the minimum will require that

$$(k^{s\bar{s}}) \left| k_s - \frac{1}{2b_+} \right|^2 = 3 \quad \rightarrow (k^{s\bar{s}})^{-1/2} = \sqrt{3} \frac{2b_+}{1 - 2b_+ k_s}\,, \tag{160}$$

where we choose to keep the precise form of the derivatives of the dilaton Kähler potential unspecified. The condition in (160) is independent of the means by which the dilaton is stabilized and is a result merely of requiring a vanishing vacuum energy in the dilaton-dominated limit. This is sometimes referred to as the *generalized* dilaton-domination scenario.[130]

The constraint of (160) is precisely what is engineered by the nonperturbative correction $f(\ell)$ and $g(\ell)$ in (61). For example, the explicit case of (133) described in Section 2.3 was able to achieve (160) with $\langle k_s \rangle = -g^2_{\rm STR}/2$. For the parameter set $\{A_0 = 3.25,\, A_1 = -1.70,\, B = 0.4\}$ the tree-level dilaton metric $k^{\rm tree}_{s\bar{s}} = \ell^2$ and corrected dilaton metric of (159) are plotted as function of ℓ in Figure 9, for arbitrary units ($m_{\rm PL} = 1$). The effect of the nonperturbative corrections is to generate a nontrivial feature in the metric near the minimum of the potential. Here this peak occurs at $\langle \ell \rangle \simeq 0.12$, for which $g^2_{\rm STR} \simeq 0.5$. The presence and importance of such a feature in the dilaton metric has been noted in this scenario by other authors.[130,131]

It is helpful to parameterize the departure that (160) represents from the tree level Kähler metric $\left\langle (k^{\rm tree}_{s\bar{s}})^{1/2} \right\rangle = \langle 1/(s+\bar{s}) \rangle = g^2_{\rm STR}/2 \simeq 1/4$ by introducing the phenomenological parameter

$$a_{\rm np} \equiv \left(\frac{k^{\rm tree}_{s\bar{s}}}{k^{\rm true}_{s\bar{s}}} \right)^{1/2} \tag{161}$$

so that the auxiliary field of the dilaton chiral supermultiplet can be expressed as

$$F^S = \sqrt{3} m_{3/2} (k_{s\bar{s}})^{-1/2} = \sqrt{3} m_{3/2} a_{\rm np} (k^{\rm tree}_{s\bar{s}})^{-1/2}. \tag{162}$$

An important property of (160) is to recognize that the factor of b_+, containing as it does a loop factor, will suppress the magnitude of the auxiliary field F^S relative to that of the supergravity auxiliary field M through the relation (162). That is, provided $K_s \sim \mathcal{O}(1)$ so that $K_s b_+ \ll 1$ we can immediately see that a Kähler potential which stabilizes the dilaton (while simultaneously providing zero vacuum energy) will *necessarily* result in a suppressed dilaton contribution to soft supersymmetry breaking. This is an unmistakable property of the generalized dilaton-domination scenario when achieved via Kähler stabilization. We will return to these issues in Chapter 5 below.

By virtue of being able to recast the results of Section 3.1 in terms of an effective chiral multiplet, we can employ the results of Kaplunovsky and Louis[127] to immediately write down the soft supersymmetry-breaking parameters of the observable sector. We will give them here to establish our notation and conventions for what follows. The tree level gaugino mass for canonically normalized gaugino fields is simply

$$M^0_a = \frac{g^2_a}{2} F^n \partial_n f^0_a, \tag{163}$$

where f^0_a is the tree-level gauge-kinetic function (here taken to be the chiral dilaton S) and the notation $F^n \partial_n X$ implies summation over all relevant chiral superfields. We define our trilinear A-terms and scalar masses for canonically normalized fields by

$$V_A = \frac{1}{6} \sum_{ijk} A_{ijk} e^{K/2} W_{ijk} z^i z^j z^k + \text{ h.c.}$$

$$= \frac{1}{6}\sum_{ijk} A_{ijk} e^{K/2} (\kappa_i \kappa_j \kappa_k)^{-1/2} W_{ijk} \hat{z}^i \hat{z}^j \hat{z}^k + \text{ h.c. }, \tag{164}$$

where $\hat{z}^i = \kappa_i^{1/2} z^i$ is a normalized scalar field, and by

$$V_M = \sum_i M_i^2 \kappa_i |z^i|^2 = \sum_i M_i^2 |\hat{z}^i|^2, \tag{165}$$

where the function κ_i is the generalization of the specific case in (82).

The precise form of the bilinear B-terms depends on how the supersymmetric μ-parameter of the Higgs potential is generated. This remains an open problem in superstring phenomenology,[10] as fundamental mass parameters are generally zero for fields in the massless spectrum. For the purposes of this section we merely give the general result. Let ν_{ij} be a (possibly field-dependent) bilinear term in the superpotential and let the Kähler potential contain a term of the Giudice-Masiero form[132]

$$K(Z^i, \overline{Z}^{\bar{i}}) = \sum_i \kappa_i |Z^i|^2 + \frac{1}{2}\sum_{ij} \left[\alpha_{ij}(Z^n, \overline{Z}^{\bar{n}}) Z^i Z^j + \text{ h.c. } \right] + \mathcal{O}(|Z^i|^3) \,. \tag{166}$$

Both are sources of masses for fields of the chiral supermultiplets Z^i

$$\mathcal{L}_M = -\sum_{ij} \left[\frac{1}{2} e^{K/2} \left(\psi^i \mu_{ij} \psi^j + \text{ h.c. } \right) + e^K |z^i|^2 \kappa^j |\mu_{ij}|^2 \right] , \tag{167}$$

where the effective μ-parameter is given by

$$\mu_{ij} = \nu_{ij} - e^{-K/2} \left(\frac{M}{3} \alpha_{ij} - \overline{F}^{\bar{n}} \partial_{\bar{n}} \alpha_{ij} \right) . \tag{168}$$

The B-term potential takes the form

$$V_B = \frac{1}{2}\sum_{ij} B_{ij} e^{K/2} \mu_{ij} z^i z^j + \text{ h.c. } = \frac{1}{2}\sum_{ij} B_{ij} e^{K/2} (\kappa_i \kappa_j)^{-\frac{1}{2}} \mu_{ij} \hat{z}^i \hat{z}^j + \text{ h.c. } . \tag{169}$$

With these conventions our tree level expressions are

$$A^0_{ijk} = \left\langle F^n \partial_n \ln(\kappa_i \kappa_j \kappa_k e^{-K} / W_{ijk}) \right\rangle \tag{170}$$

$$B^0_{ij} = \left\langle F^n \partial_n \ln(\kappa_i \kappa_j e^{-K} / \mu_{ij}) + \frac{\overline{M}}{3} \right\rangle \tag{171}$$

$$(M^0_i)^2 = \left\langle \frac{M\overline{M}}{9} - F^n \overline{F}^{\bar{m}} \partial_n \partial_{\bar{m}} \ln \kappa_i \right\rangle . \tag{172}$$

If we specialize now to the case of moduli dependence given by (82), (87) and $f_a = S$, then the tree level gaugino masses (163), A-terms (170) and scalar masses (172) become

$$\begin{aligned} M^0_a &= \frac{g_a^2}{2} F^S \\ A^0_{ijk} &= (3 - q_i - q_j - q_k) G_2(t, \bar{t}) F^T - k_S F^S \\ \left(M^0_i\right)^2 &= \frac{M\overline{M}}{9} - q_i \frac{|F^T|^2}{(t+\bar{t})^2} \,, \end{aligned} \tag{173}$$

where we have taken $F^{T_1} = F^{T_2} = F^{T_3} = F^T$ and dropped the cumbersome brackets $\langle \ldots \rangle$. For the BGW model we have $\langle F^T \rangle = 0$ and an effective $\langle F^S \rangle \neq 0$. The tree-level soft terms are therefore simply

$$M_a^0 = \frac{g_a^2}{2} F^S, \quad A_{ijk}^0 = -k_S F^S, \quad \left(M_i^0\right)^2 = m_{3/2}^2 \,. \tag{174}$$

We can be more explicit by inserting the appropriate "effective" F-term expression for F^S. Starting with the definition in (154) and taking the one-condensate approximation of (155) we can quickly recover the expressions in (148), (149) and (150) for $p_i = 0$.

3.3. *Loop corrections*

The above expressions are insufficient to adequately describe the superpartner spectrum of the BGW model, however. From (161) we note that

$$a_{\rm np} = \sqrt{3} \frac{g_{\rm STR}^2 b_+}{1 - 2k_s b_+} \ll 1 \,, \tag{175}$$

and therefore from (162) we see that if $g_{\rm STR}^2 = 1/2$ we have

$$\left| \frac{F^S}{M} \right| = \left| \frac{F^S}{3m_{3/2}} \right| = \frac{2}{g_{\rm STR}^2} \frac{a_{\rm np}}{\sqrt{3}} \simeq \frac{4a_{\rm np}}{\sqrt{3}} \ll 1 \,. \tag{176}$$

It is evident, therefore, that quantum corrections to soft supersymmetry-breaking terms suppressed by loop factors can, in fact, be comparable in size to these tree-level terms for both the gaugino mass and the trilinear A-term. In particular, loop corrections arising from the conformal anomaly are proportional to M itself and receive no suppression,[a] so they can be competitive with the tree level contributions in the presence of a nontrivial Kähler potential for the dilaton and should be included.[121,133]

In this section we aim to provide sufficient background to justify the form of these one-loop corrections to soft terms, as well as explain some notation we will need for our phenomenological analysis. More complete treatments exist in the literature.[135,121,133,129] We begin with gaugino masses which can be understood as a sum of loop-induced contributions from the field theory point of view, and terms that can be thought of as one loop stringy corrections. The field theory loop contribution is given by [121,134]

$$M_a^1|_{\rm an} = \frac{g_a^2(\mu)}{2} \left[b_a \overline{M} - \frac{1}{8\pi^2} \left(C_a - \sum_i C_a^i \right) F^n K_n - \frac{1}{4\pi^2} \sum_i C_a^i F^n \partial_n \ln \kappa_i \right] . \tag{177}$$

[a] We continue to use the popular, if not particularly precise, name for these "universal" terms.[136] The results presented here were obtained through a direct one-loop computation with supersymmetric regularization and do not appeal to notions of superconformal transformations and their possible anomalies.

As described in Section 2, one expects modular anomaly cancelation to occur through a universal Green-Schwarz counterterm with group-independent coefficient $b_{\rm GS}$ as well as possible string threshold corrections with coefficient b_a^I. The one loop contribution to gaugino masses from both terms are proportional to the auxiliary fields of the Kähler moduli, and must vanish in the vacuum of the BGW class of models. We are therefore left with only the field-theory contribution of (177) for $n = S$. Putting together the tree level gaugino masses with the loop correction gives

$$M_a = \frac{g_a^2(\mu)}{2}\left\{b_a\overline{M} + [1 - b_a' k_s]\, F^S\right\} \tag{178}$$

where the quantity b_a' is defined in (26).

To understand the form of the one-loop A-terms and scalar masses it is necessary to describe how field theory loops are regulated in supergravity, seen as an effective theory of strings. The regulation of matter and Yang-Mills loop contributions to the matter wave function renormalization requires the introduction of Pauli-Villars chiral superfields $\Phi^A = \Phi^i$, $\widehat{\Phi}^i$ and Φ^a which transform according to the chiral matter, anti-chiral matter and adjoint representations of the gauge group and have signatures $\eta_A = -1, +1, +1$, respectively. These fields are coupled to the light fields Z^i through the superpotential

$$W(\Phi^A, Z^i) = \frac{1}{2}W_{ij}(Z^k)\Phi^i\Phi^j + \sqrt{2}\Phi^a\widehat{\Phi}_i(T_aZ)^i + \cdots, \tag{179}$$

where T_a is a generator of the gauge group, and their Kähler potential takes the schematic form

$$K_{\rm PV} = \sum_A \kappa_A^\Phi(Z^N)|\Phi^A|^2, \tag{180}$$

where the functions κ_A are *a priori* functions of the hidden sector (moduli) fields. These regulator fields must be introduced in such a way as to cancel the quadratic divergences of the light field loops – and thus their Kähler potential is determined relative to that of the fields which they regulate.

The PV mass for each superfield Φ^A is generated by coupling it to another field $\Pi^A = (\Pi^i, \widehat{\Pi}^i, \Pi^a)$ in the representation of the gauge group conjugate to that of Φ^A through a superpotential term

$$W_m = \sum_A \mu_A(Z^N)\Phi^A\Pi^A, \tag{181}$$

where $\mu_A(Z^N)$ can in general be a holomorphic function of the light superfields. There is no constraint on the Kähler potential for the fields Π^A as there was for those of the Φ^A. However, if we demand that our regularization preserve modular invariance then we can determine the moduli dependence of $\mu_A(Z^N)$ as a function of the *a priori* unknown modular weight of the regulator fields Π^A. Taking the case

of an overall Kähler modulus T for simplicity we have

$$\begin{cases} \Phi^i: \quad \kappa_i^\Phi = \kappa_i = (T+\overline{T})^{-q_i}, \\ \widehat{\Phi}^i: \quad \hat{\kappa}_i^\Phi = \kappa_i^{-1}, \\ \Phi^a: \quad \kappa_a^\Phi = g_a^{-2} e^K = g_a^{-2} e^k (T+\overline{T})^{-3}, \end{cases} \tag{182}$$

and the supersymmetric mass $\mu_A(Z^N)$ in (181) is then

$$\mu_A(Z^N) = \mu_A(S)\left[\eta(T)\right]^{-2(3-q_A-q'_A)}, \tag{183}$$

with q_A and q'_A being the modular weights of the fields Φ^A and Π^A, respectively. Furthermore, we can postulate the form of the moduli dependence of κ_A for the mass-generating fields

$$\Pi^A: \quad \kappa_A^\Pi = h_A(S+\overline{S})(T+\overline{T})^{-q'_A}. \tag{184}$$

At this point the dilaton dependence in the superpotential term (181) and the functions h_A, as well as the modular weights q'_A of the fields Π^A, are new free parameters of the theory introduced at one loop as a consequence of how the theory is regulated. Given (181) we can extract the Pauli-Villars masses that appear as regulator masses in the logarithms at one loop

$$m_A^2 = e^K (\kappa_A^\Phi)^{-1/2} (\kappa_A^\Pi)^{-1/2} |\mu_A|^2, \tag{185}$$

with $m_A = (m_i, \hat{m}_i, m_a)$ being the masses of the regulator fields $\Phi^i, \widehat{\Phi}^i, \Phi^a$, respectively.

In terms of these regulator masses, the complete one-loop correction to the trilinear A-terms and scalar masses in a general supergravity theory was given elsewhere.[129] Here we will simplify things to the maximum extent in order to proceed to the phenomenology of the model. To that end, let us assume that the functions $\mu_A(Z^N)$ that appear in (181) and (185) are proportional to one overall Pauli-Villars scale $\mu_{\rm PV}$ so that $\hat{\mu}_i = \mu_a = \mu_i \equiv \mu_{\rm PV}$. This scale is presumed to represent some fundamental scale in the underlying string theory. Let us further assume that there is no dilaton dependence of the PV masses so that $h_A(S+\overline{S})$ is trivial and $\mu_{\rm PV}$ is constant. With these simplifications the complete trilinear A-term at one loop is given by

$$\begin{aligned} A_{ijk} = & \frac{1}{3} A^0_{ijk} - \frac{1}{3}\gamma_i \overline{M} - G_2(t,\bar{t}) F^T \left(\sum_a \gamma_i^a p_{ia} + \sum_{lm} \gamma_i^{lm} p_{lm} \right) \\ & - \ln\left[(t+\bar{t})|\eta(t)|^4\right] \left(2\sum_a \gamma_i^a p_{ia} M_a^0 + \sum_{lm} \gamma_i^{lm} p_{lm} A^0_{ilm} \right) \\ & + 2\sum_a \gamma_i^a M_a^0 \ln(\mu_{\rm PV}^2/\mu_R^2) + \sum_{lm} \gamma_i^{lm} A^0_{ilm} \ln(\mu_{\rm PV}^2/\mu_R^2) + \text{cyclic}(ijk), \end{aligned} \tag{186}$$

where we have defined the following combinations of modular weights from the Pauli-Villars sector

$$p_{ij} = 3 - \frac{1}{2}\left(q_i + q_j + q'_i + q'_j\right), \quad p_{ia} = \frac{1}{2}\left(3 - q'_a - \hat{q}'_i + q_i\right) \tag{187}$$

which we will refer to as "regularization weights" in reference to their origin from the PV sector of the theory.

In (186) M_a^0 and A_{ilm}^0 are the tree level gaugino masses and A-terms given in (173) and the parameters γ determine the chiral multiplet wave function renormalization

$$\gamma_i^j = \frac{1}{32\pi^2}\left[4\delta_i^j \sum_a g_a^2 (T_a^2)_i^i - e^K \sum_{kl} W_{ikl}\overline{W}^{jkl}\right]. \tag{188}$$

We have implicitly made the approximation that generational mixing is unimportant and can be neglected in (186), and that motivates the definitions

$$\begin{aligned}
&\gamma_i^j \approx \gamma_i \delta_i^j, \quad \gamma_i = \sum_{jk} \gamma_i^{jk} + \sum_a \gamma_i^a, \\
&\gamma_i^a = \frac{g_a^2}{8\pi^2}(T_a^2)_i^i, \quad \gamma_i^{jk} = -\frac{e^K}{32\pi^2}(\kappa_i\kappa_j\kappa_k)^{-1}\left|W_{ijk}\right|^2.
\end{aligned} \tag{189}$$

The scalar masses are obtained similarly and take the form

$$\begin{aligned}
(M_i)^2 = {} & (M_i^0)^2 + \gamma_i \frac{M\overline{M}}{9} - \frac{|F^T|^2}{(t+\bar{t})^2}\left(\sum_a \gamma_i^a p_{ai} + \sum_{jk}\gamma_i^{jk} p_{jk}\right) \\
& + \left\{\frac{M}{3}\left[\sum_a \gamma_i^a M_a^0 + \frac{1}{2}\sum_{jk}\gamma_i^{jk} A_{ijk}^0\right] + \text{ h.c.}\right\} \\
& + \left\{F^T G_2(t,\bar{t})\left(\sum_a \gamma_i^a p_{ia} M_a^0 + \frac{1}{2}\sum_{jk}\gamma_i^{jk} p_{jk} A_{jk}^0\right) + \text{ h.c.}\right\} \\
& - \ln\left[(t+\bar{t})|\eta(t)|^4\right]\left\{\sum_a \gamma_i^a p_{ia}\left[3(M_a^0)^2 - (M_i^0)^2\right]\right. \\
& \qquad \left. + \sum_{jk}\gamma_i^{jk} p_{jk}\left[(M_j^0)^2 + (M_k^0)^2 + (A_{ijk}^0)^2\right]\right\} \\
& + \sum_a \gamma_i^a\left[3(M_a^0)^2 - (M_i^0)^2\right]\ln(\mu_{\text{PV}}^2/\mu_R^2) \\
& + \sum_{jk}\gamma_i^{jk}\left[(M_j^0)^2 + (M_k^0)^2 + (A_{ijk}^0)^2\right]\ln(\mu_{\text{PV}}^2/\mu_R^2),
\end{aligned} \tag{190}$$

with M_i^0 being the tree level scalar masses of (173).

To put these expressions into a less cumbersome and more suggestive form, we will consider the case where the various regularization weights p_{ia} and p_{jk} can be treated as one overall parameter p. Then inserting the tree level soft terms (173) into (186) and (190) yields

$$A_{ijk} = -\frac{k_s}{3}F^S - \frac{1}{3}\gamma_i\overline{M} - p\gamma_i G_2(t,\bar{t})F^T + \tilde{\gamma}_i F^S\left\{\ln(\mu_{\text{PV}}^2/\mu_R^2) - p\ln\left[(t+\bar{t})|\eta(t)|^4\right]\right\}$$

$$+\text{cyclic}(ijk) \tag{191}$$

$$M_i^2 = \left\{ \frac{|M|^2}{9} - \frac{|F^T|^2}{(t+\bar{t})^2} \right\} \left[1 + p\gamma_i - \left(\sum_a \gamma_i^a - 2\sum_{jk} \gamma_i^{jk} \right) \left(\ln(\mu_{\rm PV}^2/\mu_R^2) - p \ln\left[(t+\bar{t})|\eta(t)|^4 \right] \right) \right]$$
$$+(1-p)\gamma_i \frac{|M|^2}{9} + \left\{ \widetilde{\gamma}_i \frac{MF^S}{6} + \text{ h.c. } \right\} + \left\{ p\widetilde{\gamma}_i G_2\left(t,\bar{t}\right) \frac{\overline{F}^T F^S}{2} + \text{ h.c. } \right\}$$
$$+|F^S|^2 \left[\left(\frac{3}{4} \sum_a \gamma_i^a g_a^4 + k_s k_{\bar{s}} \sum_{jk} \gamma_i^{jk} \right) \left(\ln(\mu_{\rm PV}^2/\mu_R^2) - p \ln\left[(t+\bar{t})|\eta(t)|^4 \right] \right) \right], \tag{192}$$

where $\widetilde{\gamma}_i$ is a shorthand notation for

$$\widetilde{\gamma}_i = \sum_a \gamma_i^a g_a^2 - k_s \sum_{jk} \gamma_i^{jk}. \tag{193}$$

The adoption of one overall regularization weight p makes it possible to identify the quantity $\ln(\mu_{\rm PV}^2/\mu_R^2) - p\ln\left[(t+\bar{t})|\eta(t)|^4\right]$ as a stringy threshold correction to the overall PV mass scale, or effective cut-off, $\mu_{\rm PV}$.[137] Let us make this identification explicit by defining

$$\ln(\mu_{\rm PV}^2/\mu_R^2) - p\ln\left[(t+\bar{t})|\eta(t)|^4\right] \equiv \ln(\tilde{\mu}_{\rm PV}^2/\mu_R^2)\,. \tag{194}$$

Specializing further to the case of generalized dilaton domination we have[b]

$$M_a = \frac{g_a^2\left(\mu_R\right)}{2} \left\{ b_a \overline{M} + \left[1 - b_a' k_s\right] F^S \right\}$$
$$A_{ijk} = -\frac{k_s}{3} F^S - \frac{1}{3}\gamma_i \overline{M} + \tilde{\gamma}_i F^S \ln(\tilde{\mu}_{\rm PV}^2/\mu_R^2) + \text{cyclic}(ijk)$$
$$M_i^2 = \frac{|M|^2}{9} \left[1 + \gamma_i - \left(\sum_a \gamma_i^a - 2\sum_{jk} \gamma_i^{jk} \right) \ln(\tilde{\mu}_{\rm PV}^2/\mu_R^2) \right]$$
$$+ \left\{ \widetilde{\gamma}_i \frac{MF^S}{6} + \text{ h.c. } \right\}. \tag{195}$$

3.4. *Superpartner spectra and fine-tuning*

With the expressions in (195) we now have a starting point for a discussion of the phenomenological implications of the BGW class of models. Here we will pause to look at some of the coarse features of the model, such as the general spectrum of superpartner masses. This will begin with a consideration of the issue of electroweak symmetry breaking and finish, ultimately, with the question of fine-tuning in this model class. In Section 5 we will take a much more detailed look at certain aspects of the model phenomenology, after we have considered how the supersymmetry breaking pattern and spectrum can change in the presence of anomalous $U(1)$ factors in Section 4.

[b]We have dropped terms of $\mathcal{O}\left(1/(16\pi^2)^3\right)$ in the scalar masses.

First let us consider the issue of overall mass scales. Returning to the expression in (176) we can see that we expect a *suppression of gaugino masses relative to scalar masses*. This is a key feature of this class of models. It can be seen explicitly by comparing (178) to $m_{3/2}^2$ using (176), for which we obtain at tree level

$$\left|\frac{M_a}{M_i}\right| = \sqrt{3}a_{\rm np}\left(\frac{g_a(\mu_R)}{g_{\rm STR}}\right)^2 . \tag{196}$$

From the definition of $a_{\rm np}$ in (175) it is clear that gaugino masses will be suppressed by a loop factor relative to scalar masses. If the chargino mass must be of order 100 GeV or higher to avoid direct search constraints, this implies that the typical size of scalars in this theory must be at least a few TeV. This model is therefore a manifestation of "loop-split" supersymmetry.[138] We will see below that this splitting is generally welcome phenomenologically, though it exacerbates certain other problems. The first of these issues is electroweak symmetry breaking (EWSB).

To adequately describe the physical masses of the superpartners in this class of theories it is necessary to consider how electroweak symmetry is broken. Dynamical breaking of the symmetry is possible is the context of the MSSM provided (a) a large top Yukawa is present, (b) there is a large range of energies between the scale of supersymmetry breaking and the electroweak scale, and (c) a supersymmetric Higgs mass (or μ-term) can be generated of the appropriate size. All conditions are compatible with the requirements and constraints of the BGW class of models, but the physics of the resulting low-energy theory depends very much on how condition (c) is obtained. For example, it can be demonstrated[139] that if the μ-parameter arises as a fundamental parameter in the superpotential and is independent of the moduli, then it is extremely difficult to achieve EWSB and a sufficiently large top quark mass in this class of theories. This is a result of the form of the supersymmetry-breaking bilinear B-term, which takes the value $B = 2m_{3/2}$ in this scenario.

Such a fundamental μ term generally would not arise from string theory, however, as the Higgs doublets are part of the massless spectrum of the theory. Instead we anticipate that the μ-parameter is dynamically generated, either via the Giudice-Masiero mechanism[132] or through the spontaneous breaking of certain additional symmetries via the *vev* of some field which is neutral under the Standard Model.[140,141] In these cases the B-parameter may take a variety of values, depending on the model. In particular, for the case of dynamical generation of the μ-parameter via singlet *vevs* with $W_\mu = \lambda_X \langle X\rangle H_u H_d$, the effective B-term is in fact a trilinear A-term. The constraints on the model arising from electroweak symmetry breaking are weaker in these more realistic cases.

For the sake of the present discussion, let us use the *requirement* of proper electroweak symmetry breaking to *determine* the values of μ and B at the electroweak scale in the usual manner. In other words, we compute the one-loop corrected effective potential $V_{1-\rm loop} = V_{\rm tree} + \Delta V_{\rm rad}$ at the electroweak scale and determine the

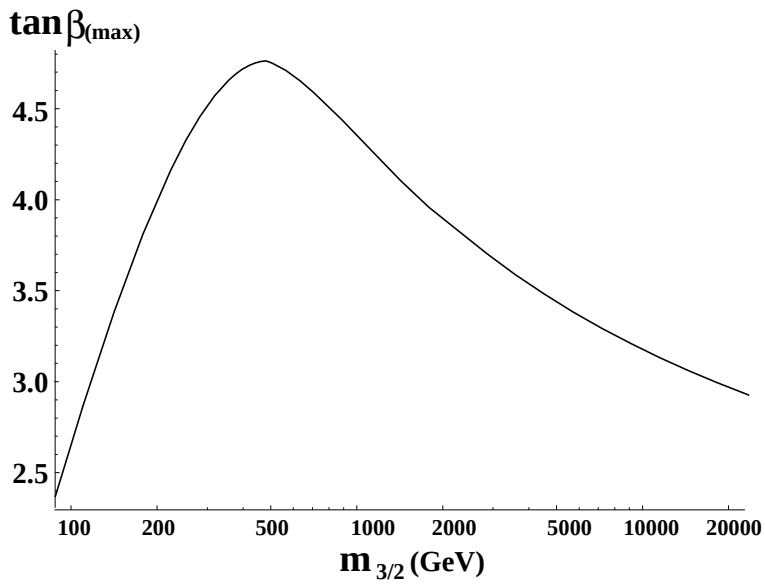

Fig. 10. **A typical constraint on** $\tan\beta$ **from the requirement of proper EWSB.** The maximum value of $\tan\beta$ consistent with EWSB and positive squark masses is displayed as a function of the gravitino mass. For this figure we have taken $b_+ = 0.08$ and $(c_\alpha)_{\rm eff} = 1$.

effective mu-term $\bar{\mu}$ via

$$\bar{\mu}^2 = \frac{\left(m_{H_d}^2 + \delta m_{H_d}^2\right) - \left(m_{H_u}^2 + \delta m_{H_u}^2\right)\tan\beta}{\tan^2\beta - 1} - \frac{1}{2}M_Z^2\,. \tag{197}$$

In equation (197) the quantities δm_{H_u} and δm_{H_d} are the second derivatives of the radiative corrections $\Delta V_{\rm rad}$ with respect to the up-type and down-type Higgs scalar fields, respectively. For the numerical results which follow we will include the effects of all third-generation particles. If the right hand side of equation (197) is positive then there exists some initial value of μ at the high-scale which results in correct electroweak symmetry breaking with $M_Z = 91.187$ GeV.

Though all scalar masses (including those of the two Higgs doublets) in the BGW class will generally have large masses at the boundary condition scale, it is well known[142] that the Yukawa structure of the MSSM is such that the up-type Higgs soft mass-squared $m_{H_u}^2$ is typically driven to negative values nevertheless. Dynamical EWSB is therefore robust in this model. However, the large scalars induce large corrections $\delta m_{H_u}^2$ and $\delta m_{H_d}^2$ which can destabilize the EWSB minimum and make $\bar{\mu}^2 < 0$. The BGW model (with the GS coupling of the matter fields p_i in (116) taken to vanish) is therefore a "focus-point" model[143] over most of its viable parameter space. The various requirements outlined above tend to push the value of $\tan\beta$ required to solve (197) to small values. This is shown in Figure 10 for the case of $b_+ = 0.08$ and $(c_\alpha)_{\rm eff} = 1$ (which we will assume from here onwards). There

is some mild dependence of the maximum value of $\tan\beta$ on these parameters, as well as on the choice of top quark pole mass assumed. Generally speaking, though, $\tan\beta \lesssim 10$ for most of the model parameter space.

