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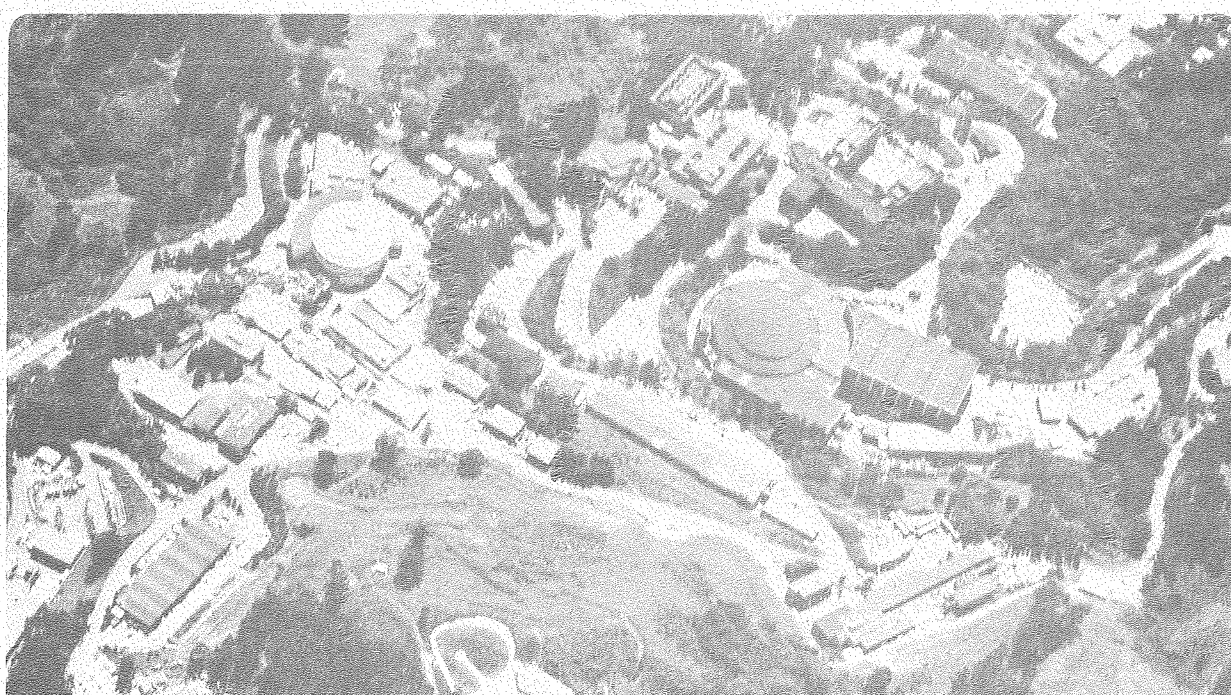

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FLAVOR SYMMETRY IN NONLEPTONIC DECAYS OF BOTTOM MESONS*

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ABSTRACT

Dominant decays of bottom mesons into two pseudoscalars are studied in the framework of the six-quark model of the weak interactions of Kobayashi and Maskawa. Some simple SU(3) relations between the amplitudes are obtained without using any dynamical assumptions.

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I. INTRODUCTION

Three narrow upsilon states $\Upsilon(9.4)$, $\Upsilon'(10.0)$, $\Upsilon''(10.3)$ have been discovered [1] in the mass spectrum of $\mu^+\mu^-$ pairs produced in hadronic collisions at Fermilab. Both Υ and Υ' have been confirmed by e^+e^- annihilation at the DORIS [2] storage ring at DESY. The mass spectrum of Υ-family suggests that they are most likely the bound states of a new quark and its antiquark. The measured leptonic width favors the charge assignment of -1/3 to the new quark. Hence the new quark is most likely to be the bottom quark b, first suggested by Kobayashi and Maskawa [3]. If this interpretation is correct, then in addition to the upsilon states, there should also exist bottom mesons containing an unpaired b or $\bar{b}$ quark. Now it seems that the production of bottom mesons has been observed by two experimental groups at CESR [4]. They reported the observation of a fourth upsilon state $\Upsilon'''(10.5)$ which has a broad resonance width (about 20 MeV). The broad width of Υ''' indicated that the fourth upsilon state is above the threshold of $B\bar{B}$ pairs. Both groups also found a dramatic increase in the yield of high-energy electrons and muons at Υ'''. They are presumed to come from weak decay of bottom mesons. The CLEO group of CESR also found a similar increase in the kaon production rate at the Υ''' peak. The increased rate provides evidence for the sequential decay $b \to c \to s$ expected from the recent estimate [5] of the generalized Cabibbo angles. The lowest-lying pseudoscalar bottom mesons B_u^-, B_d^0, B_s^0 and B_c^-, being stable against strong and electromagnetic decay, can only decay weakly [6]. Thus, a study of the decays of bottom mesons could provide us with useful information on the nature of weak interactions.

