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ENGINEERING NOTE

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| AUTHOR | DEPARTMENT | LOCATION | DATE |
|---|---|---|---|
| M. A. Green | Mechanical | Berkeley | June 30, 1981 |

PROGRAM — PROJECT — JOB
High Field Magnet Development

Two-Dimensional and Integrated Field Analysis

TITLE
# Improvement of Fermilab Dipole Magnets for ISABELLE

This engineering note describes a series of computer calculations which show how the basic Fermilab dipole magnet can be improved in order for it to be used in a storage ring similar to ISABELLE. The nominal Fermilab dipole produces a good integrated field good to about 12 parts in $10^4$ at a 1-inch radius in the magnet aperture. This engineering note suggests a design which can produce a magnetic field good to about 4 parts in $10^4$ at a 1-inch radius in the magnet aperture.

The basic Fermilab dipole was modeled using data given in Ref. 1. Since we do not have all of the data needed to describe the Fermilab dipole accurately, the basic Fermilab dipole calculated at LBL for this note must be considered an approximation of the real thing. The design used here has an integrated field good to about 12 parts in $10^4$ at a 1-inch radius. The two-dimensional dipole field has a sextupole component about 19 parts in $10^4$ at a 1-inch radius. The actual Fermilab dipole is probably not as bad as this in the two-dimensional field. In any event, the Fermilab design was checked to produce the best theoretical design within the constraints of that design. The best of these theoretical designs yielded an integrated field which was good to 11 parts in $10^4$. The base case used for comparison may not be the exact Fermilab dipole design, but it is close enough. (See Fig. 1 for the cross-section.)

Over 60 cases were run on the LBL computer program SCMAG1. This program, which is described in Ref. 2, permits one to design a dipole or quadrupole magnet with a given higher multipole field structure. (When one designs a dipole, one would like to set as many of the higher multipoles equal to zero.) The SCMAG1 code uses symmetry to set the even multipoles (N = 2, 4, 6, etc.; N = 2 is the quadrupole, N = 4 is the octopole, and so on) equal to zero in the dipole. Refs. 3, 4, and 5 discuss the role of symmetry in dipole or quadrupole magnet

IMPORTANT RADII

| | |
|---|---|
| Usefull aperture radius | = 2.540 cm |
| Inner coil, inner radius | = 3.810 cm |
| Inner coil, outer radius | = 4.608 cm |
| Outer coil, inner radius | = 4.661 cm |
| Outer coil, outer radius | = 5.459 cm |
| Inner iron radius | = 9.563 cm |

IMPORTANT ANGLES

| | |
|---|---|
| Inner coil, lower angle | = 0.17° |
| Inner coil, upper angle | = 72.54° |
| Outer coil, lower angle | = 2.00° |
| Outer coil, upper angle | = 36.50° |
| Return leg angle | = 2.00° |

CURRENT DENSITY

| | | |
|---|---|---|
| Inner coil | = | 35918. A/cm² |
| Outer coil | = | 35813. A/cm² |
| Return leg | = | 30889. A/cm² |

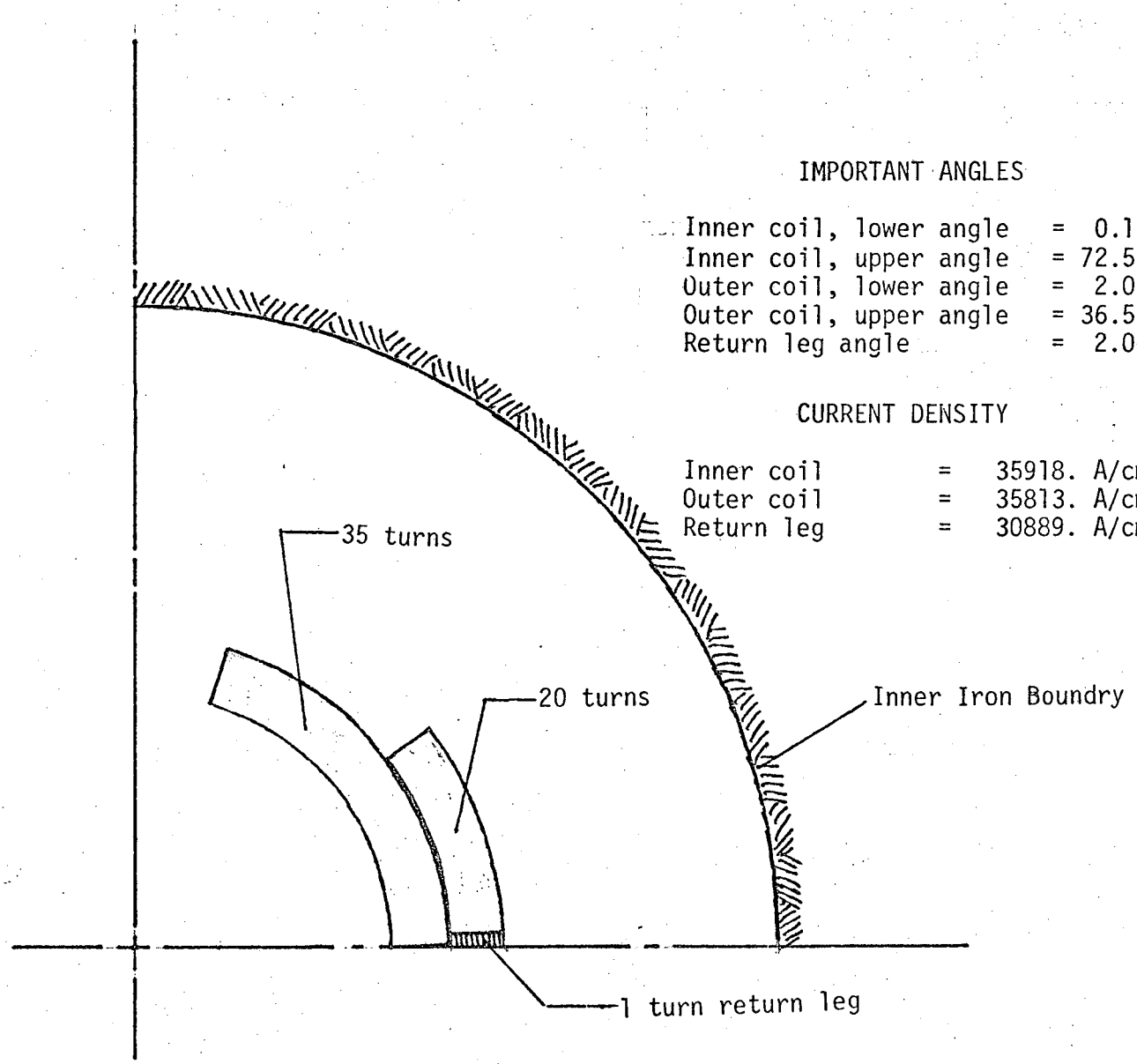



Figure 1. The Basic Fermi Lab Dipole Design with Finite Conductor (CASE 0)

design. In a dipole, the SCMAG1 code sets higher odd multipoles to zero (N = 3, 5, 7, etc.) by solving a set of simultaneous equations. These equations are usually non-linear, so a set of successive approximations is used to converge on a solution. The convergence on a solution is often difficult.