Returning to the soft supersymmetry-breaking parameters, the relation (196) implies that tree-level contributions to gaugino masses arising from the effective dilaton auxiliary field $\langle F^S \rangle$ will be of roughly the same size as those one-loop corrections proportional to the gravitino mass. There is one unique contribution proportional to the auxiliary field of supergravity, which is the first term in the gaugino mass expression in (195). These terms imply a splitting in the gaugino soft masses which will depend on the relative sizes of the beta-function coefficient b_a for the Standard Model gauge group $\mathcal{G}_a$ and that of the largest beta-function coefficient b_+ for the condensing product group. Such an outcome is not in conflict with the possibility of gauge coupling unification in this theory. Schematically, the gauge kinetic function in the superspace Lagrangian density is replaced at one loop by the expression

$$\mathcal{L} \sim \int \mathrm{d}^2\theta f_a \left(W^\alpha W_\alpha\right)_a \to \int \mathrm{d}^2\theta \left(S + \frac{1}{16\pi^2} X_a\right) \left(W^\alpha W_\alpha\right)_a \ , \tag{198}$$

where both objects S and X_a obtain $\mathcal{O}(1)$ *vevs* for their lowest scalar components (thereby producing only small corrections to gauge coupling unification). But only X_a receives an $\mathcal{O}(1)m_{3/2}$ auxiliary field *vev*; for the auxiliary field of the dilaton this auxiliary field *vev* is suppressed by a loop factor relative to the gravitino mass. The scenario we have just described, with tree-level and loop-level contributions to gaugino masses of about the same magnitude, has recently re-appeared in a number of guises.[144,145,146] Many of these examples have been engineered to provide (196), and nonuniversal gaugino masses. In the BGW class of heterotic models it is an automatic feature of Kähler stabilization. When this property arises many virtuous phenomenological properties follow, which we will investigate below.

To study the parameter space of the BGW model it is inconvenient to work in the space of hidden sector parameters $\left\{(c_\alpha)_{\rm eff}\,, \left(b_+^\alpha\right)_{\rm eff}\,, b_+\right\}$ as this defines a very narrow region in Figures 6, 7 and 8. But we note that the values of $(c_\alpha)_{\rm eff}$ and $(b_a^\alpha)_{\rm eff}$ appear only indirectly through the determination of the value of the condensate $\langle \rho_+^2 \rangle$ in (126). It is thus convenient to cast all soft supersymmetry-breaking parameters in terms of the values of b_+ and $m_{3/2}$ using equation (137).[147] While the gravitino mass itself is not strictly independent of b_+, it is clear from Figure 7 that we are guaranteed of finding a reasonable set of values for $\left\{(c_\alpha)_{\rm eff}\,, \left(b_+^\alpha\right)_{\rm eff}\right\}$ consistent with the choice of b_+ and $m_{3/2}$ provided we scan only over values $b_+ \lesssim 0.1$ for weak string coupling. This transformation of variables allows the slice of parameter space represented by the contours of Figure 8 to be recast as a two-dimensional plane for a given value of $\tan\beta$ and $\mathrm{sgn}(\mu)$.

In Figure 11 we thus exhibit the soft supersymmetry breaking parameters of (195) as a function of condensing group beta-function coefficient b_+. All masses are given relative to the gravitino mass, and we have taken $g^2_{\rm STR} = 0.5$ and chosen $\tilde{\mu}^2_{\rm PV} = \mu^2_{\rm UV}$,

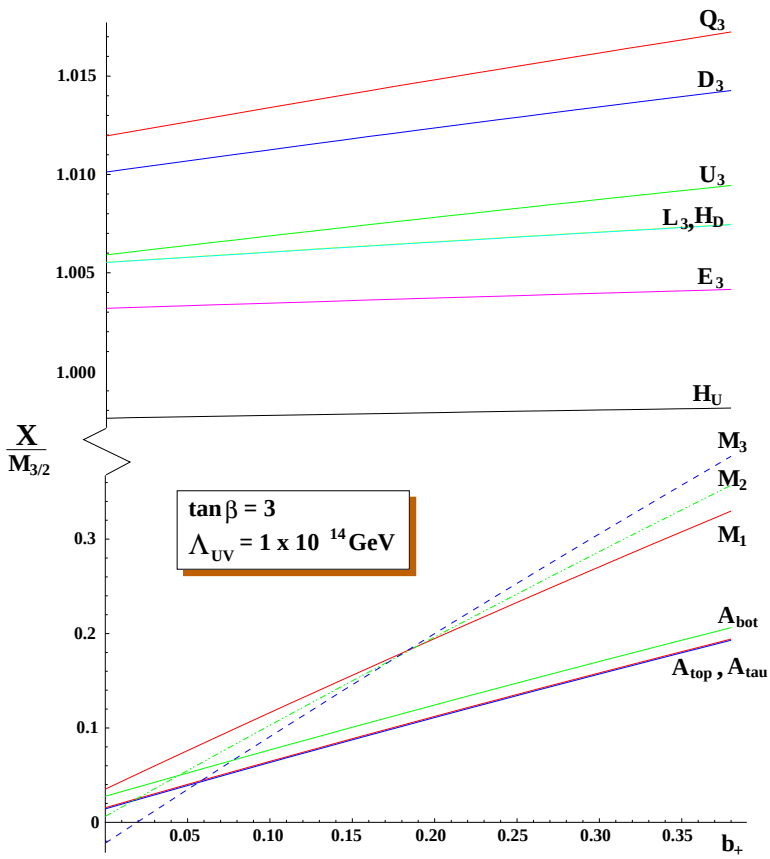

Fig. 11. **Spectrum of soft supersymmetry breaking terms in BGW model.** All values are given relative to the gravitino mass $m_{3/2}$ at a scale $\mu_{\rm UV} = 1 \times 10^{14}$ GeV as a function of the condensing group beta function coefficient b_+.

where $\mu_{\rm UV}$ is the boundary condition scale. All values tend to increase with b_+ as the relative size of $\langle F^S \rangle$ to $m_{3/2}$ increases. But note the importance of the anomaly contributions to gaugino masses in changing the *relative* sizes of the three Standard Model gaugino mass parameters. The values of the one-loop soft terms are sensitive to the choice of boundary condition scale through the logarithmic terms $\ln(\tilde{\mu}^2_{\rm PV}/\mu_R^2)$ in the A-terms and scalar masses, and through the dependence on the renormalized gauge couplings $g_a^2(\mu_R)$ in the gaugino masses. This dependence is most pronounced for the gaugino soft masses, but is milder in the regions of phenomenological viability $0 \leq b_+ \leq 0.09$ established in Section 2.3.

This dependence is demonstrated in Figure 12, where we give the gaugino soft masses as a function of b_+ over the range $0 \leq b_+ \leq 0.1$ for two choices of boundary condition scale $\mu_{\rm UV} = 2 \times 10^{16}$ GeV (solid lines) and $\mu_{\rm UV} = 1 \times 10^{14}$ GeV (dashed

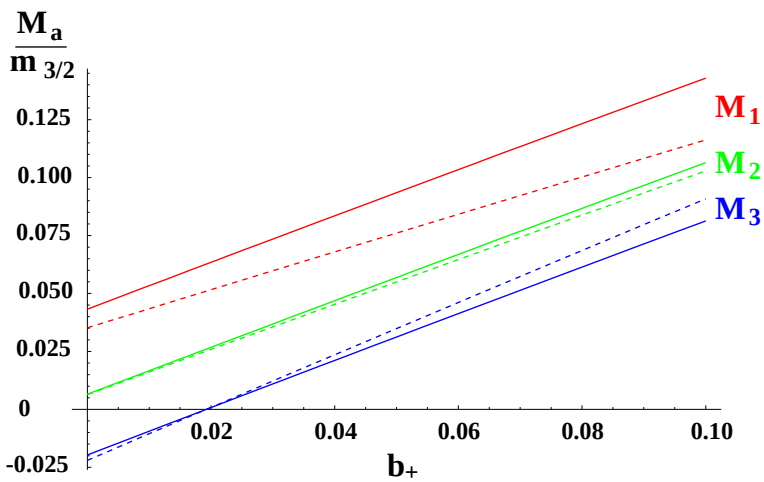

Fig. 12. **Gaugino masses in the BGW model.** Gaugino masses M_1 (top), M_2 and M_3 (bottom) are given as a function of the condensing group beta function coefficient b_+ at a scale $\mu_{\rm UV} = 2 \times 10^{16}$ GeV (solid lines) and $\mu_{\rm UV} = 1 \times 10^{14}$ GeV (dashed lines).

lines). The convergence of gaugino masses as a function of b_+ occurs at lower values for lower boundary condition scales. But over the range of phenomenological interest the hierarchy of gaugino masses is inverted relative to what occurs at low energies in unified models such as the minimal supergravity model: the gluino mass M_3 is the smallest, with the B-ino mass M_1 being the largest. Of course, these are statements that hold at the boundary condition scale – what is of interest is the hierarchy of gaugino masses at the low energy (electroweak) scale. Evolving the gaugino masses to the electroweak scale using the two-loop RG equations[18] the gluino increases in mass, but remains lighter relative to the neutralino/chargino sector than in the minimal supergravity model.

After evolving the gaugino masses M_1 and M_2 to the electroweak scale, and imposing the electroweak symmetry breaking condition (197), we can compute the physical neutralino and chargino masses. Over most of the allowed range in b_+ the B-ino is the lightest supersymmetric particle (LSP) – just as in minimal unified models – but there will be a significant component of the $SU(2)$ neutral W-ino in the wavefunction of the LSP. In fact, when the condensing group beta-function coefficient b_+ becomes relatively small (i.e. similar in size to the MSSM hypercharge value of $b_{U(1)} = 0.028$) the pieces of the gaugino mass arising from the superconformal anomaly can become equal in magnitude to part arising from the dilaton auxiliary field. Here there is a level crossing in the neutral gaugino sector. The LSP becomes predominately W-ino like and the mass difference between the lightest chargino and lightest neutralino becomes negligible. This effect is displayed in Figure 13. The phenomenology of the gaugino sector in this limit is similar to that of the minimal anomaly-mediated supersymmetry breaking (AMSB) model.[148,149,150]

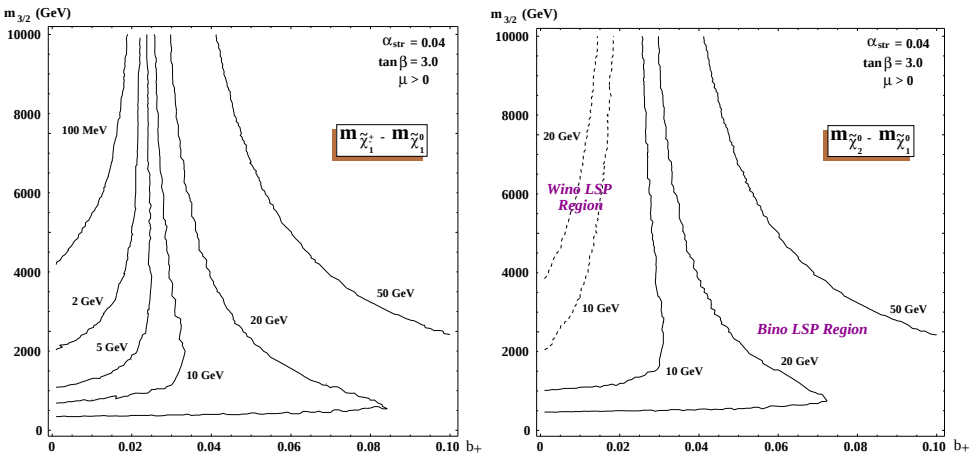

Fig. 13. **The physical gaugino sector of the BGW model**. The left panel gives the mass difference between the $\chi_1^\pm$ and the χ_1^0 in GeV. The right panel gives the difference in mass between the two lightest neutralinos χ_2^0 and χ_1^0. Note that a level crossing occurs and there exists a region in which the W-ino W_0 becomes the LSP.

One other important implication of the hierarchy in Figure 11 is the issue of fine-tuning in the electroweak sector. A widely held rule of thumb about fine-tuning states that it generally increases with the scale of the mass of squarks and sleptons. But this is only roughly true. Consider the tree-level analog of the EWSB condition in (197). This tree level relation can, in turn, be written in the following way[151]

$$M_Z^2 = \sum_i C_i m_i^2(\text{UV}) + \sum_{ij} C_{ij} m_i(\text{UV}) m_j(\text{UV})\,, \tag{199}$$

where m_i represents a generic parameter of the softly broken supersymmetric Lagrangian at an initial high scale μ_{UV} with mass dimension one, such as gaugino masses, scalar masses, trilinear A-terms and the μ parameter. The coefficients C_i and C_{ij} depend on the scale μ_{UV} and quantities such as the top mass and $\tan\beta$ in a calculable way through solving the renormalization group equations. For example, taking the running mass for the top quark at the Z-mass scale to be $m_{\text{top}}(M_Z) = 170$ GeV, the starting scale to be the grand-unified scale, and $\tan\beta = 5$ we have for the leading terms in (199)[152]

$$\begin{aligned} M_Z^2 = &-1.8\mu^2(\text{UV}) + 5.9M_3^2(\text{UV}) - 0.4M_2^2(\text{UV}) - 1.2m_{H_U}^2(\text{UV}) \\ &+0.9m_{Q_3}^2(\text{UV}) + 0.7m_{U_3}^2(\text{UV}) - 0.6A_t(\text{UV})M_3(\text{UV}) \\ &-0.1A_t(\text{UV})M_2(\text{UV}) + 0.2A_t^2(\text{UV}) + 0.4M_2(\text{UV})M_3(\text{UV}) + \ldots \end{aligned} \tag{200}$$

where the ellipsis in (200) indicate terms that are less important quantitatively and for our purposes. Note that the most important parameter is, in fact, the gluino mass as evidenced by the large value of C_3. In contrast, the scalar masses are far less important. Indeed, replacing the scalars with a universal value m_0 the relation

in (200) becomes

$$M_Z^2 = -1.8\mu^2(\text{UV}) + 0.4m_0^2 + 5.9M_3^2 - 0.4M_2^2 + \dots . \tag{201}$$

Thus, models in which gluinos are light relative to the electroweak gauginos are more likely to provide the needed cancelation of soft parameters against the supersymmetric mass term (μ) with a minimum of fine-tuning required. This feature was the driving motivation for the models of Ref. 145, which led to construction of effective theories very similar to the BGW model. We will pursue other phenomenological implications of the model in Section 5, but first we must consider how the pattern of soft supersymmetry breaking can change when the model contains an anomalous $U(1)_X$ factor.

4. Inclusion of anomalous U(1)'s

An anomalous $U(1)$, commonly denoted $U(1)_X$, is generic to effective supergravity in string theory. For example the study[113] of a large class of standard-like heterotic $\mathbf{Z}_3$ orbifold models showed that 168 of 175 models had an anomalous $U(1)_X$. The underlying theory is anomaly free, and the apparent anomaly is canceled[153,154,155] by a four-dimensional version of the Green-Schwarz term similar to the one in (102) used to cancel the modular anomaly. This leads to a Fayet-Illiopoulos (FI) term in the effective supergravity Lagrangian. Ignoring nonperturbative corrections to the dilaton Kähler potential, the D-term for $U(1)_X$ takes the form[a]

$$D_X = -2\left(\sum_A K_A q_A^X \phi^A + \xi\right), \quad K_A = \frac{\partial K}{\partial \phi^A} \quad \xi = \frac{g_{\text{STR}}^2 \,\text{Tr}\, Q_X}{192\pi^2} m_{\text{PL}}^2, \tag{202}$$

where K is the Kähler potential, q_A^X is the $U(1)_X$ charge of the (complex) scalar matter field ϕ^A, ξ is the FI term, Q_X is the charge generator of $U(1)_X$, g_{STR} is the gauge coupling at the string scale, and $m_{\text{PL}} = 1/\sqrt{8\pi G} = 2.44 \times 10^{18}$ GeV is the reduced Planck mass that we set to unity in the remainder of this section. Provided there are D-flat and F-flat directions, which is also generically the case, some number n of fields ϕ^A acquire *vevs* that break some number $m \leq n$ of gauge symmetries. The corresponding gauge supermultiplets get masses through the supersymmetric generalization of the Higgs mechanism, but local supersymmetry remains unbroken.

In renormalizable globally supersymmetric gauge theories with a spontaneously broken gauge theory, the associated vector multiplet "eats" a chiral and antichiral

[a]In this section there will be a growing number of "species" indices with which the reader must be concerned. To minimize future confusion, we here summarize the index conventions. In this section we will label generic chiral superfields with capital Latin indices (as opposed to the lower-case indices of the previous sections). Gauge groups are labeled by lower-case Latin indices from the beginning of the alphabet. At times it will be convenient to single-out an anomalous $U(1)$ factor for which the label a will be replaced with X. Matter condensates continue to be labeled by Greek letters and internal complex planes/Kähler moduli by the capital index I. Repeated indices do not imply summation – all index summations will be explicitly shown.

supermultiplet to form a massive vector supermultiplet. When these are integrated out, the effective theory below the gauge symmetry breaking scale retains manifest global supersymmetry. In the case of supergravity, we can maintain manifest local supersymmetry by promoting[156] the condition

$$D_X = 0 \tag{203}$$

to a superfield condition of which (203) is the lowest component. In effective supergravity from string theory, the chiral supermultiplets that get *vevs* have modular weights, while the vector fields are modular invariant. On the other hand the condition (203) does not break T-duality; it fixes the *vevs* of a linear combination of the invariant fields $\sum_A K_A \phi^A$. Maintaining manifest T-duality in the effective theory below the $U(1)_X$ breaking scale requires a generalization[156] of the usual gauge transformation used to remove the unphysical chiral superfields to go to unitary gauge. Finally, in supergravity from string theory, the parameter ξ in (202) is not a constant; in the classical limit $g^2_{\rm STR} = \langle 1/{\rm Re}\, s\rangle$ should be replaced by the dynamical variable $1/{\rm Re}\, s$. In other words, the scalar components of the eaten chiral multiplets acquire *vevs* that are functions of the dilaton and the Kähler moduli which themselves remain massless and are not stabilized at the scale of $U(1)_X$ breaking. When the massive modes are integrated out in a way that preserves manifest local supergravity and T-duality, the Kähler potential of the light matter fields that are charged under the broken gauge symmetries is modified. In particular their modular weights are shifted in a manner which depends on their charges under the broken gauge group. In addition the GS term (102) that cancels the modular anomaly is modified in a parallel manner such that modular anomaly cancelation is also manifest in the effective theory below the gauge symmetry breaking scale. The modified modular weights provide interesting possibilities for generating a source for the R-parity of the MSSM – or an even more restrictive discrete symmetry – that will be described in Section 5.

When supersymmetry is broken by condensation in a strongly coupled hidden gauge sector with group $\mathcal{G}_c$, dilaton stabilization is assured by the presence of D-terms – in contrast to the models without an anomalous $U(1)$ considered in Section 3 – but corrections to the dilaton Kähler potential are still needed to stabilize the dilaton at weak coupling. Several promising features of the previous models persist: enhancement of the dilaton and T-moduli masses relative to the gravitino mass, masslessness of the universal axion, and dilaton-mediated supersymmetry breaking that avoids potential problems with flavor changing neutral currents. However there are some challenges to be overcome in finding a viable scenario with an anomalous $U(1)_X$.

There is generally a large degeneracy of the vacuum associated with $U(1)$ breaking,[157,156] resulting in many massless chiral multiplets, or "D-moduli," between the $U(1)$-breaking and supersymmetry-breaking scales. Moreover, in the absence of superpotential couplings a number of these remain massless even after supersymmetry breaking. It turns out that couplings of the D-moduli to the mat-

ter condensates in the superpotential are sufficient to lift the degeneracy and give masses to the *real* parts of the D-moduli scalars as well as the fermions, while the imaginary parts of the scalars ("D-axions") remain massless in the absence of other superpotential couplings. This remaining degeneracy may be at least partially lifted by D-moduli couplings to other unconfined, $\mathcal{G}_c$-neutral chiral supermultiplets.

As has been noted by a number of authors,[158,159,160,161,55] that there is considerable tension in maintaining a vanishing cosmological constant, a positive dilaton metric and positive and acceptably small scalar masses in the observable sector on the one hand, while requiring weak coupling and acceptably large D-moduli/fermion masses on the other hand.

In Section 4.1 we illustrate the procedure for integrating out the massive vector supermultiplet while preserving manifest local supersymmetry and T-duality in the lower energy effective theory using a toy model[156] with just one broken gauge symmetry, $U(1)_X$, and just one complex scalar *vev*; in Section 4.2 we include hidden sector gaugino condensation in this toy model. Since the linear multiplet formalism for the dilaton is by far better suited to addressing these questions, we will use it in these two sections, and impose further that the modified linearity condition (42) remain true in the effective theory below the $U(1)_X$ breaking scale. The cases with any number m of broken $U(1)$'s and $n \geq m$ scalar *vevs* have been worked out in detail;[162,55] here we simply state the results. In Section 4.3 we discuss the vacuum and the moduli sector, and in Sections 4.4 and 4.5 we address observable sector and D-moduli masses, respectively, and discuss the requirements for a viable model.

4.1. *The effective theory below the $U(1)_X$ breaking scale: a toy model*

When an anomalous $U(1)_X$ is present, the effective Lagrangian at the string scale is defined by

$$\mathcal{L} = \mathcal{L}_{\rm KE} + \mathcal{L}_{\rm GS} + \mathcal{L}_{\rm th} + \sum_a \mathcal{L}_a, \tag{204}$$

where $\mathcal{L}_{\rm th}$ is the string loop threshold correction given in (105). It is convenient here to write the kinetic term (61) in the form

$$\mathcal{L}_{\rm KE} = \int {\rm d}^4\theta\, E\left[-3 + Ls(L)\right], \tag{205}$$

such that the first term contains the usual gravity and matter kinetic terms of Kähler $U(1)$ superspace, and the second term, with $2Ls(L) = 1 + f(L)$, includes the gauge kinetic term of the more conventional supergravity formulation, with the vacuum value

$$\langle s(\ell)\rangle = g_{\rm STR}^{-2} \tag{206}$$

determining the gauge coupling $g_{\rm STR}$ at the string scale.[b] The GS term (102) now includes the FI term:

$$\mathcal{L}_{\rm GS} = \int \mathrm{d}^4\theta\, EL\,(b\,G - \delta_X V_X)\,, \tag{207}$$

where V_X is the $U(1)_X$ vector superfield, G is defined by[c]

$$G = \sum_I g^I, \qquad g^I = -\ln(T^I + \overline{T}^I)\,. \tag{208}$$

The two parameters in (207) are the coefficient b, identified with $b_{\rm GS}$ of equation (102), and

$$\delta_X = -\frac{1}{48\pi^2}\mathrm{Tr}\,Q_X = -\frac{1}{2\pi^2}\sum_A C^A_{a\neq X}\,q^X_A = -\frac{1}{6\pi^2}\mathrm{Tr}\,Q^3_X, \tag{209}$$

where the last two equalities are constraints on the gauge charges of the spectrum that follow from the fact that the underlying string theory is anomaly free. The field theoretic quantum corrections $\mathcal{L}_a$ take the form

$$\mathcal{L}_a = -\int d^4\theta \frac{E}{8R}\,W^\alpha_a P_\chi B_a W^a_\alpha + \text{ h.c. }\,, \tag{210}$$

$$B_a(L, V_X, g^I) = \sum_I (b - b^I_a) g^I - \delta_X V_X + f_a(L)\,. \tag{211}$$

The full Kähler potential is

$$K = k(L) + G + \sum_A e^{G_A + 2q^X_A V_X}|\Phi_A|^2 + \mathcal{O}(|\Phi_A|^3)\,,$$
$$k(L) = \ln L + g(L), \qquad G_A \equiv \sum_I q^I_A\, g^I\,, \tag{212}$$

where q^X_A and q^I_A are $U(1)_X$ charges and modular weights, respectively. Up until now we have been working in Kähler $U(1)$ and Yang-Mills superspace, in which the Yang-Mills vector fields do not appear explicitly and chiral superfields are *covariantly* chiral, with the gauge connections implicit in all covariant derivatives. When there is an anomalous $U(1)$ the vector field appears explicitly in the GS term needed to cancel the anomaly. The vector field must also be introduced explicitly for the supersymmetric regularization[135] of ultraviolet divergences associated with an anomalous $U(1)$, which accounts for its appearance in the expression (211). When a gauge symmetry is broken, and the associated vector multiplet acquires a large mass, it is appropriate to remove its gauge connection from the covariant derivatives. In this case the vector field appears explicitly in the Kähler potential

[b]In the dual chiral formulation for the dilaton $2s(L) \to S + \bar{S} - bG + V_X\delta_x$.
[c]In this section, for simplicity, we set the parameters p_A introduced as p_i's in (116) to zero, as was done in Ref. 55. We comment on the case with $p^A \neq 0$ in Section 4.4. These parameters should not be confused with the similarly labeled parameters introduced in (187) above, nor those in (300) below.

as in (212), and the heavy degrees of freedom can easily be integrated out at the superfield level, as we outline below.

Suppose that a chiral multiplet Φ with $U(1)_X$ charge q and modular weight q^I acquires a *vev*. Setting all other matter fields to zero, the Kähler potential (212) reduces to

$$K = k(L) + G + e^{G_q + 2qV_X}|\Phi|^2, \qquad G_q = \sum_I q^I g^I, \tag{213}$$

and (in Wess-Zumino gauge) the D-term (202) is given by

$$D_X = -\frac{2}{s}\left(qe^{G_q(t)}|\phi|^2 - \frac{\ell\delta_X}{2}\right), \tag{214}$$

which vanishes at the minimum of the potential if $\phi \neq 0$ is a flat direction. Since the scalar component ϕ of Φ must get a *vev* to cancel the FI term, it is consistent to write $\phi = e^{\theta}$, where θ is a complex scalar field. Promoting this to a superfield expression, we define a chiral superfield Θ such that

$$\Phi = e^{\Theta}\,. \tag{215}$$

The Lagrangian is invariant under $U(1)_X$ gauge transformations

$$V_X \to V_X' = V_X + \frac{1}{2}\left(\Lambda + \overline{\Lambda}\right), \qquad \Phi^A \to \Phi'^A = e^{-q_A^X \Lambda}\Phi^A\,, \tag{216}$$

where Λ is a chiral supermultiplet, so we can remove the chiral supermultiplet Θ by a gauge transformation with $\Lambda = \Theta/q$:

$$V_X \to V' = V_X + \frac{1}{2q}\left(\Theta + \overline{\Theta}\right), \qquad \Phi \to \Phi' = e^{-\Theta}\Phi = 1\,, \tag{217}$$

The field V' describes a massive vector multiplet with the same number of components as a massless vector multiplet and a (complex) chiral multiplet, and gauge invariance assures that

$$\mathcal{L}\left(V_X, \Phi, \overline{\Phi}\right) \to \mathcal{L}\left(V', 1, 1\right)\,. \tag{218}$$

The expressions for K and $\mathcal{L}$ become

$$K = k(L) + G + e^{G_q + 2qV'}, \tag{219}$$

$$\mathcal{L} = \int \mathrm{d}^4\theta E\left[-3 + 2Ls(L) + L(b\,G - \delta_X V')\right] + \mathcal{L}_{\mathrm{th}} + \sum_a \mathcal{L}_a. \tag{220}$$

But note the important property that Θ – and therefore V' – is not invariant under modular transformations defined by (76) and (84):

$$V' \to V' - \frac{1}{2q}\sum_I q^I\left(F^I + \overline{F}^I\right)\;; \qquad F^I = \ln(icT^I + d)\,. \tag{221}$$

In order to integrate out the heavy degrees of freedom in a way that preserves the unbroken symmetries, therefore, we promote the vacuum condition (203) to a superfield condition. That is, we require that the *superfield*

$$q\,e^{G_q(T) + 2qV'} - \frac{\delta_X}{2}L\,, \tag{222}$$

which *is* modular invariant, vanish in the vacuum. To this end we introduce a vector superfield U with vanishing *vev* (*i.e.* all component fields are defined to vanish in the vacuum) such that this condition is maintained

$$q\, e^{G_q+2qV'} = e^{2qU}\, \frac{\delta_X}{2} L\,, \qquad \langle U\rangle = 0\,. \tag{223}$$

This corresponds to the redefinition

$$V' = U + \frac{1}{2q}\left(\ln\frac{\delta_X L}{2q} - G_q\right). \tag{224}$$

In terms of the new set of independent fields (L, U, T^I), the Kähler potential and the Lagrangian take the form

$$K = k(L) + G + e^{2qU}\,\frac{\delta_X L}{2q} \equiv K(L,U) + G\,, \tag{225}$$

$$\begin{aligned}\mathcal{L}_{\rm KE} + \mathcal{L}_{\rm GS} &= \int \mathrm{d}^4\theta E\left[-3 + 2Ls(L) + L\left(bG - \delta_X U - \frac{\delta_X}{2q}\ln\frac{\delta_X L}{2q} + \frac{\delta_X}{2q}G_q\right)\right]\\ &\equiv \int \mathrm{d}^4\theta E\left[-3 + 2L\,S(L,U) + L\sum_I b^I g^I\right]\,,\end{aligned} \tag{226}$$

where we have introduced the new coefficient

$$b^I = b + \frac{\delta_X}{2q} q^I\,. \tag{227}$$

When the superfield U is set to zero, the net effect is a modification of the functions $k(L)$ and $s(L)$ which are subject to the constraint (62), or, equivalently

$$k'(L) + 2Ls'(L) = 0\,. \tag{228}$$

These are now replaced by the functions

$$\tilde{k}(L) = K(L,0) = k(L) + \frac{\delta_X}{2q}\,L\,, \qquad \tilde{s} = S(L,0) = s(L) - \frac{\delta_X}{4q}\ln\left(\frac{\delta_X L}{2q}\right)\,, \tag{229}$$

that satisfy

$$\tilde{k}'(L) + 2L\tilde{s}'(L) = k'(L) + 2Ls'(L) = 0\,, \tag{230}$$

and the Einstein-Hilbert term remains canonically normalized. This is not the case, however, for $U \neq 0$. Moreover, there are terms linear in U that contribute to the tree-level action that must be taken into account. This can be done while maintaining $\langle U\rangle = 0$ by a redefinition of L, but an arbitrary redefinition would destroy the modified linearity condition (42) that we wish to maintain below the $U(1)_X$ breaking scale. This can be achieved by a superfield Weyl transformation[33] that leaves the product $E\,L$ invariant but modifies the Kähler potential:

$$\begin{aligned}K(L,U) &= \widehat{K} + \Delta k, \quad E = e^{-\Delta k/3}\widehat{E}, \quad L = e^{\Delta k/3}\widehat{L}\,,\\ 2\widehat{L}\,\widehat{S}(\widehat{L},U) &= 2\widehat{L}\,S(L,U) + 3\left(1 - e^{-\Delta k/3}\right)\,.\end{aligned} \tag{231}$$

The condition for a canonical Einstein term in the new Weyl basis is

$$0 = \left(\frac{\partial \widehat{K}}{\partial \widehat{L}}\right)_U + 2\widehat{L}\left(\frac{\partial \widehat{S}}{\partial \widehat{L}}\right)_U , \tag{232}$$

where the subscript on derivatives indicates that U is held fixed. The $\widehat{L}$ derivatives of Δk drop out except in an overall factor $\partial L/\partial \widehat{L}$, giving the condition

$$\begin{aligned} L - \widehat{L} = \widehat{L}\left(e^{\Delta k/3} - 1\right) &= L^2 \frac{K'(L) + 2L\, S'(L)}{3 + 2L^2\, S'(L)} \\ &= L^2 \frac{\delta_X \left(e^{2qU} - 1\right)}{2q\left[3 - L\tilde{k}'(L)\right]} = \frac{\delta_X L^2\, U}{3 - L\,\tilde{k}'(L)} + \mathcal{O}(U^2) \\ &= \frac{\delta_X \widehat{L}^2\, U}{3 - \widehat{L}\,\tilde{k}'(\widehat{L})} + \mathcal{O}(U^2)\,. \end{aligned} \tag{233}$$

We may then expand the expressions

$$\begin{aligned} \widehat{K}(\widehat{L}) &= \tilde{k}(L) + \frac{\delta_X}{2q} L\left(e^{2qU} - 1\right) - \Delta k, \\ \widehat{S}(\widehat{L}) &= \tilde{s}(L) + \frac{3}{2L\widehat{L}}\left(L - \widehat{L}\right) - \frac{\delta_X}{2} U\,, \end{aligned} \tag{234}$$

in powers of U. This gives

$$\begin{aligned} &\Delta k = 3(L - \widehat{L})/\widehat{L} + \mathcal{O}(U^2), \qquad \tilde{k}(L) = \tilde{k}(\widehat{L}) + (L - \widehat{L})\tilde{k}'(\widehat{L}) + \mathcal{O}(U^2), \\ &\tilde{s}(L) = \tilde{s}(\widehat{L}) + (L - \widehat{L})\tilde{s}'(\widehat{L}) + \mathcal{O}(U^2)\,, \end{aligned} \tag{235}$$

and, using (230), the terms linear in U drop out of (234). This means that we can just set $U = 0$ to get the effective low energy theory in the hatted basis that has a canonical Einstein-Hilbert term.[d]

Once the heavy modes have been integrated out, the Lagrangian takes the form (204) with

$$\mathcal{L}_{\rm KE} = \int \mathrm{d}^4\theta\, E\left[-3 + L\,\tilde{s}(L)\right]\,, \tag{236}$$

$$\mathcal{L}_{\rm GS} = \int \mathrm{d}^4\theta\, EL \sum_I b^I g^I\,, \tag{237}$$

and the operator B_a in (211) becomes

$$B_a(L,\, V_X,\, g^I) = \sum_I (b^I - b^I_a) g^I + \tilde{f}_a(L)\,, \quad \tilde{f}_a(L) = f_a(L) - \frac{1}{2q}\ln\frac{\delta_X L}{2q}\,. \tag{238}$$

The shift in f_a exactly cancels the effect of the shift (229) in s on the kinetic terms for the Yang-Mills fields of the unbroken gauge group, so the coupling is the same

[d]The terms quadratic in the $U(1)_X$ field strengths $W^\alpha_V = W^\alpha_{V'}$ generate terms linear in W^α_U; these involve couplings to the chiral projections of spinorial derivatives of the moduli.[163] They do not contribute to the potential as long as supersymmetry is unbroken; see (244) below.

as the string scale coupling (206), but the dilaton kinetic energy term is modified below the $U(1)$ breaking scale. The shift $b \to b^I$ matches that in $\mathcal{L}_{\rm GS}$ so that the anomaly is still canceled. The shift in $\mathcal{L}_a$ precisely corresponds to the shift in the metric for matter fields that occurs when the change in variables (224) is made. The Kähler potential is now

$$K = \tilde{k}(L) + G + \sum_A e^{G'_A} \left(\frac{\delta_X L}{2q}\right)^{q_A^X/q} |\Phi'_A|^2,$$
$$G'_A = \sum_I \left(q_A^I - \frac{q_A^X}{q} q^I\right) g^I \equiv \sum_I q_A'^I g^I \,, \tag{239}$$

where it is convenient to define the new "effective" modular weight

$$q_A'^I = q_A^I - \frac{q_A^X}{q} q^I \,. \tag{240}$$

The modified Kähler metric implies a modification of the fermion connections that induce a shift in the functions B_a; using (209),

$$\delta B_a = \frac{1}{4\pi^2} \sum_A C_a^A \frac{q_A^X}{q} \left(\ln \frac{\delta_X}{2q} - \sum_I q^I g^I\right) = \frac{\delta_X}{2q} \left(\ln \frac{\delta_X}{2q} - \sum_I q^I g^I\right) \,, \tag{241}$$

giving the correction to (211) that appears in (238). These modifications of the effective theory can have consequences for phenomenology.

