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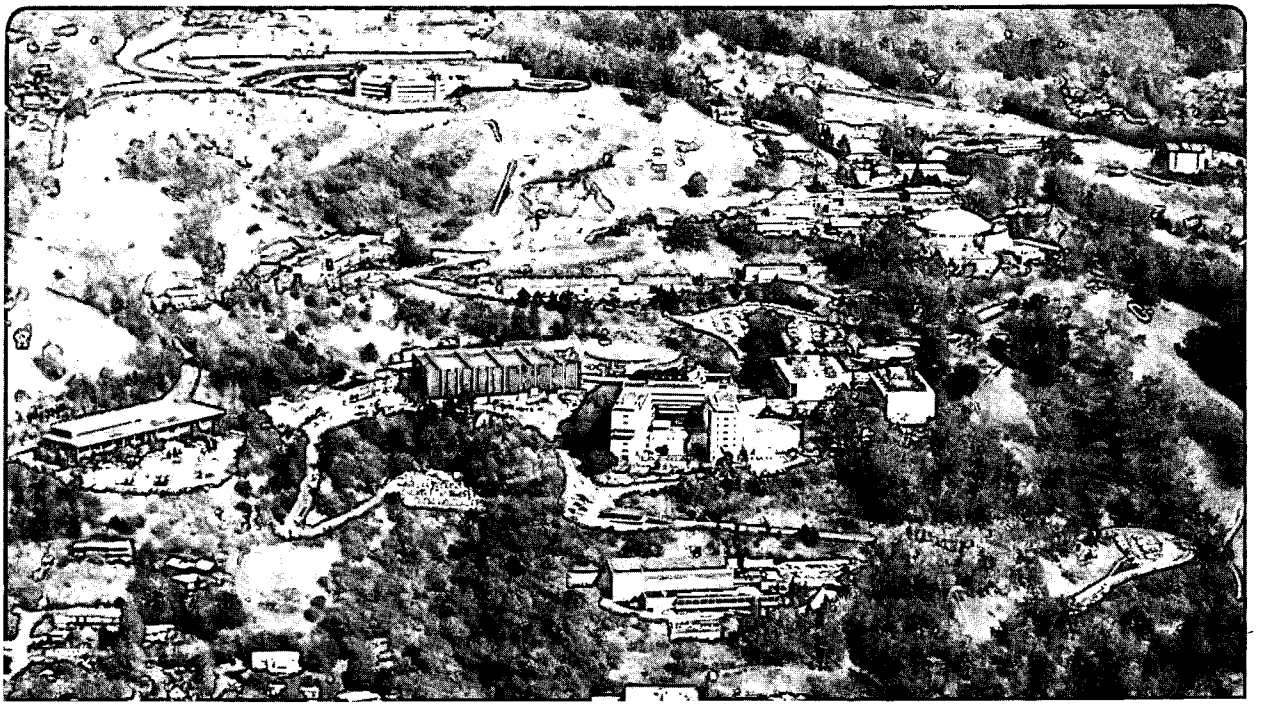

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LAWRENCE BERKELEY LABORATORY - UNIVERSITY OF CALIFORNIA
ENGINEERING NOTE

CODE	SERIAL	PAGE
PE0103	M5374	1 of 21

AUTHOR	DEPARTMENT	LOCATION	DATE
J. Singh/K. Halbach	Mechanical Engineering	Berkeley	June 13, 1979

PROGRAM — PROJECT — JOB
PEP - INJECTION

KICKER MAGNET

TITLE
PROGRAM FOR PRODUCING DESIRED MAGNETIC FIELDS IN A GIVEN REGION WITH MINIMUM ENERGY

Abstract

Field is a program that determines a configuration of conductors that produce a desired magnetic field with a minimum stored energy.[1] Presently there are two important restrictions:

1. The field constraints are specified on the mid plane
2. One has a Neumann boundry condition on the mid plane (field lines are perpendicular).

General Principles

In order to accomplish the above task the program uses a function set specified by the programmer[2] and determines its coefficients such that the field constraints are satisfied and the energy - subject to the function set chosen - is minimized. This is accomplished by solving a constrained minimization problem using Lagrange's multipliers.

---

1. This minimum is dependant on the function set chosen.
2. Currently there are three types of function sets available as a standard part of the program. These are:

   1. $F(Z) = \sum_{n=1}^{N} An \cdot (Z/Ro)^n$

   2. $F(Z) = \sum_{n=1}^{N/2} An \cdot \cos(n \cdot Z/Ro) +$

   3. $F(Z) = \sum_{n=1}^{N} An \{\mathrm{Log}((Z-Zn)/Ro) + \mathrm{Log}((Z-Zn^*)/Ro)\}$

   The above function corresponds to having filaments at location Z and Zn^* that are outside the region of interest.

General Principles (cont'd)

In order to physically realize the desired fields, it is assumed that one has steel surrounding the region of interest; $\int \vec{H}\cdot d\vec{\ell}$ is calculated along the boundary and current filaments (with $I = \int \vec{H}\cdot d\vec{\ell}$) are placed just inside of it so as to produce the tangential field components that produce the desired field on the inside of the region of interest.

Theory

In order to numerically calculate the energy as a function of the coefficients of the function set we proceed as follows:

Let F(Z) be a function of a complex variable Z with real and imaginary parts A and V, further let A and V be the vector and scalar potentials of a magnetostatic field. From the magnetostatic equations it can be shown that A and V satisfy the Cauchy-Reimann conditions and so F(Z) is an analytic function of Z.

From the relationships between the fields and the potentials it follows that:

$$B^*(Z) = i\, F'(Z) \qquad (1)$$

where $B^*(Z)$ is the complex conjugate of the magnetic induction $B(Z)$

$i = \sqrt{-1}$

$F'(Z) = \dfrac{dF(Z)}{dZ}$

The stored energy E is given by:

$$E = \frac{1}{2\mu_o}\int B(Z)\cdot B^*(Z)\, d\sigma \qquad (2)$$

Where $d\sigma$ is an area element in the Z plane.

Since $B(Z) \cdot B(Z)^*$ is a real quantity it follows that E must be real.