The SCMAG1 code was used to design the magnets so that they have both a good two-dimensional field and a good integrated field simultaneously. The Fermilab dipoles do not possess this property. The combination of good integrated field and good two-dimensional field is only really important when the magnet length is a substantial part of a betatron wave length. The Fermilab magnet length is about 2.5 percent of a betatron wavelength in the Fermilab synchrotron.

## The Search for Theoretical Solutions

The Fermilab dipole consists of two approximately equal current density layers and a block which is a return current on one side and a lower current density block on the other side. This block, which in the outer layer extends from 0° to 2.0°, is also found in all of our test cases. The current density in the return block is set to 30889 $Acm^{-2}$. In the Fermi dipole, the outer layer extends from 2.00° to 36.50°; its current density is set to 35813 $Acm^{-2}$. This current density is used in all of the outer layers of the test case dipoles as well. The inner layer of the Fermi dipole extends from 0.17° to 72.54°, and this layer current density is set to 35918 $Acm^{-2}$. All the inner layers in our test cases are set to this current density.

At first, the desired method for improving field uniformity was to keep the coils within the confines of the 72.54° and 36.50° angles. It was thought that the coils could be split into two pieces in order to achieve more degrees of freedom in the design. (Hence, more multipoles could be eliminated). By keeping the 72.54° and 36.50° angles, the Fermilab collet design could be used. Fig. 2 shows a variety of solutions that were tried. The cases which included keeping

Figure 2. Various coil configurations which have been run by SCMAG1 in order to try to improve the Fermi Lab dipole magnet

Fig. 2a Fermi Lab Dipoles Cases 0, 1, 7, and 19. Cases 7 and 19 have variable J

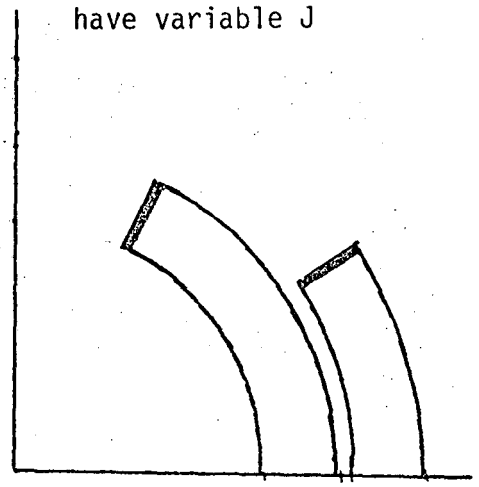


Fig. 2b Two layer-two block dipoles Cases 3, 4, 10, 11, 12, and 13

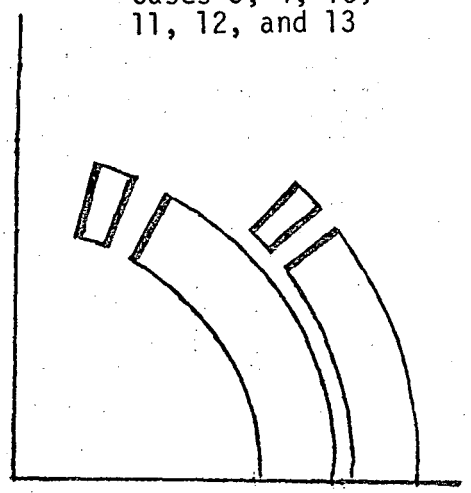


Fig. 2c Cases 6, 18, and 27.

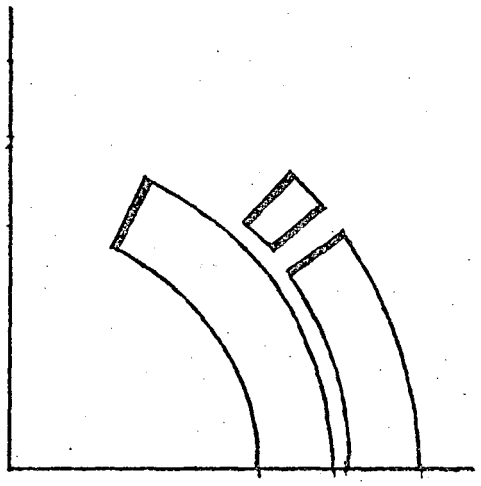
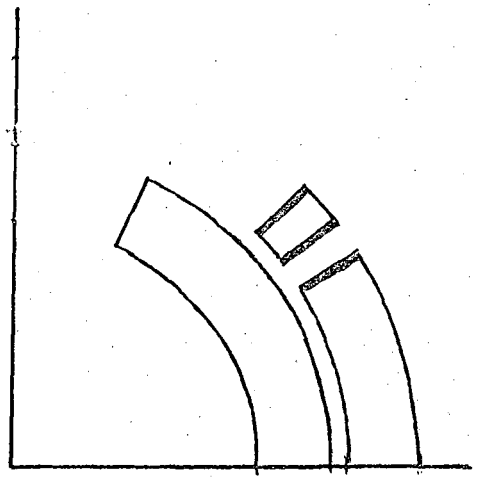


Fig. 2d Cases 5, 17, and 26.



NOTE: The heavy shaded angles are the angles to be changed by SCMAG1

Figure 2 continued

Fig. 2e Cases 8, 20, and 28.

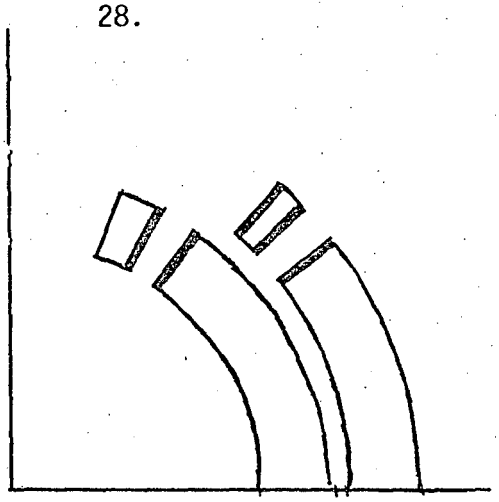


Fig. 2f Cases 2, 9, and 21.