The general properties of bottom particles have been analyzed by Ellis et al. [7] based on standard model. Our purpose here is to study the exclusive nonleptonic weak decays of bottom mesons within the framework of the standard model. The nonleptonic decays are complicated by the interplay of weak and strong interactions. There are two different approaches to attack this problem: symmetry consideration or dynamical models [8]. The two body decays of bottom mesons have been calculated by Ali et al. [9] with the help of the quark-parton model. Recent experiments [10, 11] on weak decays of D-mesons raised some doubts on the reliability of this approach [8, 12]. On the other hand, the symmetry approach is expected to provide a reliable framework for systematic study on nonleptonic weak decays. It has been particularly emphasized by Quigg [13] and Lipkin [8] in their study of charmed meson decays. In this paper, we adopt the symmetry approach and calculate the SU(3) amplitudes for bottom meson decay into two pseudoscalars.

The plan of this article is as follows. In Sec. II we present the group structure of the nonleptonic weak Hamiltonian based on the standard model of Kobayashi and Maskawa. In Sec. III, we proceed to evaluate the SU(3) amplitudes for the bottom meson decays into two pseudoscalar mesons. Several simple relations of the amplitudes (and hence the reduced decay widths) among different decay channels are then obtained. Sec. IV contains a short conclusion based on this calculation.

II. REPRESENTATION CONTENT OF THE WEAK-INTERACTION HAMILTONIAN

In the standard model of 6-quarks, the charged current is given by

$$J^\mu = \bar{u}_\alpha \gamma^\mu (1 - \gamma_5) d'_\alpha + \bar{c}_\alpha \gamma^\mu (1 - \gamma_5) s'_\alpha + \bar{t}_\alpha \gamma^\mu (1 - \gamma_5) b'_\alpha \tag{2.1}$$

where a summation over the repeated color index $\alpha = 1, 2, 3$ is understood. The weak eigenstates d', s', b' are related to the mass eigenstates d, s, b by a unitary matrix V as follows

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{2.2}$$

The unitary 3×3 mixing matrix V is usually parametrized [3] in the special form

$$V = \begin{pmatrix} c_1 & c_3 s_1 & s_1 s_3 \\ -c_2 s_1 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + c_3 s_2 e^{i\delta} \\ s_1 s_2 & -c_1 c_3 s_2 - c_2 s_3 e^{i\delta} & -c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix}, \tag{2.3}$$

where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$, and δ is the CP-violating phase parameter. To lowest order in weak interaction, the nonleptonic Hamiltonian in the limit $M_W \to \infty$ can be written as

$$\mathcal{H}_W = \frac{G}{\sqrt{2}} (J_\mu J^{\mu^+} + \text{h.c.}) \tag{2.4}$$

The bare Hamiltonian (2.4) will be modified by strong interaction effects [14]. In order to take into account the gluon corrections, it is useful to separate the product of currents into symmetric and antisymmetric pieces

$$\mathcal{H}_W = \frac{G}{\sqrt{2}} (f_+ H_+ + f_- H_-) \tag{2.5}$$

where $f_+ = f_- = 1$ in the absence of strong interactions. This separation is useful because, in the massless quark limit, the QCD Lagrangian is SU(n) symmetric for n flavors. Since H_+ and H_- behave differently under flavor transformations, they have different anomalous dimensions. Thus, when gluon corrections are included, the resulting effective Hamiltonian [14] is given by