In order to evaluate this expresssion we proceed as follows:

$$F(Z) = \sum_{n=1}^{N} a_n \, g_n(Z) = \sum_{n=1}^{N} A_n(Z) + i \, V_n(Z) \tag{3}$$

In the above expressions the a_n's must be chosen such that one has a Neumann boundary condition at the mid plane. This implies that the a_n's must be real coefficients.

$$B^*(Z) = i \sum_n a_n \, g'_n(Z) \tag{4}$$

$$B(Z) = -i \sum_n a_n (g'_n(Z))^* \tag{5}$$

So

$$E = \sum_n \sum_m \frac{1}{2\mu_o} \int_\sigma a_n \cdot a_m \, g'_n(Z) \cdot g^{*\prime}_m(Z) \, d\sigma \tag{6}$$

If one exchanges m and n one gets the complex conjugate of the above integral, however, the value of the integral must remain unhanged by this exchange, hence the integral must be real and so:

$$E = \mathrm{Re} \left\{ \sum_n \sum_m \frac{1}{2\mu o} \int a_n \cdot a_m \cdot g'_n(Z) \cdot g^*_m(Z) \, d\sigma \right\} \tag{7}$$

Let $G(Z,Z^*)$ be a function such that:

$$\frac{\partial}{\partial Z^*} G(Z,Z^*) = g'_n \cdot g^{*\prime}_m \; => G(Z,Z^*) = g'_n \cdot g^*_m \tag{8}$$

It can be shown in general that

$$\int_\sigma \frac{\partial}{\partial Z^*} G(Z,Z^*) \, d\sigma = \frac{1}{2i} \oint G(Z,Z^*) \, dZ \tag{9}$$

Hence from equations 7 and 9

$$E = \mathrm{Re} \sum_m \sum_n \{ \frac{1}{2i} \cdot \frac{1}{2\mu_o} \oint a_n \cdot a_m \; g_m^* \cdot \frac{dgn}{dZ} \cdot dZ \} \tag{10}$$

Now $$g_m^* = A_m - iV_m = g_m - 2iV_m \tag{11}$$

$$dg_n = dA_n + idV_n \tag{12}$$

so $$g_m^* \, dg_n = (g_m - 2iV_m) \cdot dg_n \tag{13}$$

$$g_m \cdot dg_n - 2i \; V_m \cdot dg_n \tag{14}$$

Since g_m and g_n are analytic functions

$$\oint g_m \cdot g_n' \, dZ = 0$$

Hence $$\oint g_m^* \; dg_n = -2i \oint V_m \; dg_n \tag{15}$$

$$= -2i \oint V_m \; (dA_n + i \; dV_n) \tag{16}$$

$$= -2i \oint V_m \; dA_n + 2 \oint V_m \; dV_n \tag{17}$$

From equation 10 and 17 this gives:

$$E = \mathrm{Re} \sum_n \sum_m \frac{a_n a_m}{2\mu_o} \{ -\oint V_m dA_n - i \oint V_m dV_n \} \tag{18}$$

$$= \sum_n \sum_m \frac{-a_n \; a_m}{2\mu_o} \oint V_m \; dA_n \tag{19}$$

But since $\oint V_m \; dA_n = -\oint A_n \; dV_m$

$$E = \sum_n \sum_m \frac{a_n \cdot a_m}{2\mu_o} \oint A_m \; dV_n \tag{20}$$

If $\underset{\sim}{a}$ is a column vector of the coefficients and m is a matrix whose elements $m_{ij} = \oint A_i \; dV_j$ (20.1)

Then we get

$$\boxed{E = \underset{\sim}{a}^T \, M \, \underset{\sim}{a}} \tag{21}$$

In order to find the coefficients vector we have the following minimization problem:

Minimize $E = a^T M a$

subject to $-i\, \underset{\sim}{a}^T \cdot \underset{\sim}{g}^{*\prime}(Z_j) = b_j$ (22)

where b_j is the field at Z_j

and $g^*(Z_j)$ is the column vector whose element are

$$g_i^*(Z_j) = \frac{dg_i^*}{dZ}(Z_j)$$

Let ℓ_k, $k = 1,2 \ldots\ldots J$ be J Lagrange multipliers, where J = # of field constraints, let L be a column vector with elements ℓ_k, then:

$$U = \underset{\sim}{a}^T M \underset{\sim}{a} + \underset{\sim}{L}^T(\underset{\sim}{P} - \underset{\sim}{B}) \tag{23}$$

Here P and B are column vectors with elements p_j and b_j.

b_j is the desired magnetic induction at point j and P_j is given by the following:

$$P_j = -i\, \underset{\sim}{a}^T \cdot g^{*\prime}(Z_j) \tag{24}$$

For U to be a minimum

$$\frac{\partial U}{\partial a_n} = 0 \qquad n = 1 \ldots\ldots N \tag{25}$$

$$\frac{\partial U}{\partial \ell_k} = 0 \qquad k = 1 \ldots\ldots J \tag{26}$$

This gives the following matrix equation.

$$\underset{\sim}{U}' = \underset{\sim}{c} \cdot \begin{pmatrix} a \\ \ell \end{pmatrix} \tag{27}$$

where U is a column vector whose element $u_j' = \frac{\partial u}{\partial aj}$ for $j \leqslant N$ (28)

and $u_j' = \frac{\partial}{\partial \ell j}$ for $N < j \leqslant N+J$ (29)

The elements of c are:

$$c_{jk} = 2\underset{\sim}{M} \quad \text{for } j \leq N \text{ and } k \leq N$$

$$c_{jk} = -i\, aj\, g^*{}_j{}'(Z_R) \quad j \leq N \text{ and } N < k \leq N+J$$

$$C_{jk} = -ia_k\, g^*{}_k\, (Z_j) - bj \quad N < j \leq N+J,\ k \leq N$$

$$C_{jk} = 0 \qquad N < j \leq N+J,\ N < k \leq N+J$$

Equation 27 is shown as a partitioned matrix equation below:

$$\begin{bmatrix} u_n' \\ u_j' \end{bmatrix} = \begin{bmatrix} M & -ia_j g_j^{*\prime}(Z_k) \\ -ia_k\, g^{*\prime}_k(Z_j) - b_j & 0.0 \end{bmatrix} \begin{bmatrix} a_n \\ \ell_J \end{bmatrix} \tag{30}$$

This gives the following solution:

$$\begin{bmatrix} a_n \\ \ell_J \end{bmatrix} = \begin{bmatrix} M & -ia_j g_j^{*\prime}(Z_k) \\ -ia_k g^{*\prime}_k(Z_j) & 0.0 \end{bmatrix}^{-1} \begin{bmatrix} 0.0 \\ b_J \end{bmatrix} \tag{31}$$

Special Cases

Let us consider the cases where the region of interest is circular and the function set chosen is:

1. $f(Z) = \Sigma a_n\, Z^n$
2. $F(Z) = a_n \log (Z - Z_n) = \log (Z - Z_n^*)$

Function set 2 above corresponds to having filaments at Z_n and Z_n^*. These must be outside the region of interest.