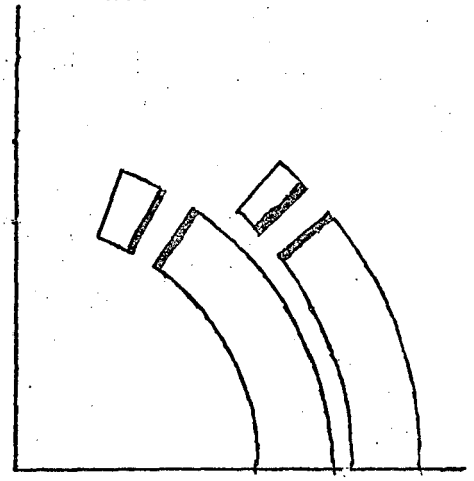


Fig. 2g Cases 22 and 29

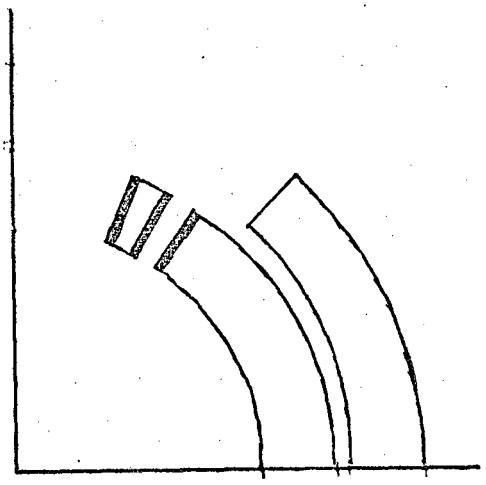
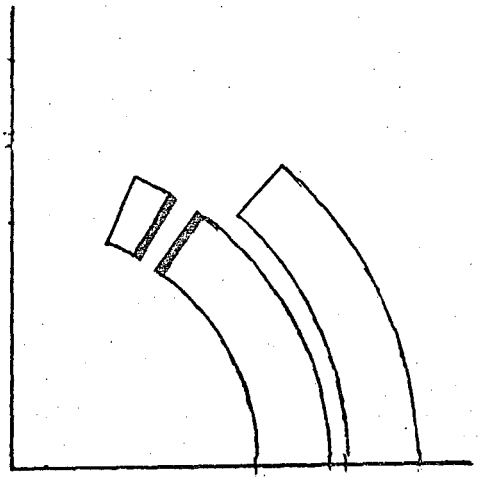


Fig. 2h Case 23



NOTE: The heavy shaded angles are the angles to be changed by SCMAG1

Figure 2 continued

Fig. 2i Cases 25 and 31

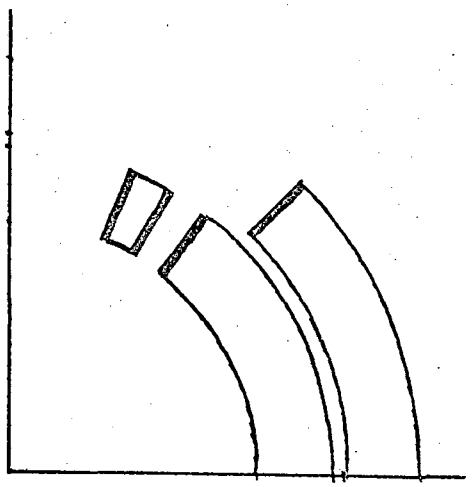


Fig. 2j Cases 24 and 30

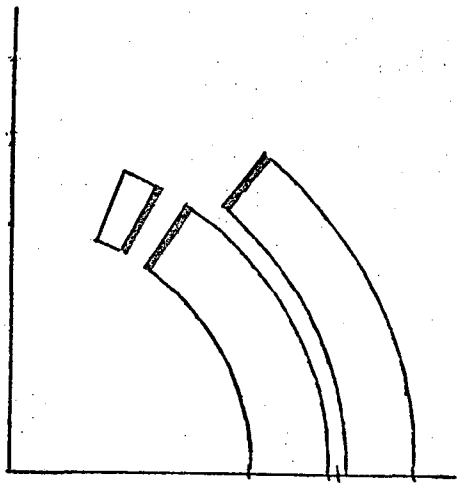


Fig. 2k Sector Dipoles
Inner Case 14
Outer Case 15
Combined are
Cases 16, 53,
and 59.

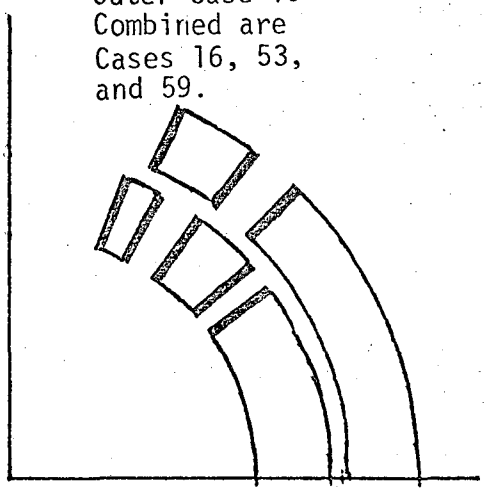
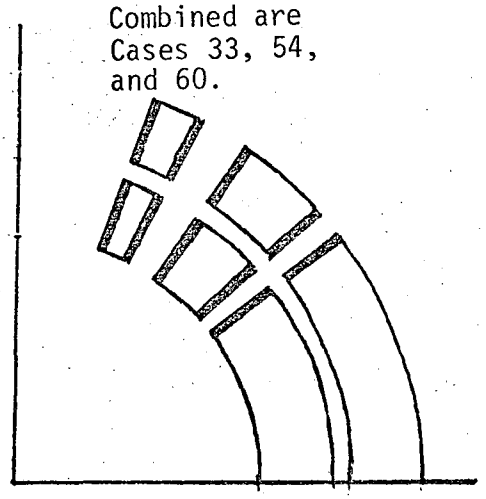


Fig. 2l Sector Dipoles
Inner Case 14
Outer Case 32
Combined are
Cases 33, 54,
and 60.



