ERNEST ORLANDO LAWRENCE
BERKELEY NATIONAL LABORATORY

Uncertainties in Air Exchange using Continuous-Injection, Long-Term Sampling Tracer-Gas Methods

Max H. Sherman, Iain S. Walker, Melissa M. Lunden

Environmental Energy Technologies Division

December 2013

Disclaimer:

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

Acknowledgment:
Funding was provided by the U.S. Dept. of Energy Building Technologies Program, Office of Energy Efficiency and Renewable Energy under DOE Contract No. DE-AC02-05CH11231; by the U.S. Dept. of Housing and Urban Development Office of Healthy Homes and Lead Hazard Control through Interagency Agreement I-PHI-01070; by the U.S. Environmental Protection Agency Office of Air and Radiation through Interagency Agreement DW-89-92322201-0 and by the California Energy Commission through Contract 500-08-061.

Uncertainties in Air Exchange using Continuous-Injection, Long-Term Sampling Tracer-Gas Methods

Max H. Sherman, Iain S. Walker, Melissa M. Lunden

ABSTRACT

The PerFluorocarbon Tracer (PFT) method is a low-cost approach commonly used for measuring air exchange in buildings using tracer gases. It is a specific application of the more general *Continuous-Injection, Long-Term Sampling (CILTS)* method. The technique is widely used but there has been little work on understanding the uncertainties (both precision and bias) associated with its use, particularly given that it is typically deployed by untrained or lightly trained people to minimize experimental costs. In this article we will conduct a first-principles error analysis to estimate the uncertainties and then compare that analysis to CILTS measurements that were over-sampled, through the use of multiple tracers and emitter and sampler distribution patterns, in three houses. We find that the CILTS method can have an overall uncertainty of 10-15% in ideal circumstances, but that even in highly controlled field experiments done by trained experimenters expected uncertainties are about 20%. In addition, there are many field conditions (such as open windows) where CILTS is not likely to provide any quantitative data. Even avoiding the worst situations of assumption violations CILTS should be considered as having a something like a "factor of two" uncertainty for the broad field trials that it is typically used in. We provide guidance on how to deploy CILTS and design the experiment to minimize uncertainties.

Introduction

Building ventilation is the primary process used to insure acceptable indoor air quality by removing pollutants from indoor sources as well as conditioning the air for occupant comfort. In many buildings, ventilation occurs by the uncontrolled leakage of air through the building envelope termed infiltration. National efforts to improve building energy efficiency have focused on reducing infiltration by making homes more airtight. In the absence of mechanical ventilation, reduced infiltration can lead to elevated concentration of pollutants indoors. The use of mechanical ventilation, however, can result in increased energy use which can offset the reduced energy losses through improved airtightness. Thus, accurate measurement of the ventilation rate, or air exchange rate, is key to assess the energy and air quality impacts of infiltration (including exfiltration). Having a reliable

estimate of building ventilation can also be necessary to characterize other indoor phenomena, such as the emission rate of contaminants indoors.

Ventilation is often expressed as an air exchange rate, where the air flow rate is normalized by the building volume. Air exchange rate varies as a function of HVAC operation, meteorological conditions, and changes in the configuration of the building envelope (e.g. windows open or closed). For buildings dominated by uncontrolled airflow through the building envelope, tracer gas techniques are the primary method used to measure air exchange rates. ASTM Standard E741 describes several methods for making tracer gas measurements. The two most common tracer gas methods are the decay rate and constant injection rate methods [Basset et al, 1981; Condon et al, 1981; Dietz et al, 1982; Grot, 1980; Harrje and Grot, 1977]. A third method uses a constant concentration approach in which the quantity of tracer emitted is varies to maintain a fixed target concentration. This allows the tracking of ventilation rate over short time scales but requires on-site tracer analysis in real time as well as computer controls for the tracer injection equipment – both of which require extra cost and complexity – hence it's limited application.

The decay rate method entails the injection of a tracer gas, mixing it to uniform concentration throughout the building and measuring the decay in real time over a few hours [Basset et al, 1981]. The method provides a robust measurement of the air exchange rate, but requires trained technicians to be onsite and only allows for the air exchange rate to be determined over short time-scales . This method assumes that the ventilation rate does not change during the experiment, so care must be taken to avoid variable weather conditions and changing building envelopes such as opening windows. Such techniques are well suited to research-grade investigations or very small sample sizes, but are often impractical for larger, more cost-constrained studies.

The constant injection method involves placing a number of emission sources, whose emission rate is well known and controlled – often using sophisticated mass flow controllers - of one or more tracer gases in a house together with samplers to measure the concentration of the gas over a period of time that can range from hours to days [Condon et al, 1980]. The time-averaged air exchange rate is determined from the volume of gas tracer emitted into the house and the concentration of that tracer measured by the sampler. Simpler methods utilize a passive technique to obtain relatively constant emission of tracer, such as the evaporation of a liquid through a controlling membrane initially developed by Brookhaven National Laboratory. This method is often called the "PFT" method because it used PerFluorocarbon Tracer gases. The defining characteristic of this technique is not the tracer gasses themselves but the fact that they use Constant Injection and Long-Term Sampling in the field. We shall refer to this technique with a more generic title of CILTS.

The CILTS method is widely used due to the small size and low cost of the(tracer gas) emission sources and samplers, the flexibility in measurement duration, and because it can be deployed using personnel with limited training. This is particularly important for applications such as field projects that require the measurement of the air exchange rate in large numbers of homes [e.g. Clayton et al., 1993; Ozkaynak et al., 1996; Weisel et al., 2005; Offermann, 2009]. There is general guidance regarding the number of (emission) sources

that should be placed based on the total area of the space [ASTM E741 2000]; sources and samplers are largely placed in a specific locations within a home based on the convenience for occupants and engineering judgment.

Due to its widespread use, it is important to have a reasonably good idea of the uncertainties associated with the CILTS method, but limited analyses exist. The factors that affect measurement uncertainty include uncertainties in the tracer emission rate, the measured tracer concentration, the time rate of change in the tracer concentration, and the spatial variability of tracer concentration within the house. Previous studies investigated some of these uncertainties [Dietz and Cole, 1982; Leaderer et al, 1985], but do not explicitly discuss the implications for the resulting air exchange rate. D'Ottavio et al, 1988, showed how to analyze the data but contains no error analysis. The objective of this work is to estimate the sources and magnitude of errors for a typical single-zone application of CILTS. These uncertainties will be examined through analysis of field data using a theoretical uncertainty analysis method developed by Sherman (1988). Based on the analysis, recommendations for reducing the errors of using the CILTS method will be listed.