For the first function type we have:

$$\Sigma\, a_n Z^n = \Sigma\, a_n\, (R_e^{\,i\phi})^n = \Sigma\, a_n R^n \cdot e^{in\phi}$$

$$= \Sigma a_n R^n \{\cos n\phi + i \sin n\phi\}$$

$$A_n = \Sigma a_n R^n \cos n\phi$$

$$V_n = \Sigma a_n R^n \sin n\phi$$

$$M_{nm} = R^{2n} \cos n\,\phi \cdot m \cos m\phi\, d\phi \quad \text{(from 20.1)}$$

for m = n the above expression = 0

for m = n $\oint \cos^2 n\phi d\phi = \Pi$

so $M_{nm} = R^{2n} \cdot n \cdot \Pi$ for n = m

$= 0$ for n = m

$$\boxed{E = \frac{1}{2\mu_o}\ \Sigma\, a_n^2 \cdot R^{2n} \cdot n \cdot \Pi} \tag{32}$$

For case 2 we have the following:

$$\text{let } I = \int \frac{1}{z-z_n} \cdot \frac{1}{z-z_m^*}\ dx\, dy \tag{33}$$

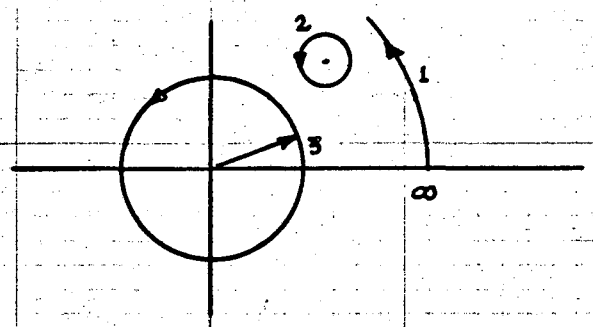
$$= \frac{1}{2i} \oint \frac{1}{z-z_n} \cdot \mathrm{Ln}\ \left(1 - \frac{z^*}{z_m^*}\right) dz \tag{34}$$

$$z = re^{i\phi},\ z^* = \frac{r^2}{z}$$

$$I = \frac{1}{2i} \oint \frac{1}{z-z_n}\ \mathrm{Ln}\ \left(1 - \frac{r^2}{z\, z_m^*}\right) dz \tag{35}$$

For large R this integral = 0

Since we have a singularity at $z\, z_m^* = r^2$, $\oint_1 = 0 = \oint_2 + \oint_3$



From the residue theorem we have

$$\frac{1}{2i}\oint \frac{1}{z-z_n} \quad \mathrm{Ln}\left(1-\frac{r^2}{zz_m^*}\right) dz = \pi\, \mathrm{Ln}\left(1-\frac{r^2}{z_n z_m^*}\right) \tag{36}$$

Hence

$$\oint_3 = -\pi\, \mathrm{Ln}\left(1-\frac{r^2}{z_n z_m^*}\right) \tag{37}$$

Since

$$E = \sum_n \sum_m \frac{a_n a_m}{2\mu_o} \int \left\{\frac{1}{z-z_n} + \frac{1}{z-z_n^*}\right\} \left\{\frac{1}{z^*-z_m^*} + \frac{1}{z^*-z_m}\right\} d\,X\,dy \tag{38}$$

From 37 and 38 one gets:

$$E = \sum_n \sum_m \frac{a_n a_m}{2\mu_o} \mathrm{Ln}\left(1-\frac{r^2}{z_n z_m^*}\right)\left(1-\frac{r^2}{z_n z_m}\right)\left(1-\frac{r^2}{z_n^* z_m^*}\right)\left(1-\frac{r^2}{z_n^* z_m^*}\right) \tag{39}$$

$$M_{nm} = \mathrm{Ln}\left(1-\frac{r^2}{z_n z_m^*}\right)\left(1-\frac{r^2}{z_n z_m}\right)\left(1-\frac{r^2}{z_n^* z_m^*}\right)\left(1-\frac{r^2}{z_n^* z_m^*}\right) \tag{40}$$

The Program

General

The Program has been written in FORTRAN version IV and resides in the Library Newmagnets on the PSS system in the update format. In order to run the program one has to access the appropriate PSS subset throuth the Fetchps card. In addition one needs to access the GRAFPACK subroutines.

Presently the following control cards are needed to run the program:

Fetchps, Newmagnets/ Oldpl/Field

Update, f, p....etc.

FTN4, R = 3.....etc.

Fetchps, GPACBN7, TVLGO/RB, AIDBN, OPNBN,CCBN,PRBN,DBN

Link, X, F, F = TVLGO,ED.

Program Inputs and Outputs

The following is a sequence of inputs to the program:

1. Problem Name. To be specified on the first input card in the 3A10 format.- any string of 30 characters or less.
2. The degree of the function set selected -N

   The # of constraints M.

   The type of potential map desired MP.

   MP = 1 for vector potential

   MP = 2 for scalar potential

   The above quantities are format free inputs.
3. The X coordinates at which fields are specified. Presently fields desired can only be specified on the mid plane. The inputs are format free.
4. Value of B at the points specified in (3). Presently the fields are assumed to be normal at the mid plane. The inputs are format free.
5. A sequence of values DX, DY, SCALE, XMN, YMN, IDEV, KMIN, LMIN, LMAX, NLIN. Where:

   DX = X increments for the grid

   DY = Y increments for the grid

   scale = scale of the const A or V plot (works only for calcomp plots)

XMN, YMN = Min X and Y that one might possibly need for the grid

IDEV = type of device on which plots are to be obtained

1 = Calcomp
2 = Printer pPlot
3 = Microfiche

In order to get more than one device add the device codes for those devices.

