

Over Voltage in a Multi-Sectioned Solenoid during Quenching

X. L. Guo, F. Y. Xu, L. Wang, M. A. Green, H. Pan, H. Wu, X. K. Liu, and A. B. Chen

Abstract—Accurate analysis of over voltage in the superconducting solenoid during a quench is one of the bases for quench protection system design. Classical quench simulation methods can only give rough estimation of the over voltage within a magnet coil. In this paper, for multi-sectioned superconducting solenoid, based on the classical assumption of ellipsoidal normal zone, three-dimension al temperature results are mapped to the one-dimension of the wire, the temperature distribution along the wire and the resistances of each turn are obtained. The coil is treated as circuit comprised of turn resistances, turn self and mutual inductances. The turn resistive voltage, turn inductive voltage, and turn resultant voltage along the wire are calculated. As a result, maximum internal voltages, the layer-to-layer voltages and the turn-to-turn voltages are better estimated. Utilizing this method, the over voltage of a small solenoid and a large solenoid during quenching have been studied. The result shows that this method can well improve the over voltage estimate, especially when the coil is larger.

Index Terms—Superconducting magnets, Over voltage, Quench Simulation, Quench Protection

I. INTRODUCTION

QUENCHING of a superconducting magnet can induce overheating, over-voltages, and in extreme cases destruction the magnet, so it is necessary to simulate the quench characteristics of superconducting magnet systems. Overheating can be better estimated by using the time varying currents and the resistance of the normal zone within the coil. Estimating over voltage requires an understanding of the voltage distribution along the wire [1], [2].

There are three types of quench simulation methods. The first method calculates the normal zone shape in the coil based on propagation velocities in three dimensions. Using this method, some codes calculate the terminal voltage [3], [4]. Other codes calculate the voltage drop on only the resistance of the normal zone [5], [6], and yet other codes calculate the voltage distribution in the coil assuming the inductance of every layer is the same [2].

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X. L. Guo, F. Y. Xu, L. Wang, H. Pan, H. Wu, X. K. Liu and A. B. Chen are with the Institute of Cryogenics and Superconductive Technology, Harbin Institute of Technology, Harbin 150001, China (e-mail: guoxinglong@hit.edu.cn). M. A. Green is with Lawrence Berkeley National Laboratory, Berkeley, CA 94720 USA (e-mail: magreen@lbl.gov).

The second method divides the coil into segments and deciding the state of each segment based on the propagation velocity [7-9]. This method can solve the voltage distribution along the wire if the segments are along the wire, but the temperature solving by this method needs a larger computing effort than that by the first method [8].

The third method is based on treating the coil as an anisotropic solid and solving the nonlinear heat differential equations governing the quench process [10-15]. This method can solve the voltage distribution in the coil, but it needs a larger computing effort than even second method [10], [12].

To the authors’ knowledge, the first one method is more diffused than the other two methods, because it is very fast and gives globally correct results. However, there is little attention to the voltage distribution and detailed over voltage analysis by the first method.

This paper is mainly devoted to the over voltage analysis within the solenoid using the first method of analysis. Based on the classic assumption of an ellipsoid normal zone, three-dimension temperature results are mapped to one-dimension along the wire. The temperature distribution and the resistance distribution along the wire are obtained. The coil is treated as turn elements connected in series, and each turn element is a combination of resistance and inductances. The resistive voltage distribution and inductance voltage distribution and the resultant voltage of these two opposite voltages distribution are obtained. The maximum internal voltage, the layer-to-layer voltage and the turn-to-turn voltage during quenching are estimated. The method in this paper can make the over voltage calculation by the first method more accurate.

I. MODELING OF QUENCH PROPAGATION

Based on the classical quench model, the normal zone in the space can be approximated as an ellipsoid. This ellipsoid spreads in three directions with different velocities until the entire magnet becomes normal. After each time step a layer is added to the normal zone – like the skins of an onion. Longitudinal propagation velocity v_l and transverse propagation velocity v_t can be calculated by the following equations [5]:

$$v_l = \frac{J}{(\gamma C)_{avm}} \sqrt{\frac{LT_s}{T_s - T_0}} \quad (1a)$$

$$v_t = \frac{(\gamma C)_{avm}}{(\gamma C)_{av}} \sqrt{\frac{k_t}{k_l}} \cdot v_l = \alpha \cdot v_l \quad (1b)$$

where J is the average current density over the unit cell, $(\gamma C)_{avm}$ and $(\gamma C)_{av}$ are the average volumetric specific heat over the metallic part of the unit cell and over the whole unit cell, L is the Lorenz number ($L=2.45 \times 10^{-8} \text{ W}\Omega\text{K}^{-2}$), T_s and T_o are the transition and operating temperature respectively. k_t and k_l are the transverse and longitudinal thermal conductivities respectively.