NOTE: The heavy shaded angles are the angles to be changed by SCMAG1

the 72.54° and 36.50° angles are illustrated in Fig. 2a, 2f, and 2h. Table 1 shows a number of cases which represent the attempts to find a solution. All cases failed except case 9, and its thin counterpart, case 21. Both of these cases yielded a central field which is less than eighty percent of the base line case. These cases were rejected.

Next, I decided to freeze the 72.54° angle but allow the 36.50° angle of the outer layer to float. Fig. 2d, 2e, and 2j illustrate this approach. None of these tries yielded a solution. (Only one mathematical solution, case 30, was found; the rest didn't converge.) Next, I decided to try solutions where the 72.54° angle could float. These cases are illustrated by Figs. 2b, 2c, 2g, and 2i. None of these solutions converged except for the two layer, two block dipole illustrated in Fig. 2b.

The two layer, two block dipole illustrated in Fig. 2b was suggested by R. Palmer of Brookhaven during a talk he gave this last spring (Ref. 6). The computer had a difficult time converging on a solution for this case, but a solution for the Fermilab geometry was finally achieved in case 13. Theoretically, this solution is very good in both the two-dimensional and integrated fields. The higher multipoles up to N = 19 are less than 1 part in $10^4$ of the dipole at Fermilab useful aperture radius of 2.54 cm. Case 13 was selected for further study in finite conductor form even though the central field is 96 percent of the baseline case and the highest angle is 5° above the 72.54° angle.

Since most of the block layer design tries were a failure, it was decided to look at sector block designs similar to ESCAR, Karlsruhe, and CERN designs. The computer immediately converged on solutions for three block inner coil plus two block outer coil (case 16, Fig. 2k), and solutions for three block inner coil plus three block outer coil (case 33, Fig. 2l). Case 16 has a worst multipole of 3.8 parts in $10^4$ at a radius of 2.54 cm. Case 33 has a multipole of 2.7 parts in $10^4$ at a radius of 2.54 cm. Both cases 16 and 33 have highest sector angles

TABLE 1

Various Cases Run on the SCMAG1 Code to Improve the Fermi Dipole

| CASE NUMBER | DESCRIPTION | DOES THE PROBLEM CONVERGE? | DOES THE SOLUTION MAKE SENSE? | REMARKS |
|---|---|---|---|---|
| | VARIATIONS OF THE BASIC FERMILAB DIPOLE MAGNET | | | |
| 0 | The Fermilab dipole as built (Fig. 1 and 2a) | yes | yes | $B_o$ = 4.33 T, 12.4 parts in $10^4$ |
| 1 | Theoretical Fermilab dipole variable angle (Fig. 2a) | yes | yes | $B_o$ = 4.37 T, 11.4 parts in $10^4$ |
| 7 | Theoretical Fermilab dipole variable J (Fig. 2a) | yes | yes | $B_o$ = 4.55 T, 12.3 parts in $10^4$ |
| 19 | Theoretical thin Fermi dipole variable J | yes | yes | same solution as case 7 |
| | MODIFICATIONS OF THE FERMI DIPOLE, THEORETICAL SOLUTIONS | | | |
| 3 | Two layer, two block, ISABELLE dimensions (Fig. 2b) | yes | yes | good to 1.1 parts in $10^4$ |
| 10 | Two layer, two block, different iron (Fig. 2b) | yes | yes | |
| 11 | Same as 10, but thin, different radii(Fig. 2b) | yes | yes | |
| 4 | Fermi two layer, two block, 1st try (Fig. 2b) | no | — | |
| 12 | Fermi two layer, two block, 2nd try (Fig. 2b) | no | — | |
| 13 | Fermi two layer, two block, 3rd try (Fig. 2b) | yes | yes | $B_o$ = 4.16 T, 0.2 parts in $10^4$ |
| 5 | Modification of Fermi (Fig. 2d) | no | — | |
| 6 | Modification of Fermi (Fig. 2c) | no | — | |
| 8 | Modification of Fermi (Fig. 2e) | no | — | |
| 2 | Modification of Fermi (Fig. 2f) | no | — | |
| 9 | Modification of Fermi (Fig. 2f), 2nd try | yes | yes | $B_o$ = 3.40 T, 7.2 parts in $10^4$ |

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DEPARTMENT Mechanical
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DATE June 30, 1981



TABLE 1 (cont.)

| CASE NUMBER | DESCRIPTION | DOES THE PROBLEM CONVERGE? | DOES THE SOLUTION MAKE SENSE? | REMARKS |
|---|---|---|---|---|
| 17 | Thin version of case 5 (Fig. 2d) | no | — | |
| 18 | Thin version of case 6 (Fig. 2c) | no | — | |
| 20 | Thin version of case 8 (Fig. 2e) | no | — | |
| 21 | Thin version of case 9 (Fig. 2f) | yes | yes | angle solution same as case 9 |
| 22 | Thin dipole (Fig. 2g) | no | — | |
| 23 | Thin dipole (Fig. 2h) | yes | no | angles clash |
| 24 | Thin dipole (Fig. 2j) | no | — | |
| 25 | Thin dipole (Fig. 2i) | no | — | |
| 26 | Close thin version of case 5 (Fig. 2d) | no | — | |
| 27 | Close thin version of case 6 (Fig. 2c) | no | — | |
| 28 | Close thin version of case 8 (Fig. 2e) | no | — | |
| 29 | Close thin version of case 22 (Fig. 2g) | no | — | |
| 30 | Close thin version of case 24 (Fig. 2j) | yes | no | angles clash |
| 31 | Close thin version of case 25 (Fig. 2i) | no | — | |
| | SECTOR BLOCK FERMI DIMENSIONS, THEORETICAL SOLUTIONS | | | |
| 14 | Inner layer with three blocks (Fig. 2k) | yes | yes | $B_o$ = 2.17 T, 5 parts in $10^4$ |
| 15 | Outer layer with two blocks (Fig. 2k) | yes | yes | $B_o$ = 2.36 T, 7 parts in $10^4$ |
| 16 | Case 14 plus case 15 (Fig. 2k) | yes | yes | $B_o$ = 4.53 T, 3.8 parts in $10^4$ |

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TABLE 1 (cont.)