KMIN = Minimum logical coordinate for which const A or V
LMIN plots are required

KMAX = Maximum logical coordinates for which const A or V
LMAX plots are required

The minimum X coordinate for the grid = XMN + (KMIN-1)*DX

Max X = XMIN + (KMAX - KMIN) * DX

Min Y = YMIN + (KMIN -1) * DY

Max Y = YMIN + (LMAX - LMIN) * DY

NLIN is the number of field lines to be plotted.

6. The major and minor radii of the ellipse RMAJ and RMIN within which lies the region of interest, and the # of divisions NDIV for computing $\oint A_m \, dv_n$ on the ellipse. Inputs are format free in the sequence RMAJ, RMIN, NDIV.

7. Minimum and maximum values of the vector or scalar potentials to be plotted. Pmin and Pmax. If Pmin = Pmax = 0.0, scaling is done internally , i.e. the max and min values of potentials are picked internally from the entire grid region. This is a reasonable thing to do if there are no field concentrations, otherwise with this option most of the field lines appear near these points and most of the rest of the picture is blank. The other scaling options are determined by the flag option described below:

Flag for the intervals of the constant potential lines to be plotted. LG.

LG = 1 for linear intervals
= 2 logrithmic intervals
= 3 linear intervals intervals, but Pmin and Pmax will be selected on the mid plane.

Region of X for which a By vs X plot is desired.

XP 1 = min X
XP 2 = max X

The sequence of input values is Pmin, Pmax, LG, XP1 and XP2.

8. Type of function set to be fitted and the normalization radius.

IFN and RO

IFN = 1 => $F(Z) = \Sigma\ (AN * Z * * N)$

= 2 => $F(Z) = \Sigma\ A_n \text{ Sin } (N*Z) + B_n \text{ Cos } (N*Z)$

= 3 => $F(Z) = \Sigma\ A_n\ (\text{Log } (Z-Z_n) + \log (Z-Z_n^*))$

Summary

The following read statements provide a summary of inputs:

1. Read 300, (Aname (IQ), IQ = 1,3)

   300 Format (3A10)

2. Read * , N, M, MP
3. Read * , (Z(I), I = 1, MN) {MN = M + N} internally determined
4. Read * , (VV(I), I = 1, MN)
5. Read * , DX, DY, SCALE, XMN, YMN, IDEV, KMIN, KMAX, LMIN, LMAX, NLIN
6. Read * , RMAJ, RMIN, NDIV
7. Read * , PMIN, PMAX, LG, XP1, XP2
8. Read * , IFN, RO

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J. Singh/K. Halbach	Mechanical Engineering	Berkeley	June 13, 1979

Program Outputs

1. List of inputs in nearly the same order as it was inputted.
2. Value of the coefficients and Lagrange multipliers. The first N values correspond to the coefficients and the next M corresponds to the Lagrange mutipliers.
3. Value of B_y at the specified points obtained by fitting the polynomial. This should check with the input.
4. Value of the energy in joules/M obtained from the matrix and coeffficients.
5. Edit. Under the headings:

x coord y coord Bx By Vec Pot Scal Pot

These are self explanatory; this calculation is done in the region bounded by KMIN, KMAX, LMIN, LMAX and at intervals of Dx and Dy.

6. Map of vector or scalar potential for the above edit region.
7. Contour values. These are the values of the constant vector/scalar potential lines plotted.
8. Map of vector and scalar potential and the current along the ellipse of interest to produce the desired fields, this appears under the heading AMPS.
9. Energy in joules/M obtained from ADV on the ellipse of interest. This is merely a check value and should correspond to that obtained in (4) above.
10. Plot of the constant scalar/vector potential map with the appropriate heading-if a printer plot was requested.
11. Plot of By vs X on the mid plane if a printer plot was requested.

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PE0103	M5374	13 OF 21

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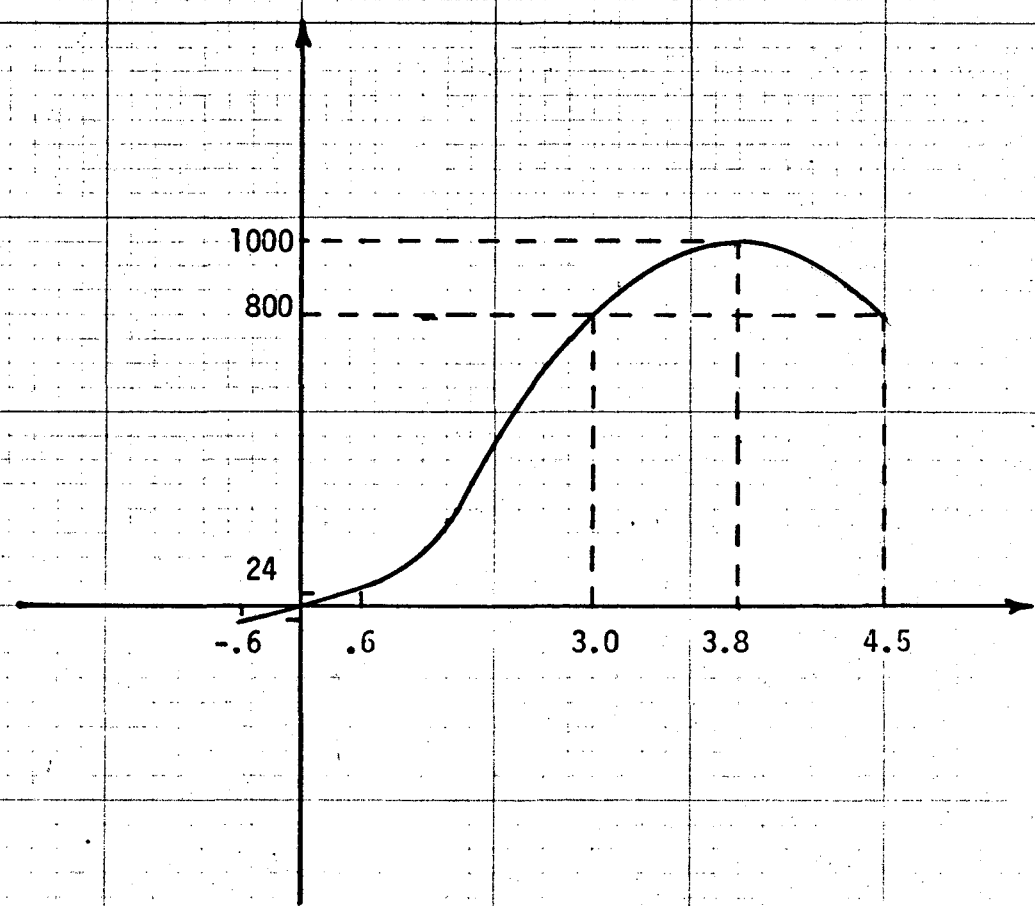
A Sample Problem (PEP Kicker)

