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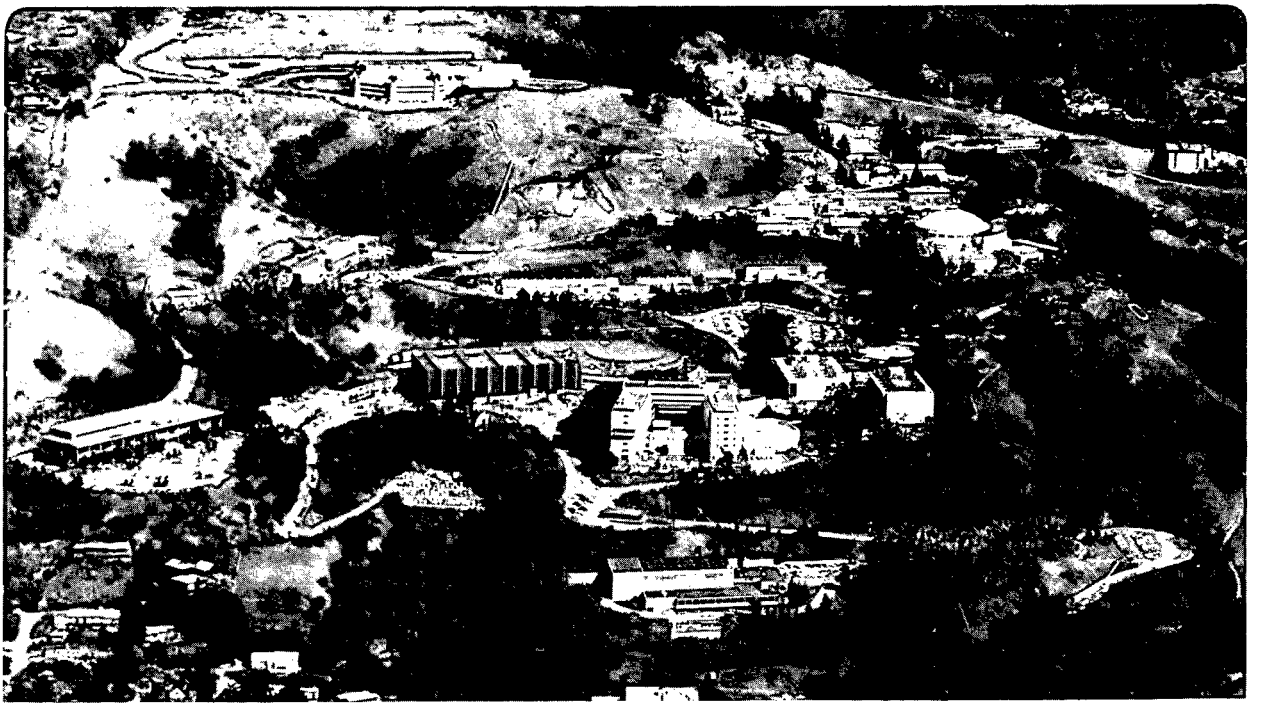

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ENGINEERING NOTE

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AUTHOR	DEPARTMENT	LOCATION	DATE
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PROGRAM — PROJECT — JOB

DOUBLET III NEUTRAL BEAM INJECTOR SYSTEM

NBIS CRYOPANELS

TITLE

HELIUM - TWO PHASE FLOW PRESSURE DROP CALCULATION OF FRICTION FACTOR "F"

SUMMARY:

The pressure drop of Helium flowing through smooth tubes is directly proportional to the "quality x" of that fluid. Assuming that temperature, mass flow rate & inside tube diameter are held constant, the pressure drop increases proportionally as the fluid goes from the saturated liquid phase to the saturated gas phase.

This Engineering Note calculates this proportionality constant, "Friction Factor F" for Helium as a function of quality for turbulent-turbulent flow from 3.8°K to 4.2°K.

