Development of an analytical solution for thermal single-well injection-withdrawal tests in horizontally fractured reservoirs

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Abstract

In this study, we have developed an analytical solution for thermal single-well injection-withdrawal tests in horizontally fractured reservoirs where fluid flow through the fracture is radial. The dimensionless forms of the governing equations and the initial and boundary conditions in the radial flow system can be written in a form identical to those in the linear flow system developed by Jung and Pruess [Jung, Y., and K. Pruess (2012), A Closed-Form Analytical Solution for Thermal Single-Well Injection-Withdrawal Tests, *Water Resour. Res*., 48, W03504, doi:10.1029/2011WR010979], and therefore the analytical solutions developed in Jung and Pruess (2012) can be applied to compute the time dependence of temperature recovery at the injection/withdrawal well in a horizontally oriented fracture with radial flow.

1. Introduction

Single-well injection-withdraw (SWIW) tracer tests have been proposed and studied as a method for characterization of fractured geothermal reservoirs. For instance, the SWIW test may be used to estimate hydraulic and thermal properties (Capuano et al., 1983; Kocabas and Horne, 1987, 1990), the number of fractures created by hydraulic stimulation and their permeability (Nalla and Shook, 2005), or heat transfer areas (Pruess et al., 2005; Pruess and Doughty, 2010). During an SWIW test, either fluid with tracers (Kocabas and Horne, 1987; Haggerty et al., 2001; Nalla and Shook, 2005; Ghergut et al., 2006, 2009; Neretnieks, 2007) or cold water (Kocabas and Horne, 1990; Kocabas, 2005, 2010; Pruess and Doughty, 2010, Jung and Pruess, 2012) is injected into a well and, after some rest period, the water is produced out of the same well. The solute or temperature recovery curves at the well are monitored during the production phase and used to infer the properties of interest.

In our previous study, we have developed a closed-form analytical solution for thermal SWIW tests in a fractured reservoir, which is highly flexible to implement various injection schemes, including multiple shut-in periods, and capable of providing a comprehensive understanding of heat transfer in fractured rocks (Jung and Pruess, 2012). A theoretical approach was employed to understand the heat transfer in a vertically oriented fracture where the flow along the fracture was linear. Here, we have extended the analytical solution to a reservoir system where a horizontal fracture intersects an injection/withdrawal well. The analytical solution developed is verified by comparisons with numerical solutions using TOUGH2 (Pruess et al., 1999). In addition, we present a

normalized form of the analytical solution, which brings out an important parameter group for interpretation of temperature recovery curves.

2. Theory

We consider a horizontally oriented fracture with uniform aperture 2*b* and porosity ϕ_f. Fluid flow through the fracture is radial, and instantaneous thermal equilibrium is assumed for fluids and rocks within the fracture. The *r* axis is in the direction of the fracture, and the *z* axis is perpendicular to the fracture-matrix interface. Cold water is injected into the fracture at a constant volumetric flow rate, *q*. Because of symmetry, we need to consider only one half of the actual model for the solution. The heat transport in the system during the injection phase can be described as follows (see "nomenclature" for notation):

$$\rho_f c_f \frac{\partial T_{f1}}{\partial t_1} + \rho_w c_w \frac{q}{2\pi r b}\frac{\partial T_{f1}}{\partial r} - \frac{k_m}{b}\frac{\partial T_{m1}}{\partial z}\bigg|_{z=0} = 0 \qquad r > r_w \tag{1a}$$

$$\rho_m c_m \frac{\partial T_{m1}}{\partial t_1} - k_m \frac{\partial^2 T_{m1}}{\partial z^2} = 0 \qquad r > r_w, \quad 0 < z < \infty \tag{1b}$$

where $\rho_f c_f = (1-\phi_f)\rho_m c_m + \phi_f \rho_w c_w$ is the specific heat of rock and fluid in the fracture per unit volume. Note that heat conduction in the radial direction is neglected.

Similar to the previous study (Jung and Pruess, 2012), we assume that the temperature is initially uniform throughout the system and the temperature of the injected fluid is constant during the injection phase. Note that the injection temperature is defined at the radial wall of the well. In addition, the temperature in the matrix at the fracture

walls is always identical to that in the fracture. Then, the initial and boundary conditions can be written as

$$T_{f1} = T_{m1} = T_0 \qquad \text{at } t_1 = 0 \tag{2a}$$

$$T_{f1} = T_{inj} \qquad \text{at } r = r_w \tag{2b}$$

$$T_{f1} = T_{m1} \qquad \text{at } z = 0 \tag{2c}$$

$$T_{m1} \to T_0 \qquad \text{as } z \to \infty \tag{2d}$$

To simplify the analysis, we define the following dimensionless variables:

$$T_{f1D} = \frac{T_0 - T_{f1}}{T_0 - T_{inj}}; \qquad T_{m1D} = \frac{T_0 - T_{m1}}{T_0 - T_{inj}} \tag{3a}$$

$$r_D = \frac{k_m \pi r^2}{\rho_w c_w q b}; \qquad z_D = \frac{z}{b}; \qquad t_{1D} = \frac{k_m t_1}{\rho_f c_f b^2} \tag{3b}$$

$$\theta = \frac{\rho_m c_m}{\rho_f c_f} \tag{3c}$$

Using the dimensionless variables, Eqs. 1a and 1b can be expressed as follows:

$$\frac{\partial T_{f1D}}{\partial t_{1D}} + \frac{\partial T_{f1D}}{\partial r_D} - \left.\frac{\partial T_{m1D}}{\partial z_D}\right|_{z_D=0} = 0 \qquad r_D > r_{wD} \tag{4a}$$

$$\theta \frac{\partial T_{m1D}}{\partial t_{1D}} - \frac{\partial^2 T_{m1D}}{\partial z_D^2} = 0 \qquad r_D > r_{wD}, \quad 0 < z_D < \infty \tag{4b}$$

The initial and boundary conditions become

$$T_{f1D} = T_{m1D} = 0 \qquad \text{at } t_{1D} = 0 \tag{5a}$$

$$T_{f1D} = 1 \qquad \text{at } r_D = r_{wD} \tag{5b}$$

$$T_{f1D} = T_{m1D} \qquad \text{at } z_D = 0 \tag{5c}$$

$$T_{m1D} \to 0 \qquad \text{as } z_D \to \infty \tag{5d}$$

The formats of the governing equations and the initial and boundary conditions are identical to those of the dimensionless governing equations (Eqs. 4a and 4b) and the initial and boundary conditions (Eqs. 5a-5d) in Jung and Pruess (2012), except that the boundary of the model on the radial axis is defined at a non-zero value of r_{wD}. To simplify the boundary condition, the dimensionless variable for the *r* coordinate can be redefined as

$$r_{SD} = r_D - r_{wD} \tag{6}$$

Upon the shift of the dimensionless variable r_D, Eqs. 4a and 4b can be rewritten:

$$\frac{\partial T_{f1D}}{\partial t_{1D}} + \frac{\partial T_{f1D}}{\partial r_{SD}} - \left.\frac{\partial T_{m1D}}{\partial z_D}\right|_{z_D=0} = 0 \qquad r_{SD} > 0 \tag{7a}$$

$$\theta \frac{\partial T_{m1D}}{\partial t_{1D}} - \frac{\partial^2 T_{m1D}}{\partial z_D^2} = 0 \qquad r_{SD} > 0, \quad 0 < z_D < \infty \tag{7b}$$

The boundary condition in Eq. 5b can also be redefined:

$$T_{f1D} = 1 \qquad \text{at } r_{SD} = 0 \tag{8}$$

Now, the dimensionless governing equations and the initial and boundary conditions in a radial geometry are identical to those defined in a linear geometry, and similar modifications can be done for the rest and withdrawal periods. Therefore, the analytical solutions developed in Jung and Pruess (2012) can be applied to compute the time dependence of temperature recovery at the injection/withdrawal well in a horizontally oriented fracture with radial flow. The detailed derivation processes are given in Appendices A and B in Jung and Pruess (2012), and the analytical solution for thermal SWIW tests involving injection, quiescent, and withdrawal phases is shown below.

$$T_{f3D}(r_{SD},t_{3D}) = \int_0^{\min(t_{3D},t_{Dq}-r_{SD})} \frac{\sqrt{\theta}\xi}{2} \frac{\exp\left(-\dfrac{\theta\xi^2}{4(t_{3D}-\xi)}\right)}{\sqrt{\pi(t_{3D}-\xi)^3}} \times T_{f3D}(r_{SD}+\xi,0)\,\mathrm{d}\xi$$

$$+\int_0^{t_{3D}} \int_0^{\min(\tau,t_{Dq}-r_{SD})} \frac{\sqrt{\theta}\xi}{2} \frac{\exp\left(-\dfrac{\theta\xi^2}{4(\tau-\xi)}\right)}{\sqrt{\pi(\tau-\xi)^3}} \cdot \theta \int_0^{\infty} T_{m3D}(r_{SD}+\xi,\eta,0) \times \frac{\sqrt{\theta}\eta}{2} \tag{9}$$

$$\times \frac{\exp\left(-\dfrac{\theta\eta^2}{4(t_{3D}-\tau)}\right)}{\sqrt{\pi(t_{3D}-\tau)^3}}\,\mathrm{d}\eta\,\mathrm{d}\xi\,\mathrm{d}\tau$$