Each incremental layer is assumed to be locally adiabatic and isothermal. Its temperature is determined by the joule heating within its volume. The temperature T of each isothermal layer can be calculated using the following:

$$(\gamma C)_{av} \frac{dT}{dt} = J^2 \rho \quad (2)$$

where t is time. ρ is the average resistivity of the coil.

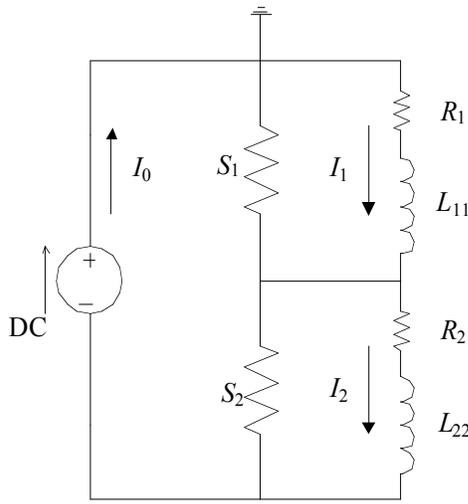


Fig. 1. The Electrical Circuit for a Two-sectioned Coil.

Fig. 1 shows the electric circuit for a two-sectioned coil. Each section is shunted and the sections are connected in series to the current supply. The electric circuit can be evaluated by the equations:

$$V_0 = \sum_{i=1}^2 (I_0 - I_j) S_j \quad (3a)$$

$$[L] \frac{dI_j}{dt} + I_j [R_j(t) + S_j] = I_0 [S_j] \quad (3b)$$

where V_0 and I_0 are the power supply voltage and current, respectively. $[L]$ is the inductance matrix of the coil sections. S_j is the shunt resistance across the section j . I_j and $R_j(t)$ are the current and resistances of the normal zone in the section j . $R_j(t)$ is not explicitly known but depend on the time evolution of the normal zone within the coil. $R_j(t)$ can be calculated by the sum of the resistance of each isothermal layer in the coil section as the following equation:

$$R_j(t) = \sum_{k=1}^n \rho_k \frac{Vol_k}{A^2} \quad (4)$$

where n is the total normal isothermal layer number in the section j . k is the isothermal layer number. A is the cross-section area per unit cell. ρ_k and Vol_k are the average resistivity and volume of the layer k .

Combining equations (2), (3), and (4), the quench characteristics in the multi-sectioned superconducting solenoid can be predicted, and the overheating can be globally estimated correctly. However, we can't get enough information to estimate the over voltage in the coil during quenching.

I. OVER VOLTAGE CALCULATION METHOD

A. Equivalent Circuit in the Coil

The coil can be treated as turn elements connected in series, and each turn element is combination of resistance and inductance. During quenching, the inductance of each turn does not change, but the resistance of each turn changes depending on the turn temperature. According the position each turn in the coil, the average temperature of each turn can be obtained by mapping the three-dimension temperature results to one-dimension along the wire.

Each turn resistance $R_i(t)$ can be calculated by the following equation;

$$R_i(t) = \rho_i \frac{l_i}{A} \quad (5)$$

where ρ_i is the turn i temperature-depending average resistivity, and l_i is the turn i length.

The inductance of each turn can be decided by the following equations [16]:

$$L_{ii} = \mu_0 R_i \left(\ln \frac{8R_i}{r_i} - \frac{7}{4} \right) \quad (6a)$$

$$L_{ij} = \mu_0 \sqrt{R_i R_j} \left[\left(\frac{2}{k} - k \right) K(k) - \frac{2}{k} E(k) \right] \quad (6b)$$

where L_{ii} is the turn i self inductance, L_{ij} is the mutual inductance between turn i and turn j . μ_0 is the magnetic permeability of free space ($\mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$). R_i and R_j are the radius of the turn loop i and j . r_i is the circular radius of the circular wires and for the rectangular wire with cross-section of height h and width w , $r = (wh/\pi)^{0.5}$. E and K are elliptic integrals, k is a geometry factor defined by R_i , R_j and the distance z between turns i and j . They are defined as;

$$K(k) = \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}} \quad (7a)$$

$$E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \phi} d\phi \quad (7b)$$

$$k = \sqrt{\frac{4R_i R_j}{(R_i + R_j)^2 + z^2}} \quad (8)$$

Through (6), the mutual inductance $L_{i,j}$ between turn i and coil section j can be evaluated by the following equation:

$$L_{i,j} = \sum_{k=1}^{N_j} L_{ik} \quad (9)$$

where N_j is the total turn number of the section j . k is the turn number, L_{ik} is the mutual inductance between turn i and k , and if i equal k then L_{ik} is the self inductance of turn i .

