

## Challenges in Determining $b$ Value in the Northwest Geysers

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### ABSTRACT

Past analyses of the Gutenberg-Richter  $b$ -value in the Geysers and other geothermal settings have revealed a deviation from the assumed linear relationship in log space between magnitude and the number of earthquakes. In this study of the Northwest Geysers, we found a gently-sloping discontinuity in the  $b$ -value curve. This is especially apparent when comparing the least-squares fit (LSQ) of the curve to the fit obtained by the maximum likelihood estimation (MLE), a widely-respected method of analyzing magnitude-frequency relationships. This study will describe the assumptions made when using each of these two methods and will also explore how they can be used in conjunction to investigate the characteristics of the observed  $b$ -value curve. To understand whether slope-fit differences in the LSQR and MLE methods is due to physical properties of the system or due to artifacts from errors in sampling, it is extremely important to consider the catalog completeness, magnitude bin size, number of events, and differences in source mechanisms for the events comprising the study volume. This work will hopefully lead to informative interpretations of frequency-magnitude curves for the Northwest Geysers, a geothermal area of ongoing high-volume coldwater injection and steam production. Through this statistical investigation of the catalog contents, we hope to better understand the dominant source mechanisms and the role of injected fluids in the creation of seismic clustering around nearly 60 wells of varying depths and injection volumes.

### INTRODUCTION

The self-similarity of earthquake processes has been used to characterize and investigate the changes in stress fields and other important aspects of the seismic cycle. One simple way to analyze this scaling is using the power-law earthquake magnitude-frequency distributions, known as the Gutenberg-Richter relationship. The exponent of this scaling is referred to as the  $b$  value and it gives a metric on the relative frequency of lower and higher magnitude earthquakes. But it has been noted that geothermal and volcanic areas often differ from this relationship. It has been found that seismicity at the Geysers is hard to fit. This study will investigate the factors that might contribute to this abnormal distribution.

The Geysers is a unique geothermal setting, with both tectonic and induced seismicity, offering an interesting and challenging environment for study. The fracture properties are changing rapidly over time and within very small areas due to active production and injection in many wells throughout the field. This induced seismicity, mostly micro-earthquakes (MEQs) of low magnitude, happen at different depths with different time lags due to various source mechanisms. Majer and Peterson [2008] report three main types of induced seismicity in the Geysers: 1) shallow, production-based with a long time lag on the order of a year; 2) deep, injection-based with a short lag time less than 2 months; and 3) deep, production-based with a short lag time, which diminished in the late 1980s.

Analytical and numerical modeling of dominant fracture processes has been used to test hypotheses of what causes these types of induced seismicity. The major hypothesized mechanisms are pore-pressure effects, thermoelastic strain, direct volume changes, and chemical changes. An increase in pore-pressure can decrease static friction and cause slip on faults with stress fields close to fracture. Fluid with high enough pore-pressures can also overcome rock strength and create new fractures. Thermal contraction of hot rocks from contact with cool fluids can also cause slight openings that decrease friction or sudden quenching from extreme temperature changes can create fractures and associated seismicity. Direct volume changes from injecting or removing fluids can also cause changes in seismicity by effecting the local stress conditions. Lastly injection of foreign fluids can cause chemical

alterations of fracture surfaces and change their coefficient of friction (Majer et al. 2007). All of these processes interact with each other and occur in conjunction with the active tectonics of the area, but their various time scales and the location of seismicity with respect to injection, as well as land deformation measurements (Mossop and Segall 1999), can help determine which are most important in each situation.