$$\mathcal{H}_W = \frac{G}{\sqrt{2}} (f_+ H_+ + f_- H_- + \ldots) \tag{2.6}$$

where the dots stand for other types of operators, and f_+, f_- are changed from the free field value $f_+ = f_- = 1$ into $f_+ < 1 < f_-$. Therefore, the antisymmetric piece H_- is enhanced relative to the symmetric piece. However, the enhancement effects are expected to be much smaller for heavier quarks [7]. The mass of the t quark is supposed to be much heavier than the mass of b quark. Therefore, from (2.1, 2.2, 2.4), the part of the Hamiltonian responsible for bottom meson decays can be written as

$$\begin{aligned}\mathcal{H}(\Delta B = 1) = \frac{G}{\sqrt{2}} \Big[& V_{13}V_{11}^{*}\{\bar{u}b,\bar{d}u\} + V_{13}V_{12}^{*}\{\bar{u}b,\bar{s}u\} \\ & + V_{13}V_{21}^{*}\{\bar{u}b,\bar{d}c\} + V_{13}V_{22}^{*}\{\bar{u}b,\bar{s}c\} \\ & + V_{23}V_{11}^{*}\{\bar{c}b,\bar{d}u\} + V_{23}V_{12}^{*}\{\bar{c}b,\bar{s}u\} \\ & + V_{23}V_{21}^{*}\{\bar{c}b,\bar{d}c\} + V_{23}V_{22}^{*}\{\bar{c}b,\bar{s}c\}\Big] \\ \equiv\ & \mathcal{H}_1 + \ldots + \mathcal{H}_8 \end{aligned} \tag{2.7}$$

where the space-time structure and the color index are omitted for brevity. We now consider the group structure of the hadronic weak current J^μ and weak Hamiltonian $\mathcal{H}_W$. The group properties in the case of $2n$ quark flavors has been sketched by Quigg [13]. Here we specialize to the case of the standard model of Kobayashi and Maskawa. In this case, the relevant group is SU(6). However, the mass of the t-quark is heavier than the mass of the b quark. Since we are interested in the decay of bottom particles, it is sufficient to consider the transformation properties of the weak Hamiltonian under the group SU(5) relating the u,d,s,c and b quarks. The weak current J_μ transforms as the adjoint representation $\underline{24}$ of SU(5). Consequently, the nonleptonic weak Hamiltonian transforms like $\underline{24} \otimes \underline{24}$ under SU(5). However, only the representation which are symmetric products of currents actually occur in the decomposition of $\underline{24} \otimes \underline{24}$. Furthermore, the singlet and the adjoint representation $\underline{24}$ are also absent in $\mathcal{H}_W$ [13]. Therefore,

$$\mathcal{H}_W \simeq \underline{75} + \underline{200} \tag{2.8}$$

Note that the representations $\underline{75}$ and $\underline{200}$ correspond to the antisymmetric and symmetric operators H_- and H_+ respectively. A direct application of the above decomposition to matrix elements of physical interest is not useful because SU(5) is badly broken. Therefore, we decompose the representations $\underline{75}$ and $\underline{200}$ with respect to SU(4) subgroups. The decompositions are given by

$$\underline{75} \supset [20'^*]_1 + \{[20]_0 + [15]_0\} + [20']_{-1}$$

$$\underline{200} \supset [10]_2 + \{[36]_1 + [4]_1\} + \{[1]_0 + [15]_0 + [84]_0\} + \{[4^*]_{-1} + [36^*]_{-1}\} + [10^*]_{-2} \qquad (2.9)$$

where we use the square brackets to denote the irreducible representations of SU(4). The subscripts of square brackets refer to the bottom quantum number B. The $|\Delta B| = 2$ pieces, i.e. [10] and $[10^*]$, do not contribute to $\mathscr{H}_W$. Note that $\mathscr{H}(\Delta B = 1)$ (2.7) transforms as [4], $[20'^*]$ and [36] under SU(4) while $\mathscr{H}(\Delta B = -1)$ transforms as $[4^*]$, [20'] and $[36^*]$. This can also be seen easily in the following way. The Hamiltonian $\mathscr{H}(\Delta B = 1)$ is a product of bottom-changing currents and bottom-conserving currents, which transform under SU(4) as [4] and [15] respectively. Therefore,

$$\mathscr{H}(\Delta B = 1) \sim [4] \otimes [15] = [4] + [20'^*] + [36] \qquad (2.10)$$

Eq. (2.9) also tells us that $[20'^*]$ is contained in $\underline{75}$ while [4] and [36] are contained in $\underline{200}$.