Uncertainties estimates for the CILTS method

All tracer gas methods use the continuity equation to calculate the air exchange rate from the measured tracer concentrations and other experimental parameters. The continuity equation for a single zone is as follows:

$$V \times \dot{C} + Q \times C = S \qquad (1)$$

where V is the zone volume (m^3), Q is the ventilation rate of the zone (m^3/h), C is the tracer concentration (g/m^3) (assuming no outdoor concentration), $\dot{C}$ is the time rate of change of tracer concentration ($g/m^3/h$) and S is the tracer emission rate (g/h). The ventilation rate generally varies as a function of time, which is directly reflected in the term for the time rate of change of tracer concentration. However, the CILTS method results in a single measurement of tracer gas concentration averaged over the time period of the experiment. Therefore, the use of the continuity equation to calculate the air exchange rate measured using the CILTS method requires that sampling time period be sufficiently long enough that the transient changes in concentration can be neglected. When this is the case, an average air exchange rate, A (1/h) can be determined from the measured tracer emission rate and the measured concentration as follows

$$Q = S / C = A \times V \qquad (2)$$

When the emitters are first placed in the building there is an additional transient period during which the tracer reaches equilibrium in the home. It is also important that the tracer sampling period either avoids this initial transient period or that the sampling period is long enough so that this transient period is inconsequential to use Eq. 2 to calculate the air exchange rate.

There are a number of errors to consider when calculating the air exchange rate using Eq. 2 with the CILTS method. The first types are instrumentation errors associated with the measurement of the tracer gas emission rate and concentration. The second types or errors are those arising from the simplified model of the continuity equation used to

interpret the data or how actual flows violate the modelling assumptions. The subsequent discussion will discuss these different sources of error in detail.

Instrumentation Error

Instrumentation error encompasses all of the errors in the directly (or indirectly) measured quantities of average emission and concentration. The contribution to the uncertainty to the calculated air exchange rate follows from Eq. 2 and can be expressed as:

$$\left(\frac{\delta Q}{Q}\right)^2 = \left(\frac{\delta A}{A}\right)^2 = \left(\frac{\delta S}{S}\right)^2 + \left(\frac{\delta C}{C}\right)^2 \tag{3}$$

There can also be an error term for the uncertainty in the volume of the space, but for this effort, we shall assume that this is small. At steady-state the volume error only affects the volumetric air flow to air change rate calculation and is, therefore, not fundamental to the measurement technique, but volume errors can be significant issues whenever converting from mass/volume flows to air changes.

Estimating the uncertainty of the average tracer emission rate, S, is a straightforward exercise. The total mass emitted is often measured gravimetrically by weighing the emitter before and after the tracer gas sampling period, and the result can be highly accurate. Without gravimetric measurements, the emission rate can be found by laboratory calibration, which will generally be less accurate. The emission rate may not be constant, but if the changes in the emission rate are not correlated with variations in the air change rate, small variations in emission rate will not affect the results provided the sampling time is long enough. We will, therefore, consider the tracer emission rate to be constant, but that assumption should be evaluated when atypical protocols are being considered. Equivalently we could consider the non-stationery emission rate to be part of the error in the emission rate measurement.

The error in the measured tracer gas concentration is due to errors in both sample collection and analysis. The analytical technique used to measure the amount of tracer gas in the sample can have precision errors due to variability in instrument response and bias errors due to imperfect calibration. The errors in sample collection are primarily due to uncertainly in the value of sampling rate and a sampling rate that may not be constant. Both of these errors are of particular importance with concentrating samplers (e.g. sorbent tubes) due to effects of changing temperature and potential sample saturation. Non-concentrating sampling techniques such as bag sampling do not experience the same issues with respect to temperature and saturation effects. There is a modelling error (discussed later) associated with assuming the concentration is spatially homogeneous at the spatial average, but there is also a measurement error associated with determining the average concentration, which is the more common problem of sampling variations. The uncertainty associated with measurement errors can be reduced by using multiple samplers in the usual manner, but the modelling error cannot, because it is a bias error.

Most experiments using the CILTS technique do not analyse the concentration data in the field, and transporting the sample to the laboratory for analysis is an opportunity for sample degradation. A thorough discussion of best practices is beyond this report, but some example issues might be beneficial: For instance, some of the sample may be lost in

transit or storage due to leakage. This is particularly important for concentrating samplers where the concentration measurement is a function of the total collected mass. For non-concentrating samplers, such as units that directly sample room air into a bag, the loss of part of the sample is less important because analysis results directly in a concentration. For concentrated samples any loss will create a negative bias. For all kinds of samples, contamination of the sample in transportation can result in error in either direction. Contamination can come from samplers and emitters being proximate or from other chemicals which might mimic or interact with a tracer.

Combining these factors results in the following expression on uncertainty due to instrumentation error,

$$\left(\frac{\delta Q}{Q}\right)^2_{instrument} = \left(\frac{\delta A}{A}\right)^2_{instrument} = \left(\frac{\delta S}{S}\right)^2_{emit} + \left(\frac{\delta C}{C}\right)^2_{collect} + \left(\frac{\delta C}{C}\right)^2_{transport} + \left(\frac{\delta C}{C}\right)^2_{analyze} \quad (4)$$

where the uncertainty in the measured tracer gas concentration, C, is expanded to include those errors arising from collection, transport, and analysis as discussed above. With the right equipment and good experimental technique, it is possible to reduce these instrumentation errors to acceptably low levels.

Model Errors

To analyse the data in the CILTS approach the following assumptions are made:
1. it is assumed that the system is in steady-state such that the concentration has had sufficient time to reach equilibrium that transient effects are unimportant
2. the parameters are stationary. (i.e., that the air exchange is truly a constant over the measurement period, and
3. the space is a single-zone and the tracer concentration is homogenous throughout the space). (Any inhomogeneity is due to an interaction between emitter placement and the actual air flow patterns. It is not necessary to know how it occurred.)
Each of these assumptions has an intrinsic error that is dependent o)n the system being measured rather than the instruments measuring that system. We will examine each of these errors individually (i.e. assuming no instrumentation or other model errors contribute) and then combine them assuming they are independent.

Steady-State Assumption Errors

The time dependent continuity equation, Eq. 1, includes the time rate of change of the tracer concentration, thus a complete solution will have include a term that accounts for the initial concentration. The CILTS method assumes that transient changes in concentration can be neglected, and so represents a source of error that depends on the difference between the initial and final concentrations.

$$\left(\frac{\delta Q}{Q}\right)_{steady-state} = \left(\frac{\delta A}{A}\right)_{steady-state} = -\frac{V}{S}\frac{\Delta C}{\Delta t} = -\frac{1}{A\Delta t}\frac{\Delta C}{C} \quad (5)$$

The bias from this error could be corrected if we knew the initial and final concentration. Since CILTS only measures the average concentration over the sampling period, we cannot correct the result without some prior knowledge of the system. For instance, if we know that the initial concentration was zero, Eq. 5 can be used recursively to correct the CILTS

result. However, as previously discussed, successful implementation of the CILTS method requires sufficiently long sampling times that these transient changes in concentration, which should make this error quite small. (We can't actually calculate that time a priori as it requires knowing the answer first, but likely a good experimenter will have an estimate that can be used to bound it.) We shall assume this is the approach taken and that any calculable biases have been taking into account.