Finally, the right hand side of the modified linearity condition (42) contains the term $(W^\alpha W_\alpha)_V$ where

$$\begin{aligned} W_V^\alpha &= -\frac{1}{4}(\overline{\mathcal{D}}^2 - 8R)\mathcal{D}^\alpha V = -\frac{1}{4}(\overline{\mathcal{D}}^2 - 8R)\mathcal{D}^\alpha V' \\ &= -\frac{1}{4}(\overline{\mathcal{D}}^2 - 8R)\mathcal{D}^\alpha U - \frac{1}{8q}(\overline{\mathcal{D}}^2 - 8R)\left(\frac{\mathcal{D}^\alpha L}{L} - \mathcal{D}^\alpha G_q\right) \\ &= W_U^\alpha - \frac{1}{8q}(\overline{\mathcal{D}}^2 - 8R)\left(\frac{\mathcal{D}^\alpha L}{L} - \mathcal{D}^\alpha G_q\right). \end{aligned} \tag{242}$$

In the hatted basis defined by the Weyl transformation (231), (242) reads

$$W_V^\alpha = \left(1 + \frac{1}{2qL(\widehat{L},U)} \frac{\partial L(\widehat{L},U)}{\partial U}\right) W_U^\alpha + W'^\alpha, \tag{243}$$

$$\int \mathrm{d}^4\theta \frac{E}{R}(W^\alpha W_\alpha)_V = \int \mathrm{d}^4\theta \frac{E}{R}(W^\alpha W_\alpha)_U \left[\left(1 + \frac{\delta_X \widehat{L}}{2q(3 - \widehat{L}\tilde{k}'(\widehat{L})}\right)^2 + O(U)\right] + \cdots, \tag{244}$$

where the ellipsis represents terms involving W'^α that are quartic in auxiliary fields and/or derivatives. This implies a renormalization of the U wave function such that

its mass is given by[156]

$$m_U^2 = \frac{1}{s(\hat{\ell})}\left(1+\frac{\delta_X \widehat{L}}{2q(3-\widehat{L}\tilde{k}'(\widehat{L})}\right)^{-2} \frac{\partial^2 \tilde{K}(\widehat{L},U)}{\partial U^2}\bigg|_{U=0,\widehat{L}=\langle\hat{\ell}\rangle}$$
$$= \left(1+\frac{\delta_X \langle\hat{\ell}\rangle}{2q(3-\langle\hat{\ell}\rangle\tilde{k}'(\langle\hat{\ell}\rangle)}\right)^{-1} \frac{q\delta_X\langle\hat{\ell}\rangle}{s\langle\hat{\ell}\rangle}, \tag{245}$$

where

$$\tilde{K} = \widehat{K} + 2\widehat{L}\widehat{S} \tag{246}$$

is the "effective" Kähler potential.

4.2. *The effective theory below the condensation scale: a toy model*

Here we consider the case where the toy model described in Section 4.1 has a strongly-coupled gauge group $\mathcal{G}_c$ in a hidden sector. The effective Lagrangian for the condensates (20) and (23) can be constructed by anomaly matching as described in Section 2. The strongly coupled Yang-Mills sector also possesses a residual global $U(1)_X$ invariance that is broken only by superpotential couplings that involve the chiral superfield Φ that gets a *vev* at the $U(1)_X$-breaking scale.[e] These couplings enter the RGE for the $\mathcal{G}_c$ gauge coupling only through chiral field wave function renormalization, which is a two-loop effect that is encoded in the expression (126) for the gaugino condensate through the appearance of the superpotential coefficients W_α as discussed in Section 2.3. We therefore impose the $U(1)_X$ anomaly matching condition

$$\sum_{\alpha,B} b_c^\alpha\, n_\alpha^B\, q_B^a = \sum_B \frac{C_c^B}{4\pi^2}\, q_B^a = -\frac{1}{2}\delta_X\,\delta_X^a\,, \tag{247}$$

which is also satisfied by (26). Note that in the last expression of (247) the quantity δ_X is the GS coefficient from (207) while δ_X^a is a Kronecker delta function which enforces $U(1)_a = U(1)_X$. The condensate superpotential now takes the form

$$W(\Pi) = \sum_\alpha W_\alpha(T^I,\Phi)\,\Pi^\alpha\,, \tag{248}$$

where T-duality and $U(1)_X$ invariance require

$$W_\alpha(T^I,\Phi) = c_\alpha \prod_I [\,\eta(t^I)]^{2\left(q_\alpha'^I-1\right)}\phi^{-q_\alpha^X/q}\,, \tag{249}$$

with $q_\alpha'^I$ defined as in (240) for the matter condensate Π^α.

When supersymmetry is broken by condensation, we can no longer assume *a priori* that $\langle D_X\rangle = 0$, and an F-term associated with the chiral field that gets a

[e]Some of these coupling could generate masses for a subset of fields that are charged under $\mathcal{G}_c$; we will comment on that case at the end of this section.

vev may also be generated. To include these we modify the field redefinitions (217) and (224) as follows:

$$\Phi' = e^{-\Theta+\Delta_\Phi}\Phi = e^{\Delta_\Phi}, \qquad V' = U + \frac{1}{2q}\left(\ln\frac{\delta_X L}{2q} - G_q\right) + \Delta_X, \tag{250}$$

with V' defined as in (217). The objects Δ_X and Δ_Φ are constant vector and chiral superfields, respectively, with vanishing fermionic components. We can make a $U(1)_X$ gauge transformation with a constant chiral superfield Λ to eliminate the scalar and auxiliary components from Δ_X, which just redefines the corresponding components of Δ_Φ:

$$\begin{aligned} 0 &= \Delta_X| = -\frac{1}{4}\, D^2\Delta_X|\,, \qquad D_X = \frac{1}{8}\,\mathcal{D}^\alpha\left(\overline{D}^2 - 8R\right)\mathcal{D}_\alpha\Delta_X\Big|\,, \\ \delta &= \Delta_\Phi|\,, \quad F = -\frac{1}{4}\, D^2\Delta_\Phi|\,, \end{aligned} \tag{251}$$

and we take

$$\langle U\rangle = 0 \tag{252}$$

as before. However we cannot set $U = 0$ before taking into account linear couplings of U that arise in the presence of the supersymmetry-breaking *vevs* (251), the matter fields Φ^M and the now nonvanishing auxiliary fields represented by the ellipsis in (244). As before, we go to the basis where the Einstein-Hilbert term is canonical, starting with the Lagrangian defined by

$$\begin{aligned} K &= k(L) + G + e^{2q(\Delta_X+U)+\Delta_\Phi+\overline{\Delta}_\Phi}\frac{\delta_X L}{2q} + \sum_A e^{G'_A}\left(\frac{\delta_X L}{2q}\right)^{q_A^X/q}|\Phi'_A|^2 \\ &\equiv K(L,M,U) + G, \end{aligned} \tag{253}$$

$$\begin{aligned} \mathcal{L}_{\rm KE} + \mathcal{L}_{\rm GS} &= \int \mathrm{d}^4\theta E\left[-3 + 2Ls(L) + L\left(b\,G - \delta_X(\Delta_X+U) - \frac{\delta_X}{2q}\ln\frac{\delta_X L}{2q} + \frac{\delta_X}{2q}G_q\right)\right] \\ &\equiv \int \mathrm{d}^4\theta E\left[-3 + 2L\,S(L,M,U) + L\sum_I b^I g^I\right], \end{aligned} \tag{254}$$

where now

$$\begin{aligned} K(L,M,U) &= \tilde{k}(L) + \frac{\delta_X L}{2q}\left(e^{2q(\Delta_X+U)+\Delta_\Phi+\overline{\Delta}_\Phi} - 1\right) + \sum_A x^A, \qquad M = \Delta, \Phi, T, \\ S(L,M,U) &= \tilde{s}(L) - \delta_X(U+\Delta_X) \qquad x^A = e^{G'_A}\left(\frac{\delta_X L}{2q}\right)^{q_A^X/q}|\Phi'_A|^2\,. \end{aligned} \tag{255}$$

We make the superfield Weyl transformation (231) and impose the condition (232) with $U \to U, M$ held fixed in (232) to obtain

$$L - \widehat{L} = L^2\left\{\frac{\delta_X\left(e^{2q(\Delta_X+U)+\Delta_\Phi+\overline{\Delta}_\Phi} - 1\right) + 2q\sum_A x'^A}{2q\left[3 - L\,\tilde{k}'(L)\right]}\right\}. \tag{256}$$

The functions $\widehat{K}(\widehat{L}, M, U)$ and $\widehat{S}(\widehat{L}, M, U)$ can be expanded as before to obtain the component Lagrangian which now includes F-terms for Φ, Φ^M and the $U(1)_X$ D-term. Here we will just describe the modifications with respect to the BGW case of Section 3.

The couplings of Φ in the condensate superpotential (248) introduce additional parameters in the potential. We define

$$p^I = -\frac{q^I}{q} \sum_\alpha b_c^\alpha q_\alpha^X \equiv p\, q^I \qquad p = -\frac{1}{q} \sum_\alpha b_c^\alpha q_\alpha^X \,. \tag{257}$$

In this toy model it follows from the $U(1)_X$ anomaly matching condition (247) that

$$p^I = b^I - b, \qquad p = \frac{\delta_X}{2q} \,. \tag{258}$$

Neglecting corrections of order Δ, the *vev* of $|\phi|^2$ is given by

$$\left\langle |\phi|^2 \right\rangle = e^{-\sum_I q^I g^I - 2qV_X} \frac{\delta_X \ell}{2q} = p\, \ell\, e^{-\sum_I p^I g^I - 2qV_X} > 0 \,, \tag{259}$$

so p and p^I are positive since the modular weights, as defined here, are generally positive.[f] Once the matter condensates have been eliminated by the equations of motion for their auxiliary fields, the equation of motion for the condensate auxiliary field F^c determines $|u_c|^2$; in particular there is a factor

$$\prod_\alpha |W_\alpha|^{2b_c^\alpha / b_c} \,. \tag{260}$$

The relation (258) holds for the p^I defined in (257) in terms of the original modular weights at the string scale, in which case

$$\sum_\alpha b_c^\alpha q_\alpha^I = \sum_B b_c^\alpha n_\alpha^B q_B^I = \sum_B \frac{C_c^B}{4\pi^2} q_B^I = b - b_c' - b_c^I, \quad \sum_\alpha b_c^\alpha = b_c - b_c', \tag{261}$$

giving

$$\sum_\alpha b_c^\alpha \left(q_\alpha'^I - 1 \right) = b^I - b_c - b_c^I \,, \tag{262}$$

which is just what one would have gotten using the modular weights obtained in unitary gauge with $q' = 0$. Now setting $\phi \to \phi' = 1$ in unitary gauge and proceeding as in Section 3, we obtain[g]

$$\bar{u}_c u_c = e^{-2b_c'/b_c} e^{\kappa - 2\widehat{S}/b_c} \prod_\alpha \left| \frac{b_c^\alpha}{4c_\alpha} \right|^{-2b_c^\alpha/b_c} \prod_I \left[2\mathrm{Re}\, t^I |\eta(t^I)|^4 \right]^{(b^I - b_c)/b_c} + \mathcal{O}(\delta), \tag{263}$$

[f] A rare exception to this statement can be found in Ref. 89.
[g] There are some errors in (3.33)–(3.39) of Ref. 55.

where $\kappa = \widehat{K} - G$ is the modular invariant part of the Kähler potential in the new basis. The Kähler moduli F-terms are now given by

$$F^I = -\frac{2\mathrm{Re}\, t^I}{1 + b^I \hat{\ell}} \frac{\bar{u}}{4} \left(b_c - b^I\right) \left[1 + 4\mathrm{Re}\, t^I \zeta(\bar{t}^I)\right]. \tag{264}$$

Note that the exponential factor in (263) contains $\widehat{S} = \tilde{s} + \mathcal{O}(\Delta)$. When re-expressed in terms of the actual string coupling that expression acquires a factor

$$e^{2(s-\tilde{s})/b_c} = (p\,\ell)^{p/b_c}\,, \tag{265}$$

which, in the weak coupling limit with $p\,\ell << 1$, amounts to a decrease in the scale of supersymmetry breaking, allowing for somewhat larger values of b_c.

To incorporate the effects of terms linear in U we need to retain terms quadratic in the components of U in the Lagrangian. Referring to (243) we have

$$\begin{aligned} D_V &= -\frac{1}{2}\mathcal{D}^\alpha W^V_\alpha \Big|_{\theta=\bar{\theta}=0} = -\frac{1}{2}\mathcal{D}^\alpha \left[\left(1 + \frac{1}{2qL(\widehat{L}, U, M)} \frac{\partial L(\widehat{L}, U, M)}{\partial U}\right) W^U_\alpha + W'_\alpha\right]\Big|_{\theta=\bar{\theta}=0} \\ &= \left[1 + \frac{1}{2q\ell(\hat{\ell}, u, m)} \frac{\partial \ell(\hat{\ell}, u.m)}{\partial u}\right] D + D', \qquad u = U|_{\theta=\bar{\theta}=0} \qquad m = M|_{\theta=\bar{\theta}=0}, \end{aligned} \tag{266}$$

where the bosonic, nonderivative part of D' is quadratic in auxiliary fields:

$$D' \sim |u_c|^2, \tag{267}$$

since the *vev* u_c of the hidden sector gaugino condensate sets the scale of supersymmetry breaking. The part of the Lagrangian that contains the auxiliary fields D is

$$\mathcal{L}(D) = \frac{s(\hat{\ell})}{2} D_V^2 + D \frac{\partial \widetilde{K}}{\partial D} = -V(D)\,. \tag{268}$$

Eliminating D by its equation of motion gives

$$\begin{aligned} V(D) &= \frac{1}{2s(\hat{\ell})} \left[A^2(\hat{\ell}, u, m) + 2s(\hat{\ell}) A(\hat{\ell}, u, m) D'\right] \\ &= \frac{1}{2s(\hat{\ell})} \left[A(\hat{\ell}, u, m) + s(\hat{\ell}) D'\right]^2 - \frac{s(\hat{\ell})}{2} D'^2, \end{aligned} \tag{269}$$

$$A(\hat{\ell}, u, m) = \left(1 + \frac{1}{2q\ell(\hat{\ell}, u, m)} \frac{\partial \ell(\hat{\ell}, u, m)}{\partial u}\right)^{-1} \frac{\partial \tilde{K}}{\partial D}. \tag{270}$$

The last term in (269) is $\mathcal{O}(|u_c|^4)$ and we may ignore it. Expanding

$$A(\hat{\ell}, u, m) + s(\hat{\ell}) D' = A_0(\hat{\ell}, m) + u A_1(\hat{\ell}, m) + u^2 A_2(\hat{\ell}, m)\,, \tag{271}$$

we note that A_0 is the u-independent part of $A + sD' = A + \mathcal{O}(|u_c|^2)$. But $\widetilde{K}$ has no term linear in U when we neglect $\epsilon \equiv \delta, x^m$. So $\partial A/\partial u \sim \epsilon, u$ and $A_0 \sim \epsilon, |u_c|^2$,

giving

$$V(D) = \frac{1}{2s(\hat{\ell})}\left[A_0 + A_1 u + A_2 u^2\right]^2 + \mathcal{O}(|u_c|^4)$$
$$= \frac{1}{2s(\hat{\ell})}\left(A_0^2 + 2A_0A_1 u + [A_1^2 + O(\epsilon, |u_c^2|)]u^2\right) + \mathcal{O}(|u_c|^4)\,. \qquad (272)$$

When order ϵ terms are included, in addition to the ϵ-dependence of $\kappa = \widehat{K} - G$ and $\widehat{S}$, the expression (263) contains a factor

$$e^{-(\delta+\bar{\delta})\sum_\alpha b_c^\alpha q_\alpha^X/q\, b_c} = e^{(\delta+\bar{\delta})p/b_c}, \qquad (273)$$

and the F-component of Φ' is given by[55]

$$F = -\frac{\bar{u}}{4}\left(\frac{\partial\tilde{K}}{\partial\delta\partial\bar{\delta}}\right)^{-1}\left[2\frac{\partial\widehat{S}}{\partial\bar{\delta}} - p - b_c\frac{\partial\widehat{K}}{\partial\bar{\delta}}\right], \qquad (274)$$

where the derivatives are taken with $\hat{\ell}$ held fixed.

In this toy model, F is of order δ. To see this, write

$$2\frac{\partial\widehat{S}}{\partial\bar{\delta}} - p - b_c\frac{\partial\widehat{K}}{\partial\bar{\delta}} = 2\left(1 + b_c\hat{\ell}\right)\frac{\partial\widehat{S}}{\partial\bar{\delta}} - p - b_c\frac{\partial\tilde{K}}{\partial\bar{\delta}} \qquad (275)$$

Taking the lowest component of the relation

$$\left(\frac{\partial\widetilde{K}}{\partial\Delta}\right)_{\widehat{L}} = \left(\frac{\partial K}{\partial\Delta}\right)_L + 2\widehat{L}\left(\frac{\partial S}{\partial\Delta}\right)_L, \qquad (276)$$

that follows[55] from (232) with $U \to \Delta$, gives

$$\left(\frac{\partial\widetilde{K}}{\partial\bar{\delta}}\right)_{\hat{\ell}} = \left(\frac{\partial K}{\partial\bar{\delta}}\right)_\ell = p\,\ell\,[1 + \mathcal{O}(\delta, u)] = p\hat{\ell} + \mathcal{O}(\epsilon, u)\,. \qquad (277)$$

The derivatives of the lowest component of (256) satisfy[55]

$$\frac{\partial\ell}{\partial\bar{\delta}} = \frac{\partial\ell}{\partial\hat{\ell}}\frac{p\,\ell^2}{3 - \ell\tilde{k}'(\ell)}[1 + O(\delta, u)] = \frac{p\,\ell^2}{3 - \ell\tilde{k}'(\ell)} + \mathcal{O}(\epsilon, u)\,, \qquad (278)$$

and, referring to (230) and (234),

$$\frac{\partial\widehat{S}}{\partial\bar{\delta}} = \frac{\partial\ell}{\partial\bar{\delta}}\left(\tilde{s}'(\ell) + \frac{3}{2\ell^2}\right) = \frac{1}{2}p + \mathcal{O}(\epsilon, u), \qquad (279)$$

so the expression in (275) is of order ϵ, u.

The full potential is given by

$$V = V(D) + \frac{|u|^2}{16}v(\hat{\ell}, \epsilon, u) + \left(\frac{\partial\tilde{K}}{\partial\delta\partial\bar{\delta}}\right)^{-1} F\overline{F} + \sum_I \frac{1 + b^I\hat{\ell}}{(t^I + \bar{t}^I)^2}\overline{F}^I F^I\,, \qquad (280)$$

with

$$\frac{\partial\tilde{K}}{\partial\delta\partial\bar{\delta}} = p\left(\ell + \frac{\partial\ell}{\partial\bar{\delta}}\right)[1 + O(\delta, u)]\,, \qquad (281)$$

and the function

$$v(\hat{\ell},\epsilon,u) = \frac{\partial_{\hat{\ell}}\widehat{K}}{\hat{\ell}}(1+b_c\hat{\ell})^2 - 3b_c^2 \tag{282}$$

is the same as in the BGW case with the replacement $k \to \widehat{K}$. It follows immediately from (246), (277) and (278) that $\partial\widehat{K}/\partial\delta$ is of order ϵ, u, that is, $\widehat{K}(\hat{\ell},m,u) = \tilde{k}(\hat{\ell}) + \mathcal{O}(x^M, \delta\epsilon, u\epsilon, u^2)$. This is just a consequence of the fact that Δ appears in $\widehat{K}$ in a linear combination with $U+\Delta_X$, and as shown in Section 4.1 there is no term linear in $U \to U+\Delta_X$ in either $\widehat{K}$ or $\widehat{S}$. $\widehat{S}$ contains a term linear in Δ that is independent of $\widehat{L}$, so all terms linear in Δ and $U+\Delta_X$ drop out of the condition (276). Expanding the potential in powers of u, it takes the form

$$V = V_0(\hat{\ell},m) + V_1(\hat{\ell},m)u + V_2(\hat{\ell},m)u^2 + \mathcal{O}(u^3), \tag{283}$$

where V_1 and V_2 are dominated by the D-term contribution (272):

$$V_1 = \frac{1}{s(\hat{\ell})}A_0A_1 + \mathcal{O}(\epsilon|u_c|^2) \sim \epsilon, \qquad V_2 = \frac{1}{2s(\hat{\ell})}A_1^2 + \mathcal{O}(\epsilon,|u_c|^2) \sim 1. \tag{284}$$

When rewritten in terms of the canonically normalized scalar v, which, to leading order, is related to u by the normalization factor found in (244), the potential (283) takes the form

$$\begin{aligned} V &= V_0(\hat{\ell},m) + \frac{1}{\sqrt{s(\hat{\ell})}}\left[A_0m_U + \mathcal{O}(\epsilon|u_c|^2)\right]v + \frac{1}{2}\left[m_U^2 + \mathcal{O}(\epsilon,|u_c|^2)\right]v^2 + \mathcal{O}(|u_c|^4) \\ &= V_0(\hat{\ell},m) - \frac{1}{2s(\hat{\ell})}A_0^2 + \frac{1}{2}\left[m_U^2 + \mathcal{O}(\epsilon,|u_c|^2)\right]v'^2 \end{aligned} \tag{285}$$

$$v' = v + \frac{A_0}{\sqrt{s(\hat{\ell})}m_U}\left[1+\mathcal{O}(\epsilon,|u_c|^2)\right] + \mathcal{O}(\epsilon|u_c|^2) + \mathcal{O}(|u_c|^4), \tag{286}$$

where m_U^2 is given in (245). The potential that results from integrating out u at tree level is obtained by setting $v'=0$ in (285). The term proportional to A_0^2, which includes the conventional D-term, cancels out and the potential $V(\ell,m)$ and is just given by the last three terms in (280) evaluated at $u=0$, up to order $|u_c|^4$ corrections. At the vacuum where $\langle x^M\rangle = \langle F^I\rangle = 0$,

$$V(\hat{\ell},\delta) = \frac{|u_c|^2}{16}\left[v(\hat{\ell}) + v_1(\hat{\ell})\delta^2\right] + \mathcal{O}(|u_c|^4), \tag{287}$$

and minimization of the potential with respect to δ gives $\delta \sim |u_c|^2$, so the potential for $\hat{\ell}$ is just the BGW potential $v(\hat{\ell})$ up to order $|u_c|^4$ corrections.

In more realistic models with n scalar *vevs* and $m \le n$ broken $U(1)$'s, the functions introduced in (255) are replaced by

$$\begin{aligned} K(L,M,U) &= \tilde{k}(L) + \sum_A k^A(L)\left(e^{\Delta\Theta_A + \overline{\Delta}\overline{\Theta}_A + 2\sum_a q_A^a(U_a+\Delta_a)} - 1\right) + \sum_M x^M, \\ S(L,M,U) &= \tilde{s}(L) - \delta_X\left(U_X + \Delta_X\right). \end{aligned} \tag{288}$$

The functions

$$k^A = |C_A|^2 \, e^{2\sum_a q_A^a h_a(L)} \tag{289}$$

are the modular invariant *vevs* generated at the $U(1)_X$ breaking scale, with h_a the L-dependent part of the shifts in the $U(1)_a$ vector superfields analogous to (250). The k^A are subject to the constraints

$$2q \sum_A q_A^a k^A = \delta_X^a \delta_X L \,, \tag{290}$$

that ensures vanishing D-terms and unbroken supersymmetry at that scale. It also ensures that the functions

$$\tilde{k}(L) = k(t) + \sum_A k^A(L), \qquad \tilde{s}(L) = s - \frac{1}{2}\delta_X h_X \,, \tag{291}$$

satisfy the Einstein-Hilbert normalization condition (230). The constant superfields Δ_{Θ_A} and Δ_a are the potential deviations of the *vevs* from their supersymmetric values.

The superpotential (248) is also modified, with now

$$W_\alpha(T,\Phi) = c_\alpha \prod_I [\,\eta(T^I)]^{2(q_\alpha^I + p_\alpha^I - 1)} \prod_A (\Phi_A)^{q_\alpha^A} \tag{292}$$

where the parameters q_α^A and p_α^I satisfy the constraints

$$\sum_A q_A^a q_\alpha^A = -q_\alpha^a, \qquad p_\alpha^I = \sum_A q_A^I q_\alpha^A \,. \tag{293}$$

The anomaly matching constraints in (261) are unchanged, and (247) and (262) are replaced by

$$\sum_\alpha b_c^\alpha q_\alpha^a = \sum_{\alpha,B} b_c^\alpha n_\alpha^B q_B^a = \sum_B \frac{C_c^B}{4\pi^2} q_B^a = -\frac{1}{2}\delta_X \delta_X^a, \tag{294}$$

and

$$\sum_\alpha b_c^\alpha \left(q_\alpha^I + p_\alpha^I - 1\right) = b - p^I - b_c - b_c^I, \tag{295}$$

where

$$p^I = \sum_\alpha b_c^\alpha p_\alpha^I \,. \tag{296}$$

Evaluating the *vevs* of $\phi_A \to \phi'_A$ in unitary gauge, (263) and (264) become

$$\bar{u}u = e^{-2b'_c/b_c} e^{\kappa - 2\widehat{S}/b_c} \prod_\alpha \left| \frac{b_c^\alpha}{4c_\alpha} \right|^{-2b_c^\alpha/b_c} \prod_I \left[2\mathrm{Re}\, t^I |\eta(t^I)|^4\right]^{(b+p^I-b_c)/b_c} + \mathcal{O}(\delta), \tag{297}$$

and

$$F^I = -\frac{2\mathrm{Re}\, t^I}{1 + b^I \hat{\ell}} \frac{\bar{u}}{4} \left(b_c - b - p^I\right) \left[1 + 4\mathrm{Re}\, t^I \zeta(\bar{t}^I)\right] . \tag{298}$$

Finally, the F-component (274) of Φ' takes the form

$$F^A = -\frac{\bar{u}}{4}\sum_B \widetilde{K}^{A\overline{B}}\left[2\widehat{S}_{\bar{B}} - p_B - \widehat{K}_{\overline{B}} b_c\right], \tag{299}$$

where as usual the indices denote derivatives with respect to chiral and anti-chiral superfields, $\widetilde{K}^{A\overline{B}}$ is the inverse of the metric derived from the "effective" Kähler potential (246), and the parameters[h]

$$p^A = \sum_\alpha b_c^\alpha q_\alpha^A, \tag{300}$$

are constrained by the anomaly matching condition (294) which implies

$$2\sum_A q_A^a p^A = 2\sum_{\alpha,A} b_c^\alpha q_\alpha^A q_A^a = -2\sum_\alpha b_c^\alpha q_\alpha^a = \delta_X \delta_X^a. \tag{301}$$

There are two distinct cases:

- Minimal models: if $n = m$, the matrix q_A^a has an inverse Q_a^A, since if m $U(1)$'s are broken, by definition the q's form a set of m linearly independent m-component vectors. The m vector superfields V_a "eat" m modular invariant combinations

$$-\frac{1}{2}\sum_A Q_a^A\left(\Theta_A + \overline{\Theta}_A + G_A\right) \tag{302}$$

of chiral superfields, with $\Phi^A = C_A e^{\Theta_A}$, resulting in a redefinition of the modular weights of the chiral superfields Φ^M:

$$q_M'^I = q^I - \sum_{a,A} q_M^a Q_a^A q_A^I, \tag{303}$$

and a corresponding shift in the coefficient of the GS term:

$$b^I = b + \frac{1}{2}\delta_X \sum_A Q_a^A q_A^I. \tag{304}$$

The constraint (293) on q_α^A can be inverted:

$$q_\alpha^A = -\sum_a Q_a^A q_\alpha^A, \qquad p_\alpha^I = -\sum_{A,a} q_A^I Q_a^A q_\alpha^a\,. \tag{305}$$

Using the anomaly matching condition (294) in the definition (296) gives

$$p^I = b^I - b \tag{306}$$

as before, because in unitary gauge $\Phi'^A = C_A$ is again modular invariant. Contracting the constraints (290) and (301) with Q_a^B gives

$$k^B = \ell p^B = \frac{1}{2}\delta_X Q_X^B. \tag{307}$$

[h]See the second footnote in 4.1.