B. Over voltage during Quenching

Taking one of the two conductor terminals where current follows out the coil as potential reference point as shown in Fig. 1, the voltage distribution along the wire can be calculated by the following equations:

$$V_0 = 0 \quad (10a)$$

$$V_{i+1} = V_i + \sum_{j=1}^{N_{sec}} L_{i,j} \frac{dI_j}{dt} + R_i I_i \quad (10b)$$

where V_i and V_{i+1} are the voltages at turn i and $i+1$ end points respectively. N_{sec} is the total section number of the coil. I_j and I_i are the current in the section j and turn i respectively. The second term and the third term in the right hand of equation (10b) correspond to the opposite inductive voltage and resistive voltage on turn i .

The maximum internal voltage V_{max} can be decided by the maximal voltage difference between any two turns in the coil. At any moment,

$$V_{max} = \left| \max \{V_i | i = 1, 2, \dots, N\} - \min \{V_i | i = 1, 2, \dots, N\} \right| \quad (11)$$

where N is the total turn number of the coil. The layer-to-layer voltage $V_{l2l, i}$ between layer i and layer $i+1$ can be decided by the maximal voltage difference between these two layers, and the peak layer-to-layer voltage V_{l2l} in the coil is the maximal value of all the layer-to-layer voltage. They can be calculated by the following equations

$$V_{l2l, i} = \max \left\{ \begin{array}{l} \max \{V_{N_{turn}(i-1)+k} | k = 1, 2, \dots, N_{turn}\} \\ -\min \{V_{N_{turn}+k} | k = 1, 2, \dots, N_{turn}\} \\ \min \{V_{N_{turn}(i-1)+k} | k = 1, 2, \dots, N_{turn}\} \\ -\max \{V_{N_{turn}+k} | k = 1, 2, \dots, N_{turn}\} \end{array} \right\} \quad (12a)$$

$$V_{l2l} = \max \{V_{l2l, i} | i = 1, 2, N_{lay} - 1\} \quad (12b)$$

where N_{turn} is the total turn number of each layer, N_{lay} is the total layer number in the coil. Turn-to-turn voltage V_{t2t} can be calculated by the following equations:

$$V_{t2t} = \max \{V_i - V_{i+1} | i = 1, 2, N - 1\}. \quad (13)$$

Through equations (11), (12), and (13), we can make a better estimation of the maximum internal voltage V_{max} , layer-to-layer voltage V_{l2l} and turn-to-turn voltage V_{t2t} during quenching.

I. RESULTS AND DISCUSSION

A. Over-voltage in a Small Experiment Solenoid

The method is applied to a two-sectioned solenoid in [17]. The parameters of the coil are listed in Table I. Each section is connected to an external protection resistor S of 0.5Ω . The initial current of the magnet system is 100A. A quench is initiated in the highest field region of Section 1. In the simulation, the power supply is a constant current source until its terminal voltage reaches a specified limit value (10V). After that, it is considered to be a constant voltage source. Quench propagation velocities are important parameters in such a model. The average propagation velocities based on equation (1) are used in the simulation. The average field is about 2.4 T at 100A, the estimated longitudinal propagation velocity $v_l = 5.2$ m s⁻¹, the ratio of transverse velocity to longitudinal velocity is 0.017.

TABLE I
PARAMETERS FOR A TWO-SECTIONED SOLENOID

	Sec. 1	Sec. 2
Inner radius (mm)	38	56
Outer radius (mm)	56	67
Length (mm)	72	72
Wire diameter (mm)	0.86	0.66
Cu to Nb-Ti Ratio	3.2	2.2
Number of Turns	1696	1798
Shunt ($_ at 4.2K$)	0.5	0.5

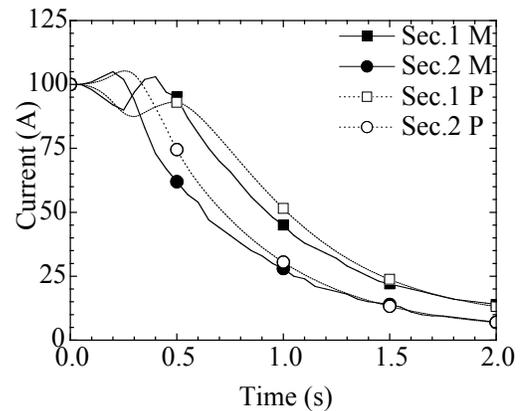


Fig. 2. The predicted and measured current decay as a function of time during a quench. (P is the predicted current; M is the measured current.)