HELIUM

TWO-PHASE FLOW PRESSURE DROP --

CALCULATION OF FRICTION FACTOR "F": 3.

FOR TURBULENT - TURBULENT FLOW -

$$C_{TT} = \left(\frac{\gamma_L}{\gamma_G}\right)^{0.1} \left(\frac{\rho_G}{\rho_L}\right)^{0.5}$$

FOR VISCOUS - VISCOUS FLOW -

$$C_{vv} \left(\frac{\gamma_L}{\gamma_G} \cdot \frac{\rho_G}{\rho_L}\right)^{0.5}$$

γ_L = liquid viscosity

γ_G = gas viscosity

ρ_L = liquid density

ρ_G = gas density

4.

T^oK	G/CM-S x 10^{-6} γ_L	G/CM-S x 10^{-6} γ_G	G/CC x 10^{-2} ρ_L	G/CC x 10^{-2} ρ_G	C_{TT}	C_{vv}
3.80	33.8	10.7	13.21	1.109	.32506	.51497
3.85	33.6	10.9	13.14	1.167	.33353	.52323
3.90	33.3	11.1	13.06	1.228	.34225	.53111
3.95	33.1	11.3	12.98	1.290	.35102	.53955
4.00	32.8	11.5	12.90	1.356	.36004	.54755
4.05	32.6	11.7	12.82	1.424	.36924	.55632
4.10	32.3	11.9	12.73	1.496	.37881	.56478
4.15	32.1	12.1	12.64	1.571	.38867	.57421
4.20	31.8	12.4	12.54	1.649	.39844	.58072
4.224	31.7	12.5	12.50	1.689	.40344	.58538

FOR TURBULENT - TURBULENT FLOW -

$$F_{TT} = \frac{1}{K}\int_0^K \frac{\left[x^{.45} + C_{TT}^{.5}(1-x)^{.45} \right]^4}{C_{TT}^2}\, dx$$

$$\text{let } y = \frac{\left[x^{.45} + C_{TT}^{.5}(1-x)^{.45} \right]^4}{C_{TT}^2}$$

$$\text{let } f(x) = \int_0^K y\, dx \qquad \text{where } K = \text{quality}$$

FOR 3.80°K

K	y	2y	f(x)	$\sum f(x)$	$\sum \frac{1}{K} f(x) = F_{TT}$
.00	1.000	------			
.02	2.791	5.582			
.04	3.775	7.550			
.06	4.632	9.264			
.08	5.423	10.846			
.10	6.169	------	.404	.404	4.040
.12	6.881	13.762			
.14	7.565	15.130			
.16	8.224	16.448			
.18	8.861	17.722			
.20	9.478	------	.787	1.191	5.955
.22	10.074	20.148			
.24	10.653	21.306			
.26	11.474	22.948			
.28	11.756	23.512			
.30	12.281	------	1.097	2.288	7.627
.32	12.788	25.576			
.34	13.592	27.184			

FOR 3.80°K

K	y	2y	f(x)	$\sum f(x)$	$\sum \frac{1}{K} f(x) = F_{TT}$
.36	13.748	27.496			
.38	14.201	28.402			
.40	14.636	------	1.356	3.644	9.110
.42	15.051	30.102			
.44	15.448	30.896			
.46	15.826	31.652			
.48	16.182	32.364			
.50	16.519	------	1.562	5.206	10.412
.52	16.833	33.666			
.54	17.126	34.252			
.56	17.396	34.792			
.58	17.642	35.284			
.60	17.864	------	1.724	6.930	11.55
.62	18.060	36.120			
.64	18.228	36.456			
.66	18.369	36.738			
.68	18.480	36.960			
.70	18.559	------	1.827	8.757	12.51
.72	18.605	37.210			
.74	18.614	37.228			
.76	18.586	37.172			
.78	18.515	37.030			
.80	18.399	------	1.856	10.613	13.266
.82	18.231	36.462			
.84	18.008	36.016			