Given: An ellipse with major radius 7.3 cms and minor radius 4.8 cms within which a field with the following characterstics has to be produced:

$$\left[B_o^{\,2} + (B_o \quad r)^2\right]^{\frac{1}{2}} \leq 2.4\ 10^{-2}\ B_1$$

where $_o$ is the field at the origin, r = .6 cm and B_1 is the field at a location 3.8 cms from the origin. The field at 3.8 ∓ .8 cm is to be within a 20% band. A field profile that satisfies this criterion is shown below:



The problem is solved for all the three types of function sets mentioned.

The inputs and selected outputs for the problem are provided below:

Card #										
1.	JACK PETERSONS KICKER MAGNET									
2.	11	6	1							
3.	-.6	0.0	..6	3.0	3.8	4.5				
4.	-24	0.0	24	800	1000	800				
5.	.5	.25	.5	-8.5	0.0	3	1	35	1	35	30
6.	8.5	5.3	180							
7.	0.0	0.0	3	-1.0	5.0					
8.	3	8.5								

Outputs

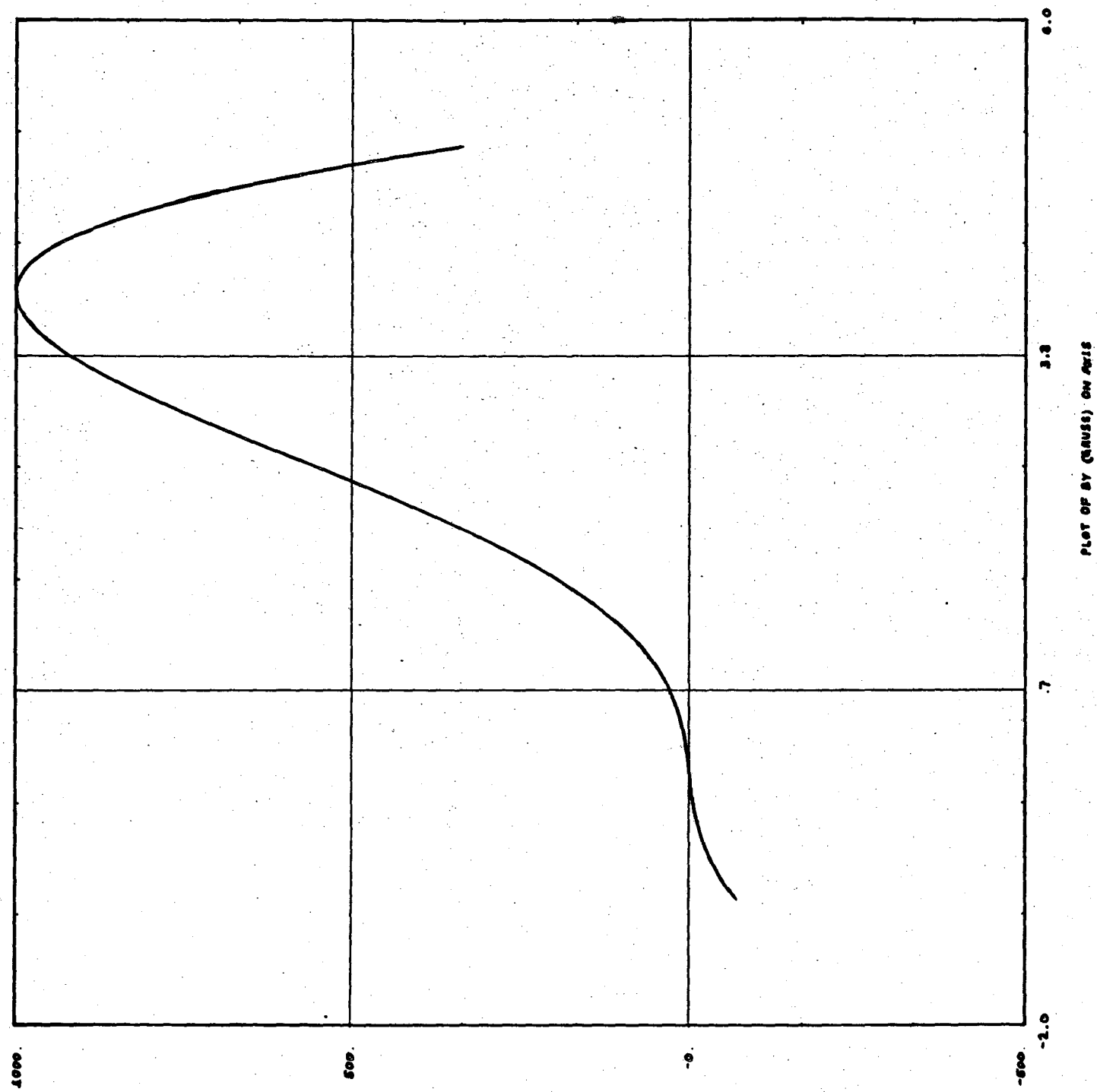
The value of coefficients for the three polynomials considered.