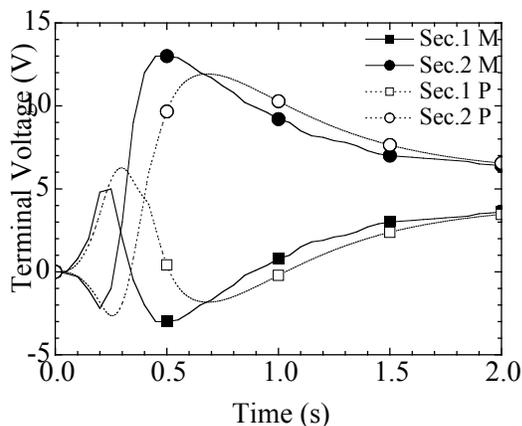


Fig. 3. The predicted and measured terminal voltage as a function of time during a quench. (M is the measured voltage; P is the predicted voltage.)

Fig.2 and Fig.3 show the current and voltage measurement results and the simulation results. At about 0.7s from the simulation results and 0.5s from the measurement results the power supply change from a constant current supply to a constant voltage supply. The measurement and simulation results show agreement. This mode is simple using only average constant propagation velocities, so the agreement is not as good as that showed in [17] using a more complex model. However, for detail over-voltage analysis in the coil this model is enough. From the experiment we only obtain the terminal results of each section, the voltage in the coil can be obtained by simulation.

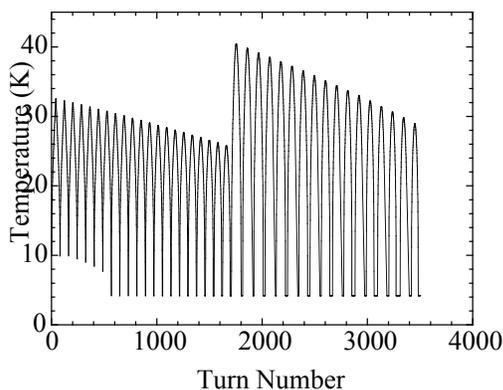


Fig. 4. Temperature distribution along the conductor within the small solenoid at $t=0.4$ s.

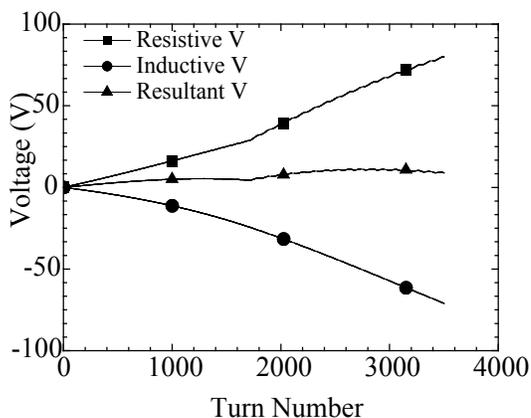


Fig. 5. The distribution of different voltages along the conductor in the small solenoid at $t=0.4$ s.

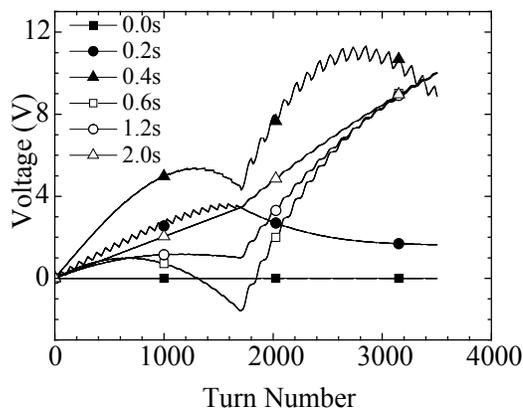


Fig. 6. The transient voltage distribution along the conductor in the small solenoid as a function of time.