Then, proceeding as for the toy model, it is straightforward to show[i] that F^A and $\partial v/\partial\delta_A$ are of order ϵ, u as in the toy model.

- Nonminimal models: if $n > m$, the matrix q^a_A is not invertible and in general $p^I \neq b^I$, because the transformation to unitary gauge does not remove the full modular weight from Φ^A; Θ'_A is a linear combination of the $n-m$ D-moduli with net modular weight $q'^I_A \neq 0$. In addition, $k^A \neq \ell p^A$ in general, and F^A does not vanish when $\delta = 0$. An exception is the case $n = Nm$ with N replicas of a minimal set of ϕ^A, with identical $U(1)_a$ charges and modular weights, that get *vevs* . They are all made modular invariant by the same transformation so $p^I = b^I$, and

$$k^A = \ell p^A = \frac{1}{2N}\delta_X Q^A_X \,. \tag{308}$$

The effective Lagrangian for the moduli sector is the same as in the minimal case, but there are $m(N-1)$ D-moduli.

Before discussing the phenomenology of these models, we note that in typical orbifold compactifications with Wilson lines there is no asymptotically free gauge group above the scale of $U(1)$ breaking. One or more asymptotically free gauge sector will emerge below that scale, provided a sufficient number of gauge-charged chiral multiplets get masses through their superpotential couplings to the Φ^A that acquire large *vevs*. In their contribution to the quantum Lagrangian (211) the effective IR cut-off is their mass, rather than the condensate scale, as in the Lagrangian (399) introduced later in Section 5.5 for supersymmetric QCD. The mass terms in the superpotential are $U(1)$ and modular invariant, containing factors analogous to the W_α in (292) that assure that $\mathcal{L}_a$ has the correct anomaly structure at the string scale. The net result is that the potential is identical to that given above, with p^I, p^A determined by the full massless spectrum at the string scale, except that the β-function coefficient b_c is the one that controls the RGE running below the $U(1)_a$ breaking scale, without the contributions from the heavy modes.[55]

4.3. *The vacuum and the moduli sector*

The potential is modular invariant, with a similar t-dependence as is found in the BGW model described in Section 3, so the moduli are still stabilized at self-dual points $t_1 = 1,\ t_2 = e^{i\pi/6}$, with $\langle F^I \rangle = 0$. The T-moduli masses are determined by the coefficients of F^I in (298). In models that satisfy (307), the squared masses are positive since $b \geq b_c$ and, using (304), (306) and (307),

$$p^I = \sum_A p^A \equiv p > 0, \tag{309}$$

because $k \sim |\Phi|^2$ and ℓ are positive. For example in a $\mathbf{Z}_3$ orbifold model of Font et al. that we will refer to as the FIQS model,[179] the three Kähler moduli are

[i]Some terms linear in δ in the F-term where inadvertently dropped in Ref. 55

approximately degenerate with $m_{\mathrm{Re}\,t}(t_i) \approx 5(6)m_{3/2}$, $m_{\mathrm{Im}\,t}(t_i) \approx 1(2)m_{3/2}$ for $i = 1(2)$.

If we set the moduli at self dual points, the potential for the dilaton becomes, after integrating out the scalar components u_a of U_a as in (283)[j]

$$V(\hat{\ell}) = \frac{|u_c|^2}{16}\left[w(\hat{\ell}) + v(\hat{\ell})\right] + \mathcal{O}(\epsilon|u_c|^2, \delta^3, |u_c|^4), \tag{310}$$

where $v(\hat{\ell}) = v(\hat{\ell}, 0)$ is given in (282), and the F contribution has

$$w = w_0 - \delta_X h'_X(\hat{\ell})(1 + b_c\hat{\ell})^2, \qquad w_0 = \sum_A \frac{(p^A + b_c k^A)^2}{k^A}. \tag{311}$$

For the minimal case characterized by (307), $w = 0$, $V = V(\hat{\ell}) + \mathcal{O}(\delta^2|u_c|^2)$, the dilaton potential is the same as in the BGW case discussed in Sections 2 and 3, except for the shift $k \to \tilde{k}$ in the dilaton metric in both the potential and in the kinetic energy term, so that the constraint (160) on the dilaton metric is unchanged. In the general case, imposing vanishing vacuum energy at leading order in $\delta \sim |u_c|^2$ gives

$$\hat{\ell}^{-1}\left(1 + b_c\hat{\ell}\right)^2 \tilde{k}'(\hat{\ell}) = 3b_c^2 - w, \tag{312}$$

and positivity[k] of the dilaton metric requires $w < 3b_c^2$. The viable region of parameter space can be explored[55] by separating out a minimal subset of the $n > m$ Φ^A with nonvanishing p^A:

$$k^A, \;\; A = 1, \ldots n, \;\; \to \;\; (k^P, k^M), \quad P = 1, \ldots, m, \quad M = 1, \ldots, n - m, \tag{313}$$

and writing

$$k^A = \ell p^A + y^A\,, \qquad \sum_A q^a_A y^A = 0\,, \tag{314}$$

where the last equality ensures that if (301) is satisfied, so is (290). The vacuum is degenerate at the $U(1)_a$ breaking scale; at the condensation scale, the (approximate) vacuum values will be those that minimize v with respect to the y^A subject to the condition in (314). If $y^A = 0$, for all A, the dilaton potential is identical to the minimal case. Defining Q^P_a as the inverse of the matrix q^a_P, the constraint in (314) reads

$$y^P = -\sum_M \zeta^P_M y^M, \qquad \zeta^P_M = \sum_a Q^P_a q^a_M, \tag{315}$$

[j]In nonminimal models with $F^A, \partial V(\hat{\ell})/\partial\delta_A \sim |u_c|^2$ instead of $\delta|u_c|^2$, it is necessary to keep order δ^3 terms in the u_a expansion of the D-terms analogous to (269). Then minimization of the potential with respect to δ gives $\delta^2 \sim |u_c|^2$, $\delta \sim |u_c| \ll 1$, $\delta^3 \sim |u_c|^3$, so the potential for $\hat{\ell}$ is still dominated by the explicit terms in (310), and δ can be ignored in the analysis of these models discussed briefly below.

[k]If the dilaton metric goes through zero, one should rewrite the theory in terms of the canonically normalized field, in terms of which the zero of the metric becomes a singularity in the potential. It is not clear that there might not be some viable region of parameter space in this case.

and we may take the y^M as independent variables. Since

$$\tilde{k}'(\hat{\ell}) = k'(\hat{\ell}) + \sum_A k'^A(\hat{\ell}) = k'(\hat{\ell}) + \delta_X \hat{\ell} h'_X(\hat{\ell}), \tag{316}$$

minimizing $V \propto v + w$ with respect to y^M amounts to minimizing w_0, giving the conditions

$$0 = \frac{\partial w_0}{\partial k^M} - \sum_P \zeta_M^P \frac{\partial w_0}{\partial k^P} = -(p^M/k^M)^2 + b_c^2 + \sum_P \zeta_M^P \left[(p^P/k^P)^2 - b_c^2\right]. \tag{317}$$

If $p^M = 0$, we require $k^M = y^M \geq 0$, and

$$\left.\frac{\partial w_0}{\partial y^M}\right|_{y=0} = b_c^2 + \sum_P \zeta_M^P \left(\hat{\ell}^{-2} - b_c^2\right). \tag{318}$$

If $\hat{\ell} b_c^2 < 1$ and $\sum_P \zeta_M^P \geq 0$, the minimum indeed corresponds to $y = 0$. However if $\sum_P \zeta_M^P < 0$, the minimum corresponds to a smaller w_0 with $y > 0$. Larger y results in smaller h'_X, so the net effect is to increase w, and positivity of the dilaton metric becomes difficult to maintain.[55]

If $p^M \neq 0$ we have instead of (318) the expression

$$\left.\frac{\partial w_0}{\partial y^M}\right|_{y=0} = \left(\sum_P \zeta_M^P - 1\right)\left(\hat{\ell}^{-2} - b_c^2\right). \tag{319}$$

In this case $y = 0$ is the minimum if and only if $\sum_P \zeta_M^P = 1$. If $\sum_P \zeta_M^P > 0$, the minimum will in general shift slightly from $y = 0$. The dilaton potential for these cases is not substantially different from the minimal case. On the other hand if $\sum_P \zeta_M^P < 0$, the situation is similar to the case with $p^M = 0$: the minimum occurs for larger k^A and larger w, such that positivity of the dilaton metric is difficult to maintain. For example in the FIQS model there are minimal flat directions in which one, two or three charge-degenerate sets of six chiral supermultiplets acquire *vevs* and break six $U(1)$'s. There are additional F-flat and D-flat directions associated with "invariant blocks" $\mathcal{B}$ of fields such that

$$\sum_{P\in\mathcal{B}} q_P^a + \sum_{M\in\mathcal{B}} q_M^a = 0. \tag{320}$$

It is clear that if we choose Φ^M that form invariant blocks with the Φ^P, at least some $\zeta_M^P < 0$. The numerical solution to the minimization equations for one such choice[55] gave $\left\langle \tilde{k}' \right\rangle < 0$, and this result is likely to be generic.

The vacuum conditions $v(\hat{\ell}) = v'(\hat{\ell}) = 0$ require $\tilde{k}' \sim \hat{\ell}\tilde{k}'' \sim \hat{\ell} b_c^2$ in the vacuum, but there is no *a priori* reason to expect $\tilde{k}'''$ to be similarly suppressed if $\hat{\ell} \sim 1$, in which case the dilaton mass is enhanced as will be discussed in Section 5.6. If there is just one hidden sector condensate the universal axion is massless at the scale of supersymmetry breaking, and is a candidate for the QCD axion, as will be discussed in Section 5.5. The fermionic superpartners of the moduli have masses similar to those of the Kähler moduli. Using the FIQS model again as an example,

it can be shown that for a minimal set of *vevs* there are two linear combinations of the three fermions χ^I that have no mixing with the dilatino, while one linear combination that is approximately an equal admixture of the χ^I, mixes with the dilatino.[55] All four masses are roughly the same as $m_{\mathrm{Re}\,t}(t_i) \approx 5\text{–}6 m_{3/2}$.

4.4. *Observable sector masses: conditions for a viable model*

In the minimal models as defined above, the tree level masses of the observable sector gauginos are identical to those in the BGW case; they are suppressed due to a factor of the dilaton metric $\tilde{k}' \sim 3b_c^2$. In nonminimal models they will be suppressed even further since the metric is suppressed further, as discussed in 4.3. When we consider chiral fields Φ^M with no couplings to the condensates we have to include their F-terms which is the same as those for the Φ^A except that $p^M = 0$:

$$\begin{aligned} V \ni \sum_M F^M F_M &= \frac{|u_c|^2}{16} \sum_M \frac{\left[x^M b_c - \left(1 + b_c \hat{\ell}\right) x'^M\right]^2}{x^M} \left[1 + \mathcal{O}(x^2)\right] \\ &= \frac{|u_c|^2}{16} \sum_M \left[x^M b_c^2 - 2b_c \left(1 + b_c \hat{\ell}\right) x'^M + 2\left(1 + b_c \hat{\ell}\right)^2 \sum_a q_M^a h_a' x'^M\right]. \end{aligned} \tag{321}$$

For fields with vanishing *vevs*, $\partial V/\partial\phi \propto \phi$ vanishes in the vacuum, the mass matrix is diagonal:

$$\begin{aligned} m_M^2 = \frac{\partial V}{\partial x^M} &= \frac{|u|^2}{16\hat{\ell}}(1 + b_c\hat{\ell})^2 \frac{\partial \widehat{K}'(\hat{\ell})}{\partial x^M} + \frac{\partial}{\partial x^M}(\widetilde{K}_{M\bar{N}} F^M \bar{F}^{\bar{N}}) + \frac{\partial}{\partial x^M}(\widetilde{K}_{A\bar{B}} F^A \bar{F}^{\bar{B}}) \\ &= \frac{|u|^2}{16}\left[b_c^2 - 4b_c(1 + b_c\hat{\ell}) \sum_a h_a' q_M^a - 2(1 + b_c\hat{\ell})^2 \sum_a h_a'' q_M^a\right] \\ &\quad + \frac{\partial}{\partial x^M}(\widetilde{K}_{A\bar{B}} F^A \bar{F}^{\bar{B}}) + \mathcal{O}(x^M). \end{aligned} \tag{322}$$

In minimal models with

$$h_a' = -\hat{\ell} h_a'' = \sum_A Q_a^A / 2\hat{\ell}, \qquad p^A = k^A/\hat{\ell} = \delta_X Q_X^A/2, \tag{323}$$

the last term in (322) is of order $\delta|u_c|^2 \sim |u_c|^4$, giving

$$m_M = m_{3/2}\left[1 + \frac{\zeta_M}{b_c^2 \hat{\ell}^2}\left(1 - b_c^2 \hat{\ell}^2\right)\right] + \mathcal{O}(|u_c|^4), \qquad \zeta_M = \sum_{a,A} Q_a^A q_M^a. \tag{324}$$

The first term in (324) is just the F-term contribution of the BGM model; the second term is expected to dominate because of the factor $b_c^{-2} >> 1$, and can give large (and in some cases negative) masses to squarks and sleptons. As discussed in Section 3.1, the gravity/dilaton mediated F-term contribution to scalar masses can be enhanced if they have couplings in the GS term. So, for example, if $p_i = b_{\mathrm{GS}} \gg b_+$ in (150) it might be possible to arrange for all masses to be positive (except possibly the Higgs) at the condensation scale, but this would involve some measure of fine

tuning. The possibility of including higher order terms in the gauge-charged chiral superfields $|\Phi|$ in the Kähler potential was considered in Ref. 55 and found to make little difference. The problem can be attenuated somewhat if b_c and/or $\hat{\ell}$ is larger than expected.[1] The factor (265) that now appears in the expression for the condensate *vev* may allow for the former, and alternate parameterizations[55] of nonperturbative string effects, based on corrections to the action rather that to the function $f(L)$ in (60) as well as perturbative[48] field theoretic quantum corrections to $k(\ell)$ can allow for a value of $\hat{\ell}$ larger than one while preserving weak coupling $s(\hat{\ell}) \leq 1$. Another difficulty with large charge-dependent contributions to observable sector scalar masses is that they can generate flavor-changing neutral currents. This can be avoided if MSSM states come in sets of three with all the same quantum numbers (*i.e.* $U(1)$ charges and modular weights as well Standard Model quantum numbers), as can happen for example in $\mathbf{Z}_3$ orbifold compactifications. The simplest solution to these problems would be to find a compactification such that $\zeta_M = 0$ for observable sector chiral multiplets, which implies strong constraints on the various $U(1)$ charges.

In nonminimal models that do not involve just charge-degenerate minimal sets, the expressions for observable sector masses are more complicated, but they are always of the form $m_M^2 = m_{3/2}^2 \left(1 + \zeta_M f(b_c\hat{\ell})/b_c^2\hat{\ell}^2\right)$. The parameter space for these nonminimal models is constrained to be rather small because of the positivity constraint on the dilaton metric, the need to avoid an overly large scalar/gaugino mass ratio and a very large axion coupling constant. Although it has recently been shown[164] that the original cosmological bounds[165,166,167] on the axion coupling can be evaded, increasing the coupling at the string scale leads to a proportional increase at the QCD scale that might further restrict the class of compactifications that allow the universal axion to be the QCD axion. We will return to this issue in Section 5.5. A possible advantage of more general nonminimal models might be that when more fields get *vevs*, more unwanted states are removed from the spectrum. On the other hand, there are other scales where masses could be generated for additional fields, such as the condensation scale itself, $\Lambda_c \sim 10^{13-14}$ GeV, and a scale $\Lambda_\nu \sim 10^{11-12}$ GeV suggested by observed neutrino masses. Indeed masses generated at one or more intermediate scales can be useful in reconciling the data with string scale unification.[168]

4.5. *Lifting the vacuum degeneracy: D-moduli masses*

In minimal models all complex scalars that vanish in the vacuum and have no couplings to matter condensates acquire masses $m = m_{3/2}$ (or somewhat larger masses if they couple to the GS term), while their fermionic superpartners remain massless, just as for the observable sector of the MSSM. In nonminimal models

[1]The point $b_c^2\hat{\ell}^2 = 1$ where the charge-dependent contribution to (324) actually vanishes is also the point where (319) vanishes identically, which is the condition for a minimal solution.

where the number n of scalar fields that get large *vevs* is larger than the number m of broken $U(1)$'s, the situation is very different. The Kähler potential in (288) for the chiral multiplets with nonvanishing *vevs* is replaced by

$$K(L,\Delta,\widehat{\Sigma}) = \tilde{k}(L) + \sum_A k^A(L)\left(e^{\widehat{\Sigma}_A+\Delta_A+\overline{\Delta}_A+2\sum_a q^a_A(U_a+\Delta_a)} - 1\right), \tag{325}$$

where the modular invariant fields $\widehat{\Sigma}_A$ are defined by[55]

$$\widehat{\Sigma}_A = \Sigma_A - \sum_{a,b,B} q^a_A x^B q^b_B N^{-1}_{ab}\Sigma_B, \qquad \Sigma_A = \Theta_A + \overline{\Theta}_A + G_A, \tag{326}$$

and satisfy

$$\sum_A q^a_A x^A(L)\widehat{\Sigma}_A(L) = 0, \qquad x^A = k^A e^{\Delta_A+\overline{\Delta}_A+2\sum_a q^a_A\Delta_a}. \tag{327}$$

They have vanishing *vevs* and are the $n-m$ uneaten Goldstone supermultiplets above the scale of supersymmetry breaking.[157,163] But when $\widehat{\Sigma} \neq 0$ we have to include them among the superfields M that appear in the superfield Weyl transformation (256) to the basis where (232) is satisfied. If we restrict ourselves to the class of "minimal" models with $n = Nm$ where N charge-degenerate sets of scalars get *vevs*, the constraints (327), that insure the absence of a linear coupling of $\widehat{\Sigma}$ to U_a, imply the additional constraint

$$\sum_A x^A(L)\widehat{\Sigma}_A(L) = 0, \tag{328}$$

with

$$x^A = k^A = Lp^A = Lk'^A = L\delta_X Q^A_X/2N, \tag{329}$$

so we may drop contractions of $\widehat{\Sigma}_A$ with any of the vectors in (329), as well as higher ℓ-derivatives of k^A. Then the expressions

$$\begin{aligned} L - \hat{L} &= \frac{L^2\sum_A k'^A(e^{\widehat{\Sigma}_A}-1)}{3 - L\tilde{k}'(L)}, \\ \widehat{K}(\widehat{L}) &= \tilde{k}(L) + \sum_A k^A(e^{\widehat{\Sigma}_A}-1) + 3\ln(\widehat{L}/L), \\ \widehat{S}(\widehat{L}) &= \tilde{s}(L) + \frac{3}{2\widehat{L}L}(L-\widehat{L}), \end{aligned} \tag{330}$$

have no terms linear in $\widehat{\Sigma}$. Expanding as before, we obtain for the effective Kähler potential

$$\tilde{k} = \widehat{K} + 2\widehat{L}\widehat{S} = \tilde{k}(\widehat{L}) + 2\widehat{L}\tilde{s}(\widehat{L}) + \frac{1}{2}\sum_A k^A(\widehat{\Sigma}_A)^2 + \mathcal{O}(\widehat{\Sigma}^3), \tag{331}$$

and, using the constraint (327)–(329)

$$\widehat{K}(\hat{\ell}) = \tilde{k}(\hat{\ell}) + \mathcal{O}[(\ell-\hat{\ell})^2] = \tilde{k}(\hat{\ell}) + \mathcal{O}(\widehat{\Sigma}^4), \tag{332}$$

where we have dropped corrections of order $\delta \sim |u_c|^2$. The D-moduli mass comes from the potential term

$$\tilde{K}_{A\overline{B}} F^A \overline{F}^{\overline{B}} = \frac{|u|^2}{32\hat{\ell}^2} \sum_A k^A (\widehat{\Sigma}_A)^2. \tag{333}$$

The modular invariant fields $\widehat{\Sigma}_A$ with $< \widehat{\Sigma}_A >= 0$ are independent of L in these models, and may be expressed[162] in terms of chiral and anti-chiral fields $D_A, \overline{D}_A$ by expanding the moduli about their *vevs* $\langle t^I \rangle$:

$$\begin{aligned}
\widehat{\Sigma}_A &= D_A + \overline{D}_A + \mathcal{O}\left(\left[\widehat{T}^I + \widehat{\overline{T}}^I \right]^2 / \left\langle t^I + \bar{t}^I \right\rangle^2 \right), \\
D_A &= \Theta_A + \frac{1}{2}\langle G_A \rangle + \left\langle \frac{\partial G_A}{\partial t^I} \right\rangle \widehat{T}^I - \frac{1}{N} \sum_{B; q_B^a = q_A^a} \left(\Theta_B + \frac{1}{2}\langle G_B \rangle + \left\langle \frac{\partial G_B}{\partial t^I} \right\rangle \widehat{T}^I \right), \\
T^I &= \left\langle t^I \right\rangle + \widehat{T}^I, \qquad \left\langle D_A + \overline{D}_A \right\rangle = 0\,.
\end{aligned} \tag{334}$$

When we re-express the chiral multiplets D_A in terms of an orthonormal set D_i subject to the constraint (328):

$$D_A = \sum_{i=1}^{n-m} c_A^i D_i\,, \qquad \sum_A k^A c_A^i = 0\,, \tag{335}$$

the Kähler metric and the squared mass matrix

$$K_{i\bar{j}} = \sum_A C_i^A k^A C_j^A, \qquad \mu_{ij}^2 = \frac{m_{3/2}^2}{b_c^2 \ell^2} \sum_A C_i^A k^A C_j^A\,, \tag{336}$$

with C_i^A the inverse of c_A^i, are diagonalized by the same unitary transformation:

$$D_i \to D_i = U_i^j D_j\,, \qquad d_i = D_i| = N_d(\sigma_i + i a_i)\,, \tag{337}$$

where the normalization constant N_d is chosen to make the kinetic energy term canonically normalized. Then the Lagrangian quadratic in the scalar D-moduli reads

$$\mathcal{L}_D = \frac{1}{2} \sum_i \left[\partial_\mu \sigma_i \partial^\mu \sigma_i + \partial_\mu a_i \partial^\mu a_i - 2 \frac{m_{3/2}^2}{b_c^2 \hat{\ell}^2} (\sigma_i)^2 \right]. \tag{338}$$

The Yukawa couplings for condensation models of the type considered here are given in Ref. 73; they generate squared masses for the D-moduli fermions $(\chi_{i\,L})_\alpha = \mathcal{D}_\alpha D_i| / \sqrt{2}$ that are equal to half the scalar squared masses up to corrections of order $b_c \hat{\ell} = m_{3/2}/m_\chi$.[55] If $\hat{\ell} \sim 1$ and $b_c \sim .03 - .04$ as suggested by the dark matter analyses to be discussed in Section 5.4 below, the scalar and fermion masses are much larger than the gravitino mass:

$$m_\chi \approx m_\sigma / \sqrt{2} \approx (24 - 30) m_{3/2}\,. \tag{339}$$

However, as discussed in Section 4.4, this hierarchy requires constraints on observable scalar $U(1)$ charges for a viable phenomenology. On the other hand, the

D-axions remain massless because the phase of $\theta = \Theta|$ is undetermined. These axions might play an interesting role in cosmology with possible contributions to dark energy or dark matter. Just as in the minimal models, in these restricted non-minimal ones there are generically many fields with nonvanishing *vevs* at the $U(1)$ breaking scale for which the complex scalars acquire masses $m = m_{3/2}$, and the fermions remain massless. This would lead to a disastrous cosmology,[169,170,171] but presumably many of these fields acquire supersymmetric masses through superpotential terms coupling them to the Φ^A that do get *vevs*, as well as to other fields that might acquire *vevs* at some intermediate scales as mentioned at the end of Section 4.4.

5. Particle physics phenomenology

5.1. *R-parity and modular invariance*

One of the challenges of string theory is to provide a mechanism for forbidding operators that violate lepton and baryon number in the low-energy effective theory. In the MSSM this is achieved by imposing a discrete symmetry called R-parity such that the unwanted operators are forbidden, while those that give masses to quarks and leptons are allowed, as is the Higgs mass term (μ-term) that is needed to produce the correct electroweak symmetry breaking pattern. We saw in Section 2.3 that the T-duality, or modular invariance, of the weakly-coupled heterotic string ensures that when SUSY is broken by gaugino condensation in a hidden sector the Kähler moduli are stabilized at self-dual points where their F-terms vanish. As a result, in the absence of other sources of supersymmetry breaking, transmission from the hidden sector to the observable sector is dilaton-mediated. Provided that the matter couplings p_i to the Green-Schwarz term (116) vanish (or are degenerate), the resulting scalar masses will be flavor-diagonal at tree level. The loop corrections are then merely RG-induced, the off-diagonal scalar masses are small and scalar-mediated flavor changing neutral current (FCNC) effects are therefore sufficiently small. We saw however in Section 4 that charge-dependent scalar masses may be generated when there is $U(1)$ anomaly cancelation by a second Green-Schwarz term. Such a term amounts to an FI term that triggers the breaking of some number m of $U(1)$'s by n nonvanishing scalar *vevs* $\langle \Phi_A^\alpha \rangle$; in this case the theory remains FCNC-free if observable sector fields that are degenerate under Standard Model charges are also degenerate under the broken $U(1)$ charges. A second consequence of modular invariance is that there is generally a residual discrete symmetry that might play the role of R-parity in the MSSM.[172]

The modular transformations given in (76) and (84) are those of the minimal subgroup of a generally larger group of modular transformations. For example in $\mathbf{Z}_3$ orbifolds with just three diagonal Kähler moduli, the largest possible symmetry

group is $[SL(2,\mathbf{Z})]^3$ with[a]

$$T^I \to \frac{a^I T^I - ib^I}{ic^I T^I + d^I}, \qquad \Phi^A \to e^{i\delta_A - \sum_I q_A^I F^I}\Phi^A, \qquad F^I = \ln\left(ic^I T^I + d^I\right),$$
$$a^I d^I - b^I c^I = 1, \qquad a^I, b^I, c^I, d^I \in \mathbf{Z} \qquad \forall\, I = 1,2,3, \tag{340}$$

under which the Kähler potential and superpotential transform as

$$K \to K + F + \overline{F}, \qquad W \to e^{-F}W, \qquad F = \sum_I F^I. \tag{341}$$

The T^I are trivially invariant under (340) with

$$a^I = d^I = \pm 1, \qquad b^I = c^I = 0, \qquad e^{F^I} = e^{in\pi}\,. \tag{342}$$

The self-dual vacua $T_{s\,d}$ are further invariant under (340) with

$$b^I = -c^I = \pm 1\,, \tag{343}$$

and for $\langle t^I \rangle = 1$

$$a^I = d^I = 0, \qquad e^{F^I} = e^{i\frac{n}{2}\pi}, \tag{344}$$

or for $\langle t^I \rangle = e^{i\pi/6}$

$$\begin{cases} a^I = b^I,\ d^I = 0, \\ d^I = c^I,\ a^I = 0 \end{cases}, \qquad e^{F^I} = e^{i\frac{n}{3}\pi}. \tag{345}$$

Thus for three moduli at self-dual points the residual symmetry group is $G_R = \mathbf{Z}_4^m \otimes Z_{\mathbf{6}}^{m'},\ m + m' = 3$.

The gaugino condensate *vevs* $\langle u \rangle \neq 0$ break this further to a subgroup with

$$e^{i\,\mathrm{Im}F} = e^F = e^{2ni\pi}, \tag{346}$$

under which $\lambda_L \to e^{-\frac{i}{2}\mathrm{Im}F}\lambda_L = \pm\lambda_L$. It is natural to identify the case with a minus sign with R-parity. This subgroup also leaves invariant the soft supersymmetry-breaking terms in the observable sector, if no other field gets a *vev* that breaks it. Consider, for example a scenario in which the μ-term comes from a superpotential term XH_uH_d, with the *vev* $\langle x \rangle = \langle X| \rangle \neq 0$ generated at the TeV scale. Then the symmetry could be broken further to a subgroup $R \in G_R$ such that $R\,X = X$, unless there is a concomitant breaking of a $U(1)$ gauge factor such that X is invariant under redefined G_R transformations that include global transformations under the broken $U(1)$. On the other hand if the μ-term comes from a Kähler potential term generated by invariant *vevs* above the scale where the moduli are fixed, there would be no further breaking until the Higgs get *vevs*. Since an invariant μ-term implies that H_u and H_d have opposite G_R charges as well as opposite weak hypercharge, the G_R transformations can be redefined to include global hypercharge transformations

[a]Here we are again neglecting mixing[95,97] among twisted sector fields of the same modular weights q_A^I with mixing parameters that depend on the integers a^I, b^I, c^I, d^I.

such that they are both invariant, and no further breaking of the discrete symmetry occurs at the electroweak scale.