For a SWIW test conducted with no quiescent time, the solution (9) can simply be rewritten as

$$T_{f3D}(r_{SD},t_{3D}) = \int_0^{\min(t_{3D},t_{Di}-r_{SD})} \frac{\sqrt{\theta}\xi}{2} \frac{\exp\left(-\dfrac{\theta\xi^2}{4(t_{3D}-\xi)}\right)}{\sqrt{\pi(t_{3D}-\xi)^3}} \cdot T_{f1D}(r_{SD}+\xi,t_{Di})\,\mathrm{d}\xi$$

$$+\int_0^{t_{3D}} \int_0^{\min(\tau,t_{Di}-r_{SD})} \frac{\sqrt{\theta}\xi}{2} \frac{\exp\left(-\dfrac{\theta\xi^2}{4(\tau-\xi)}\right)}{\sqrt{\pi(\tau-\xi)^3}} \cdot \theta \int_0^{\infty} T_{m1D}(r_{SD}+\xi,\eta,t_{Di}) \cdot \frac{\sqrt{\theta}\eta}{2} \tag{10}$$

$$\times \frac{\exp\left(-\dfrac{\theta\eta^2}{4(t_{3D}-\tau)}\right)}{\sqrt{\pi(t_{3D}-\tau)^3}}\,\mathrm{d}\eta\,\mathrm{d}\xi\,\mathrm{d}\tau$$

3. Results and Discussion

3.1. Verification of Solution

In order to verify the analytical solution, it is compared with the numerical solution using TOUGH2 (Pruess et al., 1999). Since the difference in computational effort between Eqs. 9 and 10 is significant (Jung and Pruess, 2012), here we only show the result of the analytical solution evaluation for the case of no quiescent time (t_q = 0). The parameter

values used for comparison are listed in Table 1. The fracture is modeled as a porous domain with large permeability (5×10^{-12} m^2) and porosity (50 %), considering fracture walls to be rough and allowing for the presence of minerals in the fracture itself. The grid for the fracture consists of 1,000 blocks of 1 cm radial extent each, for a total length of 10 m in the *r* direction. The matrix domain is discretized in a non-uniform way to accurately model the heat exchange between the fracture and the rock matrix; the grid spacing for the matrix gradually increases from 0.01 cm at the fracture wall to 2 m, for a total thickness of 10 m in the *z* direction, so as to be infinite-acting for the time period considered here. At the end of the fracture opposite the injection block, boundary conditions are maintained constant at their initial values. The analytical solutions show excellent agreement with the TOUGH2 simulations (see Fig. 1). Note that TOUGH2 normally updates the specific heat and density of water at every time step according to the change in temperature. Here, this functionality in TOUGH2 was turned off to compare with the analytical solution, which assumes these parameters to be constant throughout the system over time.

Table 1. Parameter values used for TOUGH2 simulation

Reservoir Properties	Value
Fracture permeability	5×10^{-12} m^2
Matrix permeability	1×10^{-18} m^2
Fracture porosity, ϕ_f	0.5
Fracture aperture, $2b$	2 cm
Well radius, r_w	0.15 m
Thermal conductivity of matrix, k_m	2.1 W/m/ºC
Specific heat of matrix, c_m	1000 J/kg/ºC
Density of matrix, ρ_m	2650 kg/ m^3
Injection/withdrawal volumetric flow rate, q	1×10^{-4} m^3/s
Temperature of injected water, T_{inj}	20 ºC
Total injection time, t_i	5 hr
Total quiescent time, t_q	0 hr
Specific heat of water, c_w	4200 J/kg/ºC
Density of water, ρ_w	1000 kg/ m^3
Initial conditions	
Pressure	200 bar
Temperature, T_0	200 ºC