| CASE NUMBER | DESCRIPTION | DOES THE PROBLEM CONVERGE? | DOES THE SOLUTION MAKE SENSE? | REMARKS |
|---|---|---|---|---|
| 32 | Outer layer with three blocks (Fig. 21) | yes | yes | $B_o$ = 2.30 T, 0.5 parts in $10^4$ |
| 33 | Case 14 plus case 32 (Fig. 21) | yes | yes | $B_o$ = 4.48 T, 2.7 parts in $10^4$ |
| 53 | Case 14 and 15; all angles to eliminate 8 poles | no | -- | |
| 54 | Case 14 and 32; all angles to eliminate 10 poles | no | -- | |
| 59 | Case 14 and 15; lengths vary to improve B dl | yes | no | solution is worse than case 16 |
| 60 | Case 14 and 32; lengths vary to improve B dl | yes | no | solution is worse than case 33 |
| | OTHER THEORETICAL TRIES | | | |
| 34 | Thick, coincident current density, 1st try | no | -- | convergence was close |
| 35 | Thick, coincident current density, 2nd try | no | -- | convergence was close |
| 36 | Inverse Fermi dipole | yes | no | angle greater than 90° |
| | Cases 13, 16, and 33 were converted to finite conductor models. | | | |
| | FINITE CONDUCTOR VERSIONS OF BEST THEORETICAL SOLUTIONS | | | |
| 37 | A version of case 14, 15 turns, lower free | yes | yes | Best $B_o$ = 2.13 T, 5.6 parts in $10^4$ |
| 38 | A version of case 14, 15 turns, lower fixed | yes | yes | N = 7 is too high |
| 39 | A version of case 14, 16 turns, lower free | yes | no | angles clash |
| 40 | A version of case 14, 16 turns, lower fixed | yes | yes | N = 7 is too high |
| 41 | A version of case 15, 23 turns, lower free | yes | yes | Best $B_o$ = 2.33 T, 7.5 parts in $10^4$ |

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TABLE 1 (cont.)

| CASE NUMBER | DESCRIPTION | DOES THE PROBLEM CONVERGE? | DOES THE SOLUTION MAKE SENSE? | REMARKS |
|---|---|---|---|---|
| 42 | A version of case 15, 23 turns, lower fixed | yes | yes | N = 5 is too high |
| 43 | A version of case 15, 24 turns, lower free | yes | no | blocks clash |
| 44 | A version of case 15, 24 turns, lower fixed | yes | yes | N = 5 is too high |
| 45 | Case 37 plus case 41 | yes | yes | $B_0$ = 4.46 T, 4.2 parts in $10^4$ |
| 46 | Two layer, two block, see case 13 | yes | no | angles clash |
| 47 | Two layer, two block, see case 13 | yes | no | angles clash |
| 48 | Two layer, two block, see case 13 | yes | no | angles clash |
| 49 | Two layer, two block, see case 13 | yes | yes | $B_0$ = 4.11 T, 8.1 parts in $10^4$, N = 7 |
| 50 | Two layer, two block, see case 13 | yes | no | angles clash |
| 51 | Two layer, two block, see case 13 | yes | yes | OK, 4.11 T, 4 parts in $10^4$, ends difficult |
| 52 | Two layer, two block, see case 13 | yes | yes | Best $B_0$ = 4.11 T, 4.2 parts in $10^4$ |
| 55 | A version of case 32, lower free | yes | no | angles clash |
| 56 | A version of case 32, lower fixed, nom. $\theta$ | yes | yes | too large N = 7 |
| 57 | A version of case 32, lower fixed, min. $\theta$ | yes | yes | Best $B_0$ = 2.30 T, 2.1 parts in $10^4$ |
| 58 | Case 37 plus case 57 | yes | yes | $B_0$ = 4.43 T, 3 parts in $10^4$ |
| 61 | Case 0 with E = $10^6$ psi in coils | yes | yes | 10.3 parts in $10^4$ added sextupole |
| 62 | Case 45 with E = $10^6$ psi in coils | yes | yes | 8.6 parts in $10^4$ added sextupole |

which are less than 72.54°. Both cases yield a central field which is 3 to 5 percent higher than the base line case. The magnetic length of these cases is somewhat longer than the base line case for a given maximum coil length.

The base line case, case 0, and the theoretical Fermi dipole case, case 1, are given as computer runs in the appendix. The theoretical solutions selected for finite conductor studies (cases 13, 16, and 33) are also given in the appendix. In theory, a large improvement is possible over the base line case (case 0) or its theoretical equivalent (case 1) using either the two layer, two block dipole (case 13) or sector block dipoles (cases 16 and 33).

Before leaving the problem of theoretical coil design, it is useful to discuss why solutions could not be found for many of the cases. In general, three types of non-convergence occurred: 1) the optimization tried to converge on a solution, but couldn't quite make it; 2) the optimization converged on a solution close to the one desired but the desired solution did not exist; and 3) the optimization did not converge at all. In this case, the optimizer often went to very large angles in order to try and find a solution.

The third type of non-convergence was the most prevalent. This does not mean there is no solution, but the program optimizer did not find a solution. The optimizer itself should be changed if solutions that are difficult to find are to be found. A suggested course of action is to put limits in the program to keep the problem within bounds. Another approach is to have the computer move the starting case so that the starting case will be in a different potential well. A third approach is to use a DFP optimizer which uses second derivatives as well as first derivatives to optimize the solution.

Some cases, such as the sector blocks, always converged. The types of cases which did not work were cases when there was a solution which already eliminated or minimized a couple of multipoles, but one further subdivided the problem to improve the solution. The higher the multipole, the harder it is to eliminate

with small changes. As an example, one can go to the basic sector block cases (cases 14, 15, and 32). If one combines, say, case 14 with 32 and tries to double the number of multipoles eliminated, the problem does not converge. (Case 54 was an example of this.) Most of the modifications I tried on the basic Fermi dipole design are examples of this type of non-convergence.

## Finite Conductor Versions of the Best Theoretical Solutions

Cases 13, 16, and 33 were selected for conversion into finite conductor magnets. The conductor assumed is keystoned Fermilab 23-strand Rutherford cable. The width of the cable with all of its barberpole wrap and insulation was assumed to be 1.52 mm based on the case 0 design. The return leg conductor on the mid-plane of the outer layer is wider. Each conductor on the inner layer is assumed to be 2.0711° wide. Each conductor on the outer layer is assumed to be 1.7228° wide. (The return leg conductor at the midplane on the outer layer is assumed to be 2.000° wide.)

The finite conductor design requires that each layer or block contains a finite number of conductors. In general, one puts the nearest number of conductors in which fits the theoretical solution. The layer or block has a midpoint angle which can be varied in order to eliminate some of the lower multipoles. Several runs of each of the theoretical cases are required in order to find the most suitable finite conductor solutions. All of the finite conductor cases converged mathematically. Most were eliminated either because they couldn't be built physically, or they had an excessive amount of a particular multipole.