The location of MEQs are a clear indicator of where fluids are causing fractures but there is also a lot of useful information in the magnitude trends of the seismicity. For example, Henderson et al. [1999] suggests that small changes in pore-pressure can lead to seismicity that locally stops further activity through a mechanism of dilatant hardening - slip of initial activity creates more pore-volume for fluid to enter, so if the rate of fluid injection is slow relative to this increase of void space the pore-pressure will decrease contributing to a higher effective strength (Scholz 2002). This process would be suggested by a lower b value, where small-magnitude earthquakes are uncommon due to hardening from a few larger earthquakes, when injection rate changes are slow. Whereas when the rate of injection increases quickly the pore-pressure will diffuse fast enough to over-come any increase in pore space and continue to trigger small events clustered around the point of injection, contributing to a lower b value with many small earthquakes. It has proven difficult to find strong trends in b value over time, so Henderson et al. [1999] compared the correlation between b value changes and the spatial clustering of earthquakes, measured with the fractal correlation dimension (D), in times of active injection versus relatively stable times. They found that before and after injection the b value had a positive correlation with fractal dimension, where there are many small earthquakes (high b) the epicenters are spread out (high D), while during injection the b value and fractal dimension had a negative correlation, showing areas of clustered (low D) low magnitude earthquakes (high b). This analysis supports the pore-pressure mechanism described, but b value independently showed little change over time and was difficult to interpret.

In this study we examine the methods of measuring the b value, including the assumptions inherent in these calculations. Each method has its own biases and tends to fit a different part of the frequency-magnitude curve. By using both methods for the data set, its possible to investigate the fit of the data to a single line. This quick and simple program has been found to expose the bend in the curve found at the Geysers. Included is a discussion of possible reasons for this bend and whether it is an aspect of the structure of the seismic cycle or is due to an error in determining the data sampled.

### **Gutenberg-Richter Relationship**

The Gutenberg-Richter relationship is expressed in the equation:

$$\text{Log}(N) = bM - a$$

where M is the magnitude of an earthquake, N is the number of earthquakes of magnitude greater than or equal to magnitude M, b (or the b value) is a constant describing the slope of the distribution or the relative frequency of large and small earthquakes, and a is another constant that gives the y-intercept or the expected number of magnitude zero earthquakes if there were no recording limitations.

When the frequency-magnitude distribution of a seismically active area is graphed in Log space, we find the linear relationship described above only within a certain magnitude range. At low enough magnitudes the stations can't accurately measure all the earthquakes that occur and high magnitudes earthquakes are more rare so that the sampling time is not large enough to capture all of them. The magnitudes at which the linear relationship no longer holds are considered the threshold magnitudes (Mmin and Mmax) and the magnitudes that follow this relationship make up the catalog completeness.

The simple maximum likelihood estimator (Utsu 1965, Aki 1965) gives the slope based only on the average magnitude within the completeness (Mav) and the minimum magnitude (Mmin):

$$b = \log e / (M_{av} - M_{min})$$

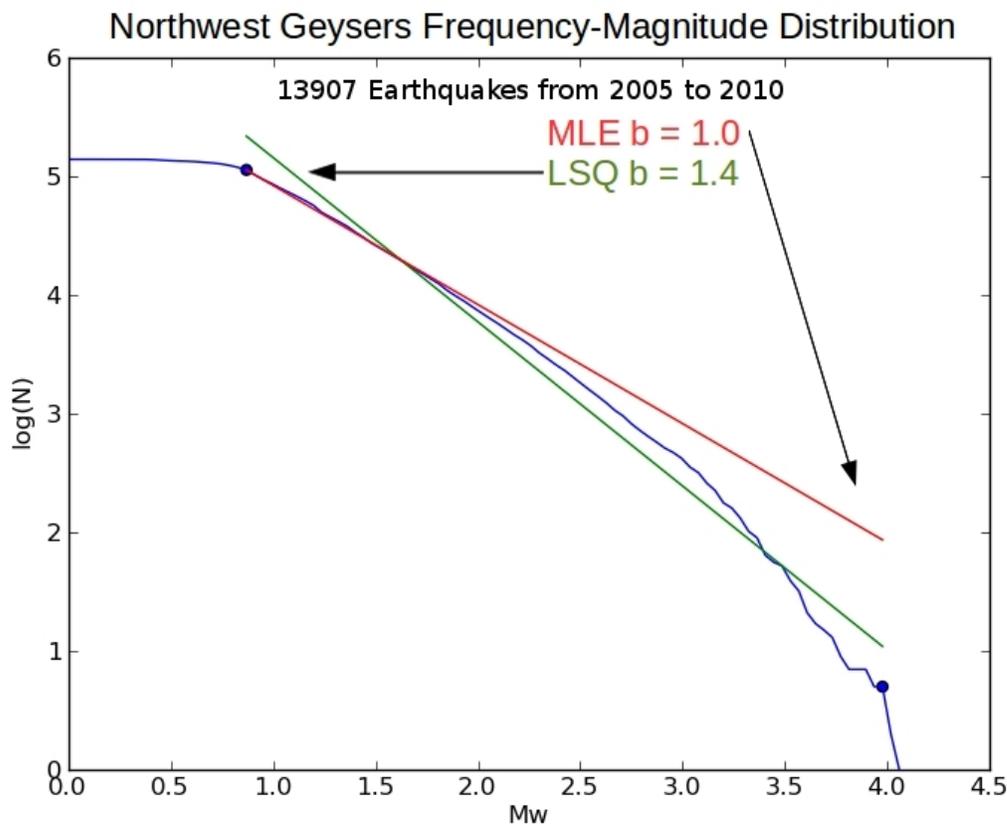
It assumes there is no maximum magnitude and is a good approximation if the completeness covers a range of at least 2 magnitude units. Page [1968] derives a version of the estimator that has a maximum magnitude ( $M_{max}$ ) good for our smaller range of completeness:

$$b = \log e / [M_{av} - (M_{min} - M_{max} e^{(-b(M_{max} - M_{min})/\log e)}) / (1 - e^{(-b(M_{max} - M_{min})/\log e)})]$$

but it must be solved numerically.

### Maximum Likelihood Estimation (MLE) vs Linear Least-squares (LSQ)

The maximum likelihood estimation of  $b$  value has some advantages over the standard linear least-squares calculation because it weights the occurrence of each earthquake equally. Since the linear Gutenberg-Richter relationship is graphed with the log of the number of earthquakes, a simple linear regression weights the data points representing fewer higher magnitude earthquakes as much as those representing the many more earthquakes at lower magnitudes when determining the linear fit. So while the least-squares method may appear to fit the line better than the maximum likelihood estimator, this doesn't take into account the bias inherent in looking at a logarithmic scale. Of course the methods give very close  $b$  values when the data (or the chosen completeness of the data) fits closely to a line, with a slope at higher magnitudes the same as that at lower magnitudes. Comparing the  $b$  values calculated using both methods can give a quick method to determine how closely the defined completeness fits a single straight line, or if it contains different slopes at different magnitude ranges



within the determined completeness.

Figure 1: Frequency-magnitude distribution of all 2005 – 2010 seismicity showing bend at low

magnitudes and how the MLE and least-squares (LSQ) methods weight different parts of the curve.

The maximum likelihood estimation was found to be more sensitive to the determination of the

minimum threshold magnitude ( $M_{min}$ ). If  $M_{min}$  is chosen too low, the maximum likelihood estimation will over-weight the lower magnitude end of the curve and will fit a slope that is more dependent on the drop off in events recorded at lower magnitudes. Since maximum likelihood gives more weight to these low magnitude points, the  $b$  value calculated will often fit that part of the curve, while least-squares will fit the high magnitude region better, because it covers a longer magnitude range and thus includes more points to be taken in the regression. The key is to know if these changes of slope at either end of the curve are real structure showing differences in the physical state of the fault, or are artifacts of measurement from an inability to measure enough of the low magnitude earthquakes or time scale too short to record enough rarer high magnitude earthquakes. For this reason it is very important to be able to determine the catalog completeness accurately in a way that depends on the measurement devices and not only on the data itself.