Constant Air Exchange Assumption Errors

The air exchange rate will most likely vary over the sampling period, so the tracer gas concentration will be varying over time. The CILTS method measures the average concentration, but the air exchange rate is inversely related to the concentration. Thus, the CILTS analysis will underestimate air exchange rather than providing in a true average air exchange rate. If the variation is small, the bias can be corrected for (See Sherman (1989a) for details), however, the bias can be intractably large if the variation in air exchange is large - as might be the case for an experiment where windows are opened and closed during the testing or the weather changes significantly. This magnitude is important when the measured *average* air exchange rate is used for energy calculations. However, the effective air change rate from CILTS is the correct air change to use for investigating the dilution of indoor contaminants.

Homogeneity Assumption Errors

The CILTS analysis assumes that the space can be treated as a single zone and that the concentration is the same everywhere in this zone. Incomplete mixing, however, can result in substantial variability in tracer concentration within the zone, resulting in a measured concentration that may not be representative of the space as a whole. In addition, the average concentration measured in the zone may not be the representative concentration needed in the CILTS analysis because the continuity equation requires that the representative concentration must be the flow-weighted average concentration of the air flowing from the space to outside.

To investigate the errors due to inhomogeneity, we have broken down the putative single zone into a set of N interacting multizone spaces. Details of the analysis are reported in the appendix. The results show that, even if the spatial average concentration could be measured with minimum uncertainty, there would be an error in the calculated average air exchange rate induced from the spatial inhomogeneity as follows:

$$\left(\frac{\delta Q}{Q}\right)^2_{spatial} = \left(\frac{\delta A}{A}\right)^2_{spatial} = N\left(\frac{\delta\ C_{rms}}{C^2}\right)^2 + N\left(\frac{\delta\ S}{S}\right)^2 \qquad (6)$$

Combining these errors results in the following expression for the uncertainty in a CILTS measurement:

$$\left(\frac{\delta Q}{Q}\right)^2_{effective} = \left(\frac{\delta A}{A}\right)^2_{effective} = \left(\frac{\delta C}{C}\right)^2_{collect} + \left(\frac{\delta C}{C}\right)^2_{transport} + \left(\frac{\delta C}{C}\right)^2_{analyze} + \left(\frac{1}{A\Delta t}\frac{\Delta C}{C}\right)^2 + N\left(\frac{\delta\ C_{rms}}{C^2}\right)^2 + N\left(\frac{\delta\ S}{S}\right)^2 \quad (7)$$

The last terms, from Eq. 6, are proportional to the number of actual zones. Note that the single-zone "emission" error has been replaced by the multizone one.

Error Analysis of CILTS data

An intensive investigation of the CILTS method was recently performed in three test homes (Lunden et al, 2012)). The tests used multiple simultaneous PFTs sampling at high spatial density in multiple configurations to evaluate the precision of the technique and to provide guidance on the best way to deploy emitters and samplers. This data set, hereafter referred to as the "Lunden data", is used to examine the errors that result from the CILTS method in the error analysis presented above. Each test house used four different PFTs with different sample densities and two different sampling methods, resulting in four separate experimental measurements of the same air exchange rate. The differences between the experiments serve to identify which factors are most important with regards to experimental uncertainty. In addition, the high spatial density of sampling locations in the experiments will help to quantify the spatial variability in tracer concentration.

The experiments were designed to investigate a range of ventilation conditions. These experimental ventilation conditions included no forced air system operation, normal operation of an air conditioner, constant operation of the forced air system fan, and other variations. The specific experimental ventilation conditions used for each of the three homes are listed in Table 1.

Estimates of precision and bias errors that are the same for each experiment are as follows:

- Emission source: The PFTs sources were the same type of emitter device for all experiments. They consisted of liquid in a glass vial with a septum through which the gas diffused. The vials were placed in dry block heaters to keep the emitters at a constant temperature. This eliminates a source of error due to changing emission rates with temperature. Given how simple and easy it is to use these block heaters are, they are highly recommended for use in CILTS experiments. The emission rate for each vial was measured gravimetrically on site using a high precision scale. The accuracy of these scales is assumed to be on the order of 1%.

- Collection and Transport: We shall assume that any precision errors due to sample collection will be reflected in any inhomogeneity of the measured concentrations, and will thus only consider bias errors. For the purposes of this analysis, we shall assume an empirically-based bias error of 3% and no transportation error.

- Analysis: The tracer gas analyser had a precision error of 5%. This uncertainty is reduced as multiple samplers are used to estimate the mean concentration. During the

analysis of the tracer gas samples, a significant bias was discovered and corrected, with no residual bias reported; Lunden (2012) has more details on this issue.

Error estimates for more conventional CILTS experiments may differ from those in Lunden data. Errors associated with emission sources can sometimes include transport and handling that will affect the precision of the measured emission rate, i.e. sources are shipped to and from experimental locations by mail can cause losses in the tracer that will result in an emission rate that may be biased high. The effect of temperature will be important for emission rates that are not determined gravimetrically. Lunden et al. (2012) found that emission rates for the LBNL vials varied by approximately 4% for every 1 °F change in temperature. The temperature dependence of other emitter types may differ, and should be characterized, but this value can provide an estimate of the importance of this error if important. Collection and transport errors that affect the tracer gas samples may occur in high-volume, mail-in, or occupant-performed experiments can be much more common, and should be carefully considered. Analysis errors are present in any experiment, and must always be methodically estimated.

Using the values listed above for the Lunden study, the errors are as follows:

$$\left(\frac{\delta Q}{Q}\right)^2 = \left(\frac{\delta A}{A}\right)^2 = \left(\frac{1}{A\Delta t}\frac{\Delta C}{C}\right)^2 + N\left(.05\right)^2_{emit} + \left(.03\right)^2_{collect} + \left(.02\right)^2_{analyser} + N\frac{\delta^2 C_{rms}}{C^2} \quad (8)$$

One of the samplers used in the experiment collected time resolved gas samples, resulting in 15 measurements of the tracer gas concentration every 24 hours. The time resolved results provide a way to estimate the magnitude of the time varying concentration. We estimate the magnitude of this variation (ΔC) as the difference in the concentrations calculated at the 95% confidence limits. Using a Δt of 24 hours and the measured average air exchange rate and concentration, the magnitude of the time resolved term in Equation 8 ranged 3% to 16% depending on the experiment. The largest values tended to occur when there was no central air handling fan operating. The use of the economizer in House 2 resulted in the largest tracer gas concentration variability with time due to the large changes in air change rate induced by this system. The average value of the time varying term for all conditions with (continuous) central forced air fan operation was 4%.