The finite conductor version of case 16 is case 45. Case 45 consists of the sum of case 37 (the best finite conductor solution for case 14) and case 41 (the best finite conductor solution for case 15). The finite conductor version of case 33 is case 58. Case 58 consists of the sum of case 37 and case 57 (the best finite conductor solution for case 32). The finite conductor version of the

Fermi-sized two layer, two block dipole (case 13) is case 52. The multipole structure of the two-dimensional and integrated field in cases 45, 58, and 52 is found in the computer printout in the appendix. These cases can be compared directly with cases 16, 33, and 13, respectively, which are also given in the appendix.

Table 2 compares cases 45, 58, and 52 with the base case (case 0). The use of finite conductor hardly affects the field uniformity which can be generated by a sector block dipole. (For example, the field error went from 3.8 parts in $10^4$ in case 16 to 4.2 parts in $10^4$ in case 45. The field error went from 2.7 parts in $10^4$ in case 33 to 3.0 parts in $10^4$ in case 58.) The two layer, two block dipole did not fare as well. (Case 13 showed 0.2 parts in $10^4$ as compared to 4.2 parts in $10^4$ in case 52.)

TABLE 2

A Comparison of the Finite Conductor Cases Studied

| Case No. | Central Field (T) | Number of Turns | Highest Angle (degrees) | Integrated Field Highest Multipole at R = 2.54 cm |
|---|---|---|---|---|
| 0 | 4.33 | 112 | 72.54 | 12.3 parts in $10^4$ |
| 45 | 4.46 | 120 | 71.72 | 4.2 parts in $10^4$ |
| 58 | 4.43 | 120 | 72.30 | 3.0 parts in $10^4$ |
| 52 | 4.11 | 108 | 79.66 | 4.2 parts in $10^4$ |

The two layer, two block dipole does not look nearly as attractive as either of the two sector block designs. The highest angle approaches 80°. This probably makes the dipole design unsuitable for use with collets of the Fermilab

type. Of the two sector block designs, case 45 is simpler. It also has a higher central field and it has a lower outer coil angle. (66.76° in case 45 versus 72.30° in case 58.)

For a full scale comparison of the improved Fermi-sized dipole with the Fermi dipole, case 0 and case 45 were run in the SCMAG2 computer code. Tables 3 and 4 compare a number of the parameters for the two designs. Fig. 3 shows the sector block design of case 45.

From Table 3, a number of advantages emerge. They are: 1) for a given current in the superconductor, the central field and integrated field are higher; and 2) for a given coil length, the magnetic length of the sector design is longer. The disadvantages of the sector design are: 1) it requires more superconductor (8 turns per magnet); and 2) considerable development work is required before the sector design can be used. The extra development is distinctly disadvantageous from the standpoint of magnet development for ISABELLE.

## The Affect of Strain and Placement Errors on Magnetic Field Uniformity

Tables 4 and the cases in the appendix illustrate that the Fermilab dipole can be improved to produce a more uniform two-dimensional and integrated field. A real improvement of magnetic field quality is only partially tied to the coil design. Two other factors come into play. The first is the placement accuracy of coils, blocks, and conductors. The second is the symmetric and asymmetric effects of magnetic coil strain on the placement of the coils and conductors.

The whole problem of field errors due to asymmetric conductor, block, and coil displacement errors is discussed in Ref. 7. For a given coil design, the effects of random conductor and coil errors can be calculated using SCMAG3. In a dipole with a 50 mm useful bore, the following random tolerance limits should be applied if random errors are kept below 3 parts in $10^4$.

TABLE 3

A Comparison of Finite Conductor Sector Block Dipole with the Base Line Fermilab Dipole

| | Base Fermi Magnet Case 0 | Sector Block Magnet Case 45 |
|---|---|---|
| Design Aperture Radius (cm) | 2.54 | 2.54 |
| Inner Coil Radius (cm) | 3.81 | 3.81 |
| Iron Radius (cm) | 9.563 | 9.563 |
| Coil Length (m) | 6.226 | 6.226 |
| Coil Design Current (A) | 4350 | 4350 |
| Central Induction (T) | 4.328 | 4.456 |
| Integrated Induction (Tm) | 26.519 | 27.362 |
| Magnetic Length (m) | 6.127 | 6.141 |
| Peak Induction in the Coil (T) | 4.71 | 4.86 |
| Magnet Inductance (H) | 0.046 | 0.051 |
| Number of Turns | 112 | 120 |
| Stored Magnet Energy at Design Current (J) | $4.38 \times 10^5$ | $4.81 \times 10^5$ |
| Superconductor Matrix Current Density ($Am^{-2}$) | $5.12 \times 10^8$ | $5.12 \times 10^8$ |
| $EJ^2$ one coil $JA^2m^{-4}$ | $1.15 \times 10^{23}$ | $1.26 \times 10^{23}$ |

TABLE 4

Two-Dimensional and Integrated Field Multipole Ratios at R = 2.54 cm for the Base Fermi Dipole (Case 0) and the Sector Block Magnet Design (Case 45) Described in Table 3

| Multipole Number | Multipole Ratio: Two-Dimensional Field, Case 0 | Two-Dimensional Field, Case 45 | Integrated Field, Case 0 | Integrated Field, Case 45 |
|---|---|---|---|---|
| 1 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 3 | 0.00187 | --- | --- | --- |
| 5 | 0.00000 | --- | -0.00013 | -0.00010 |
| 7 | 0.00073 | -0.00000 | 0.00072 | -0.00010 |
| 9 | -0.00123 | -0.00041 | -0.00123 | -0.00042 |
| 11 | 0.00037 | -0.00000 | 0.00037 | 0.00006 |
| 13 | -0.00008 | -0.00026 | -0.00008 | -0.00026 |
| 15 | 0.00000 | 0.00009 | 0.00000 | 0.00009 |
| 17 | -0.00000 | 0.00002 | 0.00000 | 0.00002 |
| 19 | -0.00000 | -0.00001 | -0.00000 | -0.000001 |
| Central Induction (T) | 4.328 | 4.456 | --- | --- |
| Integrated Induction (Tm) | --- | --- | 26.519 | 27.362 |