The transformation property (89) of the Dedekind η-function is not fully general. More precisely it reads

$$\eta(T^I) \to e^{i\delta_I} e^{\frac{1}{2}F(T^I)}\eta(T^I), \quad F(T^I) = F^I, \quad \delta_I = \delta_I(a^I, b^I, c^I, d^I)\,. \tag{347}$$

The constant phases[7] δ_I are commonly dropped in the literature because they do not appear in the scalar potential, and it follows from T-duality that they can be re-absorbed[8] into the transformation properties of the twisted sector fields by removing the phases δ_A in (340). This was implicitly done in writing (84), following the commonly used convention. When these phases are taken into account the constraints obtained by imposing T-duality covariance on superpotential terms of the form

$$W = \prod_A \Phi^A \prod_I \eta(T^I)^{2\left(\sum_A q_A^I - 1\right)}, \tag{348}$$

coincide with selection rules[174] that follow from the discrete symmetries of orbifold compactification.

Consider for example a $\mathbf{Z}_3$ orbifold with untwisted sector fields U_A^I, and twisted sector fields T_A and Y_A^I with modular weights

$$\left(q_{IA}^J\right)_U = \delta_I^J, \qquad \left(q_A^I\right)_T = \left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right), \qquad \left(q_{IA}^J\right)_Y = \left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right) + \delta_I^J\,. \tag{349}$$

Allowed superpotential terms trilinear in matter fields are of the form[8] $U_1U_2U_3$ and $(T)^3$. When the appropriate Dedekind η factors in (348) are included, all such terms are covariant provided $\delta_U = 0$, $\delta_T = -\frac{2}{3}\delta = -\frac{2}{3}\sum_I \delta_I$. If we further impose $\delta_{Y^I} = -\frac{2}{3}\delta - 4\delta_I$, the superpotential terms (348) constructed from the monomials

$$U_1U_2U_3, \quad T^3, \quad U_IY^IT^2, \quad U_IY^IU_JY^JT, \quad U_IY^IU_JY^JU_KY^K, \tag{350}$$

are covariant under (340) and (347). Higher dimensional monomials can be constructed by adding factors of invariant monomials. For example

$$\eta^6\Pi, \qquad \Pi = Y^1Y^2Y^3, \qquad \eta = \prod_I \eta_I, \tag{351}$$

is invariant. The group (340) of duality transformations on T^I is generated[173] by $T^I \to 1/T^I$ with $\delta(0,1,-1,0) = \pi/4 \bmod 2\pi$, and $T^I \to T^I - i$ with $\delta(1,1,0,1) = \pi/12 \bmod 2\pi$. Therefore $\delta = n\pi/12$ is a rational number, and invariant operators can be constructed from products of covariant operators (348) multiplied by η:

$$\eta^{2m}\prod_{i=1}^m W_i, \qquad 2m\delta = m'\frac{\pi}{6} = 2\pi n\,. \tag{352}$$

These potential terms are consistent with the selection rules;[174] they are further restricted by additional selection rules and gauge invariance. The invariant operators (351) and (352) can also be used to construct terms in the Kähler potential.

For the subgroup defined by (342) - (346), $i\delta = -\frac{1}{2}F = -in\pi$, the superpotential is invariant, as are the monomials in (350). Therefore any product of them could appear in the superpotential or Kähler potential in the effective theory below the scale where the T-moduli are fixed and supersymmetry is broken (*e.g.* through quantum corrections and/or integrating out massive fields) with perhaps additional *vevs* that are invariant under some G_R generated at that scale.

Superpotential terms of dimension three can be generated from higher order terms when some fields acquire *vevs*. In models with an anomalous $U(1)_X$, with m $U(1)$'s broken by $n = Nm$ *vevs* as discussed above, the modular weights are modified in the same way as in (303) for the minimal case $n = m$. Then for a term in the superpotential (348) with some $\langle \Phi^A \rangle \neq 0$:

$$\begin{aligned} W &= \prod_M \Phi^M \prod_A \langle \Phi^A \rangle \prod_I [\eta(T^I)]^{2\left(\sum_M q^I_M + \sum_B q^I_A - 1\right)} \\ &= \prod_M \Phi'^M \prod_A \langle \Phi^A \rangle \prod_I [\eta(T^I)]^{2\left(\sum_M q'^I_M - 1\right)}, \end{aligned} \tag{353}$$

because W is also $U(1)_a$ invariant: $\sum_M q^a_M + \sum_A q^a_A = 0$.[b] In order to make T-duality fully manifest below the $U(1)$-breaking scale, we have to redefine the transformation (340) on Φ^A by including a global $U(1)_a$ transformation such that Φ^A is fully invariant, and

$$\Phi'^M \to e^{i\delta'_M - \sum_I q'^I_M F^I} \Phi^M, \qquad \delta'_M = \delta_M - \sum_{A,\,a} q^a_M Q^A_a \delta_A \,. \tag{354}$$

A priori we expect that $\langle \Phi^A \rangle \sim 0.1$, so that couplings arising from high dimension operators in the superpotential are suppressed.[c] We would like to have one large coupling (Q_3, T^c, H_u) which should correspond to one of the dimension-three operators in (350). Until recently, most models studied were $\mathbf{Z}_3$ orbifold compactifications with all quark doublets in the untwisted sector.[87,175,176,177,88,178,113] In this case we should take T^c and H_u in the untwisted sector as well, and require $q^a_{Q_3} + q^a_{T^c} + q^a_{H_u} = 0$. That is, if we identify the Q_I generation index with the modulus index, we can have, *e.g.*, $T^c = T^c_2$, $H^u = H^u_1$. Then to suppress the $Q_2 C^c H^u$ and $Q_2 U^c H^u$ couplings we require $C^c, U^c \notin U_3$, so one of these must be in the twisted sector T. These requirements can be met in the FIQS model[179] mentioned in Section 4. This model was analyzed in Ref. 172 where it was found not to produce the desired R-parity. The subgroup G_R that leaves the self-dual points invariant has $i\delta_I = -\frac{1}{2}F^I$, and after the redefinitions (353) and (354) the MSSM chiral multiplets transform with phases $e^{ni\pi/33}$. However these phases combine in the superpotential terms in such a way that baryon number violating couplings cannot be forbidden. Moreover the symmetry must be broken to a smaller group to allow mass terms

[b]Recall from Section 4 that q^a_M is the charge of field Φ^M with respect to the Abelian factor $U(1)_a$, and similarly for q^a_A.
[c]The factors multiplying these terms can in fact be rather large.[180,101]

for all quarks and leptons and to generate a μ-term; once this is done all gauge invariant trilinear terms are allowed.

We can turn the question around and ask what are the constraints on the $U(1)$ charges that allow for R-parity in a given compactification. Taking $\mathbf{Z}_3$ models as an example, assume that, as in the FIQS model, Q_I, T^c, and H^u are in the untwisted sector, all other MSSM superfields are in the twisted sector T, and in each sector fields degenerate in MSSM charges are also degenerate in $U(1)$ charges. The last condition, which assures the suppression of FCNC, implies that fields in the twisted sector T that are degenerate in MSSM charges are further degenerate in R-parity. Then for all the observed Yukawas to be generated the Q_I must all have the same R-parity, which is compatible with their having the same $U(1)$ charges only if the R-parity group is further constrained to have[172]

$$F^I = 2n^I i\pi \qquad \forall\ I. \tag{355}$$

Assuming further that there are no standard model singlets in the untwisted sector, the R-parity transformations take the form

$$\begin{aligned}
\Phi^m &\to e^{2i\pi\beta^m}\Phi^m, \\
\beta^{U_J^m} &= \sum_I n_I \left(\frac{1}{3} \sum_{a,A\in T,Y} q_m^a Q_a^A - \sum_{a,A\in Y_I} q_m^a Q_a^{Y_I^A} \right) \\
\beta^{T^m} &= \sum_I n_I \left[\frac{1}{3} \left(\sum_{a,A\in T,Y} q_m^a Q_a^A - 1 \right) - \sum_{a,A\in Y_I} q_m^a Q_a^{Y_I^A} \right].
\end{aligned} \tag{356}$$

In addition, requiring that mass terms be allowed for all quarks implies that the untwisted field $T^c \in U$ has the same R-parity as $U^c, D^c \in T$, which entails a relation between their (different) $U(1)$ charges. We are left with the following set of phases:

$$\begin{aligned}
\beta^{H^u} &= -\beta^{H^d} = \gamma, \qquad \beta^Q = \beta + 2\gamma, \qquad \beta^{U^c} = -\beta - 3\gamma, \\
\beta^{D^c} &= -\beta - \gamma, \qquad \beta^L = \alpha - \gamma, \qquad \beta^{E^c} = -\alpha + 2\gamma.
\end{aligned} \tag{357}$$

With these phases all MSSM superpotential terms are allowed, while QD^cL and LLE^c are forbidden provided $\alpha \neq n$, and $U^cD^cD^c$ is forbidden provided $3\alpha + 5\beta \neq n$. Below the scale of electroweak symmetry breaking we redefine R-parity as discussed above:

$$\beta^m \to \beta'^m = \beta^m - 2Y_m\gamma, \tag{358}$$

so that $\beta'^{H^u} = \beta'^{H^d} = 0$. In contrast with the conventional definition of R-parity, higher dimension operators that generate B and L violation can also be suppressed. For example, the dimension-four superpotential operator $\tilde{U}^c\tilde{U}^c\tilde{D}^cE^c$, allowed by conventional R-parity, leads to dimension-five operators in the effective Lagrangian that may be problematic[181] even if these couplings are Planck- or string-scale suppressed, given the current bounds on the proton decay rate. This problem is easily

evaded in the current context; for the choice of phases considered above this requires $3\beta + \alpha + 3\gamma \neq n$. The stability of the lightest neutralino is assured at the same level as proton stability since its decay products would have to include an odd number of Standard Model fermions and hence violate B and/or L.

There are many other possibilities even within the context of $\mathbf{Z}_3$ orbifolds, such as cases with all MSSM particles in the twisted sector.[182] The FIQS model, which is the best studied of the $\mathbf{Z}_3$ models, cannot reproduce the observed Standard Model Yukawa textures,[101] quite apart from the issue of providing an R-parity. More generally it is difficult to get the standard $SU(5)$ normalization of weak hypercharge in these models,[113] and different compactifications, such as recently proposed $\mathbf{Z}_6$-based orbifolds,[80,81,82,183] with the first two generations of quark doublets in the twisted sector, would be interesting to analyze in this context.

Finally we remark that it is not actually necessary for the moduli to be stabilized at their self-dual points for R-parity to be a symmetry of the superpotential. This is because the superpotential is a sum of monomials that are products of matter fields with coefficients that are functions of the moduli only. These functions are invariant at the self-dual points, which means that products of matter fields are invariant by themselves *provided* $F^I(T^I)$ is replaced by $F^I(T^I_{s\,d})$ in the transformation property of Φ_A in (340). In other words, there is a $(\mathbf{Z}_4 \otimes \mathbf{Z}_6)^3$ subgroup of the group of modular transformations that coincides with a subset of the orbifold selection rules, which have also been considered a potential source of R-parity. The use of residual discrete symmetries from T-duality allow these selection rules to be rephrased in terms of the explicit symmetries of the effective supergravity theory, and combining them with $U(1)$ symmetries to obtain an unbroken discrete symmetry at each stage of gauge symmetry-breaking generates noninteger charges that make it possible to exclude higher dimension operators as well as the dimension-four operators excluded by conventional R-parity.

5.2. *General flavor changing processes*

The issue of R-parity is clearly critical to a viable low energy phenomenology. By eliminating terms odd under R-parity, the contribution of superpartners to FCNC processes will generally be small if the size of the scalar masses is fairly large. But the previous statement assumes that soft scalar masses themselves are diagonal in the flavor basis. That is, that all flavor-changing processes involving squarks and sleptons are proportional to Yukawa couplings. To a good approximation this is true in the BGW model: scalar masses are generally large and universal at tree level, with loop corrections violating this universality only at the few percent level. In fact, the constraints on the off-diagonal elements of the scalar mass matrices weaken very quickly as the overall scale of the masses exceeds 1 TeV, though the imaginary parts of these same masses can remain tightly constrained.[184]

Naively, therefore, we might conclude that the BGW class of theories is among those that are largely immune to the supersymmetric FCNC problem. But it is often

said that the staring point for a discussion of flavor in the context of supergravity should be one of arbitrary off-diagonal scalar masses – not the diagonal tree-level masses we derived in Section 3. The argument runs something like this. One can imagine *a priori* operators of the form

$$\int \mathrm{d}^4\theta \frac{\overline{X}X}{m_{\mathrm{PL}}^2}\overline{Q}^{\bar{i}}Q^j, \tag{359}$$

where X is a Standard Model singlet that is presumably a hidden sector field. If it participates in supersymmetry (SUSY) breaking, then it will generate off-diagonal soft-masses. In the absence of a rule for how this X couples to Standard Model matter (such as via gauge charges in gauge mediation) we must assume that different flavors can be treated differently. In other words, there is no symmetry argument as to why operators of the form of (359) which mix flavors should be absent.

But by "symmetry" what is typically being considered is a *gauge* symmetry. Yet the operator in (359) may admit a *geometrical* interpretation. Let us rewrite things to make this more apparent

$$\int \mathrm{d}^4\theta \frac{R_{\bar{i}j\bar{k}\ell}}{M_{\mathrm{PL}}^2}\overline{X}^{\bar{k}}X^{\ell}\overline{Q}^{\bar{i}}Q^j\,, \tag{360}$$

where the tensor $R_{\bar{i}j\bar{k}\ell}$ is the curvature tensor formed from the field-reparameterization connection

$$R^i_{jk\bar{m}} = D_{\bar{m}}\Gamma^i_{jk}; \quad \Gamma^i_{jk} = K^{i\bar{n}}\partial_j K_{k\bar{n}} = K^{i\bar{n}}\partial_j\partial_k\partial_{\bar{n}}K\,, \tag{361}$$

where $D_{\bar{m}} = K_{\ell\bar{m}}D^{\ell}$ is a covariant derivative with respect to field reparameterization and $K_{\ell\bar{m}}$ is the Kähler metric. Thus, while an understanding of the form of this tensor in (360) may not be possible in terms of the gauge quantum numbers of the fields involved, an understanding in terms of the isometries of the manifold defined by the chiral superfields of the theory may indeed exist. This point has been emphasized recently for string-based effective supergravity theories [185,186].

To address the question of what constraints are needed to avoid experimentally excluded FCNC effects, we first note that the tree potential of an effective supergravity theory includes a term

$$V_{\mathrm{tree}} \ni e^K K_i K_{\bar{\jmath}} K^{i\bar{\jmath}}|W|^2\,. \tag{362}$$

So prior to any discussion of large loop-induced contributions to flavor-changing operators it is necessary to ensure their absence at the tree level. The observed suppression of FCNC effects thus constrains the Kähler potential already at the leading order – to a high degree of accuracy we require that[187]

$$K_i K_{\bar{\jmath}} K^{i\bar{\jmath}} \not\ni \left\langle f(X,\bar{X})\right\rangle \phi^a_f \bar{\phi}^{\bar{a}}_{f'\neq f}, \tag{363}$$

where f, f' are flavor indices, a is a gauge index, ϕ^a_f any standard model squark or slepton, and X is a singlet of the Standard Model gauge group. For example, in the

no-scale models that characterize the untwisted sector of orbifold compactifications, we have

$$K_i K_{\bar{\jmath}} K^{i\bar{\jmath}} = 3 + K_S K_{\bar{S}} K^{S\bar{S}}, \tag{364}$$

which is safe, since K_S is a function only of the dilaton. The twisted sector Kähler potential (83) is flavor diagonal and also safe. But the higher order corrections to (83) could be problematic if some $\phi^a = X^a$ have large *vevs* (*i.e.* within a few orders of magnitude of the Planck scale). Thus phenomenology requires that we forbid couplings of the form $\phi^a_f \phi^{\bar{a}}_{f' \neq f} |\phi^{a'}_{f''}|^2 X^{b_1} \cdots X^{b_n}$, $n \leq N$, where N is chosen sufficiently large to make the contribution $\left\langle X^{b_1} \cdots X^{b_n} \right\rangle$ to the scalar mass matrix negligible.

In addition to these higher-order terms, the quadratically divergent one-loop corrections generate a term

$$V_{1-\text{loop}} \ni e^K K_i K_{\bar{\jmath}} R^{i\bar{\jmath}} |W|^2, \qquad R^{i\bar{\jmath}} = K^{i\bar{k}} R_{\bar{k}l} K^{k\bar{\jmath}}. \tag{365}$$

where $R_{i\bar{\jmath}}$ is the Kähler Ricci tensor. Since the Ricci tensor involves a sum of Kähler Riemann tensor elements over all chiral degrees of freedom, a large coefficient may be generated, proportional to the number of chiral superfields N_χ. For example, for an untwisted sector U with three untwisted moduli T^i and Kähler potential as in (81) we get

$$R^n_{i\bar{\jmath}} = (N_n + 2) K^n_{i\bar{\jmath}}. \tag{366}$$

While this contribution is clearly safe, since the Ricci tensor is proportional to the Kähler metric, the condition that the tree potential be FCNC safe does not by itself ensure that (365) is safe in general. For this we require in addition the absence of Kähler potential terms of the form $\phi^a_f \bar{\phi}^{\bar{a}}_{f' \neq f} |\phi^{a'}_{f''}|^4 (X^b)^{n \leq N}$. On the other hand, if the Kähler metric is FCNC safe due to an *isometry*, the same isometry will protect the Ricci tensor from generating FCNC.

There is a large class of models in which FCNC are suppressed independently of the details of the structure of the Kähler potential, provided the moduli t^I are stabilized at self dual points. The supersymmetric completion of the potential in any given order in perturbation theory yields (in the absence of D-term contributions) the scalar squared mass matrix

$$(m^2)^i_j = \delta^i_j m^2_{3/2} - \left\langle \widetilde{R}^i_{jk\bar{m}} \right\rangle \widetilde{F}^k \overline{\widetilde{F}}^{\bar{m}}, \tag{367}$$

where $\widetilde{R}^i_{jk\bar{m}}$ is an element of the Riemann tensor derived from the fully renormalized Kähler metric, and $\widetilde{F}^i$ is the auxiliary field for the chiral superfield Φ^i, evaluated by its equation of motion using the quantum corrected Lagrangian. Since the latter is perturbatively modular invariant, the Kähler moduli t^i are still stabilized at self-dual points with $\left\langle \widetilde{F}^{t^i} \right\rangle = 0$. Classically we have $R^a_{bs\bar{s}} = 0$ where the indices a, b refer to gauge-charged fields in the observable sector. This need not be true at

the quantum level. For example, if the quantum correction to the Kähler potential includes a term[188,189,190,191]

$$\Delta K = \frac{1}{32\pi^2}\mathrm{STr}\Lambda_{\rm eff}^2 \ni \frac{cN_\chi}{32\pi^2}e^{\alpha K}\,, \tag{368}$$

with α and c coefficients which depend on the nature of the Pauli-Villars regulating sector, we get

$$\left\langle \widetilde{R}^a_{bs\bar{s}} \right\rangle = \delta^a_b \frac{cN_\chi}{32\pi^2}\alpha^2 e^{\alpha K}\left(K_{s\bar{s}} + \alpha K_s K_{\bar{s}}\right), \tag{369}$$

which is flavor diagonal, and therefore FCNC safe.

A more substantive comparison can be made by considering a particular Kähler potential. Take the case of

$$K = g(M,\overline{M}) + \sum_a f_a(M,\overline{M})|\Phi^a|^2 + \frac{1}{4}\sum_{ab} X_{ab} f_a f_b |\Phi^a|^2|\Phi^b|^2 + \mathcal{O}(|\Phi|^3), \tag{370}$$

$$g(M,\overline{M}) = -\sum_i \ln(T^i + \overline{T}^i), \qquad f_a(M,\overline{M}) = \prod_i (T^i + \overline{T}^i)^{-q^i_a}\,. \tag{371}$$

The term proportional to X_{ab} may give rise to potentially dangerous off-diagonal elements through the second term in (367). The form of these terms will depend on how the quadratically divergent parts of $\widetilde{R}^i_{jk\bar{m}}$ are regularized. Returning to the discussion of loop corrections in supergravity from Section 3.3 we recall that there were two sets of Pauli-Villars (PV) fields necessary to regulate loops involving light matter fields. These fields were labeled Φ and Π and were coupled through a supersymmetric mass term as in (181). We can relate the field-dependent effective cut-off for the quadratic divergences to this PV mass via the relation

$$\Phi^I, \Pi^I \;:\quad e^{(1-\alpha_\alpha)K} K^{i\bar{i}} |\beta_\alpha \mu|^2 \equiv (\beta_\alpha)^2\,\Lambda_\alpha^2\,, \tag{372}$$

where i labels the light (observable sector) chiral superfield Z^i and α labels the PV regulating fields associated with Z^i. Note that $\Lambda_{\rm PV} \simeq m_{\rm PL} = 1$ and β is assumed to be an $\mathcal{O}(1)$ parameter. If the supersymmetric Pauli-Villars regulator masses are independent of the dilaton, then in the BGW model the potential off-diagonal scalar masses take the form

$$(m^2)^i_j = 3m^2_{3/2}\beta^2_\alpha \Lambda^2_\alpha \delta^i_j \left(\sum_k X_{jk} + \sum_I q^I_j\right)(1-\alpha_\alpha)\,(2-\alpha_\alpha)\,. \tag{373}$$

Therefore, even in this particularly simple case of dilaton domination there is a potential for sizable FCNCs since the summation in the first term of (373) runs over all fields which participate in the quartic coupling of (370).[192] However, the presence of this off-diagonal scalar mass contribution depends on the parameters α_α, which are determined by Planck-scale physics. In particular, the contribution vanishes completely – independent of the values of the modular weights or the values of X_{ab} – provided $\alpha = 1$ or 2.

The issue of whether supergravity effects induce large FCNCs therefore depends on the physics of the UV completion of the theory. In this case that UV completion

is heterotic string theory itself. The manner in which the Planck-scale theory softens ultraviolet divergences is reflected in the Kähler potential factors α_A. As argued in Section 5.1, there are good reason to believe that the higher-dimension operators involving X_{ab} in (370) *are not arbitrary*, but are in fact tightly constrained. Ultimately the issue of FCNCs thus becomes an issue of how flavor physics is encoded in string models in the first place.[10]

5.3. *Collider signatures*

The relatively low masses for the gauginos in this class of models should make them easily accessible at hadron colliders. With the reduction in the gluino mass it is possible to greatly increase the accessibility of the gluino at Tevatron energies. This suggests that these models can be probed significantly in the short term even before the LHC data taking begins. It was for this reason that the Kähler-stabilized models of the weakly-coupled heterotic string were included in a set of benchmark models for the Tevatron constructed in Ref. 193.

As described in Section 3.4, a convenient parameter space for the BGW class of models is $\{\tan\beta, m_{3/2}, b_+\}$ and $\mathrm{sgn}(\mu)$. The three benchmark points chosen in Ref. 193 were defined by the set $\{\tan\beta, m_{3/2}, a_{\rm np}\}$, but we can replace $a_{\rm np}$ with b_+ through (175). The boundary condition scale was taken to be $\Lambda_{\rm UV} = 2\times 10^{16}$ GeV as this is a common convention in the literature and makes for easier comparisons with previous results. The specific values were

$$\text{Case A}: \quad \{\tan\beta,\ m_{3/2},\ b_+\} = \{10,\ 1500\ \text{GeV},\ 0.152\} \tag{374}$$

$$\text{Case B}: \quad \{\tan\beta,\ m_{3/2},\ b_+\} = \{5,\ 3200\ \text{GeV},\ 0.063\} \tag{375}$$

$$\text{Case C}: \quad \{\tan\beta,\ m_{3/2},\ b_+\} = \{5,\ 4300\ \text{GeV},\ 0.038\}\ . \tag{376}$$

The last two cases correspond to beta-function coefficients $b_+ = 5/8\pi^2 = 0.063$ and $b_+ = 3/8\pi^2 \simeq 0.038$. The former could result from a condensation of pure $SU(5)$ Yang-Mills fields in the hidden sector. The latter case could be obtained either from a similar condensation of pure $SU(3)$ Yang-Mills fields or from the condensation of an E_6 hidden sector gauge group with 9 **27**'s condensing in the hidden sector.[d] The first case is just on the edge of where realistic gravitino masses can be obtained from some pair of values for $\{(c_\alpha)_{\rm eff}, (b_+^\alpha)_{\rm eff}\}$. It corresponds to a condensing group beta-function coefficient of $b_+ = 12/8\pi^2 \simeq 0.152$, which could result from a hidden sector condensation of pure E_6 Yang-Mills fields.

From these values and the expressions in (195) it is possible to construct the entire superpartner spectrum, assuming the MSSM field content and proper electroweak symmetry breaking. The results are give in Table 1. For comparison we also give the physical spectrum for a unified model of the minimal supergravity type, for which the unified scalar mass is taken to be $m_0 = 100$ GeV, the unified gaugino mass is taken to be $m_{1/2} = 250$ GeV, and the unified trilinear coupling A_0

[d]This is precisely the case marked with the circle in Figure 8.

Table 1. Sample Spectra for benchmark points. All masses are in GeV.

Point	A	B	C	mSUGRA
$\tan\beta$	10	5	5	10
$m_{\chi_1^0}$	78	93	91	98
$m_{\chi_2^0}$	122	132	110	182
$m_{\chi_1^\pm}$	120	132	110	181
$m_{\tilde{g}}$	471	427	329	582
$\widetilde{B}\,\%\|_{\rm LSP}$	89.8 %	98.7 %	93.4 %	99.9 %
$\widetilde{W}_3\%\|_{\rm LSP}$	2.5 %	0.6 %	4.6 %	0.0 %
m_h	114.3	114.5	116.4	112.0
m_A	1507	3318	4400	381
m_H	1510	3329	4417	382
μ	245	631	481	332
$m_{\tilde{t}_1}$	947	1909	2570	392
$m_{\tilde{t}_2}$	1281	2639	3530	571
$m_{\tilde{c}_1}$, $m_{\tilde{u}_1}$	1553	3254	4364	528
$m_{\tilde{c}_2}$, $m_{\tilde{u}_2}$	1557	3260	4371	547
$m_{\tilde{b}_1}$	1282	2681	3614	501
$m_{\tilde{b}_2}$	1540	3245	4353	528
$m_{\tilde{s}_1}$, $m_{\tilde{d}_1}$	1552	3252	4362	527
$m_{\tilde{s}_2}$, $m_{\tilde{d}_2}$	1560	3261	4372	553
$m_{\tilde{\tau}_1}$	1491	3199	4298	137
$m_{\tilde{\tau}_2}$	1502	3207	4308	208
$m_{\tilde{\mu}_1}$, $m_{\tilde{e}_1}$	1505	3207	4309	145
$m_{\tilde{\mu}_2}$, $m_{\tilde{e}_2}$	1509	3211	4313	204

is taken to vanish. This is model point B of Battaglia et al.[194], which is very nearly the Snowmass point 1A.[195] A few initial comments are in order. First we note that the μ-parameter for the three BGW points is quite small relative to typical scalar masses. This is the focus-point effect at work, alluded to earlier. Note also the departure of these models from the typical mSUGRA relation $m_{\chi_1^0} \simeq 0.5 m_{\chi_1^\pm}$, with the significant W-ino component of the LSP (indicated by the size of $\widetilde{W}_3\%|_{\rm LSP}$ versus $\widetilde{B}\ \%|_{\rm LSP}$). Finally, the three BGW points have scalars which are much larger than those mSUGRA models (such as Snowmass point 1A) which are typically used for collider studies. This impacts on both the production of superpartners as well as the branching fractions of these superpartners to "typical" SUSY-indicating final states.

Figure 14 shows naive estimates of numbers of events of various signatures in 2 fb^{-1} integrated luminosity at Tevatron center-of-mass energies for the models of Table 1. The signature of these models was calculated using `PYTHIA`,[196] but only at the generator level. No geometric or kinematic cuts, no triggering efficiencies are applied, no jet clustering is performed, tau leptons are not decayed, etc. The

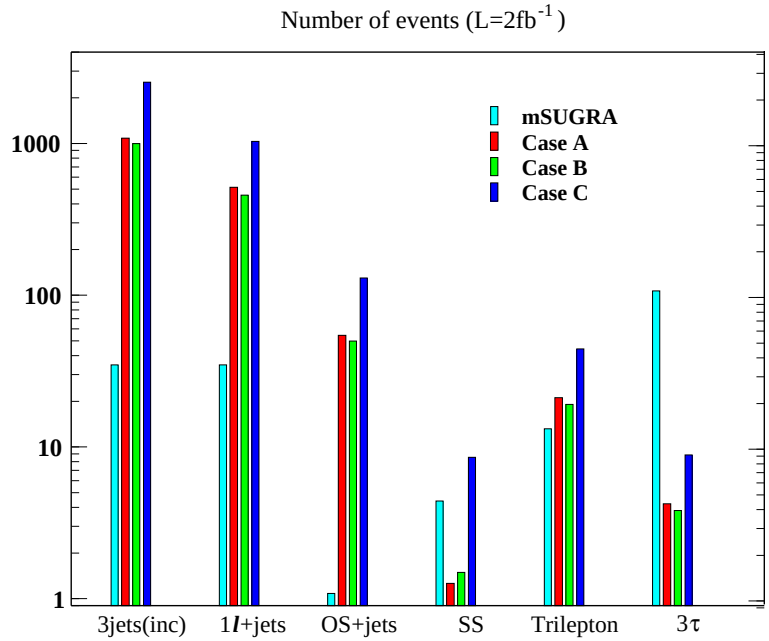

Fig. 14. **Number of superpartner events of different signatures for models of Table 1 at the Tevatron.** These numbers are based on counting topologies from `PYTHIA` at the parton level with no kinematic or geometric cuts. Note the logarithmic scale in numbers of events. Descriptions of each signature type are given in the text.

event numbers are only meant to illustrate the generic features of each model and demonstrate the experimental challenges. All events have missing transverse energy as we assume an intact R-parity. The signature set is as follows

(1) Inclusive multi-jets $n_{\rm jets} \geq 3$,
(2) One lepton plus $n_{\rm jets} \geq 2$,
(3) Opposite sign (OS) dileptons plus $n_{\rm jets} \geq 2$,
(4) Same-sign (SS) dileptons plus any number of jets,
(5) Trileptons plus any number of jets,
(6) Three taus plus any number of jets [before decaying the taus].