FOR 3.80°K

K	y	2y	f(x)	$\sum f(x)$	$\sum \frac{1}{K} f(x) = F_{TT}$
.86	17.719	35.438			
.88	17.355	34.710			
.90	16.900	------	1.779	12.392	13.769
.92	16.331	32.662			
.94	15.611	31.222			
.96	14.666	29.332			
.98	13.310	26.620			
1.00	9.464	------	1.462	13.854	13.854 *

FOR 4.0°K

.00	1.00	------			
.02	2.664	5.328			
.04	3.577	7.114			
.06	4.329	8.658			
.08	5.036	10.071			
.10	5.698	------	.379	.379	3.79
.12	6.327	12.654			
.14	6.928	13.856			
.16	7.505	15.010			
.18	8.060	16.120			
.20	8.595	------	.719	1.098	5.49
.22	9.111	18.223			
.24	9.609	19.219			
.26	10.090	20.181			
.28	10.554	21.108			
.30	11.000	------	.983	2.081	6.938

FOR 4.0°K

K	y	2y	f(x)	$\sum f(x)$	$\sum \frac{1}{K} f(x) = F_{TT}$
.32	11.430	22.860			
.34	11.843	23.689			
.36	12.239	24.478			
.38	12.618	25.236			
.40	12.980	------	1.202	3.283	8.208
.42	13.324	26.648			
.44	13.650	27.301			
.46	13.959	27.918			
.48	14.249	28.498			
.50	14.520	------	1.379	4.662	9.323
.52	14.771	29.543			
.54	15.003	30.005			
.56	15.214	30.427			
.58	15.403	30.806			
.60	15.570	------	1.509	6.171	10.285
.62	15.714	31.428			
.64	15.834	31.668			
.66	15.929	31.857			
.68	15.997	31.994			
.70	16.037	------	1.586	7.757	11.081
.72	16.047	32.095			
.74	16.026	32.052			
.76	15.971	31.942			
.78	15.879	31.758			
.80	15.747	------	1.596	9.353	11.692

FOR 4.0°K

K	y	2y	f(x)	$\sum f(x)$	$\sum \frac{1}{K} f(x) = F_{TT}$
.82	15.571	31.141			
.84	15.345	30.689			
.86	15.062	30.125			
.88	14.715	29.429			
.90	14.228	------	1.530	10.883	12.075
.92	13.764	27.527			
.94	13.108	26.216			
.96	12.259	24.517			
.98	11.056	22.112			
1.00	7.714	------	1.224	12.107	12.091 *

FOR 4.2°K

.00	1.00	------			
.02	2.534	5.068			
.04	3.338	6.676			
.06	4.025	8.050			
.08	4.649	9.299			
.10	5.231	------	.353	.353	3.53
.12	5.779	11.558			
.14	6.300	12.601			
.16	6.798	13.596			
.18	7.275	14.550			
.20	7.732	------	.653	1.006	5.03
.22	8.172	16.343			
.24	8.594	17.187			

FOR 4.2°K

K	y	2y	f(x)	$\sum f(x)$	$\sum \frac{1}{K} f(x) = F_{TT}$
.26	8.999	17.998			
.28	9.388	18.776			
.30	9.761	------	.878	1.884	6.28
.32	10.118	20.236			
.34	10.459	20.918			
.36	10.785	21.570			
.38	11.095	22.190			
.40	11.389	------	1.061	2.945	7.362
.42	11.667	23.334			
.44	11.929	23.857			
.46	12.174	24.348			
.48	12.402	24.805			
.50	12.614	------	1.203	4.148	8.30
.52	12.807	25.615			
.54	12.983	25.966			
.56	13.141	26.281			
.58	13.279	26.557			
.60	13.397	------	1.304	5.452	9.087
.62	13.495	26.990			
.64	13.572	27.144			
.66	13.626	27.253			
.68	13.658	27.315			
.70	13.664	------	1.358	6.810	9.728
.72	13.645	27.290			
.74	13.598	27.197			