IMPORTANT ANGLES

| | |
|---|---|
| Inner coil, 1st block, lower angle | = 0.50° |
| Inner coil, 1st block, upper angle | = 31.57° |
| Inner coil, 2nd block, lower angle | = 36.05° |
| Inner coil, 2nd block, upper angle | = 52.62° |
| Inner coil, 3rd block, lower angle | = 63.44° |
| Inner coil, 3rd block, upper angle | = 71.72° |
| Outer coil, 1st block, lower angle | = 2.21° |
| Outer coil, 1st block, upper angle | = 41.84° |
| Outer coil, 2nd block, lower angle | = 51.25° |
| Outer coil, 2nd block, upper angle | = 66.76° |
| Outer coil, return leg angle | = 2.00° |

CURRENT DENSITY

| | | |
|---|---|---|
| Inner coil | = | 35918 A/cm² |
| Outer coil | = | 35813 A/cm² |
| Return leg | = | 30889 A/cm² |

IMPORTANT RADII

| | |
|---|---|
| Usefull aperture radius | = 2.540 cm |
| Inner coil, inner radius | = 3.810 cm |
| Inner coil, outer radius | = 4.608 cm |
| Outer coil, inner radius | = 4.661 cm |
| Outer coil, outer radius | = 5.459 cm |
| Inner iron radius | = 9.563 cm |

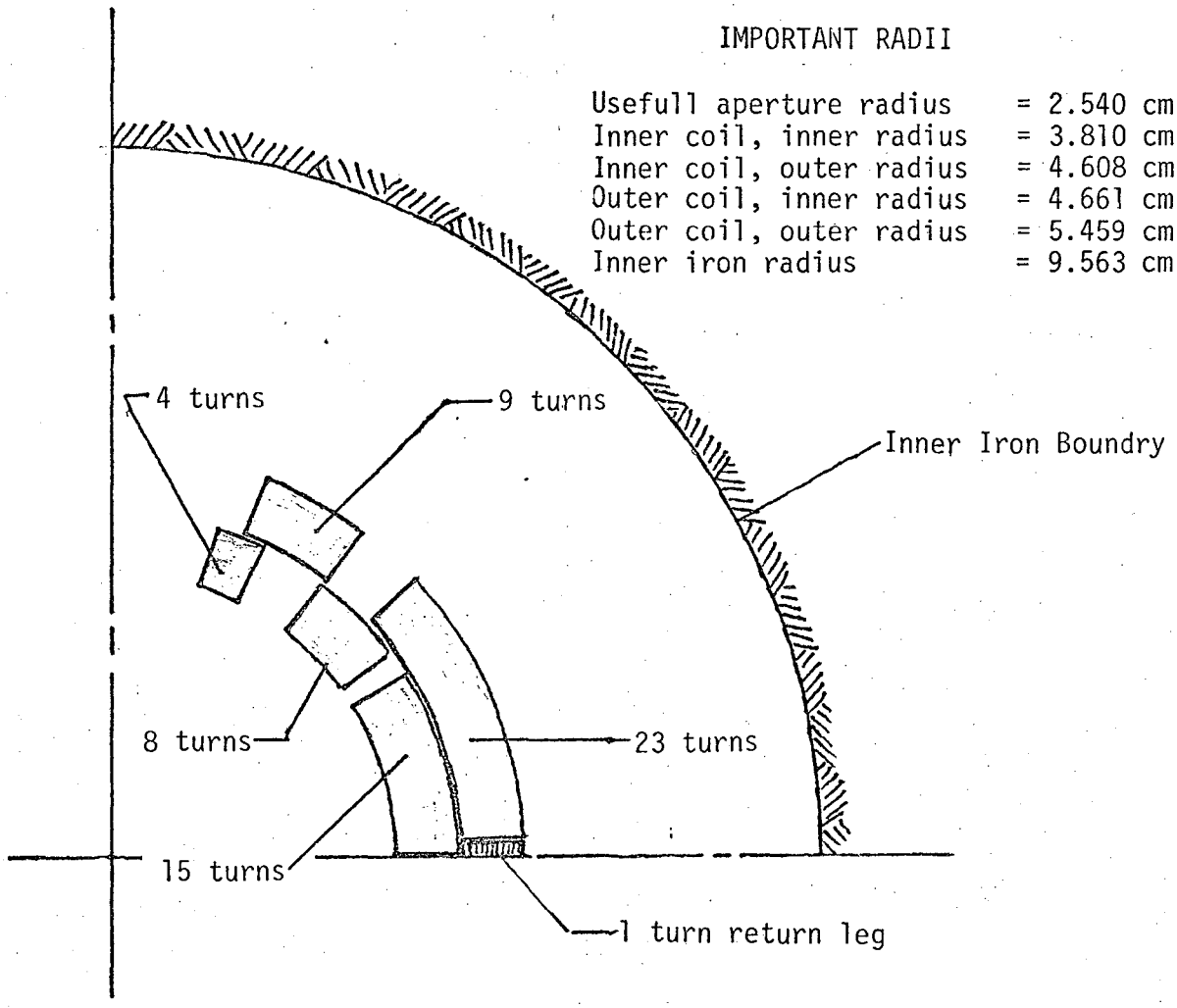



Figure 3. The Three Block Inner Coil and Two Block Outer Coil with Finite Conductor (CASE 45)

| | |
|---|---|
| Single 4350 A conductor | ≅ 0.5 mm |
| Sector block 20 block magnet | ≅ 0.07 mm |
| Coil half placement 4 parts | ≅ 0.04 mm |
| Placement in iron* | ≅ 0.02 mm |

Symmetric field errors, such as those caused by strain in a magnetic field, are also discussed in Ref. 7. I used the SCMAG2 code to calculate the magnetic forces within the coil. Once the forces are calculated, stress the radial, and azimuthal direction can be estimated. Within the Fermi dipole (case 0) peak azimuthal stresses of $4.8 \times 10^7$ $Nm^{-2}$ (7000 psi) are calculated. Similar stresses are found in the sector block design (case 45). If the coil is not prestressed so that the pole angle is allowed to move, considerable strain can occur within the coil.

The angle change of various key angles in case 0 and case 45 was calculated. No prestress was assumed. The coil structure modulus was assumed to be $7 \times 10^9$ $Nm^{-2}$ ($10^6$ psi). Runs 61 and 62 (see the appendix) estimated the field structure which would be generated at a current of 4350 A at a radius of 2.54 cm in the Fermi dipole (case 0) and its improvement (case 45). The magnetic field changes which result from magnetic strain are given in terms of multipole ratios. Table 5 shows ratios for N = 3, 5, 7, and 9. At an elastic modulus of $7 \times 10^9$ $Nm^{-2}$ ($10^6$ psi) the sextupole error approaches 10 parts in $10^4$. Proper prestress of the coil will reduce the error considerably but it appears that a minimum coil modulus of $1.4 \times 10^{10}$ $Nm^{-2}$ ($2.0 \times 10^6$ psi) is needed.