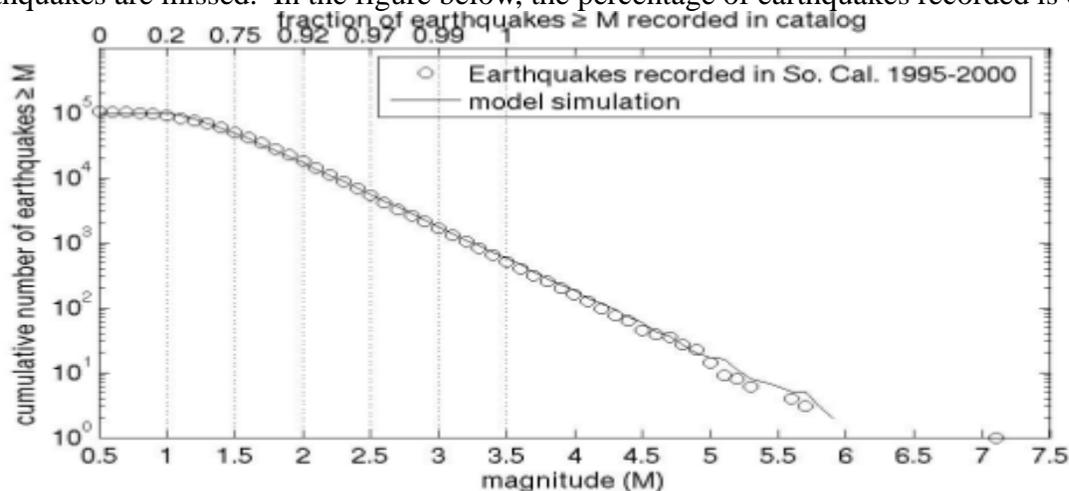
### Catalog Completeness

The catalog completeness is often determined by assuming the Gutenberg-Richter relationship and finding at what magnitudes the data differs from a line. This is often done by eye, or a simple program, to find the magnitude with greatest slope change from the line. Using the same assumption, some have tested the stability and behavior of the  $b$  value or maximized the goodness of fit while varying  $M_{min}$  to find an optimal magnitude range (Marsan 2003, Wiemer and Wyss 2000, Cao and Gao 2002).

Woessner and Wiemer [2005] also assumed a linear fit for the completeness range but found models to fit the incomplete part and find the best range to fit. The completeness has also been calculated by Rydelek and Sacks [1989] comparing day-to-night ratios of measured earthquakes, assuming that the noise is greater in the day, or signal-to-noise ratios to determine what magnitudes can be reliably measured (Kværna et al. 2002). A recent thesis (Bachman 2007) attempted to determine the spatial completeness of all of California, by using the entire catalog to determine the magnitude earthquakes they expect to be able to measure in each location. The data from the Geysers were dense and differed from the Gutenberg-Richter relationship, so they threw off the calculation and had to be excluded from the state-wide analysis. These methods are based on instrument characteristics, and can be used to determine if data that doesn't fit the linear assumption should be kept, but have their own assumptions about the noise and are too difficult and time consuming to be practical in most regions.

*There is a published discussion with Rydelek and Sacks on the Wiemer and Wyss [2000] method that is focused on this problem, if you can determine the completeness based on the linear section of the curve or whether there is any real evidence that there is deviations from the GR relationship at low magnitudes. Wiemer's later paper used the method of modeling the incomplete part of the curve to show its not unfairly excluded, but it seems like this circular argument is still being debated.*

Felzer 2007 shows that determining the completeness by eye can be deceiving. By using earthquake population simulations and setting a gradient of how many of each magnitude of earthquake can be recorded, she shows that the frequency-magnitude distribution looks linear even when many earthquakes are missed. In the figure below, the percentage of earthquakes recorded is on the top axis.



This simulation technique is also used to determine that a minimum of 2000 events are required to measure b values with 0.05 confidence errors 98% of the time. Other common errors and suggestions for more accurately evaluating b values are outlined in this report. For example, she recommends using the magnitude rounding for bin resolution to avoid further binning.

### **Deviations from Gutenberg-Richter?**

The assumption of the Gutenberg-Richter relation to exclude the part of the curve which doesn't fit is problematic if there is real earthquake scaling information lost in this section of the distribution. Lombardi [2003] shows that if the data (in that case mainshocks) doesn't follow the assumed exponential distribution the simple maximum likelihood estimator shown above will give a significantly different b value that doesn't show if the earthquake mechanisms are the same. Instead, Lombardi maximizes the more general log-likelihood function without assuming the Gutenberg-Richter law and finds a b value for the mainshocks that matches that of the rest of the earthquakes. This paper uses a statistical technique that is unique to mainshocks, treated as the largest variable in the data set of all earthquakes, but it describes the same problem seen in the analysis of the Geysers.