It might be useful to exploit the SU(4) symmetry for bottom meson decays since the masses of bottom mesons are much heavier than those of charmed mesons and ordinary mesons. This deserves a further study. In this paper, only the SU(3) symmetry of the weak Hamiltonian will be exploited. The $\Delta B = 1$ part of the Hamiltonian (2.7, 2.10) can be decomposed with respect to SU(3) into:

$$[36] \supset (6)_{-1} + \{(15)_0 + (3)_0\} + \{(1)_1 + (8)_1\} + (3^*)_2$$

$$[20'^*] \supset (3^*)_{-1} + \{(6^*)_0 + (3)_0\} + (8)_1$$

$$[4] \supset (3)_0 + (1)_1 \qquad (2.11)$$

where the subscripts refer to the charm quantum number. It is easy to see that the singlet and $(3^*)_2$ piece of (2.11) are absent in the actual $\Delta B = 1$ Hamiltonian (2.7). Both the analysis [5] of mixing angles and the recent experimental result [15] indicated that the weak decay of a b quark is dominated by the $b \to c$ transition over the $b \to u$ transition. This implies the dominant terms in (2.7) are $\mathscr{H}_5$ and $\mathscr{H}_8$, which are proportional to $V_{23}V_{11}^*$ and $V_{23}V_{22}^*$ respectively. In the following we shall restrict ourselves to the dominant decays of bottom mesons. In this approximation,

$$\mathscr{H}(\Delta B = 1) \simeq \frac{G}{\sqrt{2}} [V_{23}V_{11}^*\{\bar{c}b, \bar{d}u\} + V_{23}V_{22}^*\{\bar{c}b, \bar{s}c\}] = \mathscr{H}_5 + \mathscr{H}_8 \qquad (2.12)$$

From (2.12), one can easily deduce that the first term $\mathscr{H}_5$ transforms as an SU(3) octet and satisfies the following selection rules

$$\Delta B = \Delta C = 1, \ \Delta Y = 0, \ |\Delta\vec{I}| = -\Delta I_3 = 1 \qquad (2.13)$$

The second term $\mathscr{H}_8$ transforms as an SU(3) triplet and satisfies the following selection rules

$$\Delta B = 1, \quad \Delta C = 0, \quad \Delta Y = -\frac{2}{3}, \ |\Delta\vec{I}| = \Delta I_3 = 0 \qquad (2.14)$$

They are denoted as $(8)_1$ and $(3)_0$ respectively in (2.11). From (2.9, 2.11), we can see that both representations <u>75</u> and <u>200</u> of SU(5) contain these two pieces.

III. AMPLITUDES FOR BOTTOM MESON DECAY

Let us now consider the decays of $B = -1$ bottom mesons B_u^-, B_d^0, B_s^0 and B_c^-. We shall focus our attention to the decay of a bottom meson into two pseudoscalar mesons because the relatively small number of independent matrix elements in these decays enables us to infer simple relations among the amplitudes. The relevant matrix element is of the form $\langle PP|\mathscr{H}|P_b\rangle$, where P_b denotes a pseudoscalar bottom meson and $\langle PP|$ denotes a pair of bottomless pseudoscalar mesons in the final state. Note that Bose symmetry requires the final state to be symmetric in SU(4) indices. That means the final state can only occur in the representations [1], [15], [20] and [84] of SU(4). Their SU(3) contents are given by

$$\begin{aligned}
[1] &\supset (1)_0 \\
[15] &\supset (3^*)_1 + \{(8)_0 + (1)_0\} + (3)_{-1} \\
[20] &\supset (6)_1 + (8)_0 + (6^*)_{-1} \\
[84] &\supset (6^*)_2 + \{(3^*)_1 + (15^*)_1\} + \{(1)_0 + (8)_0 + (27)_0\} \\
&\quad + \{(3)_{-1} + (15)_{-1}\} + (6)_{-2}
\end{aligned} \qquad (3.1)$$