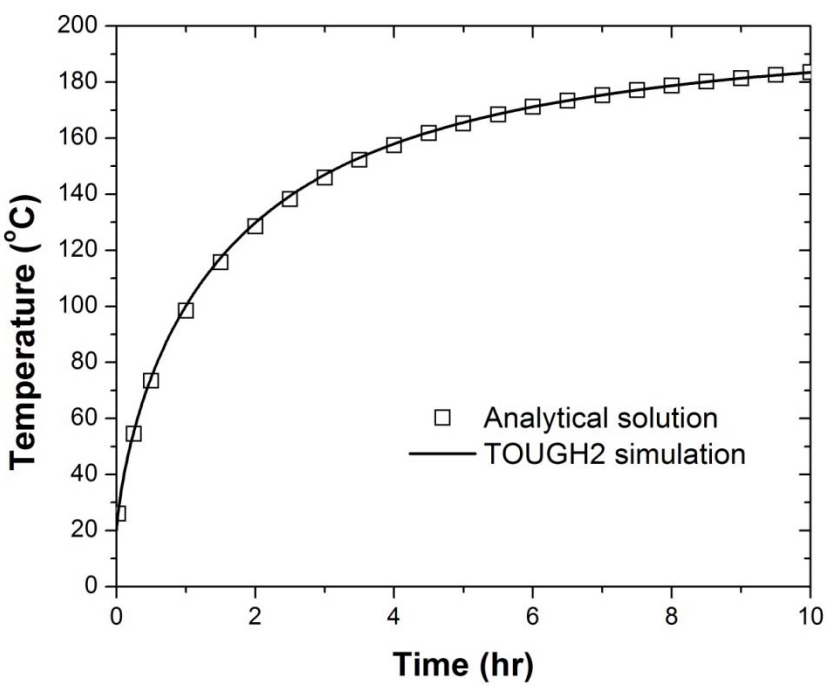

Figure 1. Comparison of the analytical solution with the numerical (TOUGH2) simulation. Temperature return profiles at the well during the withdrawal period.

3.2. Normalized Form of Solution

The analytical solutions developed for the linear and radial flow systems can be normalized by dividing the dimensionless time variable by the dimensionless form of the total injection time t_{Di} or the total quiescent time t_{Dq}. Here, we only show the normalized form of the analytical solution (10) by t_{Di}, since the analytical solution (9) can be similarly transformed by t_{Dq}. After normalization by t_{Di} at r_{SD} = 0, Eq. 10 becomes

$$T_{f3D}(t_{3DN}) = \int_0^{\min(t_{3DN},1)} \frac{\sqrt{\theta t_{Di}}\,\xi}{2} \frac{\exp\left(-\dfrac{\theta t_{Di}\xi^2}{4(t_{3DN}-\xi)}\right)}{\sqrt{\pi(t_{3DN}-\xi)^3}} \cdot T_{f1D}(\xi,1)\,\mathrm{d}\xi$$
$$+ \int_0^{t_{3DN}} \int_0^{\min(\tau,1)} \frac{\sqrt{\theta t_{Di}}\,\xi}{2} \frac{\exp\left(-\dfrac{\theta t_{Di}\xi^2}{4(\tau-\xi)}\right)}{\sqrt{\pi(\tau-\xi)^3}} \cdot \theta t_{Di} \int_0^{\infty} T_{m1D}(\xi,\eta,1) \cdot \frac{\sqrt{\theta t_{Di}}\,\eta}{2} \tag{11}$$
$$\times \frac{\exp\left(-\dfrac{\theta t_{Di}\eta^2}{4(t_{3DN}-\tau)}\right)}{\sqrt{\pi(t_{3DN}-\tau)^3}}\,\mathrm{d}\eta\,\mathrm{d}\xi\,\mathrm{d}\tau$$

where

$$t_{3DN} = t_{3D} / t_{Di}$$

$$T_{f1D}(\xi,1) = \mathrm{erfc}\frac{\sqrt{\theta t_{Di}}\,\xi}{2\sqrt{1-\xi}} U(1-\xi)$$

$$T_{m1D}(\xi,\eta,1) = \mathrm{erfc}\frac{\sqrt{\theta t_{Di}}\,(\xi+\eta)}{2\sqrt{1-\xi}} U(1-\xi)$$

As shown in the analytical solution (11), the product of θ and t_{Di} is the only parameter affecting temperature return curves. The importance of these parameters has been pointed out in Jung and Pruess (2012), and a sensitivity analysis for the key parameters was conducted. For the analytical solution with a rest period, therefore, t_{Dq} is

the additional parameter that has an impact on return temperatures. In addition, using these normalized analytical solutions helps reduce errors in numerical calculations of this multi-dimensional improper integral.

Nomenclature

b = half fracture aperture, m

c_f = average specific heat of rock and fluid in fracture, J/kg/ºC

c_m = specific heat of rock matrix, J/kg/ºC

c_w = specific heat of water, J/kg/ºC

k_m = thermal conductivity of rock matrix, W/m/ºC

q = volumetric flow rate, m^3/s

r = distance along flow direction, m

r_w = well radius, m

t = time, seconds

t_i = total injection time, seconds

t_q = total quiescent time, seconds

T = temperature, ºC

T_{inj} = temperature of injected fluid, ºC

T_0 = initial reservoir temperature, ºC

z = distance normal to flow direction, m

ϕ_f = porosity of fracture

ρ_f = average density of rock and fluid in fracture, kg/m^3

ρ_m = density of rock matrix, kg/m^3

ρ_w = density of water, kg/m^3

θ = dimensionless parameter in (3c) ($\theta = \rho_m c_m / \rho_f c_f$)

Subscripts

f = fracture

m = matrix

w = water

D = dimensionless

N = normalization

S = shift

1 = injection phase

2 = quiescent period

3 = withdrawal phase

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