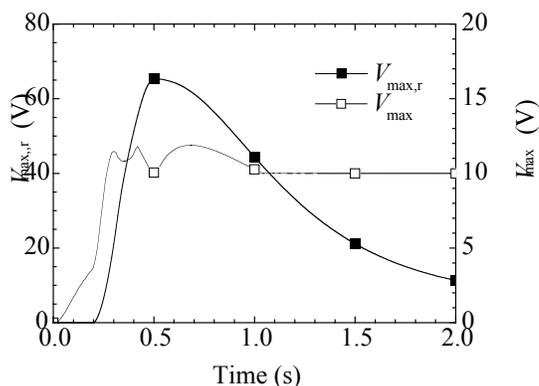


Fig. 7. The maximum internal voltage in the small solenoid calculated using a different method of calculation

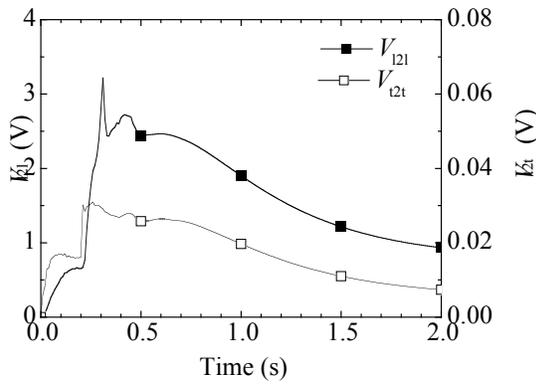


Fig.8. Layer-to-layer voltage and turn-to-turn voltage in the small solenoid

Fig. 4 shows the temperature distributions in the coil along the conductor in the small solenoid at $t = 0.4$ s. At this time, the two sections have both quenched, but not the entire coil has become normal. Each saw tooth has a sharp peak corresponding to the element in the mid-plane, and there are a total of 38 peaks, one for each layer in this coil. The element with the highest temperature in the layer was the first to go normal in that layer.

Fig. 5 shows the resistive voltage, inductance voltage and resultant voltage of these two opposite voltages distribution in the small solenoid along the conductor at $t = 0.4$ s. The resistive and inductance voltage are both as high as 70V, but the maximum of the resultant voltage is only about 11 V. We can obtain from Fig. 5 that if the resistive voltage is used to estimate the maximum internal voltage in the coil, this will result in large over-estimate of maximum internal voltage.

Fig. 6 shows the transient voltage distribution along the conductor in the small solenoid. The terminal voltage obtained from Fig. 6 corresponds to the simulation voltage result showed in Fig. 3. After about 1.2 s the peak voltage in the coil is decided by its terminal voltage of 10V. There is a bulge in each section voltage distribution result, because of the normal zone resistive voltage in each section. Like the temperature results, there is a saw tooth peak voltage in each layer. We can find from Fig. 6 the peak voltage is not only decided by the terminal voltage, it usually is decided by the voltage differences within the coil.

Fig.7 shows the maximum internal voltage in the small solenoid by using two different calculation methods. Using the voltage on the normal zone resistance in each section, the peak maximum internal voltage, $V_{\max,r}$ is about 65V. Using the maximum voltage difference between any two turns, the maximum internal voltage V_{\max} is only about 12V. V_{\max} is about 18% of $V_{\max,r}$.

Fig. 8 shows the layer-to-layer voltage V_{12l} and turn-to-turn voltage V_{12t} in the small solenoid. The peak layer-to-layer voltage is about 3.2V, and peak turn-to-turn voltage is about 0.03V. The V_{12l} and V_{12t} are important information for magnet design. However, classical simulation method can not give good estimation of these voltages.

B. Over-voltage in a Large Solenoid

The method is applied to a large solenoid called coupling solenoid used in the MICE project [18]. Table II shows the parameters of the large solenoid. The large solenoid is protected by eight-section with a pair of back-to-back R620 cold diodes and a resistor about 0.02Ω across each section. Each section has 12 layers. The mandrel acts as a shorted secondary circuit inductively coupled with each of the coil section and absorbs energy from the coil during quench. Also the mandrel will be heat up and speed up the quench process through quench back. Multi-section combined with quench back composes the large solenoid passive quench protection system. The initial current of the magnet system is 210A. The back-to-back diodes are modeled as a resistor of $0.01 \text{ m}\Omega$. The quench start point is the same as used in the small solenoid simulation. The power supply terminal voltage is ignored during quench. The average field is about 2.5 T at 210A, the estimated longitudinal propagation velocity v_l is 4.7 m s^{-1} , the ratio of transverse and transverse velocities to longitudinal velocity is 0.017 and 0.015 respectively.