The very large number of jet events with missing energy, relative to the Snowmass benchmark, is the result of the much lighter gluino for this model. The signatures are most pronounced for point C with the lightest gluino mass. In general, the SUSY signature space is dominated by gluino production and decay for the BGW class of models.[197]

This is only the beginning of a meaningful analysis. A study of the backgrounds must of course be done to be sure any given channel is detectable, but models with hundreds of events are presumably detectable for the first two signatures, and

models with tens of events for the rest. The same-sign dilepton channel has smaller backgrounds: even a handful of clean events may constitute a signal. Furthermore, despite the fact that the event numbers in Figure 14 are based on unsophisticated estimates, taking ratios of such inclusive signatures should be robust under more detailed analyses. For example, the ratio of OS dilepton + jets events to trilepton events should be roughly 5 : 2 in the BGW model, independent of the other parameters, as a result of the predominance of gluino pair production over squark production.[193]

The potential utility of using combinations of inclusive observables to separate classes of models was explored in Ref. 9, where point C of (376) was studied in much greater detail for the LHC environment. Consider the situation after 10 fb^{-1} of integrated luminosity (*i.e.* one year at $10^{33}\mathrm{cm}^{-2}\mathrm{sec}^{-1}$) at the LHC. Looking at just the most inclusive signatures – those of Figure 14 – plus crude kinematic information, can separate the Kähler-stabilized models from alternative SUSY-breaking paradigms. For example, in Table 2 we collect the predictions for numbers of events with missing transverse energy in excess of 100 GeV, at least two jets (each with transverse energy above 100 GeV) and various final state topologies for Standard Model processes as well as various paradigms of new supersymmetric physics. We also include the peak in the effective mass distribution, where m_{eff} is defined as

$$m_{\mathrm{eff}} = E^T_{\mathrm{miss}} + \sum_{\mathrm{jets}} E^T_{\mathrm{jet}} \,. \tag{377}$$

In the table, model A is the Snowmass point SPS 1A (only slightly modified from the unified model in Figure 14) with parameter set $m_0 = 200$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -800$ GeV, $\tan\beta = 10$ and positive μ. Model B is also a unified model, but in this case with very heavy scalars to achieve a "focus point" model similar in nature to the BGW class. The parameter set for this model is $m_0 = 2150$ GeV, $m_{1/2} = 300$ GeV, $A_0 = 0$ GeV, $\tan\beta = 10$ and positive μ. Model D is a strongly-coupled heterotic string model without D_5-branes,[198,199,200,201,202] which we include here for comparison to our Kähler stabilized model. More examples were considered in the original paper of Reference 9.

Table 2. Number of events in excess of the Standard Model prediction for different signatures. For each channel the Standard Model baseline is given in the first column. Subsequent columns give the excess beyond this baseline for selected models described in the text.

Channel	SM	A	B	BGW Point C	D
Jets ($\times 10^3$)	100.0	59.5	0.7	31.7	6.6
1ℓ ($\times 10^3$)	13.0	17.1	0.5	7.3	1.7
OS ($\times 10^3$)	7.0	5.7	0.2	2.0	0.6
SS	20	1332	99	504	160
3ℓ	60	737	97	204	77
$m_{\mathrm{eff}}^{\mathrm{peak}}$ (GeV)	-	812	1140	838	1210

We note that each model can be "discovered" above the Standard Model background estimation in at least one channel (assuming a very naive estimate of $\sqrt{N_{\rm SM}}$ for a measure of the approximate experimental error in the observations). What is more, the models can be distinguished from one another – this is particularly important for using models of string physics as interpreters of possible LHC signals. The ability of inclusive signatures to distinguish the Kähler-stabilized heterotic string model from other SUSY models was tested using global fits to simulated data.[9]. For example, the minimal supergravity point given by

$$\tan\beta = 10 \quad m_{1/2} = 380 \quad m_0 = 600 \quad A_0 = 0 \quad \mathrm{sgn}(\mu) > 0. \tag{378}$$

was used to simulate 50,000 events for LHC center-of-mass energies using `PYTHIA`. The inclusive observables of Table 2 were then computed. Also computed was the SUSY contribution to the anomalous magnetic moment of the muon $a_\mu^{\rm SUSY}$ and to the rate for $b \to s\gamma$ processes. In an effort to fit the resulting "data," the same simulation was performed on an ensemble of minimal supergravity models with $A_0 = 0$ but varying the parameters $\tan\beta$, m_0 and $m_{1/2}$. For each resulting set of collider + indirect observables, a χ^2-fit was performed to determine how well the test point reproduced the target data. Not surprisingly, the best-fit χ^2 corresponded to a point very close to (378) in the ensemble, namely the case $\tan\beta = 10$, $m_{1/2} = 380$ GeV and $m_0 = 500$ GeV with a minimum chi-squared of $(\chi^2)_{\rm min} = 1.7$ with three degrees of freedom. But would another model fit this data equally well?

When an ensemble of experiments from the BGW model was constructed, by simulating 50,000 events at each point in a mesh over the three-dimensional parameter space defined by $\left\{\tan\beta, m_{3/2}, b_+\right\}$, the resulting best fit point had the parameters $\left\{\tan\beta, m_{3/2}, b_+\right\} = \left\{5\,, 2750\ \mathrm{GeV}\,, b_+ = 8/8\pi^2\right\}$ which corresponds to $\left(\chi_T^2\right)_m = 2.8$ with five degrees of freedom. It would appear that the nonuniversal model is doing a fairly adequate job of reproducing the inclusive signatures! But when additional kinematic data is taken into account the two can clearly be distinguished. For example, when we include the peak in the $m_{\rm eff}$ distribution and the peak in the invariant mass distribution $m_{\ell\ell}$ of opposite-sign dilepton events we obtain $\left\{m_{\rm eff}^{\rm peak}, m_{\ell\ell}^{\rm peak}\right\} = \{1360\ \mathrm{GeV}, 92\ \mathrm{GeV}\}$ for the true "data," which was well reproduced by the best-fit minimal supergravity point. In contrast, the best-fit Kähler-stabilization model point produced $\left\{m_{\rm eff}^{\rm peak}, m_{\ell\ell}^{\rm peak}\right\} = \{987\ \mathrm{GeV}, 58\ \mathrm{GeV}\}$. Even allowing for relatively large uncertainties in the measurement of these quantities these are without question measurable differences.

Is it reasonable to ask how well do the above results hold when trying to distinguish two models across their entire parameter space (as opposed to trying to fit a single point)? Doing so requires expanding the list of observables to consider to include additional kinematic information and asymmetries in the LHC event rates. Kane et al.[203] have considered the parameter space of several prominent string constructions that have been studied in the literature, including the Kähler stabilization model reviewed here. They conclude that models of the BGW class

can be distinguished from the other string-based effective theories over most of its parameter space. The key observables are

(1) The number of "clean" multi-lepton events
(2) The total rate of dilepton events with at least two jets
(3) The fraction of dilepton + jet events for which at least one of the jets is tagged as a b-jet
(4) The charge asymmetry in events in which there is a single high-p_T lepton
(5) The number of events with a single (hadronically-decaying) tau
(6) The peak in the missing E_T distribution over all events with at least 100 GeV of missing E_T

Let us consider just a couple of examples of how these observables help to distinguish this class of models from others. Events with high-p_T leptons but with no jets over 100 GeV in transverse energy are called clean events. They arise from direct production of neutralino/chargino pairs, which certainly has a sizable cross-section in the BGW class. However, the mass difference between the lightest chargino and lightest neutralino is typically small. The resulting leptonic decay products therefore tend to be soft and thus the number of clean leptonic events will be very sensitive to the p_T threshold demanded of the leptons. To be included in the data sample in the study of Kane et al. a minimum p_T of 10 GeV was demanded for electrons and muons. Therefore there are essentially no clean dilepton/trilepton events in the BGW model when LHC-motivated cuts are used.

Associated production of squarks with gluinos can give rise to a charge asymmetry in events with a single high-p_T lepton. At the LHC, the produced squark is most likely to be an up-type squark as the initial state is asymmetric between up and down quarks. These squarks will decay preferentially to a positive chargino, which then decays to a positively charged lepton and missing energy. When squarks are relatively light one therefore expects to see more events with jets and a single high-p_T lepton as well as an asymmetry in favor of positive charges for these leptons. The heavy squarks of the BGW model reduce this production rate and give rise to fewer events and almost no asymmetry. By combining observations such as these it should be possible to separate models from one another, as well as eliminate classes of models when confronting them with actual signals at the LHC.

5.4. *Dark matter*

Another important experimental arena in which nonuniversal gaugino masses can play a significant role is in the thermal production of cold dark matter in the form of stable LSP neutralinos.[204,205] One of the prime virtues of supersymmetry as an explanation of the hierarchy problem is that it tends to also provide a solution to the dark matter problem as a nearly automatic consequence of R-parity conservation. But models in which gaugino masses are universal at some high scale tend to predict too large a relic neutralino density for generic parameter choices.[206,207] The reason

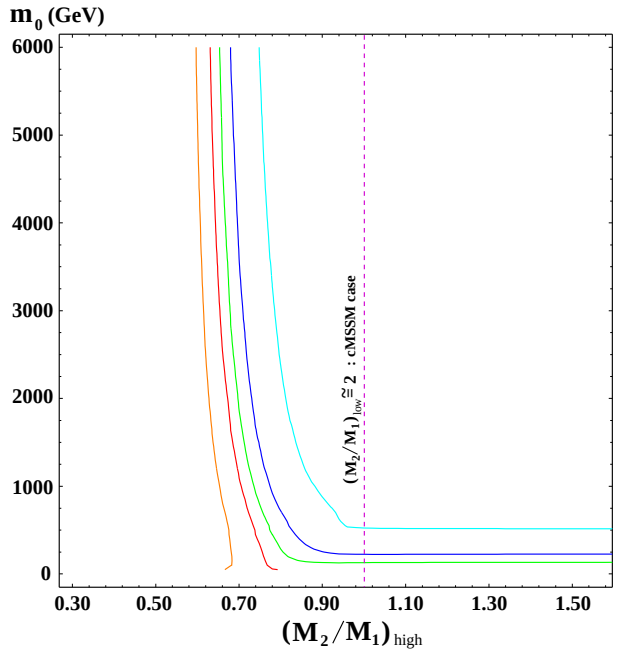

Fig. 15. **Preferred dark matter region for nonuniversal gaugino masses.** Contours of $\Omega_\chi h^2$ of 0.01, 0.1, 0.3, 1.0 and 10.0 from left to right, respectively, are given as a function of the ratio of $SU(2)$ to $U(1)$ gaugino masses M_2/M_1 at the high scale. The unified model is recovered where the two masses are equal at the high scale, as has been indicated by the dashed line. In this plot we have set $M_3 = m_{1/2} = 200$ GeV.

is not hard to understand. For neutralinos of a mass of approximately 100 GeV, the thermally produced density of particles after reheating is far larger than that required to account for the nonbaryonic dark matter. But as the universe cools relic neutralinos must find one another and annihilate into light fields (such as leptons) before the rate of annihilation falls below the expansion rate of the cosmos. Typically, weak-scale interaction rates are very close to the needed annihilation rate as a first approximation. But on closer examination it becomes clear that annihilation of neutralinos into leptons will only be efficient if diagrams involving t-channel exchange of sleptons contribute significantly to the total rate. For models with slepton masses much above 100 GeV this rate quickly drops, particularly when the LSP is predominantly B-ino like, resulting in far too much relic density at freeze-out.

In the BGW class of models all scalar fields are very massive, so *a priori* we would expect a very significant problem for the relic neutralino density. But the rather simplified description from the previous paragraph is remarkably sensitive to the wave-function of the LSP. Let us adopt the following parameterization for

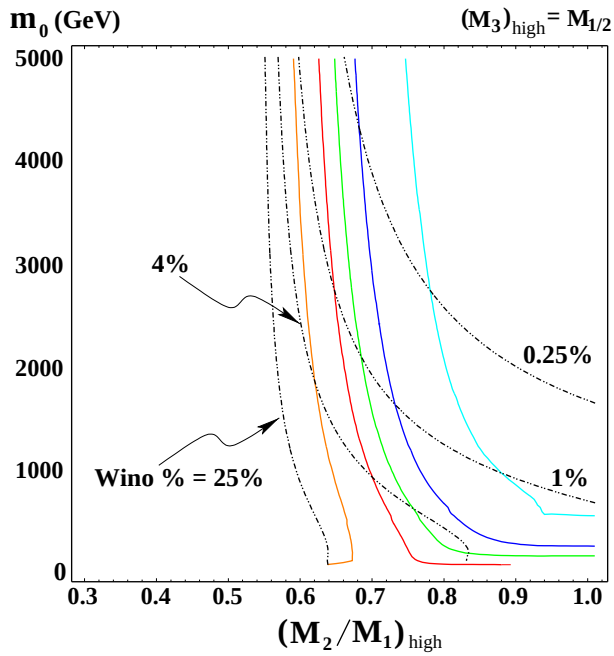

Fig. 16. ***W*-ino content of the LSP for cosmologically preferred regions.** Contours of constant relic density are given by the solid lines for $\Omega_\chi h^2 = 0.01$, 0.1, 0.3, 1, and 10 from left to right. The dotted lines are curves of constant wino content for $N_{12}^2 = 0.25$, 0.04, 0.01 and 0.025 from left to right.

this wave-function

$$\chi_1^0 = N_{11}\tilde{B} + N_{12}\tilde{W} + N_{13}\tilde{H}_d^0 + N_{14}\tilde{H}_u^0 \,, \tag{379}$$

which is normalized to $N_{11}^2 + N_{12}^2 + N_{13}^2 + N_{14}^2 = 1$. In unified models (with low to moderate $\tan\beta$) the LSP is overwhelmingly Bino-like, which is to say that $N_{11} \simeq 1$. If we relax the GUT relationship between the gaugino masses but still remain in the large $|\mu|$ limit (low $\tan\beta$) then we will continue to have a predominantly *gaugino-like* LSP ($N_{11}^2 + N_{12}^2 \simeq 1$) with the relative values of N_{11} and N_{12} governed by the relative values of M_1 and M_2. Decreasing M_2 relative to M_1 at the electroweak scale increases the wino content of the LSP until ultimately $M_1 \gg M_2$ and $N_{11} \simeq 0$, $N_{12} \simeq 1$. The B-ino component of the neutralino couples with a U(1) gauge strength whereas the wino component couples with the larger SU(2) gauge strength, thus enhancing its annihilation cross section and thereby lowering its relic density. As N_{12} is increased more SUSY parameter space should open up for correct dark matter abundance until eventually annihilation becomes too efficient and we are left with no neutralino dark matter at all.

Another important effect of the deviation of gaugino masses from GUT relations is the increasing importance of co-annihilations between the LSP and other light

neutralinos and charginos in the early universe. In unified models like minimal supergravity co-annihilation is only important in limited regions of parameter space – through resonant annihilation diagrams involving the CP-odd neutral Higgs or from co-annihilation diagrams involving the stau.[208,209,210] But in the Kähler-stabilized models the mass difference between the lightest neutralino and the next-to-lightest neutral gaugino, or lightest chargino, can be relatively small. This was demonstrated in the plots of Figure 13. When mass differences between gauginos reach a few GeV, additional coannihilation processes become efficient at removing relic LSPs from the early universe. Such processes include $\chi_1^\pm \chi_1^0 \to f f'$ (such as $e^\pm \nu_e$), $W^\pm \gamma$, and $W^\pm Z$.

These additional processes were studied in the context of nonuniversal gaugino masses in References 211 and 212. There it was found that for particular special ratios of the soft Lagrangian parameters M_2/M_1 at the high-energy input scale the relic abundance of neutralinos achieves the observationally preferred values of $\Omega_\chi \mathrm{h}^2 \simeq 0.1 - 0.2$ *independent of the scalar fermion masses*. More specifically, we fix an overall mass scale for the gauginos by a value of $m_{1/2} \equiv \min(M_1, M_2)$ and then allow the larger to vary according to the ratio (M_2/M_1). Once the gluino mass is determined in relation to $m_{1/2}$ the parameters at the low-energy scale can be found through RG evolution. The preferred ratio of (M_2/M_1) at the high scale for the resulting relic density of the LSP is then only a mild function of the value of the gluino mass relative to $m_{1/2}$.

The results of scanning over a range of values $(M_2/M_1)_{\mathrm{high}}$ at the high scale is given in Figure 15, where contours of constant relic density $\Omega_\chi \mathrm{h}^2$ are given. Cosmological observations therefore suggest a preferred value of this ratio in the range $0.6 \leq (M_2/M_1)_{\mathrm{high}} \leq 0.85$. In Figure 16 we overlay the contours of constant W-ino fraction in the LSP wavefunction. The impact of additional coannihilation channels and increased annihilation rates is felt even at relatively small admixtures of $SU(2)$ gaugino to the predominantly B-ino LSP.

In the BGW class of models this ratio is determined through (195) by the beta-function coefficients of the Standard Model electroweak gauge groups, the beta-function coefficient b_+ of the condensing gauge group, and the dilaton *vev*

$$\frac{M_2\left(\mu_{\mathrm{UV}}\right)}{M_1\left(\mu_{\mathrm{UV}}\right)} = \frac{g_2^2\left(\mu_{\mathrm{UV}}\right)}{g_1^2\left(\mu_{\mathrm{UV}}\right)} \frac{\left(1 + b_2' \ell\right) - \left(b_2/b_+\right)\left(1 + b_+ \ell\right)}{\left(1 + b_1' \ell\right) - \left(b_1/b_+\right)\left(1 + b_+ \ell\right)} \,. \tag{380}$$

It is therefore possible to map the results of Figure 16 directly onto the parameter space of this class of models. The result is shown in Figure 17. The feature along the $\widetilde{W}_3\%|_{\mathrm{LSP}} = 1\%$ contour is the Higgs annihilation resonance. The recent data from the WMAP experiment favors a nonbaryonic dark matter relic density of $\Omega_\chi \mathrm{h}^2 \simeq 0.127$,[213] which translates into an $\mathcal{O}(1\%)$ W-ino fraction in the LSP wave-function for large scalar masses. This is a natural outcome of the Kähler-stabilization mechanism for reasonable hidden sector configurations.

In Figure 18 we reproduce the parameter space of Figure 8, highlighting the region in which the Yukawa parameter takes the values $0.1 \leq (c_\alpha)_{\mathrm{eff}} \leq 10$ and the

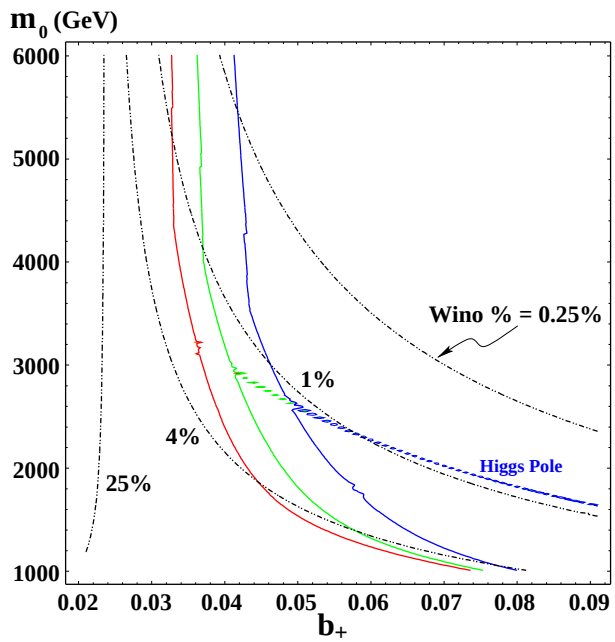

Fig. 17. **Preferred dark matter region in the BGW model.** Contours of constant relic density are given as a function of $M_0 = m_{3/2}$ and b_+ by the solid lines. Moving outward from the left are contours of $\Omega_\chi h^2$ =0.1, 0.3 and 1.0. The dotted lines give the value of the LSP wino content (25%, 4%, 1%, 0.25% from left to right).

resulting gravitino mass is between 100 GeV and 10 TeV. Achieving the proper LSP thermal relic density singles out a very specific subset of the model space. The dotted points in Figure 18 represent the subset of parameter choices in a scan over 25,000 possible combinations of $\{b_+, (b_a^\alpha)_{\rm eff}, (c_\alpha)_{\rm eff}\}$ which (a) give rise to gravitino masses between 100 GeV and 10 TeV, (b) yield a particle spectrum consistent with experimental bounds, and (c) yield a realistic relic density. Those that give rise to the WMAP value for $\Omega_\chi h^2$ tend to cluster around the value of $b_+ \simeq 3/8\pi^2$ – the benchmark point of (376) used in the collider studies. We will see below that this same value of b_+ has intriguing implications for the axion sector of the model as well.

5.5. *The axion sector*

Banks and Dine[78] pointed out some time ago that in a supersymmetric Yang Mills theory with a dilaton chiral superfield that couples universally to Yang-Mills fields

$$\mathcal{L}_{YM} = \frac{1}{8}\sum_a \int d^2\theta\, S(W^\alpha W_\alpha)_a + \text{ h.c. }, \tag{381}$$

there is a residual R-symmetry in the effective theory for the condensates of a strongly coupled gauge sector, provided that (a) there is a single condensation

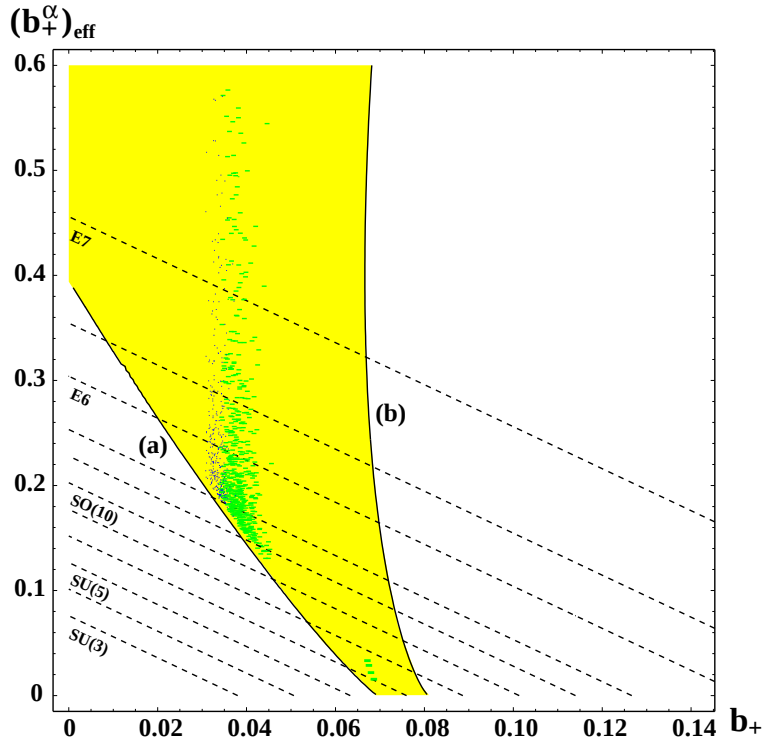

Fig. 18. **Preferred dark matter region in hidden sector configuration space.** This plot illustrates the dark matter parameter space in terms of the gauge group and matter content parameters of the hidden sector. The fine points on the left have the preferred value $0.1 \leq \Omega_\chi h^2 \leq 0.3$ and the coarse points have $0.3 < \Omega_\chi h^2 \leq 1.0$. The swath bounded by lines (a) and (b) is the region in which the $0.1 \leq (c_\alpha)_{\rm eff} \leq 10$ and the gravitino mass is between 100 GeV and 10 TeV. The dotted lines are the possible combination of gauge parameters for different hidden sector gauge groups.

scale governed by a single β-function, (b) there is no explicit R-symmetry breaking in the strongly coupled sector, (c) the dilaton S has no superpotential couplings, and (d) the Kähler potential is independent of $\mathrm{Im}\, S$. The latter two requirements are met in effective supergravity obtained from the weakly-coupled heterotic string, and explicit realizations of this scenario are found in the BGW model described in Sections 2 and 3, and generalizations thereof to include an anomalous $U(1)_X$ that are discussed in Section 4.

The R-symmetry transformations on the gauginos λ_a and chiral fermions χ^A

$$\lambda_a \to e^{\frac{i}{2}\alpha}\lambda_a, \qquad \chi^A \to e^{-\frac{i}{2}\alpha}\chi^A, \tag{382}$$

leave the classical Lagrangian (381) invariant, but are anomalous at the quantum level:

$$\Delta\mathcal{L}_{YM} = \frac{i\alpha}{8}\sum_a b'_a \int d^2\theta (W^\alpha W_\alpha)_a + \text{ h.c. }, \tag{383}$$

with b'_a given in (26). In the case that there is a single simple condensing gauge group $\mathcal{G}_c$, the symmetry can be restored by an axion shift

$$a = \mathrm{Im}\, s \to a - ib'_c \alpha\,. \tag{384}$$

If this gauge group becomes strongly coupled at a scale

$$\Lambda_c \sim e^{-1/3b_c g_0^2} \Lambda_0\,, \tag{385}$$

with b_a as defined in (6), then the effective theory[49,19,53,40] below that scale will have the same anomaly structure as the underlying theory. A potential is generated for the dilaton $\mathrm{Re}\, s$, but not for the axion.

If the gauge group is not simple $\mathcal{G} = \prod_a \mathcal{G}_a$, the R-symmetry is anomalous but no mass is generated for the axion as long as there is a single condensate. In the two-condensate case with β-functions $b_2 \ll b_c$ in dilaton stabilization models[44,55] the axion acquires a small mass

$$m_a \sim (\Lambda_2/\Lambda_c)^{\frac{3}{2}} m_{\frac{3}{2}}\,. \tag{386}$$

In the context of the BGW models, we saw in Sections 2 and 3 that a viable scenario for supersymmetry breaking occurs if a hidden sector gauge group $\mathcal{G}_c$ condenses with $b_c \approx .036$, $\Lambda_c \sim 10^{13}$ GeV and $m_{3/2} \sim$ TeV. Then if there is no additional condensing gauge group other than QCD, the universal axion is a candidate Peccei-Quinn axion. The result (386) would suggest an axion mass

$$m_a \sim 10^{-9}\mathrm{eV}\,. \tag{387}$$

However, the result (386) cannot be directly applied to the QCD axion since QCD condensation occurs far below the scale of supersymmetry breaking and heavy modes need to be correctly integrated out. Moreover, the result (386) was obtained under the assumptions that in the hidden sector there are no additional spontaneously-broken nonanomalous symmetries such as the chiral flavor $SU(N)$ of QCD, and no gauge invariant dimension-two operators such as quark mass terms. Under these assumptions two-condensate hidden sector models have a point of enhanced symmetry where the condensing gauge sectors $\mathcal{G}_a$ have the same beta-function coefficients b_a, and the axion mass is proportional to $|b_1 - b_2|$.

To investigate whether the string axion can be the QCD axion, we need to consider the case where the second condensing gauge group $\mathcal{G}_Q$ is $SU(N_c)$ with a $U(N)$ flavor symmetry for quark supermultiplets. In this case the point of enhanced symmetry occurs for[214]

$$b_c = \frac{N_c}{8\pi^2}. \tag{388}$$

As a result, the standard relation between the axion mass and its Planck scale coupling constant is modified in this class of models due to a contribution to the axion-gluon coupling that appears below the scale of supersymmetry breaking when gluinos are integrated out. Put differently, *the axion coupling constant is different*

above and below the scale of supersymmetry breaking. The axion mass vanishes at the point of enhanced symmetry; for QCD with $N_c = 3$, this occurs for

$$b_c = \frac{3}{8\pi^2} = .038, \tag{389}$$

which is again in the preferred range $.3 \leq b_c \leq .4$ found earlier. As a consequence, the axion mass is suppressed and higher dimension operators[78,215] might lead to strong CP violation. If on the other hand the string axion acquires a mass as in (386) from multiple hidden sector condensates, it cannot be the QCD axion. The possibility of detecting both types of axions has been analyzed[216] following the procedures developed by Fox, Pierce and Scott[164] for the case of the string axion with the standard relation[217] between its mass and its string-scale coupling. In all of these cases the string-scale axion coupling, which in these models is of the order of the reduced Planck mass, is outside the conventional cosmological bound.[165,166,167] Yet this can be evaded[164] by reducing relic axion production with a sufficiently small initial misalignment angle and/or late entropy release. More specifically, if the classical dilaton Kähler potential is used, the axion coupling constant is approximately[164] $F_a \simeq 10^{16}$ GeV. When string nonperturbative corrections are invoked to stabilize the dilaton, they have the effect of enhancing both the dilaton mass and the axion coupling constant:[e] $F_a \sim (1/\sqrt{3}b_c) \times 10^{16}$ GeV $\sim 10^{17}$ GeV.

To understand the origin of the enhanced symmetry points, first consider a supersymmetric model with condensing gauge group $\mathcal{G}_c \otimes \mathcal{G}_Q$. Although this is not a realistic model for QCD, it has the advantages that all the symmetries are manifest and the effective Lagrangian is highly constrained by supersymmetry. In the absence of a superpotential there is a classical $[U(1)]^3$ symmetry defined by (382), the chiral $U(1)$ transformations

$$Q \to e^{i\beta} Q, \qquad Q^c \to e^{i\beta} Q^c, \tag{390}$$

introduced in Section 1.2 and

$$\Phi_c^A \to e^{i\gamma} \Phi_c^A, \tag{391}$$

all of which can be made nonanomalous by an axion shift. The superpotential[f] $W(\Pi)$ for the hidden sector condensates Π_c^α that is needed for condensation to occur breaks this symmetry down to $[U(1)]^2$, with the parameter γ restricted such that

$$\Pi_c^\alpha \to e^{i d_c^\alpha \gamma} \Pi_c^\alpha = e^{i\alpha} \Pi_c^\alpha, \tag{392}$$

[e]In Ref. 126 it was incorrectly stated that the axion coupling constant was suppressed by these effects.

[f]This may reflect a potential $W(\Phi)$ in the classical theory and/or arise from nonperturbative QFT effects.

and the anomaly coefficient of the hidden sector condensate U_a is simply given by the β-function coefficient:

$$b_c = b'_c + \sum_\alpha b_c^\alpha = \frac{1}{8\pi^2}\left(C_c - \tfrac{1}{3}\textstyle\sum_A C_c^A\right), \tag{393}$$

for dimension-three matter condensates. If there is a second hidden sector with the same condensate structure, the residual classical symmetry is just $U(1)$, and the anomaly cannot be canceled by an axion shift unless the beta functions are the same. On the other hand if the only other condensate is the SUSY QCD one and the quarks are massless, there is a nonanomalous $U(1)$ defined by (382) and (390) with

$$\alpha\, b_c = \frac{1}{8\pi^2}\left[\alpha\,(N_c - N) + 2\beta N\right]. \tag{394}$$

If there are no massless quarks, flavor-chiral $U(1)$ symmetry is broken, and there is no longer the freedom to choose the R-parity of Q; in this case the classical R-symmetry has $\beta = \alpha/2$, and it is anomalous at the quantum level unless

$$b_c = \frac{N_c}{8\pi^2} = \frac{b'_Q N_c}{N_c - N}. \tag{395}$$

The effective meson Lagrangian for this toy model has been worked out explicitly.[214] The Kähler moduli sector is essentially unchanged with respect to the BGW case, and we will neglect it here. In the absence of quark masses the scalars and the flavor singlet pseudoscalar get masses of the order of the gravitino mass, which is the same as for the BGW model if $|u_c| \gg |u_Q|$, and the flavor adjoint pseudoscalars $\mathbf{a}$ as well as the axion a are massless. When (flavor invariant and generally complex) quark masses are turned on, these acquire masses in the ratio

$$\frac{m_a}{m_{\mathbf{a}}} \approx \sqrt{\frac{3v}{N}}(8\pi^2 b_c - N_c), \qquad v = \langle Q^c Q\rangle. \tag{396}$$

The axion mass vanishes at the symmetry point.