Thus the error for the Lunden study becomes approximately the following:

$$\left(\frac{\delta Q}{Q}\right)^2 = \left(\frac{\delta A}{A}\right)^2 = 0.0034 + N\left(0.0001 + \frac{\delta^2 C_{rms}}{C^2}\right) \quad (9)$$

If we disregard the time varying term and assume that the space is truly a homogeneous single zone, the air exchange rate resulting from the CILTS method as deployed by Lunden et al (2012) would have an uncertainty of 6%. Assuming a value of 4% for the time varying term increases the uncertainty to 7%. This uncertainty estimate represents the minimum uncertainty in the measured air exchange rate. It is highly unlikely that the tracer gas concentration in a home would ever be homogeneous. The extent to which spatial homogeneity contributes to the uncertainty can be assessed for the Lunden data due to the relatively high spatial density of samplers deployed in their experiments.

Spatial Homogeneity Errors in the Lunden Data

House 1

House 1 in the experiments conducted by Lunden et al (2012) is a 93 m^2 (1000 ft^2) single story house with a simple, compact floor plan. Four different PFTs were deployed, each with a different spatial distribution of emitters, and in some cases, samplers. The tracer gases were over-sampled (i.e., using more locations) compared to a typical CILTS measurement to allow a better estimate of the spatial variation. The number of emitters for each PFT is specified in Table 2. Table 2 also lists the average air exchange rate for the house calculated using each PFT as well as the spatial coefficient of variance. In their report, Lunden et al. (2012) divided the space into nine zones. Some of the zones are small enough to ignore or sufficiently well coupled to be considered a single zone. As a result, we shall assume four zones in our error calculations, recognizing that this may be an under estimate. The air handler in house 1 turns over the air seven times per hour.

In the test in which the central air handler was not run the average air exchange rate from the four tracers was 0.5 ACH with a standard deviation from the different tracer approaches of 13%. The four tracer gasses all showed a spatial variation of 16%-22% with an average of 20%. Using Equation 10 to estimate the error we expect an uncertainty of 40%. This value is much larger than the 6 to 7% uncertainty due to all other sources of error and bias, showing that the heterogeneity in the measured tracer concentration dominates the overall uncertainty of the measured air exchange rate. The air change rate measured with this data has an unknown bias, but the standard deviation of 13% between the four tracer gases is well within the 40% estimate of the overall uncertainty. Thus if only that standard deviation were considered as a measurement of the error, the total uncertainty would be under-estimated.

The average air exchange rate for the experiment with constant central forced air fan operation was 0.87ACH with a standard deviation of 29%. This value is higher than with the air handler off, and may be due to the contribution of duct leakage. There is also a larger standard deviation between the four tracers. The average spatial variation, 12%, was smaller for this condition, but had a larger range of values. The larger range of spatial variation is largely due to the results from tracer 3, which had only two emitters in the house. Discounting this value, the total uncertainty we would expect in this test is 18% but because of the outlier we see almost 25%.

Lunden attributes the outlier to the fact that the concentrations analyzed were low and close to the detection threshold for the analyzer. This type of error can happen because of the difficulties of knowing the air exchange rate and therefore the required emission rate as well as the appropriate number and location of emitters and samplers before starting the experiments. This problem is particular to these passive measurements that lack the instant feedback from real time measurements.

House 2

House 2 was a 325 m^2 (3480 ft^2) ranch style home with a long narrow floor plan. This house had two central forced air systems and with them both operating the air was

circulated 4.3 times per hour. The number of emitters for each PFT is specified in Table 3. The house was divided into nine zones. Unlike the more compact configuration of house 1, these zones do not combine as easily and nine may be an under-estimate. In the error calculations, we shall use nine as the physical number of zones. The average air exchange rate for the house calculated using each PFT as well as the spatial coefficient of variance is listed in Table 3.

During economizer operation, one of the central systems supplied air from outside at the airflow rate used by the central system in normal recirculation mode – in this case about 3 ACH. This is a much higher flow than natural infiltration or most mechanical ventilation. For the three experimental conditions, it appears that the results from tracer 3, which has only one emitter, has a higher spatial variability than that observed for the other three tracers. (One emitter for a house this size would not be good practice, but we wished to explore its effect.) This is similar to the results for house 1, where there was a significantly higher spatial variability for one of the experimental conditions for the tracer with only two emitters.

In Normal Operation, the average measured ACH from the 4 tracer gasses is 0.27 ACH with a standard deviation of 10%. The coefficient of variation was the same (15%) for three tracers , and almost double this at 28% for tracer 3, that only has one emitter. Using the average coefficient of variation (CV) of 18% and nine zones results in an ACH total uncertainty of 55%. Continuous fan operation reduced the CVs for tracers 1-3 but not for tracer 4, and had twice the effect for tracer 3 that had the biggest CV. This indicates that the fan operation can help a lot to reduce mixing errors, but not in all cases. The concentration variation for the home with continuous fan operation is between 12% and 22%, with an average of 15%, leading to an overall estimated uncertainty of 45%, while the measured air change is 0.42 ACH with a standard deviation of 9%. In economizer mode, the economizer operates at times when outdoor air will cool the home. This leads to times of very high air change rate when the economizer is operating and much lower air change rate at other times. The resulting concentration variation is between 10% and 25% with an average of 20%, leading to an estimated uncertainty of 60%. The measured air change rate is 0.29 ACH with a standard deviation of 7%. This apparent reduction in variability in ACH in economizer mode when we know the time varying ventilation is large is an indicator that other factors, such as weather changes and opening of windows, can have as big an influence on variability and errors as well known and characterized ventilation changes. In addition, the reduction in air change rate observed here, when we know the economizer significantly increases ventilation rates, is another indicator of the uncertainties in the PFT method – particularly the underprediction of average air change rates when air change rates are not constant.

We note again that the standard deviations of our measured values are significantly smaller than our estimated uncertainty. If all one cared about were repeatability this would indicate that our error estimate was too large, but our error estimate includes errors caused by model violations—in particular the fact that the average concentration may not be the same as the exfiltration weighted concentration - and all the tracers have these errors. An example of this from the House 2 experiments was that a couple of rooms on the windward side had slightly open windows. This resulted in a net flow of air across the

house leaving the windward rooms at lower concentrations. These rooms likely had less exfiltration and the samples from these rooms should have been weighted less. Thus we might expect a positive bias in the results, i.e., the experiment overestimated the air change rate.

The increased mixing due to central forced air heating and cooling system air handler operation reduces the variability in concentrations from zone to zone. It also shows increased air exchange - probably due to leaky ducts. The improvement in homogeneity is most noticeable for the experiments that had the fewest number of emitters.