FOR 4.2°K

K	y	2y	f(x)	$\sum f(x)$	$\sum \frac{1}{K} f(x) = F_{TT}$
.76	13.522	27.044			
.78	13.414	26.828			
.80	13.271	------	1.353	8.163	10.204
.82	13.091	26.181			
.84	12.867	25.735			
.86	12.596	25.192			
.88	12.268	24.536			
.90	11.874	------	1.268	9.431	10.479
.92	11.396	22.791			
.94	10.807	21.613			
.96	10.053	20.107			
.98	9.001	18.002			
1.00	6.1439	------	1.005	10.436	$\boxed{10.436}$ *

The following numerical integration technique was used to calculate the above friction factors "F_{TT}" at constant temperatures: 5.

TRAPEZOIDAL RULE -

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(x_o) + 2\,f(x_1) + 2\,f(x_2) + \ldots 2\,f\,(x_n-1) + f(x_n)\right]$$

where $h = \frac{b-a}{n}$ = equal intervals of total length

IN THIS NOTE $\int_a^b f(x)\,dx = f(x)$; $h = .02$

EXAMPLE: @ 3.8°K integrating from 0.0 → 0.1

f(0)	= 1.000
2f (.02)	= 5.582
2f (.04)	= 7.550
2f (.06)	= 9.264
2f (.08)	=10.846
f (.10)	= 6.169
SUM	=40.41

$$\int_0^{.1} f(x)\,dx = \frac{.02}{2}\,(40.41) = .404 = f(x)$$

Integrating from 0.0 to 1.0 quality, the following values of friction factor F_{TT} are calculated:

Temp. °K	3.80	4.00	4.20
F_{TT}	13.854	12.107	10.436

OBSERVATION: As temperature increases, friction factor F_{TT} decreases.

FOR VISCOUS - VISCOUS FLOW:

$$F_{vv} = \frac{1}{K}\int_0^K \left[\frac{x^{8/22} + Cvv^{8/11}\,(1 - x)^{8/22}}{Cvv^2}\right]^{22/8} dx$$

Using the same method as shown above, the following viscous - viscous friction factors F_{vv} were calculated:

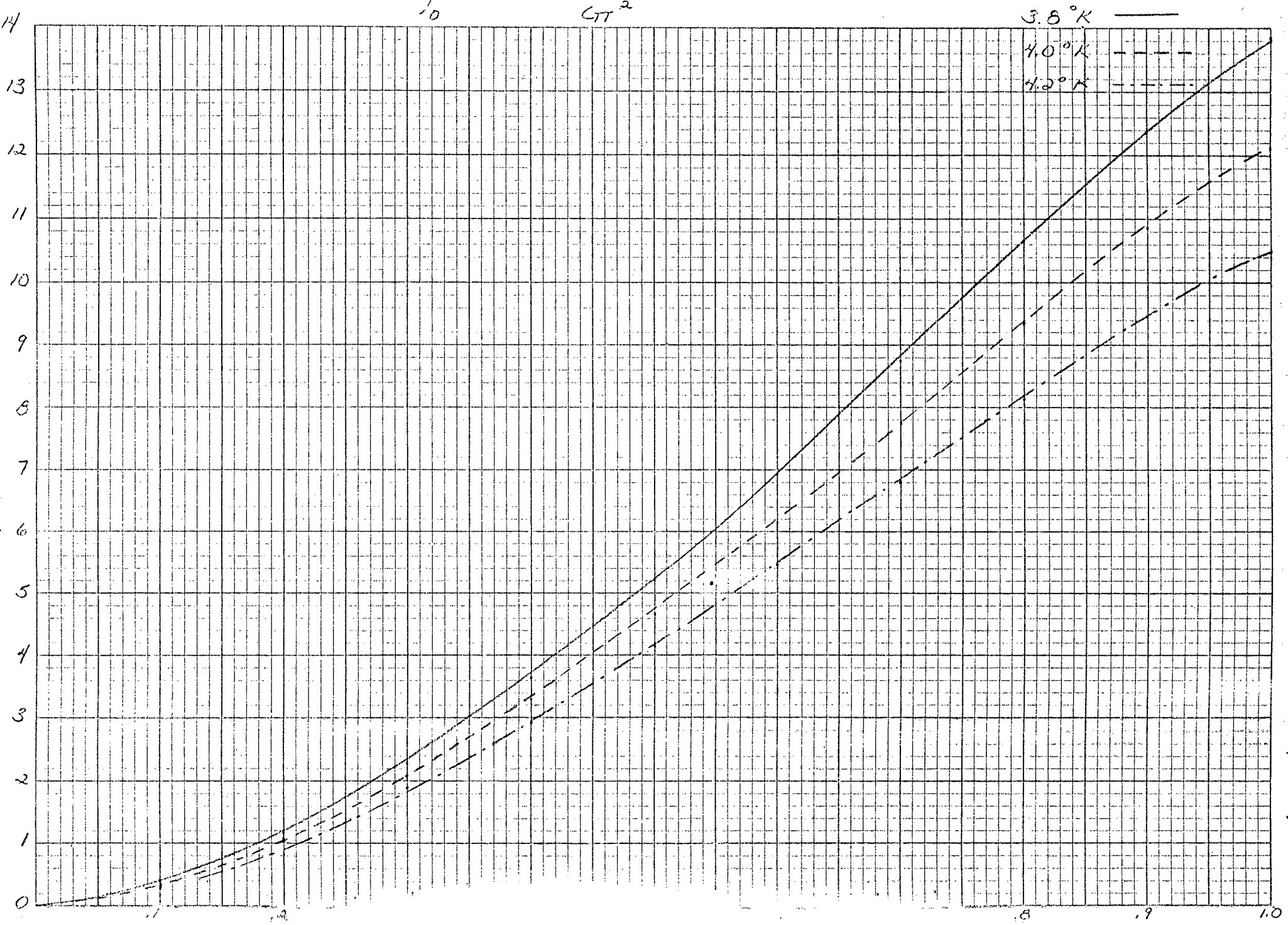
Integrating from 0.0 to 1.0 quality