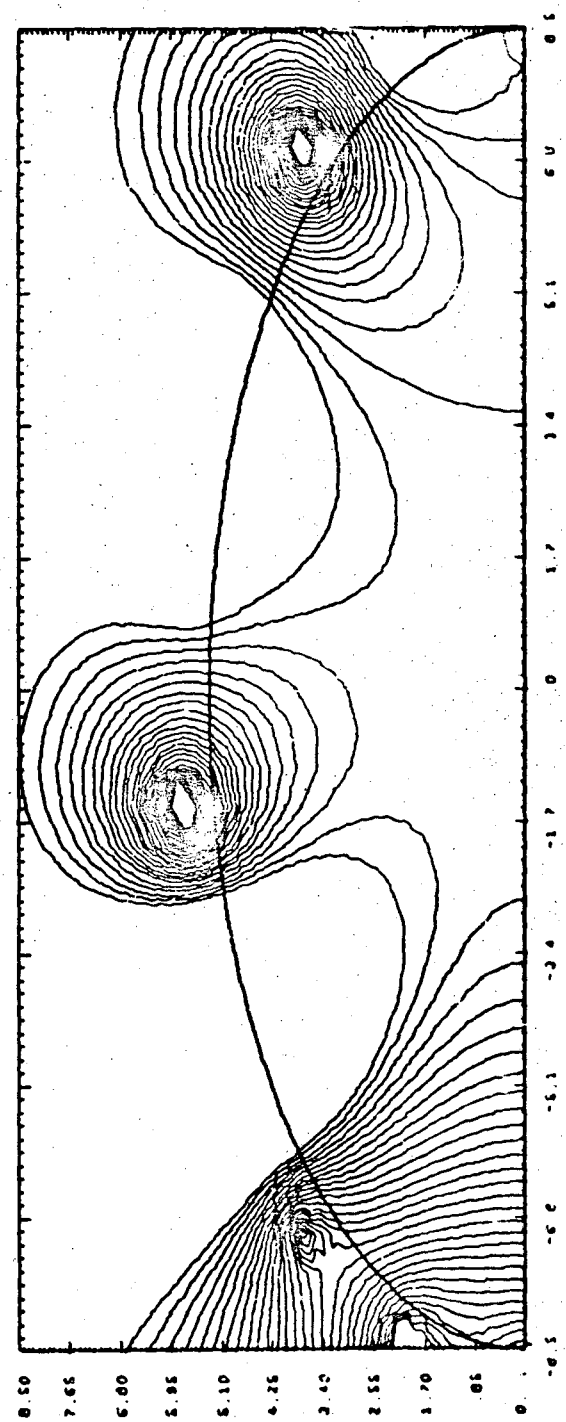
#	1	2	3
1	$-1.989\ 10^{-8}$	$-4.494\ 10^{6}$	$3.235\ 10^{3}$
2	$-7.971\ 10^{2}$	$2.828\ 10^{6}$	$-1.433\ 10^{4}$
3	$-1.171\ 10^{2}$	$-1.262\ 10^{6}$	$2.737\ 10^{4}$
4	$-6.606\ 10^{4}$	$3.723\ 10^{5}$	$-1.165\ 10^{4}$
5	$1.390\ 10^{4}$	$-5.668\ 10^{4}$	$-6.353\ 10^{3}$
6	$1.418\ 10^{5}$	$1.0887\ 10^{6}$	$3.035\ 10^{4}$
7	$2.896\ 10^{4}$	$-1.426\ 10^{-}$	$-4.588\ 10^{4}$
8	$-1.176\ 10^{5}$	$1.005\ 10^{6}$	$1.256\ 10^{4}$
9	$-4.852\ 10^{4}$	$-5.065\ 10^{5}$	$1.119\ 10^{7}$
10	$3.966\ 10^{4}$	$1.489\ 10^{5}$	$5.710\ 10^{3}$
11	$2.121\ 10^{4}$	$-2.019\ 10^{4}$	$-1.118\ 10^{7}$

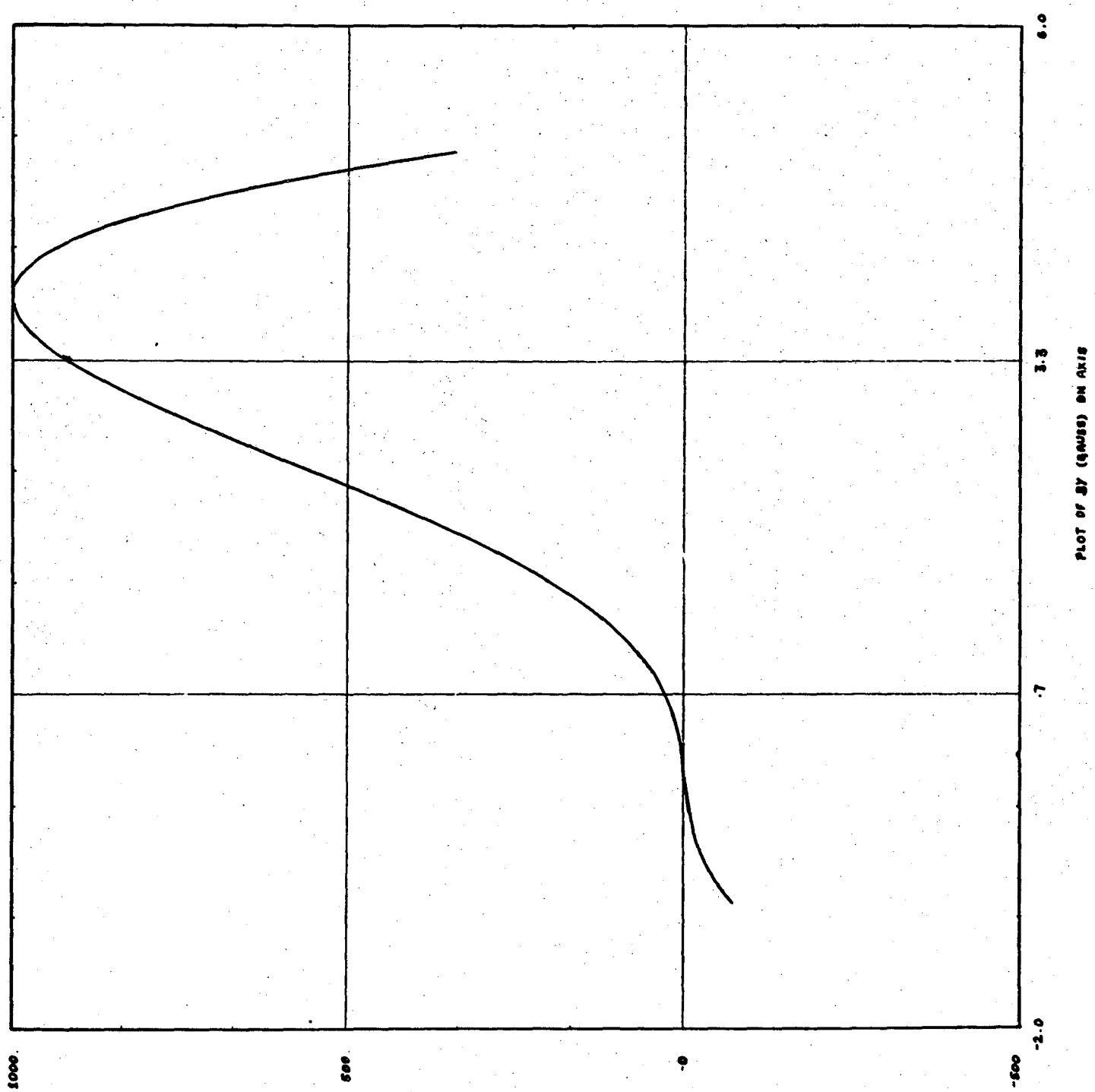
Energy (Joules/M)