House 3

House 3 experiments were designed to evaluate the effects of different distributions of emitters. House 3 was a 237 m^2 (2540 ft^2), 3 story, open-plan house. Unlike houses 1 and 2, the tracer gas emitters were placed differently in house 3. Each floor had a unique tracer associated with it in order to better identify distribution patterns. A fourth tracer was evenly distributed. The number of emitters for each PFT and the floor location are specified in Table 4. Operation of the air handler fan introduced 3.4 ACH of internal mixing. House 3 was divided into 12 zones within the space spread over 3 floors. Because of the large stack effect in this home, there are generally much larger differences in tracer gas concentration from floor to floor than between most rooms within a single floor. Since the spatial variability is driven by vertical stratification in the house and each floor is open-plan, we shall use the three floors as the number of zones.

If we look at the case where the tracer was emitted everywhere the air exchange was 0.26 with an estimated error of 40% when the air handler was not running and 0.3 with an estimated error of 29% when the air handler was running. (Based on CVs of 23% and 17% respectively.) Since the natural ventilation air change rate was stack dominated for this house we would expect that the single tracer emitted only on the lower floor would give results similar to the tracer emitted everywhere. The air exchange for this single tracer was 0.26±47% with the air handler off and 0.32±9% with the air handler on, which confirms that this is indeed the case. By contrast if we use the data from the tracer injected only on the upper floor the result is quite different: 1.3±211% with the air handler off and 0.98±124% with the air handler on. The very large positive bias is the result of there being very little third floor tracer on the lower two floors due to the internal stack driven airflow from the lower to upper floors. These results indicate that if sampler locations are poorly chosen the errors are so large that the results are not useable. Again, we have the problem for passive methods that we do not know a priori (or even during the experiment) that these errors are occurring. The best we can do to minimize this problem it to emit and sample tracers on all floors of buildings and in more than one location per floor.

Additional experiments were performed in house 3 with interior doors closed and a kitchen exhaust on the second floor operating. With no air handler fan, this mode of operation showed the biggest special variation for each tracer with a range of CV from 68% to 196% and a mean of 114%. The estimated error increased to 118% for the tracer emitted everywhere and for the third floor tracer the error increased to over 300%. With

the air handler fan operating these errors decreased only marginally to 107% and 253% respectively. This result shows that even with the air handler operating it could not overcome the compartmentalization due to closed doors combined with highly non-uniform exfiltration.

Mixing and Emitter Density

We can use the complete dataset to get some idea of whether mixing and/or density of emitters can have a significant impact on the CILTS errors. We know these errors are dominated by spatial concentration inhomogeneities and so anything that reduces those will reduce the error.

Figure 1 plots the spatial CV as a function of source emitter density. Different symbols indicate whether the air handler was on or off or ran intermittently. The size of the emitters and air handlers varied from experiment to experiment so quantitative estimates of the impacts are problematic but we can see some general trends. The clearest signal is that running an air handler reduces inhomogeneity and thus improves the accuracy of the measurement significantly. Using a central air handler may not always be possible, however, due to absence of a central unit or because operating the unit would change the desired experiment.

If one cannot mix with an air handler (or similar device) then there is a trend toward reduced inhomogeneity with increased emitter density. As discussed earlier it may be desirable to have higher emission rates in some zones than another, but for a given emission rate more physically separate emitters is generally preferred.

Applications to other experiments

In the Lunden data there were many redundant measurements, allowing for the uncertainty to be determined with a higher degree of confidence. More typical execution of the CILTS method will result in sparser data. For instance, there may be only a few emitters and perhaps only a single sampler. These experiments will require estimates of quantities like the spatial inhomogeneity, making it difficult to calculate a credible estimate for the uncertainty of the resulting air exchange rate. The uncertainty will depend strongly on the number of zones and assumed heterogeneity. For instance, using the same instrumentation and experimental error estimates as in Lunden et al. (2012), Equation 10, a 4 zone structure with a 20% homogeneity in tracer concentration results in a 40% uncertainty in the calculated air exchange rate. A home with 7 zones with an assumed 30% inhomogeneity will result in an 80% uncertainty.

The issue of whether a factor of 2 is good enough or 10% is not good enough depends on how the number is to be used. When the air exchange is used as an input to a calculation of, for example, energy or contaminant emission rate any error in the air exchange will propagate through and limit the certainty of the final answer. If the value is to be used as part of a large statistical sample, a lot more error can be tolerated than if it is being used to answer a question specific to that particular building. This would be particularly true if were an issue of compliance with some code, standard, or program.

Recommendations for CILTS Applications

CILTS can be an accurate and precise method for determining air exchange when the system being measured matches the model assumptions—in particular that the air exchange and source emission are constant and that the system is in fact a single, isolated, well-mixed zone. Such a situation may occur in the laboratory or field studies with low air exchange rates and high internal mixing (e.g., due to operating a central forced air heating or cooling system air handler).

However, CILTS is most used in homes where we know the assumptions are violated to at least some non-trivial degree. The uncertainties associated with these violations can be minimized by careful experimental design and deployment. The recommendations below will reduce the uncertainty of the CILTS result:

- *Emission Rate*: The emitters typically used by CILTS are passive emitters whose rate changes slowly over time, but more importantly is a function of temperature. The emitters should be placed in a temperature controlled environment to keep their emission rate constant during the experiment. The emitters should be calibrated for each experiment or they should be gravimetrically weighed before and after each experiment such that the total amount of emission is determined.
- *Emitter Deployment*: Emitters should be deployed in proportion to the local infiltration to improve homogeneity. Of course the infiltration is not actually known; so this becomes a judgment by the experimenter. In many instances the best strategy is to deploy them evenly around the perimeter on all floors of the building. In instances where we know the air flow patterns, such as in the winter in a stack-dominated building, we know that the infiltration will predominantly happen in the lower parts of the building; so our emitter deployment should be predominantly in the lower parts.
- *Sampler Deployment*: As with the emitters, if we have no a priori knowledge, the samplers should be deployed evenly throughout the building. However if we can predict some of the air flow patterns in advance the emitters should be placed near areas of exfiltration. For example, if there is a prevailing wind direction, the samplers should preferentially be placed away from the windward side.
- *Sampler Number*: We recommend using a sampler for every 250-300 sq.ft. (25-30 sq. m) of floor area. An advantage of using multiple samples in addition to improving special averaging is that we can use the results to improve uncertainty estimates based on the standard deviation of the sampler results
- *Mechanical Mixing*: When additional mixing (e.g. by use of air handler) can be applied it will improve homogeneity and reduce uncertainty. Care must be taken, however, to assure that the mixing does not change the system being measured. If the duct system connected to the air handler is leaky, for example, use of the air hander to provide additional mixing may increase the air exchange.
- *Experiment Duration*: To avoid issues from initial equilibrium transient effects a good practice is to deploy emitters for 24 hours before sampling begins. If this cannot be done, the integration time for CILTS must be at least 24 hours and preferably longer. The integration time, however, should not be so long that the fundamental flow paths have changed—for example going from a stack dominated to wind dominated pattern. In such a case there will be a bias to the results and the estimate of the uncertainty from the spatial concentration variance will be under estimated.