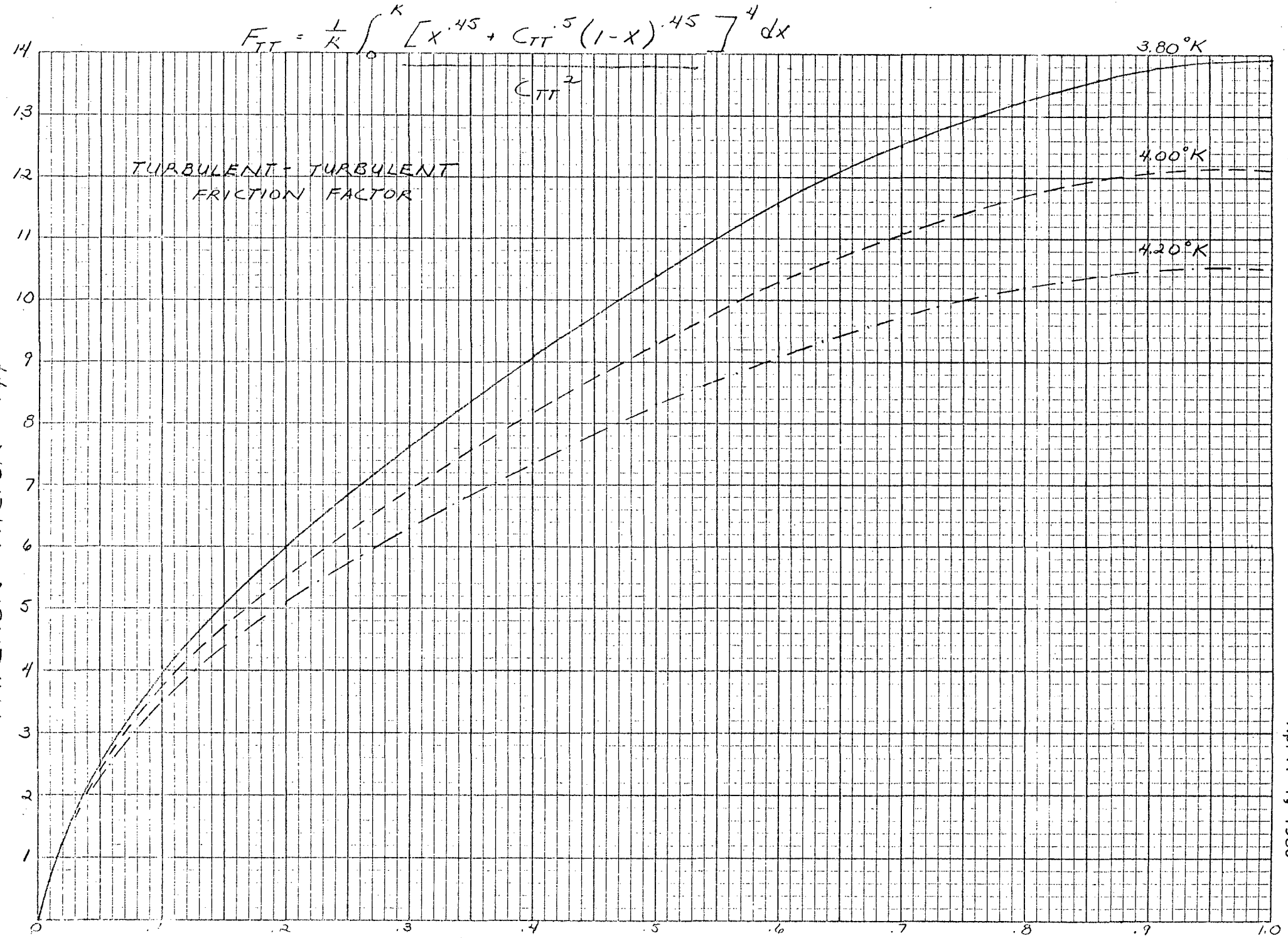
Temp ^oK	3.80	4.00	4.20
F_{vv}	5.922	5.487	5.057