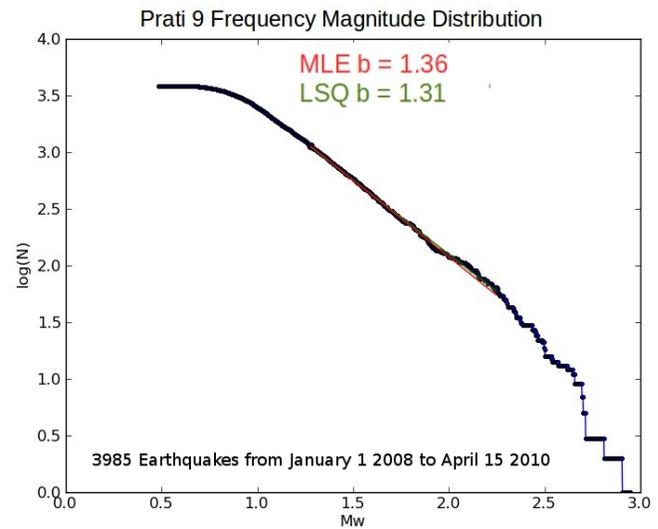
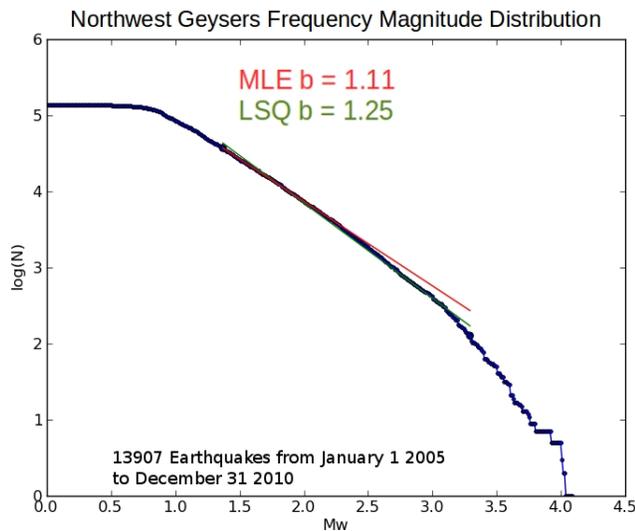
There is evidence that there may be a real deviation from the Gutenberg-Richter relationship at small magnitudes due to minimum source dimensions or surface measurement limits on high frequencies (Aki 1987). Ellsworth claimed to be seeing this at very low magnitudes at SAFOD, which are borehole measurements that aren't effected by surface attenuation. The drop-offs described above are at higher magnitudes. There have also been claims that geothermal and volcanic swarms are more likely to deviate from the Gutenberg-Richter relation or at least show much lower b values than other tectonic regions. Bachman [2007] shows that the events in the Geysers cluster fluctuate more than the rest of the California catalog, assuming this is due to geothermal activity.

### **Identifying Earthquake Populations**

Besides determination of catalog completeness, the spatial and temporal confines must be chosen to best examine how the b value depends on time and space. It is assumed by using the Gutenberg-Richter law that a particular fault configuration has self-similar ruptures, and that the slope of this power-law tells us something about the physical conditions under which the earthquakes are happening. So it is very important that we isolate the time and location of a specific configuration to accurately compare the changes and start to postulate what physical mechanisms are changing with injection into and production from the various wells. In this case we'd like to make inferences about clusters of induced earthquakes around production and injection wells, to determine how the geology is reacting to these human influences. The spatial correlation between seismic clusters and the point of injection or production in a well must be determined to view how the seismicity changes with injection and production properties – flow rate, temperature, pressure, etc.