Conclusions

We have analysed the results from oversampled field experiments with multiple gases and sampler/emitter locations and combined them with an error analysis to show that the CILTS method can have an uncertainty of 6-15% under ideal conditions. Ideal conditions include quality calibration of experimental equipment, correct placement of samplers and emitters relative to air flow patterns in the building, and a constant ventilation rate. All these things are generally impossible to achieve in a typical field experiment and real laboratory analyses; thus we should not expect to get close to ideal results.

Deviations from ideal conditions include several issues related to effective sampling. Overall the most important factor about the system is the degree of mixing (i.e. how closely it is a single, well-mixed zone). It is not sufficient to measure the average concentration correctly as spatial inhomogeneities themselves introduce additional uncertainties. The experimental data suggests that even with optimum emitter and sampler placement, CILTS uncertainties of 20-25% should be expected when no special provisions are made for mixing. The amount of (intermittent or continuous) mixing needed depends on the air exchange and also the air flow patterns, but most household central heating and cooling systems operating to provide roughly 5 ACH of internal mixing should be adequate for most experiments. Of course adding mixing can create its own biases, by changing the system being measured.

When the infiltrating and exfiltrating flows are not evenly distributed around the parts of the building errors increase. The induced errors can, in principle, be mitigated by careful placement of the samplers (near exfiltrating areas) and the emitters (near infiltrating areas). This requires that those patterns persist through the experiment and that the experimenter knows what the pattern is.

Variations in the air exchange during the experiment will result in a negative bias on the inferred average air change rate independent of the issues of mixing and the need to change the optimal deployment. CILTS measures the effective air exchange not the average air exchange. The effective air exchange is the value relevant for dilution and most IAQ purposes, but not for energy purposes.

In general CILTS is not a very good method for estimating air exchange when there are large intermittent air exchanges going on (e.g., through open windows). In most circumstances it will be practically impossible to deploy samplers and emitters to accommodate this situation and it is unlikely that sufficient mechanical mixing can be supplied to minimize its impact. CILTS is best deployed over a period of time where the weather conditions are stable such that the air exchange is reasonably constant.

The typical use of CILTS is in high-volume or low cost situations where it is deployed by technicians (or even occupants) who are not highly trained in its application. Very often no prior estimate of the air exchange (rate or pattern) has been made. Under these more typical (and less certain) conditions, one might consider CILTS to provide results in the range of a “factor of 2” of the right answer.

References

ASTM, E741-00 (2000) Standard test method for determining air change in a single zone by means of a tracer gas dilution, American Society for Testing Materials, West Conshocken, PA.

Bassett, M.R., Shaw. C-Y, and Evans. R.G. (1981) An appraisal of the sulfur hexafluoride decay techniques for measuring air infiltration rates in buildings, ASHRAE Trans. 87, 361-373.

Clayton, C. A., Perritt, R. L., Pellizzari, E. D., Thomas, K. W., Whitmore, R. W., Wallace, L. A., Ozkaynak, H., and Spengler, J. D. (1993) Particle Total Exposure Assessment Methodology (PTEAM) study: distributions of aerosol and elemental concentrations in personal, indoor, and outdoor air samples in a southern California community. J. Exposure Analysis Environ. Epidemiol., 3, 227-250.

Condon, P.E., Grimsrud, D.T., Sherman, M.H., and Kamerud, R.C. (1981) An automated controlled-flow air infiltration measurement system, in Building air Change Rate and Infiltrations Measurements, C.M Hunt, J.C. King, and H.R. Treschel, eds., pp. 60-72. ASTM STP 710, American Society for Testing Materials, Philadelphia, PA.

Dietz, R.N. and Cote, E.A. (1982) Air infiltration measurements in a home using a convenient perfluorocarbon tracer technique. Environment International, 8, 419-433.

D'Ottavio, T.W., Senum, G.I., and Dietz, R.N. (1988). Error analysis techniques for perfluorocarbon tracer derived multizone ventilation rates. Building and Environment, 23(3), 187-194.

Fisk, W.J., Faulkner, D., and Hodgson, A.T. (1993) "The Pollutant Control Index: A New Method of Characterizing Ventilation in Commercial Buildings." *Proceedings of Indoor Air'93: The 6th International Conference on Indoor Air Quality and Climate*, vol. 5, pp. 9-14, July 4 - 8, 1993, Helsinki, Finland. Published by Indoor Air'93, Helsinki. LBL-33685.

Grot, R.A. (1980) A low-cost method for measuring air infiltration rate in a large sampler of dwellings, in Building Air Change Rate and Infiltration Measurements, C.M. Hunt, J.C. King and H.R. Treschel, eds., pp. 50-59, ASTP STP 719, American Society for Testing and Materials, Philadelphia, PA.

Harrje, D. and Grot, R. (1977). Automated air filtration measurement and implications for energy conservation, in *Proceedings of the International Conference on Energy Use Management*, Vol. 1, R.A. Fazzalare and C.B. Smith, eds., pp. 457-464, Pergamon Press, New York.

Klepeis, N.E., Nelson, W.C. et al, (2001) J. Exposure Anal. Environ. Epidemil. 11, 231-252.

Leaderer, B.P., Schaap, L., and Dietz, R.N. (1985) Evaluation of the perfluorocarbon tracer technique for determining infiltration rates in residences. Environ. Sci. Tech., 19, 1225-1232.

Lunden, M., Faulkner, D., Heredia, E., Cohn, S., Dickerhoff, D.J., Noris, F., Logue, J., Toshifumi, H. Singer, B. and Sherman, M. 2012. *Experiments to Evaluate and Implement Passive Tracer Gas Methods to Measure ventilation Rates in Homes.*

Offermann, F. J. (2009) *Ventilation and Indoor Air Quality in New Homes*. California Air Resources Board and California Energy Commission, PIER Energy-Related Environmental Research Program. Collaborative Report. CEC-500-2009-085.

Ozkaynak, H., Xue, J., Spengler, J., Wallace, L., Pellizzari, E., and Jenkins, P. (1996) Personal Exposure to Airborne Particles and Metals: Results from the Particle Team Study in Riverside, California. Journal of Exposure Analysis and Environmental Epidemiology, 6 (1), pp. 57-77.

Sherman, M.H., "Analysis of Errors Associated with Passive Ventilation Measurement Techniques," Building and Environment 24 (No. 2): 131-139, 1989a. LBL-23088.

Sherman, M.H. , "Uncertainty in Air Flow Calculations Using Tracer Gas Measurements", Building and Environment, 24(4), pp. 347-354, 1989b. LBL-25415

Sherman, M.H., "On Estimation of Multizone Ventilation Rates from Tracer Gas Measurements," Building and Environment 24 355-362, 1989c. LBL-25772.