As discussed in Sec. II, only the dominant decay modes with $\mathscr{H}$ approximated by $\mathscr{H}_5 + \mathscr{H}_8$ will be considered. Let's denote the uncharmed ($C = 0$) bottom mesons B_u^-, B_d^0, B_s^0 by $\tilde{P}_b$, which transforms as (3^*) under SU(3). Since $\mathscr{H}_5$ transforms like an SU(3) octet, only three independent reduced matrix elements E,F,G defined as follows

$$
\begin{aligned}
E &= \langle (6) \;||\; (8) \;||\; (3^*) \rangle \\
F &= \langle (3^*) \;||\; (8) \;||\; (3^*) \rangle \\
G &= \langle (15^*) \;||\; (8) \;||\; (3^*) \rangle
\end{aligned}
\tag{3.2}
$$

will occur in the matrix elements $\langle PP|\mathcal{H}_5|\tilde{P}_b\rangle$. Note that the final states appearing in the representation (15^*) can have isospin equal to either 3/2 or 1/2. Only I = 1/2 final states are allowed in the representations (6) and (3^*). Since B_c^- is an SU(3) singlet, the final state in $\langle PP|\mathcal{H}_5|B_c^-\rangle$ must be an SU(3) octet, which may be formed from the following diffrent products of SU(3) multiplets $(3) \otimes (3^*)$, $(1)_{\eta_c} \otimes (8)$, $(8) \otimes (8)$, $(1)_{\eta'} \otimes (8)$. The corresponding reduced matrix elements are denoted by K,K',K" and K''' respectively.

Now let us consider the matrix element $\langle PP|\mathcal{H}_8|P_b\rangle$. Since $\mathcal{H}_8$ transforms like an SU(3) triplet, the final state $\langle PP|$ in the decay of charmless bottom mesons $\tilde{P}_b$ can only be SU(3) octet or singlet. The octet can be formed from the products $(3^*) \otimes (3)$ or $(1) \otimes (8)$ and the corresponding reduced matrix elements are denoted as M and M'. Those for the singlet are denoted as N,N',N" corresponding to the final state singlet contained in $(3^*) \otimes (3)$, $(1)_{\eta_c} \otimes (1)_{\eta'}$, and $(1)_{\eta_c} \otimes (1)_{\eta_c}$ respectively. The final state for the matrix element $\langle PP|\mathcal{H}_8|B_c^-\rangle$ must be an SU(3) triplet, which may be obtained from the products $(3) \otimes (8)$, $(3) \otimes (1)_{\eta'}$, and $(1)_{\eta_c} \otimes (3)$. The corresponding reduced matrix elements are denoted as Q,Q',Q".

The SU(3) amplitudes [16] for the dominant decays of B_u^-, B_d^0, B_s^0 and B_c^- into two pseudoscalar mesons are collected in Table 1 and Table 2. Here η is regarded as a member of the pure pseudoscalar

octet, η' is regarded as an SU(3) singlet, and η_c is treated as another SU(3) singlet.

First consider the decays listed in Table 1. The amplitudes for these decays are proportional to $V_{23}V_{11}^*$. Using the general expressions given in Table 1 one finds

$$
\begin{aligned}
&\sqrt{2}\, A(B_d^0 \to D^0\pi^0) + A(B_d^0 \to D^+\pi^-) \\
&= A(B_s^0 \to F^+\pi^-) + A(B_s^0 \to D^0K^0) \\
&= A(B_u^- \to D^0\pi^-)
\end{aligned}
\tag{3.3}
$$

$$
A(B_c^- \to \pi^-\pi^0) = 0 \tag{3.4}
$$

$$
A(B_c^- \to \pi^-\eta) = -\sqrt{2/3}\, A(B_c^- \to K^-K^0) \tag{3.5}
$$

Eq. (3.3) implies that the reduced decay rates [17] satisfy the following inequalities

$$
\begin{aligned}
&\sqrt{2\Gamma(B_d^0 \to D^0\pi^0)} - \sqrt{\Gamma(B_d^0 \to D^+\pi^-)} \leqslant \sqrt{\Gamma(B_u^- \to D^0\pi^-)} \\
&\leqslant \sqrt{2\Gamma(B_d^0 \to D^0\pi^0)} + \sqrt{\Gamma(B_d^0 \to D^+\pi^-)}
\end{aligned}
$$

and

$$
\begin{aligned}
&\sqrt{\Gamma(B_s^0 \to F^+\pi^-)} - \sqrt{\Gamma(B_s^0 \to D^0K^0)} \leqslant \sqrt{\Gamma(B_u^- \to D^0\pi^-)} \\
&\leqslant \sqrt{\Gamma(B_s^0 \to F^+\pi^-)} + \sqrt{\Gamma(B_s^0 \to D^0K^0)}
\end{aligned}
\tag{3.6}
$$