1	3.151	10^3
2	3.111	10^3
3	1.123	10^4

The plots of constant vector potential lines and By vs X on mid plane are attached for the three cases.


1000.
500.
-0.
-500.
-1.0
.7
3.3
6.0
PLOT OF BY (GAUSS) ON AXIS


PROBLEM NAME-JACK PETERSONS KICKER MAGNET
FUNCTION TYPE- LOG(Z-ZN)-LOG(Z-ZN+1)
PLOT OF CONST VECT POT.
LINEAR INTERVAL ON MID PLANE
TOTAL STORED ENERGY= 1.122303E+04J/M
8.50
7.65
6.80
5.95
5.10
4.25
3.40
2.55
1.70
.85
0
-6.5
-5.2
-3.9
-2.6
-1.3
0
1.3
2.6
3.9
5.2
6.5


1000
500
-0
-500
-2.0
-.7
3.3
6.0
PLOT OF BY (GAUSS) ON AXIS

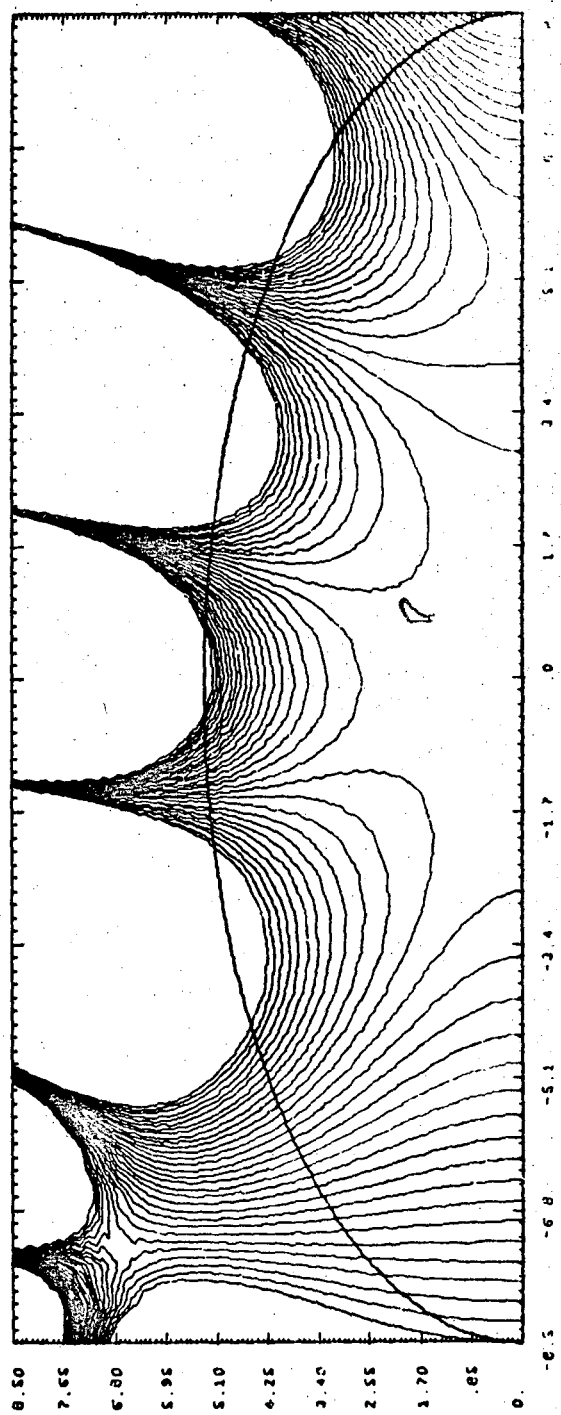