The above toy model is not a realistic model for QCD, but we can modify it in several ways to make it more closely resemble the MSSM while keeping manifest supersymmetry of the effective Lagrangian. In the MSSM only $n < N$ chiral supermultiplets have masses below the condensation scale $u_Q^{1/3} \sim \Lambda_Q$, while $m = N - n$ chiral supermultiplets have masses M_A above that scale. The latter decouple at scales below their masses, which explicitly break the nonanomalous $U(N)_L \otimes U(N)_R$ symmetry to a $U(n)_L \otimes U(n)_R$ symmetry if $m = N - n$ quarks are massive. They do not contribute to the chiral anomaly at the $SU(N_c)$ condensation scale. To account for these effects we replace b'_N in (27) by

$$b'_n = (N_c - n)/8\pi^2, \tag{397}$$

and replace the second term in (27) by

$$\frac{b_Q^\alpha}{8}\int \mathrm{d}^4\theta \frac{E}{R} U_Q \left[\ln(\det \mathbf{\Pi}_n) - \sum_{A=1}^{m} \ln M_A\right] + \text{ h.c. }, \tag{398}$$

where $\mathbf{\Pi}_n$ is an $n \times n$ matrix-valued composite operator constructed only from light quarks. This result can be formally obtained from (27) by integrating out the heavy quark condensates as follows. As the threshold M_A is crossed, set $\det \mathbf{\Pi}_{n+A} \to \pi^A \det \mathbf{\Pi}_{n+A-1}$ and take the condensate $\pi^A \sim Q^A Q^c_A$ to be static: $K(\mathbf{\Pi}_{n+A}) \to K(\mathbf{\Pi}_{n+A-1})$. Then including the superpotential term $W(\pi^A) = -M_A \pi^A$, the equation of motion for F^A gives $\pi^A = e^{-K/2} u_Q / 32\pi^2 M_A$, giving (397) and (398) up to some constant threshold corrections. The flat SUSY analog of (33) is now

$$W = \frac{1}{4} U_Q \left\{ g_0^{-2} + b'_n \ln(U_Q) + b^\alpha_Q \left[\ln(\det \mathbf{\Pi}) - \sum_{A=1}^{m} \ln M_A \right] \right\}, \tag{399}$$

and we recover (34)–(35) with now

$$\Lambda_Q = e^{-1/3b_n g^2} \prod_{A=1}^{m} M_A^{b_3/3b_n}, \qquad 3b_n = \frac{3N_c - n}{8\pi^2} = 3b'_n + 2n\, b_3\,, \tag{400}$$

which corresponds to running $g^{-2}(\mu)$ from $g^{-2}(1) = g_0^{-2}$ to $g^{-2}(\Lambda_Q) = 0$ using the β-function coefficient $(3N_c - n - A)/8\pi^2$ for $m_A \le \mu \le m_{A+1}$, again in agreement with the results of nonperturbative flat SUSY analyses.[57]

The anomaly matching conditions in the toy model correctly reproduce[44] the running of g^{-2} from the string scale to the condensation scale provided supersymmetry is unbroken above that scale. This is not the case for QCD, and the effective "QCD" Lagrangian (27) or (398) is not valid below the scale Λ_c of supersymmetry breaking. The arguments of the logs are effective infra-red cut-offs. For gauginos and squarks, they should be replaced by the actual masses, as was done in (398) for quark supermultiplets with masses above the QCD condensation scale. The gaugino and squark mass terms are given by

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} \frac{|m_{3/2}|}{F_a^2 b_c} \left(e^{i\omega_c} \bar{\lambda}_R \lambda_L + \text{ h.c. } \right) - m_{3/2}^2 |\tilde{q}|^2, \tag{401}$$

where F_a is the axion coupling constant

$$\frac{1}{F_a^2} = 2\langle K_{s\bar{s}} \rangle \approx \frac{3}{2} b_c^2, \tag{402}$$

as defined by its coupling to the gauge fields above the condensation scales

$$\mathcal{L} \ni -\frac{a}{4F_a} \sum_a (F \cdot \widetilde{F})_a\,. \tag{403}$$

The phase ω_c of the static condensate u_c in (401) is determined by the equations of motion in terms of the axion

$$\omega_c = -\frac{a}{F_a b_c} + \phi\,, \tag{404}$$

where the constant phase ϕ includes the phases of the quark mass and of the meson condensate. The mass terms (401) are invariant under (382) which is spontaneously broken by the vacuum value $u_c \neq 0$, but remains an exact (nonlinearly realized) symmetry of the Lagrangian, since the anomaly can be canceled by an axion shift

as long as QCD nonperturbative effects can be neglected. Since U_c transforms the same way as U_Q, an effective theory with the correct anomaly structure under (382) and (390) is obtained by using (398) and replacing the first term of (27) by

$$\frac{1}{8}\int \mathrm{d}^4\theta \frac{E}{R} U_Q \left[b_1 \ln(e^{-K/2}U_Q) + b_2 \ln(e^{-K/2}U_c)\right] + \text{ h.c. }, \tag{405}$$

provided

$$b_3 = \frac{1}{8\pi^2} = b_Q^\alpha\,, \qquad b_1 + b_2 = \frac{N_c - n}{8\pi^2} = b'_n\,, \tag{406}$$

and we can choose b_1 and b_2 to better reflect the correct infrared cut-offs for squarks and gauginos. The potential is modified, but its qualitative features are the same; in particular the axion mass is unchanged since it depends only on $b_1 + b_2 = b'_n$. The scalar components of the composite chiral superfields U_Q and $\mathbf{\Pi}$ are composed of gauginos and squarks that get large masses proportional to $m_{3/2}$, while the true light degrees of freedom are the quarks and gauge bosons. The corresponding composite operators are the F-components F_Q, $\mathbf{F}$ of U_Q, $\mathbf{\Pi}$; these were eliminated by their equations of motion to obtain an effective Lagrangian for the scalars. We can trade the former for the latter by inverting these equations. Then setting everything except the light pseudoscalars at their vacuum values, to leading order in $1/m_{\rm PL}$ and the quark mass

$$m_q = e^{i\delta} m, \tag{407}$$

one obtains the identification

$$\overline{\mathbf{F}}_n \approx e^{i\delta}\left(c_1 e^{-i\mathbf{P}} - c_0\right), \quad \mathbf{P} = -\frac{\sqrt{2}\mathbf{a}}{F_\pi} - \frac{8\pi^2 b_c - N_c}{b_c n}\frac{a}{F_a}, \quad F_\pi \approx 2\sqrt{v}, \tag{408}$$

and the effective potential for the light pseudoscalars takes the form

$$V = c\,\mathrm{Tr}\,\mathbf{F}_n + \text{ h.c. }, \tag{409}$$

which is the standard result in QCD if $\mathrm{Tr}\,\mathbf{F}_n$ is identified with the quark condensate. To check that this identification is correct, we note that under the Kähler $U(1)$ transformation (382) and the transformation (390) on the n light quark supermultiplets, the anomalies induce a shift in the Lagrangian

$$\delta\mathcal{L} \ni -\frac{1}{4}\left[\alpha b_c (F\cdot\widetilde{F})_c + (\alpha b'_n + 2n\beta)(F\cdot\widetilde{F})_Q\right], \tag{410}$$

which is canceled in the nonanomalous case (394) by a shift in (403) due to the axion shift

$$a \to a - \alpha b_c F_a\,. \tag{411}$$

This gives

$$\mathbf{F}_n \to e^{i\alpha b_{\mathbf{F}}}\mathbf{F}_n, \qquad b_{\mathbf{F}} = \frac{8\pi^2 b_c - N_c}{n}, \tag{412}$$

which matches the phase transformation of the quark condensate

$$\chi_L \chi_L^c \to e^{i\alpha b_\chi} \chi_L \chi_L^c \,, \qquad b_\chi = 2\frac{\beta}{\alpha} - 1 = \frac{8\pi^2 b_c - N_c + n}{n} - 1 = b_{\mathbf{F}} \,. \tag{413}$$

For $n = 2$ we identify the factor $\exp(i\sqrt{2}\mathbf{a}/F_\pi)$ with the operator $\Sigma = e^{2i\pi^i T_i/F_\pi}$ of standard chiral Lagrangians, where π^i are the canonically normalized pions, and T_i is a generator of $SU(2)$. That is, we identify a_i with π_i giving

$$m_a = \frac{|8\pi^2 b_c - N_c|}{b_c n} \frac{\sqrt{n} F_\pi}{F_a \sqrt{2}} m_\pi \approx \frac{\sqrt{3}|8\pi^2 b_c - N_c| F_\pi}{2\sqrt{n}} m_\pi \,, \quad F_\pi \approx 93 \text{ MeV} \,. \tag{414}$$

The result (414) appears to differ from the standard result by a factor $1 - N_c/8\pi^2 b_c$. However, F_a is the axion coupling to Yang-Mills fields *above* the scale of supersymmetry breaking. When the gluinos of the supersymmetric extension of the Standard Model are integrated out, a term is generated that modifies the axion coupling strength to $(F\widetilde{F})_Q$ by precisely that factor. This can be seen in two ways.

First, note that in the absence of the light quark mass the Lagrangian is invariant under (382) together with (390) with $\beta = \alpha/2$ for the heavy quark supermultiplets (ϕ^A, χ^A), and

$$\beta = \frac{1}{2} + \frac{8\pi^2 b_c - N_c}{2n}, \tag{415}$$

for the light quark supermultiplets (ϕ^i, χ^i). In order to keep this approximate symmetry manifest in the low energy effective theory, we can redefine the fields so as to remove the ω_c-dependence from all terms in the Lagrangian for the heavy fields that do not involve the light quark mass m:

$$\begin{aligned} \lambda_a &= e^{i\omega_c/2} \lambda'_a, \qquad \phi^A = e^{i\omega_c/2} \phi'^A, \qquad \chi^A = \chi'^A, \\ \phi^i &= e^{i\beta\omega_c} \phi'^i, \qquad \chi^i = e^{i\gamma\omega_c} \chi'^i, \qquad \gamma = \frac{8\pi^2 b_c - N_c}{2n}. \end{aligned} \tag{416}$$

The primed fields are invariant under the nonanomalous symmetry, and when expressed in terms of them, the potential and Yukawa couplings have no dependence on ω_c when $m \to 0$. This ensures that any effects of integrating out the heavy fields will be suppressed by powers of m/M_A, $m/m_{3/2}$ relative to the terms retained.

However, these transformations induce new terms in the effective Lagrangian. First, because the transformation (416) with ω_c held fixed is anomalous, it induces a term

$$\mathcal{L}' \ni \Delta\mathcal{L} = -\frac{\omega_c b_c}{4} (F \cdot \widetilde{F})_Q \,. \tag{417}$$

Second, there are shifts in the kinetic terms. The ones that concern us here are the shifts in the fermion axial connections A_m

$$\Delta A_m^\lambda = -\frac{1}{2} \partial_m \omega_c \,, \qquad \Delta A_m^{\chi^i} = -\gamma \partial_m \omega_c \,. \tag{418}$$

Quantum corrections induce a nonlocal operator coupling the axial connection to $F \cdot \widetilde{F}$, at scales $\mu^2 \sim \Box \gg m_\lambda^2$, through the anomalous triangle diagram

$$\mathcal{L}_{\rm qu} \ni -\frac{1}{4}(F \cdot \widetilde{F})_Q \frac{1}{\Box}\left(\frac{N_c}{4\pi^2}\partial^\mu A_\mu^\lambda + \frac{n}{2\pi^2}\partial^\mu A_\mu^{\chi^i}\right). \tag{419}$$

The contribution to (419) from the shift (418) exactly cancels the shift (417) in the tree level Lagrangian, leaving the $aF\widetilde{F}$ S-matrix element unchanged by the redefinition (416). However at scales $\mu^2 \ll m_\lambda^2$, we replace $\Box \to m_\lambda^2$ in the first term of (419) because the contribution decouples, but the analogous contribution (417) to the tree Lagrangian $\mathcal{L}'$ remains in the effective low-energy Lagrangian. This is a reflection of the fact that the classical symmetry of the unprimed variables, without a compensating axion shift, is anomalous. The gluino contribution to that anomaly is not canceled by the gluino mass term, because the gluino mass does not break the symmetry. Its phase ω_c is undetermined above Λ_{QCD} and transforms so as to make the mass term invariant.

To see that the gluino contribution to the anomaly does not decouple, we write the (unprimed) gaugino contribution to the one-loop action as

$$S_1 = -\frac{i}{2}{\rm Tr}\ln(i\not{D} + m_\lambda) = S_A + S_N, \tag{420}$$

where

$$S_A = -\frac{i}{2}{\rm Tr}\ln(i\not{D}) \tag{421}$$

is mass-independent and contains the gaugino contribution to the anomaly

$$\delta\mathcal{L} \ni \delta S_A = -\frac{\alpha N_c}{32\pi^2}(F \cdot \widetilde{F})_Q\,. \tag{422}$$

The mass-dependent piece

$$S_N = -\frac{i}{2}{\rm Tr}\ln(-i\not{D} + m_\lambda) + \frac{i}{2}{\rm Tr}\ln(-i\not{D}) \tag{423}$$

is finite and therefore nonanomalous. A constant mass term would break the symmetry and the contribution from S_N would exactly cancel that from S_A in the limit $\mu/m_\lambda \to 0$. However it clear that S_N is invariant under (382) because the gaugino mass is covariant. On can show[214] by direct calculation that gaugino loops give the contribution (422) under a nonanomalous $U(1)$ transformation in the limit $m_\lambda \gg \mu$, which in this limit arises only from the phase of the mass matrix. This implies that the effective low energy theory must contain a coupling

$$\mathcal{L}_{\rm eff} \ni \mathcal{L}_{\rm anom} = -\frac{\omega_c N_c}{32\pi^2}(F \cdot \widetilde{F})_Q, \tag{424}$$

which is precisely the term that is generated by the redefinitions in (416).

Below the scale of QCD condensation standard effective chiral Lagrangian techniques can be used[214] to recover (414), or, taking $n = 2$ and allowing for $m_u \neq m_d$,

$$m_a \approx 2m_\pi \frac{F_\pi\sqrt{z}}{f(1+z)}, \qquad z = \frac{m_u}{m_d}, \tag{425}$$

where

$$\frac{na}{32\pi^2} f^{-1} = \left(1 - \frac{N_c}{8\pi^2 b_c}\right) F_a^{-1}, \tag{426}$$

in agreement[g] with the result[217] of Srednicki. The coupling f is the low energy axion coupling constant as defined by the convention

$$\mathcal{L} \ni -\frac{na}{32\pi^2 f} \sum_b (F \cdot \widetilde{F})_b, \qquad f = \frac{nF_a}{8\pi^2}, \tag{427}$$

used by Fox *et al.*[164] The axion coupling to photons is important in direct axion searches. A similar shift in the axion coupling to $SU(2)_L$ gauge fields is generated when the W-inos are integrated out, with $N_c = 2$ in (424). Then the couplings $f_{\gamma,g}(\mu)$ for the canonically normalized gauge fields as measured at a scale $\mu < m_\lambda$ are in the ratio

$$\frac{f_\gamma}{f_g} = \frac{g_3^2(\mu)}{e^2(\mu)} \frac{8\pi^2 b_c - 3}{8\pi^2 b_c - 2\sin^2\theta(\mu)}, \tag{428}$$

which could be very different from the value at $\mu > m_\lambda$ which is just the ratio of the fine structure constants. In addition there is an induced $\gamma W a$ coupling for $\mu < m_\lambda$.

If use the preferred value $b_c = .036$, we are very close to the symmetric point for $N_c = 3$: $8\pi^2 b_c = 2.84$, so we get an (accidental) suppression of the axion mass. In particular, if $b_c \ell \ll 1$ and $n = 2$ we obtain

$$m_a \approx 5 \times 10^{-13} \text{ eV}, \tag{429}$$

which raises the issue of the importance of higher dimension operators that might contribute to the axion potential and destroy the solution to the strong CP problem.[78] Indeed, exactly at the point of enhanced symmetry $b_c = 8\pi^2 N_c$, the nonanomalous symmetry with (415) does not include a chiral transformation on the quarks, and one loses the solution to the CP problem. The axion decouples from the quarks in the effective Lagrangian, and its *vev* cannot be adjusted to make the quark mass matrix real in the $\theta = 0$ basis.

There is no reason to expect that nature sits at this point, but if the axion mass is very small one should worry about other sources of an axion potential. The contribution of higher dimension operators was studied in Reference 215 in the context of modular-invariant gaugino condensation models. Modular invariance severely restricts the allowed couplings; the leading contribution to the axion mass takes the form

$$m_a'^2 \approx \frac{9}{4} b_c p^3 |u|^2 k' \lambda |\eta^2 e^{-K/2} u|^p \tag{430}$$

where λ is a dimensionless coupling constant, and p is the smallest integer allowed by T-duality. An orbifold compactification model with three complex moduli and an

[g]The coupling constant f used by Srednicki[217] is a factor two larger than the one defined in (427) and used by Fox *et al.*[164]

$[SL(2,\mathbf{Z})]^3$ symmetry has $p = 12$, and, with the values of the various parameters used above and $\lambda \approx 1$, one finds $m'_a \approx 10^{-63}$eV, which is completely negligible. However if the symmetry is restricted, for example, to just $SL(2,\mathbf{Z})$ one has $p = 4$ and the contribution from (430) is of the order of (429). The axion potential takes the form

$$\begin{aligned} V(a) &= -f^2 m_a^2 \cos(a/f + \phi_0) - f'^2 m_a'^2 \cos(a/f')\,, \\ \frac{1}{f'} &= \frac{p}{b_c}\frac{1}{F_a} = \frac{p\,n}{8\pi^2 b_c - N_c}\frac{1}{f}\,, \end{aligned} \tag{431}$$

where we have absorbed constant phases in a and/or in ϕ_0 so as to make the coefficients negative. The strong CP problem is avoided if for some value of b_c the vacuum has $\langle\bar{\theta}\rangle = \langle n(a/f + \phi_0)/2\rangle < 10^{-9}$ for any value of ϕ_0. For values of b_c in the preferred range $.3 \leq b_c \leq .4$ this does not occur. For example for $b_c = .036$ with $p = 4$ and $f'/f \approx 1/50$, this requires $f'^2 m_a'^2/f^2 m_a^2 < 10^{-10}$, whereas evaluating (431) gives $f'^2 m_a'^2/f^2 m_a^2 \approx 4 \times 10^{-4}$ in this case. A numerical analysis shows that the CP problem is avoided provided $p \geq 5$, that is, provided the T-duality group is not the minimal one, which is in fact the case for most orbifold compactifications of the weakly-coupled heterotic string.

The reasoning leading to (430) is similar to that used in Section 5 in the discussion of R-parity. In the language of Kähler $U(1)$ supergravity, superpotential terms must have Kähler $U(1)$ weight 2, where chiral fields Z^A have weight 0 and the Yang-Mills superfield strength W_α has weight 1. Thus the following terms with at least one factor $W^\alpha W_\alpha$ are allowed

$$\mathcal{L}_{SP} = \frac{1}{2}\int \mathrm{d}^4\theta F W^\alpha W_\alpha \mathcal{F}(e^{-K/2} W^\alpha W_\alpha,\, Z^A) + \text{ h.c. }. \tag{432}$$

Invariance under the T-duality transformations (340) restricts the function $\mathcal{F}$ to the form

$$\mathcal{F} = \mathcal{F}(\eta^2 e^{-K/2} W^\alpha W_\alpha, \eta^A \Phi^A), \quad \eta = \prod_I \eta_I, \quad \eta^A = \prod_I \eta_I^{2q_I^A}, \quad \eta_I = \eta(T^I). \tag{433}$$

Consider first terms with no Φ^A-dependence; since for a general transformation (340) $\delta_I = n_I \pi/12$, the only invariant superpotential is of the form:

$$\mathcal{L}_{HW} = \frac{1}{2}\int \mathrm{d}^4\theta \frac{E}{R} W^\alpha W_\alpha \mathcal{F}(\eta^2 e^{-K/2} W^\alpha W_\alpha) + \text{ h.c. }, \quad \mathcal{F}(X) = \sum_{n=1} \lambda_n X^{12n}. \tag{434}$$

If the $[SL(2,\mathbf{Z})]^3$ symmetry implied by (340) were instead restricted, say to just $SL(2,\mathbf{Z})$, with a_I, b_I, c_I, d_I, independent of I as in (76) and (84), then the phase of η is $3\delta_I = n\pi/4$, and lower dimension operators would be allowed: $\mathcal{F}(X) = \sum_{n=1} \lambda_n X^{4n}$. These give the estimate in (430).

In addition to the operators in (433) chiral superfields with zero chiral weight can be constructed using chiral projections of any functions of chiral fields. Operators of this type were found[218] in (2,2) orbifold compactifications of the heterotic string

theory with six dynamical moduli. In the class of models considered here we can construct zero-weight chiral superfields of the form

$$\mathcal{F} = e^{-(p+n)K/2}(W^\alpha W_\alpha)^p \eta^{2(p+n)} \prod_{i=1}^{n} (\overline{\mathcal{D}}^2 - 8R) f_i[|\eta_I|^4 (t^I + \bar{t}^I)] \tag{435}$$

that are modular invariant provided $(p+n)\sum_I \delta = m\pi$. Since $\langle F^I \rangle = 0$, the corresponding terms in the potential at the condensation scale are proportional to[h] $|u|^p (m_{\frac{3}{2}})^{n+1}$, so for fixed $p+n$ one is trading factors of $|u|$ for factors of $m_{3/2} \sim 10^{-2}|u|$, and these contributions to the axion mass will be smaller than those in (430).

We may also consider operators with matter fields that have nonvanishing *vevs*. Since $e^{-K/2}W^\alpha W_\alpha$ transforms like the composite operators $U_1U_2U_3$ constructed from untwisted chiral superfields, the rules for construction of a covariant superpotential including this chiral superfield can be directly extracted from the discussion in Section 5.1 of modular-invariant superpotential terms in the class of $\mathbf{Z}_3$ orbifolds considered here. They take the form of (433) with

$$\mathcal{F}_{pnq} = \Pi^q (e^{-K/2}W^\alpha W_\alpha)^p \eta^{2(p+n)} \prod_{\alpha=1}^{n} W_i, \qquad (p+n)\sum_I \delta_I = m\pi, \tag{436}$$

where $\Pi = Y^1Y^2Y^3$ is the product of twisted sector oscillator superfields introduced in Section 5.1, and W_i is any modular covariant zero-weight chiral superfield that is a candidate superpotential term (subject to other constraints such as gauge invariance). For example, the superpotential terms for matter condensates could contribute to this expression. However the equations of motion for the auxiliary fields of these condensates give $W_i \sim m_{3/2}$ for these terms, so again they are less important than the contribution in (430).

Most $\mathbf{Z}_3$ orbifold compactifications of the type considered here have an anomalous $U(1)$ gauge group, with the anomaly canceled by a Fayet-Illiopoulos D-term, as discussed in Section 4. A number n of scalars ϕ^A acquire *vevs* along an F- and D-flat direction such that $m \leq n$ $U(1)_a$ gauge factors are broken at a scale Λ_D that is close to the Planck scale. *A priori* there might be gauge- and modular-invariant monomials of the form (436) with considerably larger *vevs* than those in (434), and no modular covariant, gauge invariant superpotential term W_i, so that the direction $\phi^A \neq 0$ is F-flat. However if $m = n$, there is no gauge invariant monomial $\prod_A (\phi^A)^{p_A}$. Gauge invariance requires

$$\sum_A p_A q_A^a = 0 \qquad \forall a, \tag{437}$$

[h]The coefficients of the nonpropagating condensate superfield auxiliary fields vanish by their equations of motion.

where q_A^a is the $U(1)_a$, charge of ϕ^A. If $m = n$ these are linearly independent and form an $m \times m$ matrix with inverse Q_a^A; then (437) implies

$$p_A = 0 \qquad \forall A. \tag{438}$$

Similarly, for the chiral projection of a monomial $\prod_A (\phi^A)^{p_A+q_A}(\bar{\phi}^{\bar{A}})^{q_A}$ gauge invariance still requires (437) and (438), so any such monomial can be written in the form

$$f(T^J, \overline{T}^{\overline{J}}) \prod_A \left[|\phi^A|^2 \prod_I (T^I + \overline{T}^{\overline{I}})^{-q_I^A} \right]^{q_A} . \tag{439}$$

It is the modular invariant composite fields $|\phi^A|^2 \prod_I (T^I + \overline{T}^{\bar{I}})^{-q_I^A}$ that acquire large *vevs*; any coefficients of them appearing in overall modular-invariant operators are subject to the same rules of construction as the operators in (435). The same considerations hold if N sets of fields ϕ_i^A with identical $U(1)_a$, charges $(q_A^i)^a = q_A^a$, $i = 1, \ldots, N$ acquire *vevs*.

In the general case with $n > m$ one cannot rule out the above terms. However in this case part of the modular symmetry is realized nonlinearly on the $U(1)_a$-charged scalars after $U(1)_a$-breaking. Monomials of the above type would generate mixing of the axion with massless "D-moduli" that are Goldstone particles associated with the degeneracy of the vacuum at the $U(1)_a$-breaking scale, requiring a more careful analysis.

5.6. *Early universe physics*

So far in this section we have focused on phenomena relevant for low-energy observations in the late-time universe. Yet string theory is meant to provide a single framework for understanding *all* phenomena, including the physics of the early universe. Therefore, a string-derived effective supergravity model – to the extent that it is a complete description of the underlying string degrees of freedom – should provide such things as an inflaton candidate, a baryogenesis mechanism and a source for the dark energy in the universe. And it should do these things while allowing for the successful predictions of the Big Bang Nucleosynthesis (BBN) theory. In this section we will concern ourselves with some of these topics where a definite statement can be made in the context of the BGW class of models.

Scalar field inflation has long been the leading paradigm for understanding the horizon and flatness problems of the big bang cosmology.[219,220] All supersymmetric theories provide many such scalar fields – and string-derived models have still more. The latter include the moduli fields that have been our focus throughout this work. These fields carry no Standard Model quantum numbers and provide an interesting possibility to realize the "sterile" field models that are common in inflation theories. Unfortunately, it has long been appreciated that maintaining adequate flatness of the potential to achieve slow-roll is difficult in supergravity models of scalar fields

in an expansionary universe.[221] Higher-order Kähler potential terms are the most troublesome since they do not benefit from nonrenormalization theorems.

In this section we will consider a hybrid inflation scenario suggested by the BGW class of theories, in which some field other than the inflaton is displaced from its vacuum value and thereby generating most of the potential during inflation. The necessary linear and bilinear terms in the superpotential will be the result of some fields getting *vevs* when an FI D-term is driven to small values, while preserving SUSY and some flat directions. To be more specific, consider a theory of untwisted matter having a Kähler potential given by (81) from Section 2. Let us assume that during some period of inflation all matter from twisted sectors have vanishing *vevs*. If we further assume that the limit $\langle (z^i)^I \rangle \ll \langle t^I \rangle$ during inflation, then it is permissible to regard the scalar fields

$$x^I = t^I + \bar{t}^I - \sum_i |(z^i)^I|^2 \tag{440}$$

as the low energy degrees of freedom. Theories described in this manner are of the flat no-scale type and enjoy a Heisenberg symmetry.[222,223] Typically such a model has a minimum only in the limit as $x^I \to \infty$, but the presence of FI D-terms of the form (202)

$$D_X = -2\left(\sum_i K_i q_i^X (z^i)^I + \xi\right), \quad K_i = \frac{\partial K}{\partial (z^i)^I} \quad \xi = \frac{g_{\rm STR}^2 \,{\rm Tr}\, Q_X}{192\pi^2} m_{\rm PL}^2, \tag{441}$$

can prevent this runaway behavior because of the nontrivial x^I dependence of the metric K_i. The GS term will preserve this Heisenberg invariance provided that it also depends on the moduli only through the combination x^I. This is to say, $p_i = b_{\rm GS}$ in (116) for the untwisted fields.

Other terms in the superpotential which involve the twisted sector fields will explicitly break the Heisenberg invariance, but these are small effects during inflation by assumption. For the scalar components of the twisted sector fields we can define a quantity

$$X_i \equiv \prod_I (x^I)^{-q_i^I} |z^i|^2 \tag{442}$$

such that (the scalar part of) the Kähler potential reads

$$K = \ln(\ell) + g(\ell) - \sum_I \ln x^I + \sum_i X_i \,. \tag{443}$$

Therefore, near the origin in scalar field space, we can write the Kähler metric for arbitrary matter field z^i as

$$K_i = \prod_I \bar{z}^i (x^I)^{-q_i^I} \,; \quad K_{i\bar{\jmath}} = \prod_I \delta_{i\bar{\jmath}} (x^I)^{-q_i^I} \,. \tag{444}$$

As for the superpotential, we assume as always that its form is dictated by the requirements of modular invariance. Written in terms of scalar fields it is given by

$$W = \sum_m \lambda_m \prod_i (z^i)^{n^i_m} \prod_I \eta(t^I)^{2(\sum_i n^i_m q^I_i - 1)} \,, \tag{445}$$

which is the equivalent to what was considered in (125) for the hidden sector matter condensates. The integers n^i_m are nonnegative. Note that this implies

$$\frac{\partial W}{\partial t^I} \equiv W_I = 2\zeta(t^I) \left(\sum_i q^I_i z^i W_i - W \right) \,, \tag{446}$$

and t^I are stabilized at one of the two self-dual points.