Sherman, M.H., "Tracer Gas Techniques for Measuring Ventilation in a Single Zone," Building and Environment 25 (No. 4): 365-374, 1990. LBL-29328.

Weisel, C.P., Zhang, J., Turpin, B.J., Morandi, M.T., Colome, S., Stock, S.H., Specktor, D.M and others. (2005) Relationships of Indoor, Outdoor, and Personal Air (RIOPA): Part 1. Collection Methods and Descriptive Analyses. HEI Research Report 130, NUATRC Research Report 7, Health Effects Institute, Boston, MA.

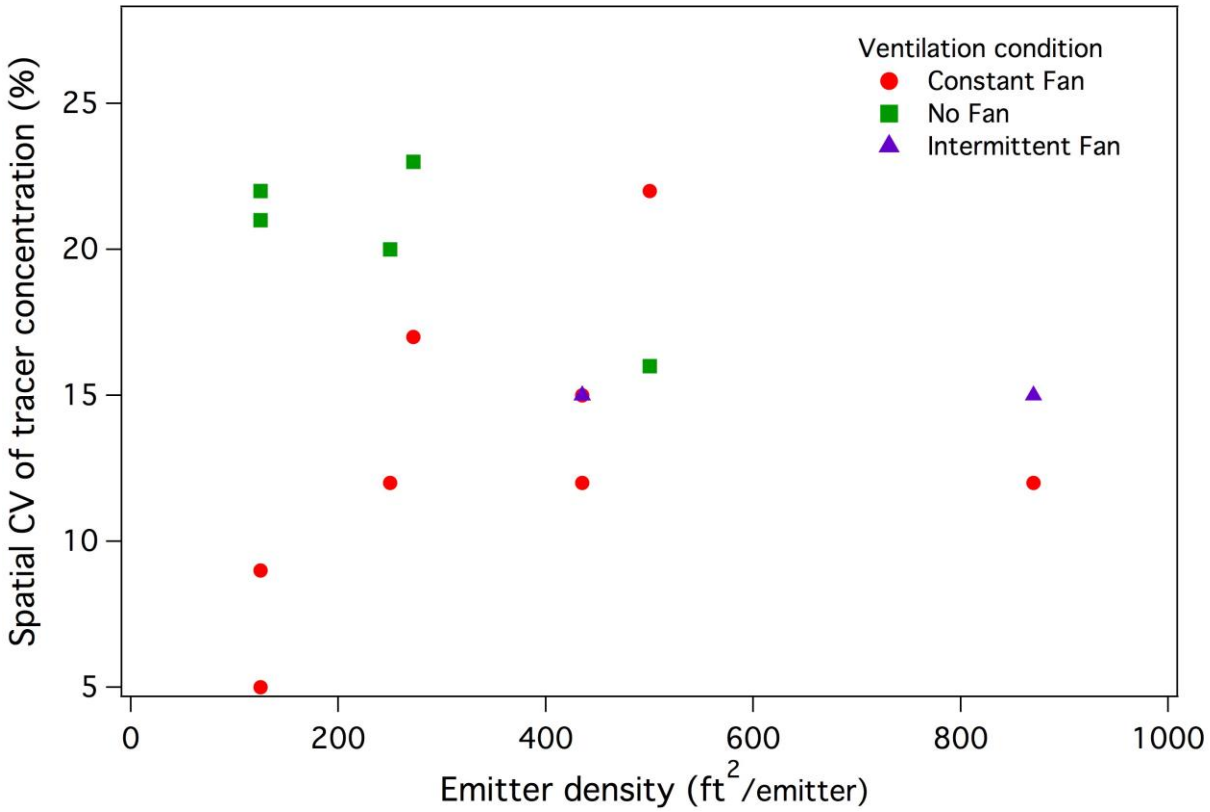

Figure 1: The variability of tracer concentration (CV, %) as a function of emitter density for the three houses. The markers indicate different ventilation conditions: constant fan use (circles), no fan use (squares), and intermittent fan use (triangles).

Table 1: Experimental Ventilation conditions performed at the three houses.

	Experiment	Ventilation Condition
House 1	1	No forced air system operation
	2	Constant forced air system fan
House 2	1	Normal operation of air conditioner
	2	Air conditioner use with constant fan operation
	3	Normal operation of air conditioner with economizer
House 3	1	No forced air system operation
	2	No forced air system operation but with constant kitchen exhaust fan. Internal doors closed.
	3	Constant forced air system fan and kitchen exhaust fan use. Internal doors closed.
	4	Constant forced air system fan

Table 2: Results from House 1, including the number of emitters for each tracer and the air exchange rate, spatial coefficient of variation in the PFT concentration, and the error in the ACH calculation due to this spatial concentration.

		No Fan			Continuous Fan		
	# of emitters	ACH (hr-1)	Spatial CV (%)	Error (%)	ACH (hr-1)	Spatial CV (%)	Error (%)
Tracer 1	4	0.48	20	41	0.81	12	25
Tracer 2	8	0.47	21	42	0.76	5	12
Tracer 3	2	0.60	16	33	1.23	22	44
Tracer 4	8	0.43	22	44	0.62	9	19
Average		0.5 ± 13%	20 ± 2	40	0.87 ± 29%	12 ± 6	25

Table 3: Results from House 2, including the number of emitters for each tracer and the air exchange rate, spatial coefficient of variation in the PFT concentration, and the error in the ACH calculation due to this spatial concentration.

		Normal Operation			Continuous Fan			With Economizer		
	# emit	ACH (hr-1)	CV (%)	Error (%)	ACH (hr-1)	CV (%)	Error (%)	ACH (hr-1)	CV (%)	Error (%)
Tracer 1	8	0.29	15	46	0.46	12	37	0.301	21	63
Tracer 2	4	0.27	15	46	0.39	12	37	0.282	10	31
Tracer 3	1	0.31	28	84	0.45	22	66	0.305	24	72
Tracer 4	8	0.24	15	46	0.37	15	45	0.253	25	75
Ave		0.27± 10%	18± 5	55	0.42± 9%	15± 4	46	0.29± 7%	20± 6	60

Table 4: Results from House 3, including the number of emitters for each tracer and the air exchange rate, spatial coefficient of variation in the PFT concentration, and the error in the ACH calculation due to this spatial concentration.