PROBLEM NAME-JACK PETERSONS KICKER DAGNET

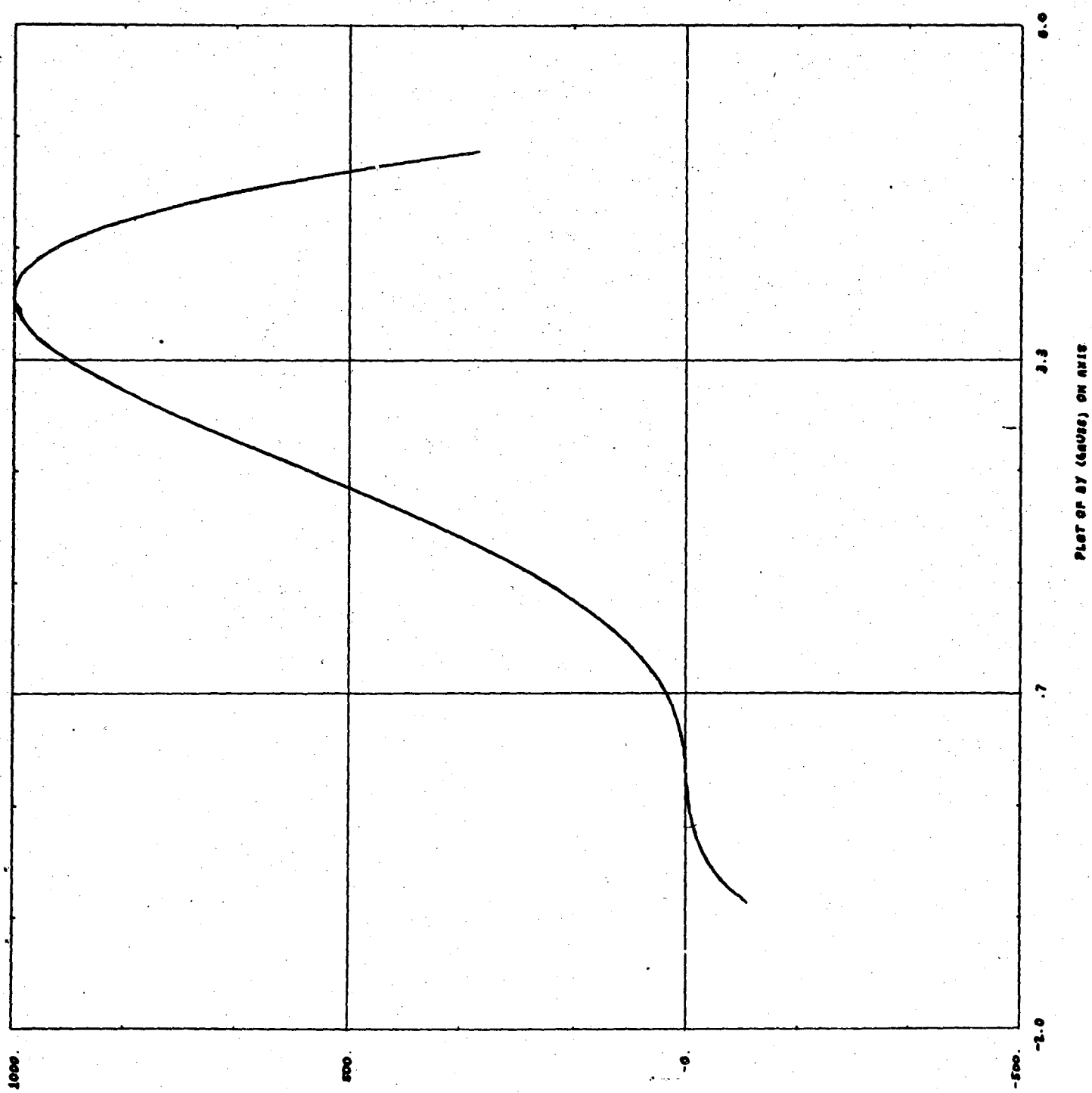
FUNCTION TYPE- AN*SIN(NZ)+BN*COS(NZ)

PLOT OF CONST VECT POT.

LINEAR INTERVAL ON MID PLANE

TOTAL STORED ENERGY= 3.110899E+03J/M




1000.
500.
-0.
-500.
-2.0
.7
3.3
6.0
PLOT OF BY (GAUSS) ON AXIS

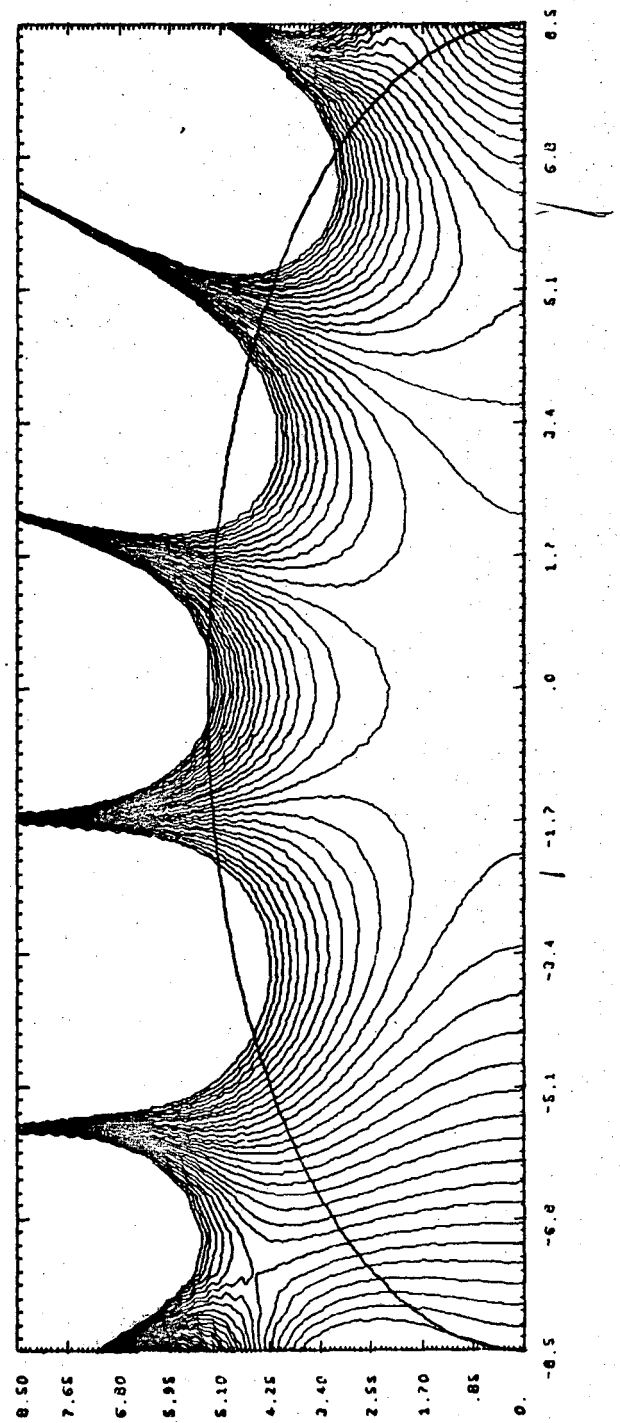
PROBLEM NAME-JACK PETERSONS KICKER MAGNET

FUNCTION TYPE- ANNZAAN

PLOT OF CONST VECT POT.

LINEAR INTERVAL ON XID PLANE

TOTAL STORED ENERGY= 3.151274E+03J/M



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TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720