TABLE II
PARAMETERS FOR THE LARGE SOLENOID

Parameter	Value
Inner radius (mm)	750
Outer radius (mm)	860
Length (mm)	285
Wire dimension (mm^2)	$1.00^2; 1.65$
Cu to Nb-Ti Ratio	4:1
Number of Turns	166
Number of Layers	96

Fig. 9 shows the transient voltage distribution along the conductor in the large coil. Similar to the small solenoid result, there is a voltage bulge in each section because of the normal zone in each section. The back-to-back diodes and small shunt resistor across each section limit the voltage drop of each section terminals to almost zero. The large over-voltage only appears within the coil.

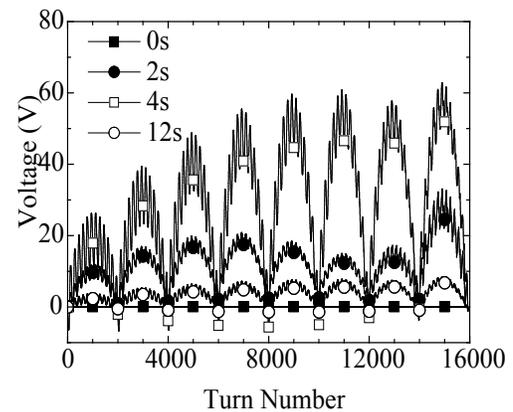


Fig. 9. Transient voltage distribution along the conductor in the large solenoid

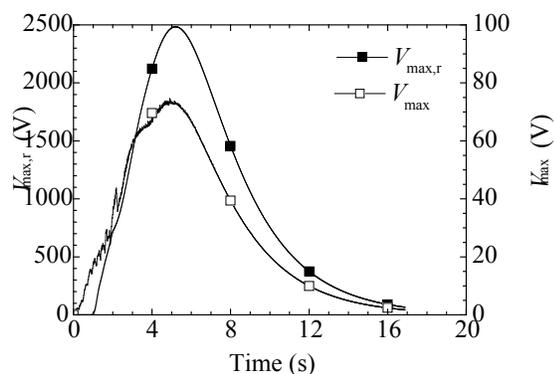


Fig.10. The maximum internal voltage within the large solenoid calculated using different calculation methods

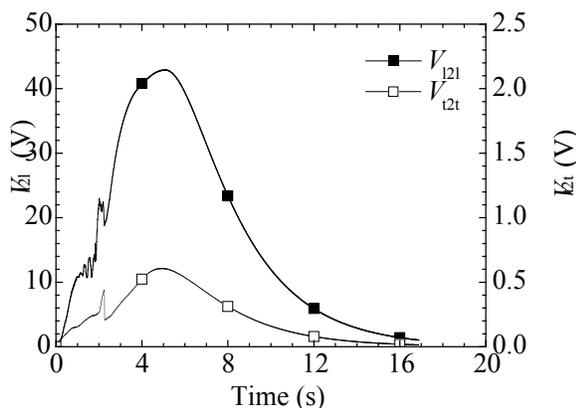


Fig.11. Layer-to-layer and turn-to-turn voltages in the large solenoid

Fig.10 shows the maximum internal voltage in the coil calculated using different calculation method. Using the voltage on the normal zone resistance in each section, the peak maximum internal voltage $V_{\max,r}$ is about 2500V. Using the maximum voltage difference between any two turns, the peak maximum internal voltage V_{\max} is only about 75V. V_{\max} is about 3% of $V_{\max,r}$. Compared with the simulation results of the small solenoid as shown in Fig. 7, it is found that the difference of maximum internal voltage, voltage calculated using different methods increases with the solenoid size. This is mainly due to the increased coil resistance, as it turns normal.

Fig. 11 shows the layer-to-layer voltage V_{12l} and turn-to-turn voltage V_{12t} in the large solenoid. The peak layer-to-layer voltage is about 43V, and peak turn-to-turn voltage is about 0.6V.

II. CONCLUSION

We have present an exactly over-voltage analysis method in the multi-sectioned solenoid based on the classical quench simulation method. By this method the maximum internal voltage, layer-to-layer voltage and turn-to-turn voltage in the

solenoid can be better estimated. The calculation results show that when the maximum internal voltage is estimated by the resistive voltage drop on the normal zone, the maximum internal voltage is often over estimated. This amount of the over estimate increases with the solenoid size. The calculation method in this paper can aid in the design of a superconducting solenoid, especially when a multi-section quench protection system design is employed. It is hoped that there is a better understanding of the voltage distribution within the multi-sectioned solenoid that is quenching.

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