Now if we are concerned with cases in which $\langle V \rangle \gg u^2$ – that is, cases in which the vacuum energy is much greater than the size of the eventual gaugino condensates – then the terms involving the condensates u can be neglected in the scalar potential of the BGW model. This leaves us with two sources for the scalar potential: possible D-terms from an anomalous $U(1)$ and derivatives of the superpotential in (445) with respect to the chiral matter. Let us take a very simple ansatz. Assume that each individual term in (445) vanishes (which is to say that at least one scalar field in each term in (445) has vanishing *vev*). Also assume that all W_i vanish except for one i in the untwisted sector. Without loss of generality assume that this field is associated with $I = 3$ so that $i = C3$. Finally, assume that all matter field values are much smaller than one in Planck-scale units. The potential in this limit is

$$V = \frac{\ell e^{g(\ell)}}{(1 + b_{\rm GS}\ell) x^1 x^2} |W_{C3}|^2 \,. \tag{447}$$

Let us take W_{C3} to depend only on x^1 and x^2 so that the moduli are stabilized but a flat direction persists for t^3 as well as matter fields in the $I = 3$ sector. The inflaton could be identified with a particular combination of these fields.[i]

To achieve something like this simplified scenario one might imagine that the superpotential that arises from the string compactification has a form

$$W = \lambda \left[\eta(t^1)\eta(t^2) \right]^{-2} z^{C3} \prod_{i \neq C3} z^i \prod_I \left[\eta(t^I) \right]^{-2q^I_i} \,. \tag{448}$$

Then to obtain the desired form of W_{C3} we must suppose that during inflation there are nonzero and modular-invariant *vevs* for certain z^i's:

$$\left\langle |z^i|^2 \prod_I (x^I)^{-q^I_i} \right\rangle = c_i \left\langle \ell^{\,d_i} \prod_I \left[x^I |\eta(t^I)|^4 \right]^{p^I_i} \right\rangle \tag{449}$$

where c_i is a constant. But in fact *vevs* of precisely this form are indeed induced at the anomalous $U(1)_X$ scale, as was discussed in Section 4. Simply consider the expression in (441) with $g^2_{\rm STR} = 2\ell/1 + f(\ell)$ and use (444) to find the form for K_i.

[i]In fact, the canonically normalized inflaton field φ can be found (up to a phase) in this simple case by inverting the relationship $|z^3_C| = \sqrt{t^3 + \bar{t}^3}\,\tanh(\varphi/\sqrt{2})$.[226]

This yields a *vev* of the form in (449) with $p_i^I = 0$ and $d_i = 1$. Once these *vevs* are integrated out of the theory the moduli dependence of (447) is then simply

$$V \propto \prod_I \left[|\eta(t^I)|^4 x^I\right]^{n^I} , \quad n^{1,2} = \sum_i (p_i^{1,2} q_i^{1,2}) - 1 , \quad n^3 = \sum_i (p_i^3 + q_i^3) . \tag{450}$$

To achieve flat directions and a realistic inflaton it is necessary that at least one of the n^I vanish. If the remaining n^I are negative then the corresponding moduli are stabilized at $t^I = e^{i\pi/6}$. The dilaton dependence of (445) is then

$$V \propto \frac{e^{g(\ell)} \ell^d}{1 + b_{\rm GS} \ell} , \quad d = 1 + \sum_i d_i \tag{451}$$

which slightly modifies the minimization conditions for the dilaton when $d \neq 1$, though weak-coupling solutions with the domain of attraction can be found.[224,225,226]

Note that this scenario actually generates a precisely flat potential. Ending inflation therefore requires a perturbation on the scenario. These could come from (a) having some terms in W that do not vanish during inflation, (b) having additional fields which contribute to (445) beyond our one untwisted field, and/or (c) assuming that the D-term is not forced to vanish but instead driven to very small but nonvanishing values. The resulting theory will be a hybrid inflation model which is a variant on that of Ref. 227. Flat directions involving the untwisted sector are lifted by mass terms with a typical size that is $|m_{(z^i)^I}|^2 \sim m_{t_I}^2$ provided the field *vevs* satisfy $\langle |(z^i)^I|^2 \rangle \lesssim \langle {\rm Re}\, t^I \rangle$. This contribution is generally negative and much smaller than that induced by loop effects.[223] If either is the dominant source of nonvanishing slope for the inflaton potential then one expects the spectral index $n = 1 + 2m^2/V$ to be very close to unity.

Finally, we must worry about the overall scale of the inflationary potential, which is constrained by the Cosmic Background Observer (COBE) normalization to have $V^{1/4} \lesssim 10^{-2} m_{\rm PL}$. This will require that the superpotential (448) that gives rise to (445) involve nonrenormalizable operators. The scale of the potential is given by

$$V = \lambda \Lambda^{-2n} \xi_D^{2(2+n)} \tag{452}$$

where $\Lambda^2 = g_{\rm STR}^2 m_{\rm PL}^2$ and λ is a ratio of dimensionless couplings in the superpotential. The mass dimension of the term that contains z_C^3 is $3 + n$. We see, therefore, that

$$V^{1/4} \sim \lambda^{1/4} g_{\rm STR} \left(\frac{{\rm Tr}\, Q_X}{192\pi^2} \right)^{(2+n)/4} m_{\rm PL} \tag{453}$$

which implies $n = 1$ or $n = 2$ is necessary to match the COBE normalization.

After inflation ends the moduli must end up at the minima of the scalar potential. This can be problematic for moduli whose potential is generated by nonperturbative effects. In particular, for the dilaton the scalar potential after inflation is generated by gaugino condensation and is quite steep for large field values (see

Figure 4, with only a small barrier separating the nontrivial minimum from the vacuum with vanishing gauge coupling. One might worry that the dilaton field will "overshoot" the desired minimum at the end of inflation.[228] However, recent work demonstrates that this is not the case with the nonperturbative corrections (60) employed in the BGW model.[229,230,226] If during an expansionary period the dilaton begins at some point in its scalar potential which corresponds to a regime of strong coupling, then over a wide range of initial field values it will enter a quasi-scaling regime as it evolves towards its (weak-coupling) true minimum. This is in spite of the rather steep potential set up by gaugino condensates (and Kähler stabilization). This expansionary period must be generated by some other field in the theory, however. In Ref. 229 the minimum number of e-foldings $N_{\rm min}$ required such that the dilaton enters the scaling regime for $m_{3/2} = 1$ TeV, $g^2_{\rm STR} = 1/2$ and a radiation-dominated universe was found to be $N_{\rm min} = 11$. It was therefore claimed in that work that the dilaton in the BGW model should enter its scaling regime and therefore end up at the global minimum after inflation (without overshooting) provided the dilaton scalar is significantly more massive than the gravitino.

This is not the only instance in which the masses of the moduli fields play an important role in the physics of the early universe. We generally expect re-heating after inflation to produce states whose masses are less than or on the order of the reheat temperature $T_{\rm RH}$. Of particular interest are those scalar fields which have no classical potential. Being flat directions, these scalars are likely to take large field values away from their eventual minima. As the universe cools, oscillations of these fields about their minimum-energy configurations will generally produce too much energy density to be consistent with the known age of the universe unless their masses are impossibly small.[169,170,171] They therefore must decay, but as these particles interact with the observable sector only via Planck-suppressed operators, their resulting lifetime is generally quite long. When they decay, light elements produced via BBN will be dissociated. It is therefore necessary that the decay of these moduli fields reheat the universe once again, with a reheat temperature this time of order the BBN scale of 1 MeV. A simple computation reveals that the needed $T_{\rm RH}$ can be achieved provided the moduli have masses of $\mathcal{O}(10$ TeV) or higher.[171,231]

In the BGW class of models the corrections to the dilaton metric result in an enhancement of the dilaton scalar mass relative to the that of the gravitino. The masses obey the relation

$$\frac{m_\ell}{m_{3/2}} \simeq \frac{1}{b_+^2} \gg 1\,. \tag{454}$$

For the case of the E_6 condensates with 9 fundamentals of matter this implies $m_\ell \sim 10^3 m_{3/2} \sim 3000$ TeV. This is sufficient to keep the dilaton in the domain of attraction for its post-inflation potential while simultaneously avoiding the cosmological problem for this modulus.

More problematic are the Kähler moduli. Their masses can be found from the

second derivative of the scalar potential

$$\frac{\partial^2 V}{\partial (t^I)^2} \simeq \frac{1}{32\ell^2} \sum_{ab} \rho_a \rho_b \left[\frac{\pi^2}{9} \frac{\ell^2}{(1+b_{\rm GS}\ell)} (b-b_a)(b-b_b) \right] \simeq \rho_+^2 \frac{\pi^2}{288} \frac{(b-b_+)^2}{1+b_{\rm GS}\ell} \,, \quad (455)$$

from which we extract the normalized mass-squared

$$m_{t_I}^2 \simeq \left\langle \rho_+^2 \frac{\pi^2}{36} \frac{(b-b_+)^2}{1+b_{\rm GS}\ell} \right\rangle \,. \quad (456)$$

When $b_{\rm GS} > b_+$, as when $b_+ = 3/8\pi^2$ and $b_{\rm GS} = b_{E_8} = 30/8\pi^2$, one can enhance the Kähler modulus mass by an order of magnitude relative to the gravitino mass. But this is roughly the largest such an enhancement can be. For more realistic scenarios we expect $b_{\rm GS}$ to be similar in magnitude to the value of b_+. However, when there are anomalous $U(1)$ factors the equation of motion for the auxiliary fields of the Kähler moduli are modified to the form of (264) from Section 4. In this case the Kähler moduli masses are enhanced relative to the those given above for the same values of $b_{\rm GS}$ and b_+. We conclude that the masses of the moduli in the BGW class of models are likely to be sufficiently large to avoid the cosmological moduli problem,[j] but that the actual resolution of the problem is dependent on the parameters that arise from the underlying string construction.

Conclusion – Where do we go from here?

In this review we have tried to take the reader on a largely self-contained exploration of one particularly well-understood corner of the moduli space of M-theory. We began by introducing the notion of string moduli, reviewing the manner in which nonperturbative field theory effects can be employed to produce a potential for their scalar components. As these scalar fields determine all dimensionless parameters in the low-energy effective supergravity Lagrangian, this is clearly the heart of any phenomenological treatment of string theoretic models. The well-known shortcomings of the traditional treatments of moduli stabilization were demonstrated: the need for multiple condensates and the difficulty in finding a minimum with vanishing vacuum energy. The BGW class of models remedies both problems by utilizing nonperturbative corrections to the Kähler potential of the dilaton, thereby generating an acceptable level of supersymmetry breaking with vanishing vacuum energy. As with all such methods of supersymmetry breaking, some degree of tuning between the various parameters of the theory must be employed.

The importance of dealing with the above issues cannot be overstated. Without achieving a minimum for the overall scalar potential such that the vacuum energy is negligible, no truly meaningful statements about supersymmetry breaking, the superpartner spectrum or phenomenology can be made. By negligible we will mean small on the scale of particle physics experiments, *i.e.* significantly smaller than the

[j]Recall that typical gravitino masses are already in the multi-TeV range, so enhancement factors of 5 to 10 may be sufficient.

electroweak scale. Given any particular mechanism to achieve $\langle V \rangle \simeq 0$, the degree to which we can achieve precisely vanishing vacuum energy is a function of the amount of fine-tuning we can engineer in the model. It is to be expected – and is generally the case in explicit examples – that any such mechanism for achieving vanishing *vev* for the scalar potential will have some sort of "back-reaction" on the observable sector particle physics. In other words, if one simply assumes that $\langle V \rangle \simeq 0$ by some unspecified mechanism then it is natural to wonder whether this mysterious sector truly plays no role in determining the low-energy phenomenology of the model in question. The BGW class of weakly-coupled heterotic string models has the virtue of being forthright in addressing the problem. Since the Kähler stabilization has real effects on the resulting phenomenology (mostly good ones, we hasten to add) it is perfectly reasonable as string *phenomenologists* to study this mechanism. Comparing eventual data to the resulting "complete" theory then in part tests this mechanism for stabilizing the moduli and achieving appropriate vacuum energy. We do not know, in general, how a small vacuum energy arises from any quantum theory, let alone a string-based one. Therefore other mechanisms should be sought out, but a framework which envisions no such mechanism must eventually make only empty statements about Nature.

Starting from the context of weakly-coupled heterotic string theory an effective supergravity theory was built in Section 2 to describe the dynamics of the low energy four-dimensional world. The form of this effective Lagrangian was guided by the principle of target space modular invariance. This symmetry acts as a classical symmetry of the supergravity Lagrangian. The underlying string theory informs us that it should be a good symmetry to all orders in the string perturbation expansion. Therefore we ought to ensure that it remains an intact symmetry to all orders in quantum field theory. The anomalies associated with modular invariance can be remedied by terms arising from the string theory itself: Green-Schwarz counterterms and threshold corrections to gauge coupling constants. We saw that these are easiest to implement when the dilaton is packaged in the linear multiplet. This is no surprise since the degrees of freedom of the string are precisely those of the linear multiplet and the genus-counting parameter is the real object ℓ. The drawback is that the axion sector is slightly more difficult to discuss and less familiar than the dual pseudoscalar treatment.

Modular invariance proves to be a powerful constraint. Preserving it when integrating out heavy matter at the anomalous $U(1)$ scale can imply a relation between string selection rules, modular invariance and an intact R-parity. This, in turn, has implications for stable relics (dark matter) and the issue of rapid proton decay. In conjunction with the Kähler corrections for the dilaton it led to a vacuum solution in which Kähler moduli are fixed at self-dual points where their auxiliary fields vanish. It is the dilaton, therefore, which communicates supersymmetry breaking to the fields of the observable sector. This is an example of the (generalized) dilaton domination scenario. In Section 3 we demonstrated that at tree-level scalar masses

and trilinear A-terms are universal. The field-theory loop corrections are small and the higher-order terms in the Kähler potential may preserve the resulting FCNC suppression provided certain well-motivated isometries exist from the compactification. Gaugino masses are generally an order of magnitude smaller than scalars since the same mechanism which allows for an acceptable vacuum tends to dramatically alter the Kähler metric of the dilaton (whose effective auxiliary field determines the size of gaugino masses). The model therefore predicts scalar masses to be in the multi-TeV range, with implies a further suppression of superpartner-mediated FCNC and CP-violating effects. The relatively light gluino implies reduced fine-tuning in the electroweak sector despite the heavy scalars. In Section 5 we saw that it also has profound implications for the signature of this class of models at hadron colliders. The fact that tree level gaugino masses are similar in size to certain loop-induced terms gives rise to a mixed modulus/anomaly-mediation scenario for gauginos. Achieving the right scale of superpartner masses tended to imply a very particular set of possible hidden sector configurations. These just so happen to be configurations in which the relic abundance of (stable) neutralinos is naturally in the right range to account for the non-baryonic dark matter as suggested by the WMAP experiment. Furthermore, the range of parameters singled out by these considerations also happens to coincide with the range in which the model-independent axion can be the QCD axion which solves the strong-CP problem. Finally we note that (modulo the issues of Section 4) the physical masses of the scalar moduli tend to be significantly larger than that of the gravitino, perhaps by as much as an order of magnitude. Since avoiding the direct search constraints on light gauginos will tend to imply $m_{3/2}$ of several TeV, this should significantly mitigate the cosmological problems associated with the moduli. The above results are promising, but require specific parameter choices from the string model. This is not necessarily a bad thing – it demonstrates that low-energy phenomenology has an impact on the viability of a certain string framework! This is the essence of string phenomenology.

Though this class of theories is arguably the most complete string model in the literature, there are many topics that were not addressed. Most significantly is the issue of initial conditions, alluded to in the Introduction. We have assumed the minimal field content for a supersymmetric version of the Standard Model. It is by no means clear whether this should be the goal of string model-builders. For example, we expect on general principles that no fields in the massless spectrum of the superstring will have a supersymmetric mass. Therefore we expect the Higgs μ-parameter and any potential Majorana mass for right-handed neutrinos to arise only dynamically from the *vev* of some fields from *outside* the MSSM field content. We have addressed neither issue within this review, and indeed string theory is largely silent on these important issues.[k] We also assumed that the tree level

[k]Some examples specific to the weakly-coupled heterotic string have been studied.[232,233,234,235,236,237,238,239]

Kähler metric for chiral matter can be made diagonal in the flavor basis. This is the standard supersymmetric flavor problem which all theories, whether string-based or otherwise, share. In some string constructions, such an assumption can be directly examined, but in many constructions the relation between the flavor indices of chiral matter and the underlying compact geometry is obscure. Other important issues are the question of CP-violating phases in the soft supersymmetry-breaking Lagrangian, the possible presence of charge and color-breaking minima, the mechanism for baryogenesis, and the issue of creating an explicit model for inflation.

Let us finally address the very place we began this investigation: the weakly-coupled $E_8 \otimes E_8$ heterotic string. Despite the many clear phenomenological advantages of this starting point, other string constructions have moved to the fore in recent years. In part this is due to their ability to achieve the same outcomes of the weak-coupling heterotic string: $N = 1$ supersymmetry, chiral matter, a natural hidden sector, anomaly cancelation, and (sometimes) gauge coupling unification. But primarily it is because of the issue that has so motivated this concluding section – the issue of supersymmetry breaking and vanishing vacuum energy. Very promising mechanisms now exist in other corners of the M-theory landscape for addressing these problems.[47] Of significance is the fact that a very large number of solutions with gaugino condensation and background flux are possible, each such solution giving rise to a vacuum with a slightly different value of $\langle V \rangle$ for the geometrical moduli. The numerical coefficients in the gaugino condensation part of the effective scalar potential are ultimately discrete numbers, determined by the underlying string theory, just as in the examples considered in this review. But the part of the effective scalar potential generated by the non-trivial fluxes can be treated as effectively continuous in these contexts. This allows for a very finely-grained instrument for tuning the resulting vacuum-energy – a "knob" which is explicitly lacking in the context of weakly-coupled heterotic models.[240,241,242,243,244]

In the current context we have not sought to use the nonperturbative corrections represented by the expansion in (60) to tune $\langle V \rangle$ to match the comparative small value implied by recent supernova data. Nevertheless, the results we achieve requires tuning the various coefficients in (60). Just like the coefficients in (26) that appear in the effective Lagrangian for the condensates, we should expect that these parameters can not be changed by infinitesimally small amounts. It remains an open question, therefore, as to whether the nonperturbative effects in (60) can truly generate a potential with vanishing *vev* in realistic heterotic string compactifications. At the moment it is unclear whether analogs exist in the heterotic context for these flux compactification scenarios, though treatments of a more phenomenological spirit have given rise to interesting results.[245] The nonperturbative Kähler potential corrections included in the BGW model have two virtues. They are known to exist and take a well-defined functional form. Yet it may be argued that it is inconsistent from the point of view of effective field theory to include nonperturbative contributions prior to examining the relevant *perturbative* contributions.[187,246] Consideration of perturbative corrections to the Kähler potential has already proven

fruitful in other string frameworks.[247,248,249,250,251]

This is an exciting time for string phenomenology. The rapid progress being made in understanding the dynamics of string theory in various corners of the M-theory landscape is about to be joined by data from a number of forthcoming experiments. The domain in which these two themes intersect is the effective supergravity Lagrangian describing string moduli, their interaction with matter and their stabilization. Unlike bottom-up models constructed to describe only one set of phenomena (such as the process of electroweak symmetry breaking), string-based models have the burden and the opportunity to describe much more. Classes of string models with some claim to this level of "completeness" are growing, though many challenges remain ahead of us. The question of whether the post-LHC era will be a golden age for string phenomenology will largely depend on how well we address these challenges and the deepening of the vertical integration between phenomenologists and formal theorists.

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Appendix A. An overview of Kähler $U(1)$ superspace

The action for the coupling of matter fields to supergravity is described in the conventional superspace approach[34] by the quantity

$$\mathcal{L}_{\rm kin} = -3 \int E\, e^{-\frac{1}{3}K(\Phi,\bar{\Phi})} \tag{A.1}$$

where E denotes the super-determinant of the super-vielbein E_M^A and the measure is understood to be $\mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}^2$. The component expression of (A.1) contains the kinetic terms for the supergravity multiplet as well as the matter multiplets Φ. However, this component expression yields the correct normalization for the Einstein term of the gravity action only after a field-dependent re-scaling of the component fields.[252] One must assign the transformation property under Kähler transformations to these re-scaled fields (not the superfields that appear in the superspace Lagrangian).

Through a combination of classical symmetries of the supergravity Lagrangian – specifically Kähler transformations and super-Weyl (or Howe-Tucker) re-scalings

– it is possible to absorb the exponential factor into the super-determinant

$$\mathcal{L}_{\rm kin} = -3 \int E' \,. \tag{A.2}$$

In other words, in this new frame the kinetic Lagrangian is nothing more than -3 times the volume of superspace. The Kähler $U(1)$ formalism incorporates these transformations into the structure group of superspace,[33] thereby dispensing with the need for conformal compensators and/or Weyl re-scalings of the component field Lagrangian. Kähler transformations are now understood at the *superfield* level and the Einstein term automatically has the canonical normalization.

The structure group of Kähler U(1) geometry contains the usual Lorentz transformations as well as an additional chiral $U(1)$. This additional chiral $U(1)$ acts on a superfield Φ of chiral weight w as $\Phi \to \Phi \exp\left(-\frac{i}{2}\, w {\rm Im}\, F\right)$, where F is a superfield. To incorporate this transformation into the structure of superspace one constructs a connection A_M with which one forms a covariant derivative in superspace with respect to this additional symmetry operation. The superfield A_M associated with this transformation has the following components

$$A_\alpha = \frac{1}{4}\mathcal{D}_\alpha K, \qquad A^{\dot\alpha} = -\frac{1}{4}\mathcal{D}^{\dot\alpha} K \tag{A.3}$$

$$A_\mu = \frac{1}{4}\left(K_i \partial_\mu \varphi^i - K_{\bar\jmath}\partial_\mu \overline{\varphi}^{\bar\jmath}\right) + \frac{i}{8} K_{i\bar\jmath}\overline{\sigma}_\mu^{\dot\alpha\alpha}\chi^i_\alpha \overline{\chi}^{\bar\jmath}_{\dot\alpha}\,, \tag{A.4}$$

where $\varphi = \Phi|_{\theta=\bar\theta=0}$ and $\chi_\alpha = \mathcal{D}_\alpha \Phi|_{\theta=\bar\theta=0}$. We see that the gauge field of a Kähler $U(1)$ transformation is a composite object made up of the various chiral fields in the theory. The effect of a Kähler U(1) transformation is simply to shift the vector part of the connection as

$$A_\mu \to A_\mu - \partial_\mu\left(-\frac{i}{2}{\rm Im}\, F\right) \tag{A.5}$$

in analogy to Abelian gauge theory.

One can now use this symmetry to remove the superpotential and Yang-Mills kinetic terms from the F-density part of the Lagrangian (those terms that involve integration over only half of superspace) and recast the entire superspace Lagrangian in the form of D-densities (integration over all of superspace). The Kähler potential only appears implicitly, through the connections in the covariant derivatives used to obtain the component-field expression. The Kähler $U(1)$-invariant kinetic terms for the matter fields are now interpreted as the "FI term" for the Kähler potential. The single expression (A.2) contains the kinetic terms for the entire supergravity/matter system. In the chiral formulation of the dilaton (or in the absence of a dilaton) the kinetic terms for the Yang-Mills sector must be introduced through an F-density expression as in (11). In the formalism of the modified linear multiplet, however, these terms are also incorporated in a single expression of the form (A.2), though the expression (A.2) is generalized to the form

$$\mathcal{L}_{\rm kin} = -3 \int E\, F(\Phi, \overline{\Phi}, L)\,, \tag{A.6}$$

as was done in (44) of Section 1.3.

To obtain the component expression from any superfield Lagrangian density in Kähler $U(1)$ superspace, one may employ the chiral density method. This is the locally supersymmetric generation of the F-term construction in global supersymmetry. Consider a superspace expression of the form $\mathcal{L} = \int E\Omega$, where Ω is a real superfield that has weight $w_\Omega = 0$. Using integration by parts in $U(1)$ superspace we can rewrite this expression as

$$\mathcal{L} = \int \frac{E}{2R}\mathbf{r}; \quad r = -\frac{1}{8}(\overline{\mathcal{D}}^2 - 8R)\Omega\,. \tag{A.7}$$

The component Lagrangian can now be expressed in terms of this quantity $\mathbf{r}$ via the following rule

$$\frac{1}{e}\mathcal{L}_{eff} = -\frac{1}{4}\overline{\mathcal{D}}^2\mathbf{r}|_{\theta=\bar\theta=0} + \frac{i}{2}(\bar\psi_\mu\bar\sigma^\mu)^\alpha\mathcal{D}_\alpha\mathbf{r} - (\bar\psi_\mu\bar\sigma^{\mu\nu}\bar\psi_\nu + \overline{M})\mathbf{r}|_{\theta=\bar\theta=0} \;+\; \text{h.c.}\,, \tag{A.8}$$

where $\overline{M} = -6R^\dagger|_{\theta=\bar\theta=0}$ is the supergravity auxiliary field whose *vev* determines the gravitino mass via $9m_{3/2}^2 = \langle |M|^2\rangle$. More details on the procedure can be found in Reference 73 where the complete component Lagrangian for the most general model of the BGW class is given.

As an example, let us consider an extension of the simple model from Section 1 with Lagrangian density

$$\mathcal{L}_{\text{eff}} = \int \mathrm{d}^4\theta\, E\,\{\,-2 \,+\, f(L) \,+\, bL\sum_I g^I \,+\, bL\ln(e^{-K}\overline{U}U/\mu^6)\,\}\,. \tag{A.9}$$

That is, we consider the theory defined by (66) but extended to include Kähler moduli and universal anomaly cancelation. The Green-Schwarz term of (116) is taken to have vanishing p_i so that $V_{\text{GS}} = \sum_I g^I \equiv G$, and $b_{\text{GS}} = b$ for the single condensing gauge group. To apply (A.8) we must work with the quantities

$$\begin{aligned}\mathbf{r} \;&=\; -\frac{1}{8}(\overline{\mathcal{D}}^2 - 8R)\{\,(\,-2 \,+\, f(L)\,) \,+\, bLG \,+\, bL\ln(e^{-K}\bar{U}U/\mu^6)\,\},\\ \bar{\mathbf{r}} \;&=\; -\frac{1}{8}(\mathcal{D}^2 - 8R^\dagger)\{\,(\,-2 \,+\, f(L)\,) \,+\, bLG \,+\, bL\ln(e^{-K}\bar{U}U/\mu^6)\,\}.\end{aligned} \tag{A.10}$$

If we are interested in the purely bosonic part of the component Lagrangian then we must compute the first and last terms of (A.8). Computing $\overline{M}\mathbf{r}|_{\theta=\bar\theta=0} +$ h.c. is straight-forward, but the first term in (A.8) requires more care. In particular we will need to concern ourselves with quantities such as $(\mathcal{D}^2R + \overline{\mathcal{D}}^2R^\dagger)$ and $(\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot\alpha}X^{\dot\alpha})$, where

$$X_\alpha = -\frac{1}{8}(\overline{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha K\,; \quad X^{\dot\alpha} = -\frac{1}{8}(\mathcal{D}^2 - 8R^\dagger)\mathcal{D}^{\dot\alpha}K\,. \tag{A.11}$$

In our simple example these quantities are related in the following manner

$$\left(L\frac{\mathrm{d}g(L)}{\mathrm{d}L} + 1\right)(\mathcal{D}^2R + \overline{\mathcal{D}}^2R^\dagger) + (\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot\alpha}X^{\dot\alpha}) = \Delta \tag{A.12}$$

where the bosonic components of Δ are

$$\begin{aligned}\Delta|_{\theta=\bar\theta=0} = & -\frac{1}{\ell^2}(\ell^2 g''-1)\partial^\mu\ell\,\partial_\mu\ell + \frac{1}{\ell^2}(\ell^2 g''-1)v^\mu v_\mu + 2\nabla^m\nabla_m k \\ & + 4\sum_I \frac{1}{(t^I+\bar t^I)^2}\partial^\mu\bar t^I\,\partial_\mu t^I - \frac{4}{9}(\ell^2 g''-\ell g'-2)\overline{M}M \\ & + \frac{4}{9}(\ell^2 g''+2\ell g'+1)b^a b_a - 4\sum_I \frac{1}{(t^I+\bar t^I)^2}\overline{F}^I F^I \\ & - \frac{4}{3\ell}(\ell^2 g''+\ell g')v^\mu e_\mu{}^a b_a - \frac{1}{2\ell}(\ell g'+1)(F_U+\overline{F}_{\bar U}) \\ & - \frac{1}{6\ell}(2\ell^2 g''-\ell g'-3)(\,u\overline{M} + \bar u M\,) - \frac{1}{4\ell^2}(\ell^2 g''-1)\bar u u\,. \end{aligned} \tag{A.13}$$

In the above we have defined

$$g' = g'(\ell) = \left.\frac{\mathrm{d}g(L)}{\mathrm{d}L}\right|_{\theta=\bar\theta=0}; \quad g'' = g''(\ell) = \left.\frac{\mathrm{d}^2 g(L)}{\mathrm{d}L^2}\right|_{\theta=\bar\theta=0}, \tag{A.14}$$

the field v_μ is the vector component of the linear multiplet (38), and $b_a = -3G_a|_{\theta=\bar\theta=0}$ is an auxiliary field of the supergravity multiplet which is constrained to vanish in the vacuum. The complete bosonic component Lagrangian for (A.9) is given by[72]

$$\begin{aligned}\frac{1}{e}\mathcal{L}_B = & -\frac{1}{2}\mathcal{R} - \frac{1}{4\ell^2}(\ell g'+1)\partial^\mu\ell\,\partial_\mu\ell - (1+b\ell)\sum_I \frac{1}{(t^I+\bar t^I)^2}\partial^\mu\bar t^I\,\partial_\mu t^I \\ & + \frac{1}{4\ell^2}(\ell g'+1)v^\mu v_\mu + \frac{1}{9}(\ell g'-2)\left[\overline{M}M - b^a b_a\right] + (1+b\ell)\sum_I \frac{1}{(t^I+\bar t^I)^2}\overline{F}^I F^I \\ & + \frac{1}{8\ell}\{\,f + 1 + b\ell\ln(e^{-k}\bar u u/\mu^6) + 2b\ell\,\}(F_U+\overline{F}_{\bar U}) \\ & - \frac{1}{8\ell}\{\,f + 1 + b\ell\ln(e^{-k}\bar u u/\mu^6) + \frac{2}{3}b\ell(\ell g'+1)\,\}(\,u\overline{M} + \bar u M\,) \\ & - \frac{1}{16\ell^2}(1+2b\ell)(\ell g'+1)\bar u u \\ & - \frac{i}{2}b\ln(\frac{\bar u}{u})\partial^\mu v_\mu - \frac{i}{2}b\sum_I \frac{1}{(t^I+\bar t^I)}(\,\partial^\mu\bar t^I - \partial^\mu t^I\,)v_\mu. \end{aligned} \tag{A.15}$$

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