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According to (3.4), the decay $B_c^- \to \pi^-\pi^0$ is forbidden. For B_c^- decay into $\pi^-\eta$ and K^-K^0, Eq. (3.5) predicts the relation

$$3\Gamma(B_c^- \to \pi^-\eta) = 2\Gamma(B_c^- \to K^-K^0) \tag{3.7}$$

Now let us consider the decays listed in Table 2. The amplitudes for these decays are proportional to $V_{23}V_{22}^*$. By looking at Table 2, many equalities can be predicted:

$$\Gamma(B_u^- \to D^0F^-) = \Gamma(B_d^0 \to D^+F^-)$$

$$\Gamma(B_u^- \to \eta_c K^-) = \Gamma(B_d^0 \to \eta_c \bar{K}^0)$$

$$\Gamma(B_s^0 \to D^0\bar{D}^0) = \Gamma(B_s^0 \to D^+D^-)$$

$$2\Gamma(B_c^- \to \bar{D}^0K^-) = 2\Gamma(B_c^- \to D^-\bar{K}^0) = 3\,\Gamma(B_c^- \to F^-\eta) \tag{3.8}$$

In addition, the amplitudes for B_s^0 decay into F^+F^-, D^+D^- satisfy the sum rule

$$A(B_s^0 \to F^+F^-) + A(B_s^0 \to D^+D^-) = A(B_u^- \to D^0F^-) \tag{3.9}$$

which implies the triangle inequalities

$$\sqrt{\Gamma(B_s^0 \to F^+F^-)} - \sqrt{\Gamma(B_s^0 \to D^+D^-)} \leqslant \sqrt{\Gamma(B_u^- \to D^0F^-)}$$

$$\leqslant \sqrt{\Gamma(B_s^0 \to F^+F^-)} + \sqrt{\Gamma(B_s^0 \to D^+D^-)} \tag{3.10}$$

It should be noted that many of the above relations are simply results of an SU(2) symmetry instead of an SU(3) symmetry. For instance, the relation between B_d^0 and B_u^- decays of (3.3) and the relation (3.4) can be obtained from the $\Delta I = 1$ rule (2.13) of $\mathcal{H}_5$. And, all of (3.8) except for the second relation of the last one are consequences of the $\Delta I = 0$ rule (2.14) of $\mathcal{H}_8$. Therefore, they should hold much more accurately than the other SU(3) relations.

It is important to emphasize that the predictions through the symmetry approach are more general than those obtained from dynamical models based on the selection of certain types of diagrams. For example, the selection rules of neutral bottom meson decay have been obtained by Rosen [18] assuming the W-exchange dominance. For B_d^0 decay into $D^+\pi^-$, $D^0\pi^0$, they predict

$$\Gamma(B_d^0 \to D^+\pi^-) = 2\Gamma(B_d^0 \to D^0\pi^0) \tag{3.11}$$

In our symmetry approach, we have

$$\sqrt{2}A(B_d^0 \to D^0\pi^0) + A(B_d^0 \to D^+\pi^-)$$

$$= A(B_u^- \to D^0\pi^-) \tag{3.12}$$

The decay $B_u^- \to D^0\pi^-$ can only proceed through W-radiation diagrams while the decays $B_d^0 \to D^0\pi^0$, $D^+\pi^-$ can proceed both through the W-radiation and W-exchange diagrams. If the W-exchange diagram is dominant, (3.12) becomes

$$\sqrt{2}A(B_d^0 \to D^0\pi^0) + A(B_d^0 \to D^+\pi^-) = 0 \qquad (3.13)$$

which implies the prediction (3.11) by Rosen [18]. However, the W-radiation diagram is supposed to be important for bottom meson decay. Therefore, (3.11) is not expected to be valid while our prediction (3.12) holds without involving any dynamical assumption.