Without being confident that the time and space sampled correspond to a single configuration, it's possible that any structure seen in the deviations from linearity of the frequency-magnitude curve are due to a combination of multiple curves with different slopes. This is also difficult to determine at the Geysers because of the close configuration of injection and production wells. There appears to be a non-trivial correspondence between seismic density and the location of injection or production. This includes possible coupling effects from multiple wells sufficiently close to an area of seismicity. There are also many unknown variations in geological structure that control the location of injected fluids and thus the fault configurations that need to be isolated.

Shown below is the frequency-magnitude distribution of the entire Northwest Geysers and that of a cluster of MEQs around the actively injecting Prati 9 well.



### Comparison to b Value Studies in Tectonic, Volcanic, Geothermal, and Laboratory Settings

B values have been calculated to be about one world-wide and for various tectonic settings. This follows from a hypothesized theory to derive the Gutenberg-Richter relationship from physical principles (Felzer 2007). The derivation starts with the assumption that the forces (stress) pushing the fault to grow to a larger faulting area, and thus larger magnitude, scale linearly with the faulting area while the frictional forces stopping the earthquake from growing are constant with faulting area. Using this model the probability an earthquake grows to a certain area, or the number expected to go to that size (N), is inversely proportional to the faulting area (A):  $N \propto 1/A$ .

With this reasoning, no matter the fault properties, the relative number of earthquakes of different sizes will follow this relationship, and since the moment magnitude (M) is related to the faulting area in the relation:  $A \propto 10^M$  then  $N \propto 10^{-M}$ ,

which is the Gutenberg-Richter relationship with a b value of 1.

Mathematically, since the global catalog shows this power law relationship and is assumed to be the super-position of power-law relationships representing smaller areas of seismicity, the smaller areas should have the same exponent, namely  $b = 1$  for all areas. This is due to the fact that the sum of power laws is another power law only if all the exponents are the same. This has led to the claim of Felzer and others that b value for every tectonic setting should be one. To this end she has shown how calculations finding otherwise could be due to unaccounted errors. This is an interesting theory, still being debated, but it hasn't been extended to cases of volcanic and geothermal clusters or induced seismicity, and there is some reason to believe there are at least periods of higher b values in these settings.

Frequency-magnitude distributions have also been investigated for volcanic and geothermal regions, and large anomalies have been found. At first it was found that these regions just had much higher b values than that found in purely tectonic settings, but when looked at with higher spatial resolution, it was discovered that there were actually very localized areas of high b values within regions of normal b values of about 1, and that these specific areas were correlated with other geophysical and geodetic data showing magma chambers (Wiemer and Wyss 2002). These high b value areas were also interpreted with laboratory findings to make inferences of how physical processes in these areas might

effect the frequency-magnitude distribution of seismicity. Many of these processes could also be active in the induced seismicity we see at the Geysers. Scholz [1968] found that lower ambient stresses were responsible for higher  $b$  values measured for fractures in the lab, and Mogi [1962] found structural heterogeneity of the samples caused higher  $b$  values. Warren and Latham [1970] measured the magnitude-frequency distribution of thermally induced fractures in the lab, and found “kinks or knees” in distribution indicating  $b$  value variation from mean at certain magnitude ranges, similar to the deviations seen at the Geysers. This might suggest that the seismicity at the Geysers is dominated by thermal fracture, which has been suggested by other studies (Stark 2003). Warren and Latham also found greater  $b$  values when the thermal gradient was greater. All these effects are possible at the Geysers, which makes it difficult to make a unique interpretation based only on  $b$  value analysis, but characterization of the frequency-magnitude distribution might provide a restraint on the problem.

### **Conclusions**

The induced seismicity in the Northwest Geysers differs from the Gutenberg-Richter relationship found in tectonic settings. Our program uses two methods (Maximum Likelihood Estimation and Linear Least-squares) to fit the frequency-magnitude distribution, giving a quick metric for how much the curve varies from a single line. This situation highlights the importance of determining the catalog completeness independent of the shape of the data. Once a trusted completeness is determined, the changes in frequency-magnitude distribution and  $b$  value temporally and spatially correlated to changes of production and injection can be used to better understand the dominant fracture mechanisms at work.

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