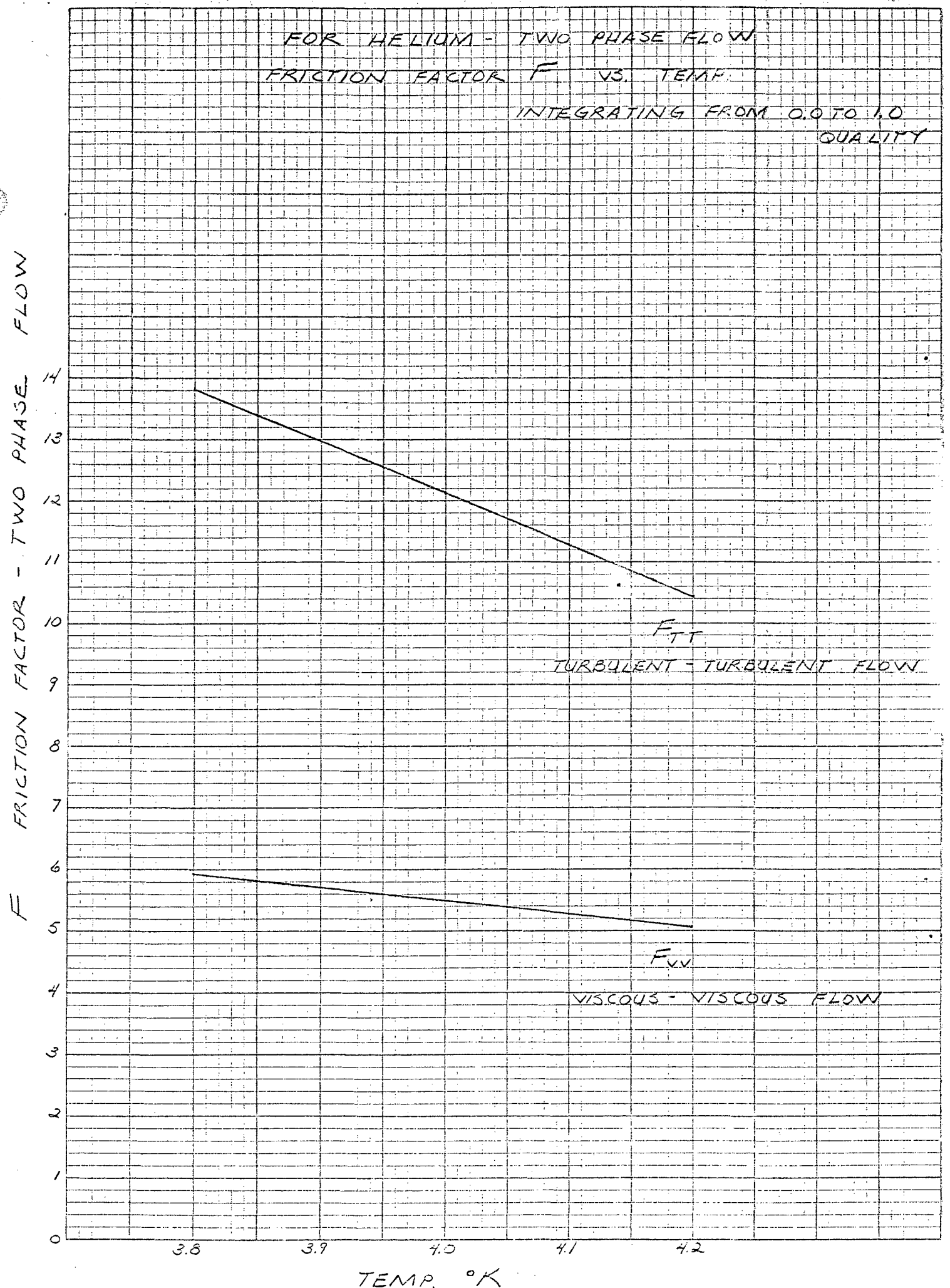
OBSERVATION: Turbulent - turbulent friction factor F_{TT} is greater than viscous - viscous friction factor F_{vv} at a given temperature; $F_{TT} \geq F_{vv}$

SQUARED 5 X 5 TO THE HALF INCH AS-0311-01

$$f(x) = \int_0^K \frac{\left[x^{.45} + C_{TT}^{.5}(1-x)^{.45}\right]}{C_{TT}^{2}}\,dx$$




$F_{TT} = \frac{1}{K}\int_0^K \left[x^{.45} + C_{TT}^{.5}(1-x)^{.45}\right]^4 dx$
C_{TT}^2
TURBULENT - TURBULENT
FRICTION FACTOR
3.80°K
4.00°K
4.20°K
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
.2
.3
.4
.5
.6
.7
.8
.9
1.0


FOR HELIUM - TWO PHASE FLOW
FRICTION FACTOR F VS. TEMP.
INTEGRATING FROM 0.0 TO 1.0 QUALITY
F FRICTION FACTOR - TWO PHASE FLOW
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
F_{TT}
TURBULENT - TURBULENT FLOW
F_{VV}
VISCOUS - VISCOUS FLOW
3.8
3.9
4.0
4.1
4.2
TEMP. °K

REFERENCES

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4. "Thermosphysical Properties of Helium-4 from 2 to 1500K with Pressures to 1000 Atmosphers," NBS Technical Note 631, November 1972.

5. Protter/Morrey, College Calculus with Analytical Geometry, Addison-Wesley, 1972, p. 544-545.

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