			No Fan			No Fan w Exhaust			Cont Fan w Exhaust			Continuous Fan		
	# emit	Floor	ACH (hr-1)	CV (%)	Error (%)	ACH (hr-1)	CV (%)	Error (%)	ACH (hr-1)	CV (%)	Error (%)	ACH (hr-1)	CV (%)	Error (%)
Tracer 1	3	1	0.41	54	93	1.01	81	142	0.59	19	33	0.32	10	17
Tracer 2	3	2	0.26	27	47	0.63	111	192	0.47	29	50	0.32	5	9
Tracer 3	3	3	1.30	122	211	0.54	196	339	0.62	146	253	0.98	72	125
Tracer 4	9	All	0.26	23	41	0.26	68	118	0.35	62	107	0.30	17	29
Ave			0.56± 78%	57± 40	98	0.52± 59%	114± 50	197	0.51± 21%	64± 50	111	0.48± 60%	26± 26	45

APPENDIX:

Multizone Bias in Single Tracer

In the main body of this paper (e.g. eqs. 1-3) we develop the equations for determining the air exchange from steady-state tracer gas measurements assuming a single, well-mixed zone. We know from field measurements (e.g. Lunden 2012) that a real building is virtually never a single, well-mixed zone. Using those equations would then cause a systematic error (i.e. bias or modeling error) that may not be apparent from a simple error analysis. This appendix derives the bias due to this assumption.

Although we know the whole building is not really a single, well-mixed zone, we can assume that it can be broken down into a set of *N* well-mixed zones that communicate with each (and outside). Sherman(1989c and 1989b) develops the general case and we use that nomenclature unless otherwise specified.

With *N* independent tracer gases, it is possible to simultaneously determine the N^2 independent flows and their uncertainties, but the CILTS case is more limited. We are considering only a single tracer gas and we only wish to find to total air exchange with the outside. Without a priori knowledge of zonal flows this can only be done in steady state using a constant-concentration technique (i.e. where the concentration in each zone is made to be the same by adjusting the emission rate of tracer gas in each zone accordingly). In such a case we can find the desired air exchange as follows:

$$Q_o = S_o / C_o \tag{A.1}$$

where C_o is the concentration everywhere and the other "o" subscripts represent single-zone totals. In equation A.2 S_i is the injection rate in zone *i* necessary to produce that concentration and S_o is the total injection rate:

$$S_o = \sum_i S_i \tag{A.2}$$

Equation A.1 is just a restatement of the single zone equation (Eq. 1), but the assumptions that it is derived under will allow us to better estimate biases. (Note that the subscript "o" is used here to designate whole-building values, but is dropped in the main body of the paper.) The constant concentration assumption would in fact allow us to determine the *in*filtration of outside air into each zone

$$I_i = S_i / C_o \tag{A.3}$$

where I_i is the infiltration rate in each zone necessary to produce that concentration the sum of these infiltration flows is the total value we seek. The sum of the infiltration (or exfiltration) rate in all zones is the total:

$$Q_o = \sum_i I_i = \sum_i E_i \tag{A.4}$$

where E_i is the exfiltration of air to outside from zone i

In the actual CILTS experiment though, this is not particularly useful because we are not controlling the injection rate in every zone to rigorously maintain a constant concentration. These expressions are, however, useful because the provide guidance on how best to deploy the emitters to get close to constant concentration. That is, one should deploy the CILTS emitters proportional to any a-priori knowledge one has about the infiltration rate into that zone.

Because we are not rigorously controlling the concentration at a single level, we need to know how to measure "the" concentration (i.e. is the representative concentration, C_o, the physical averaged concentration) used in our analysis. The multizone continuity equation provides the answer (again from Sherman 1989c) that the right concentration to use in the simple equation is the exfiltration-weighted average concentration:

$$C_o = \sum_i (E_i / Q_o) C_i \tag{A.5}$$

Conceptually, again, this is a useful expression because it indicates we should deploy samplers proportion to the local *ex*filtration, but since we don't know that values quantitatively, we cannot use this expression directly. If we posit as a practical matter that the exfiltration is proportional to the volume of the space represented by the concentration we can use a common operational definition of "the" concentration by using volume weighting:

$$C_o \approx \sum_i (V_i / V_o) C_i \tag{A.6}$$

Regardless of how reasonable a choice is made there will be bias in our definition of C_o, since we don't know the exfiltration a priori. This bias comes from the nature of the experiment itself and not the measurement errors from sources such as instrumentation discussed in the main body. It is, therefore, present in every zonal measurement and the best we can do is include it in our uncertainty estimate.

To see what effect these multizone considerations will have on the determination of air exchange we care about, we can use Equation 21 from Sherman (1989c) to estimate the uncertainty for a multizone, constant-concentration measurement:

$$\delta^2 Q_{multizone} = \left(\frac{N}{C_o}\right)^2 \sum_i \left(\delta^2 S_i + V_i^2 \delta^2 \dot{C}_i + E_i^2 \delta^2 C_i\right) \tag{A.7}$$

The first of the three terms in parenthesis reflects uncertainties in the measured source emission in each zone. In a true, constant-concentration experiment these might be quite

complicated, but for CILTS we assume this is a fixed value and we can express the error term in terms of the error of the total emission rate:

$$\sum_i \left(\delta^2 S_i\right) = \frac{\delta^2 S_o}{N} \tag{A.8}$$

This is a reasonable assumption, but is surely never correct. The error could be either higher or lower, but we can use this expression to make a reasonable estimate.

In a true constant-concentration experiment, the last two of the three terms should be insignificant because the concentration in every zone should be being held constant. In a CILTS experiment the middle term should still be insignificant because the period of the experiment is long enough to minimize it. We shall ignore it moving forward, but care should be taken not to make a CILTS experiment so short (e.g. less than a day in most typical houses) that that term becomes significant.

We must investigate the last term because the concentration in any one zone is neither constant nor necessarily centered on C_o. We do not know the exfiltration distribution, but if we assume it is not correlated to the concentration variations we can bound the size of the last term by looking at the cases where the exfiltration is concentrated:

$$0 < \sum_i \left(E_i^2 \delta^2 C_i\right) < Q_o^2 \delta^2 C_{i,\max} \tag{A.9}$$

A reasonable intermediate to choose is when the exfiltration is evenly distributed (by volume) in all zones using the same weighting as we did for the concentration:

$$\sum_i \left(E_i^2 \delta^2 C_i\right) \approx Q_o^2 \frac{\sum_i \left(V_i^2 \delta^2 C_i\right)}{V_o^2 N^2} = \frac{Q_o^2 \delta^2 C_{rms}}{N} \tag{A.10}$$

Where δC_{rms} is the volume-weighted, root-mean square deviation of the measured concentrations around C_o.

Putting this all together we can get an estimate for the uncertainty of our result due to the fact that the experimental configuration is truly multizone.

$$\frac{\delta^2 Q_{multizone}}{Q_o^2} = N \frac{\delta^2 S_o}{S_o^2} + N \frac{\delta^2 C_{rms}}{C_o^2} \tag{A.11}$$

This uncertainty increases roughly with the square root of the number of zones in the building. This is the number of actual well-mixed zones in the building; it may or may not be related to the number of samplers or emitters used and thus needs to be estimated independently.

The root-mean-squared concentration deviation also refers to the actual number of zones, but in this case multiple samplers deployed around the building may give a reasonable estimate of its value. Operationally, these definitions are the ones we use in the body of the paper.