IV. CONCLUSIONS

In this paper, the dominant decays of bottom mesons into two pseudoscalars are calculated by using a flavor-symmetry approach. In contrast to the case of charmed meson decays, in which a large number of different amplitudes contribute to any given process and consequently no simple relations exist for the Cabibbo-favored decays [13], some simple SU(3) relations(3.3 - 3.10) are obtained for dominant bottom meson decays. These simple relations could provide useful tests for the nature of weak interactions if the two-body decays of bottom mesons can be seen in future experiments. For completeness, one should also calculate the SU(3) amplitudes for Cabibbo-suppressed decay modes, involving all the terms in (2.7) other than $\mathscr{H}_5$ and $\mathscr{H}_8$. However, the branching ratios for these suppressed decay modes are small. Therefore, they are unlikely to be seen experimentally in the near future. A useful extension of this work would be to exploit the SU(4) symmetry for bottom meson decays since the masses of bottom mesons are much heavier than those of charmed mesons and ordinary mesons. This deserves a further study. Perhaps, a more realistic approach is to study the soft pion theorem for nonleptonic weak decays of bottom mesons [19]. After this work was essentially completed I learned that Zeppenfeld [20] has carried out a similar calculation. To the extent that the weak Hamiltonian can be approximated by (2.12), our results agree with those of Zeppenfeld. However, the treatment given here differs from the work of Zeppenfeld in several respects. First, we focus our attention upon the group theoretical structure of the weak Hamiltonian such that an immediate generalization to SU(4) symmetry can be easily carried out. Second, the relations (3.3 - 3.10) among the amplitudes and reduced decay rates have not been obtained or discussed explicitly in [20]. Third, the decay of B_c^- is completely omitted in [20]. Since B_c^- transforms like an SU(3) singlet, only one independent reduced matrix element contributes to any given process for B_c^- decay as displayed in Table 1 and Table 2. Consequently, very simple relations such as (3.7) and the last equation in (3.8) can be obtained for B_c^- decays.

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TABLE CAPTIONS

Table 1. Amplitudes proportional to $V_{23}V_{11}^{*}$ for bottom meson decay

Table 2. Amplitudes proportional to $V_{23}V_{22}^{*}$ for bottom meson decay

TABLE 1.

Decay mode	Amplitude
$B_u^- \to D^0\pi^-$	G
$B_d^0 \to D^+\pi^-$	$(2E + 3F + 3G)/8$
$D^0\pi^0$	$\sqrt{2}(-2E - 3F + 5G)/16$
$D^0\eta$	$\sqrt{6}(-2E + F + G)/16$
$D^0\eta'$	$\sqrt{6}\,F'/4$
F^+K^-	$(2E - 3F + G)/8$
$\eta_c D^0$	$\sqrt{6}\,F''/4$
$B_s^0 \to F^+\pi^-$	$(E + G)/2$
D^0K^0	$(-E + G)/2$
$B_c^- \to \eta_c\pi^-$	K'
D^-D^0	K
$\pi^-\pi^0$	0
$\pi^-\eta$	$K''/\sqrt{5}$
$\pi^-\eta'$	K'''
K^-K^0	$-\sqrt{3/10}\,K''$

TABLE 2.

Decay mode	Amplitude
$B_u^- \to D^0F^-$	M
$\eta_c K^-$	M'
$B_d^0 \to D^+F^-$	M
$\eta_c \bar{K}^0$	M'
$B_s^0 \to F^+F^-$	(2M + N)/3
$D^0\bar{D}^0$	(- M + N)/3
D^+D^-	(M - N)/3
$\eta_c \eta$	$\sqrt{2/3}$ M'
$\eta_c \eta'$	$N'/\sqrt{3}$
$\eta_c \eta_c$	$N''/\sqrt{3}$
$B_c^- \to \bar{D}^0K^-$	$\sqrt{6}$ Q/4
$D^-\bar{K}^0$	$-\sqrt{6}$ Q/4
$F^-\eta$	Q/2
$F^-\eta'$	Q'
$\eta_c F^-$	Q"