From Table 5, it is clear that in order to use the field improvement possible in the sector block design, one must increase the modulus of the coil.

*This is a quadrupole error which can be corrected after magnetic measurement.

TABLE 5

Elastic Coil Deformation

Errors in Case 0 and Case 45 Due to Magnetic Forces

When the Coil Structure has an Elastic Modulus of $7 \times 10^9$ $Nm^{-2}$ ($10^6$ psi).

| Multipole Number | Multipole Ratio: Fermi Dipole Case 61 - Case 0 | Multipole Ratio: Sector Dipole Case 62 - Case 45 |
|---|---|---|
| 3 | 0.00103 | 0.00086 |
| 5 | -0.00044 | -0.00010 |
| 7 | 0.00017 | 0.00008 |
| 9 | -0.00006 | 0.00001 |

Increasing the structure elastic modulus also helps reduce the random conductor, coil, and block placement errors. It is clear that additional development work is needed in order to improve the quality of the field in the Fermi-type dipole magnets.

## REFERENCES


1. "Design Report 1979, Superconducting Accelerator," published by the Fermi National Accelerator Laboratory.

2. M. A. Green, "Magnetic Field Programs, Instructions for the Use of SCMAG1, SCMAG2, SCMAG3, and SCMAG4," LBL Engineering Note M5692, February 1981.

3. K. Halbach, "Fields and First Order Perturbation Effects in Two-Dimensional Conductor Dominated Magnets," Nucl. Instr. and Meth., 78 (1970) p. 185-198.

4. M. A. Green, "The Elimination of Higher Multipoles in Air Core Dipoles and Quadrupoles," LBL Engineering Note M4299, December 1969.

5. M. A. Green, "The Elimination of Higher Multipoles in Two-Dimensional and Integrated Fields of Conductor-Dominated Dipole and Quadrupole Magnets with Iron Shells," LBL Engineering Note M4373, January 1971.

6. C. Taylor, private communication; and view graphs presented by R. Palmer of Brookhaven National Laboratory for a magnet with improved field.

7. M. A. Green, "Field Aberrations in Superconducting Dipole and Quadrupole Magnets for a Superconducting Synchrotron", Kernforschungszentrum Karlsruhe Externer Bericht 3/71-7, September 1971 (revised June 1972), Karlsruhe, West Germany.

## APPENDIX

This appendix shows the computer printouts for cases 0, 1, 13, 16, 33, 45, 52, 58, 61, and 62. Some of the cases have the SCMAG1 code change parameters to get the desired multipole structure in both the two-dimensional and integrated field. In other cases, the integrated field is the only field which has higher multipoles eliminated. There are also a number of cases where the field and integrated field structures are calculated for a given geometry.

A sample case is shown on the next page. This case (case 41) has SCMAG1 eliminate two multipoles (N = 3 and N = 5) in the two-dimensional field. One multipole (N = 3) is eliminated in the integrated field. The first group of data is the input parameters to the problem. The meaning of the symbols NT, R1, R2, A1, A2, SLEN, XJ, A3, and MK are explained in Table 6 and Fig. 4 which come from Ref. 2. There are three lines of data starting with NT = 1 or NT = 2 which describe the magnet coils. Round, equal length ends are assumed. XLEN is the straight section length of the coils between the ends. The definition of A1, A2, R1, and R2 is given in Fig. 4 depending on the value of NT. The three lines below the lines which have NT = 1 or 2 start with NT = -0. These lines have a 1 in the A1 or A2 column. This tells one that that value is changed to optimize the two-dimensional field multipole structure. In the line which starts with NT = -0, a 2 in the XLEN column states that the length of that coil straight section can be changed in order to improve the multipole structure of the integrated field.

The second group of data in the sample run is the iterated values of the two-dimensional field multipole coefficients. This only is printed when there is a two-dimensional optimization. The third group of data in the sample run is two-dimensional and integrated field multipole solutions. This is printed in all cases. The data applies to the final solution for the coil. The fourth group

TITLE BLOCK

TEST CASE 41, BLOCK DIPOLE WITH FERMI BORE, SEE CASE 15

END TYPE

RO = USEFULL APERATURE RADIUS

RIIRON = IRON SHELL INNER RADIUS

INPUT PARAMETERS

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 20 | 4 | 2.5400 | 9.5630 | -0. | -0. | |
| 1 | -0. | -0 | -0. | -0 | -0. | -0 | -0. |
| 1 | -0. | -0. | -0. | -0. | -0. | | |

← desired multipole values N=3, 5, 7 . .

| NT | R1 | R2 | A1 | A2 | XLEN | XJ | A3 | MK |
|---|---|---|---|---|---|---|---|---|
| 2 | 4.6610 | 5.4590 | 22.1900 | 19.8122 | 590.0000 | 35813. | .1500 | 1 |
| 2 | 4.6610 | 5.4590 | 59.2765 | 7.7526 | 590.0000 | 35813. | .1500 | 2 |
| 1 | 4.6610 | 5.4590 | 0. | 2.0000 | 590.0000 | 30889. | .1500 | 3 |
| -0 | -0. | -0. | 1.0000 | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | 1.0000 | -0. | 2.0000 | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |

to be changed by program 2D field

To be changed by program integrated field

ITERATED VALUES OF THE MULTIPOLE COEFFICIENTS (2 DIMENSIONAL FIELD)

| N=3 | N=5 |
|---|---|
| -.002814 | -.000003 |
| .000011 | -.000002 |
| .000000 | -.000000 |

ONLY TWO MULTIPOLES CAN BE CHANGED IN 2D FIELD
ONLY ONE MULTIPOLE CAN BE CHANGED IN INTEGRATED FIELD

TWO DIMENSIONAL AND INTEGRATED FIELD MULTIPOLE SOLUTIONS

| N | 2D FIELD | INTG. FIELD |
|---|---|---|
| 1 | -2.33472 | -13.99764 |
| 3 | -.00000 | -.00000 |
| 5 | .00000 | .00266 |
| 7 | .00002 | .00089 |
| 9 | .00181 | .01061 |
| 11 | -.00036 | -.00215 |
| 13 | -.00006 | -.00035 |
| 15 | .00003 | .00020 |
| 17 | -.00000 | -.00000 |
| 19 | -.00000 | -.00001 |

OUTPUT MAGNET PARAMETERS

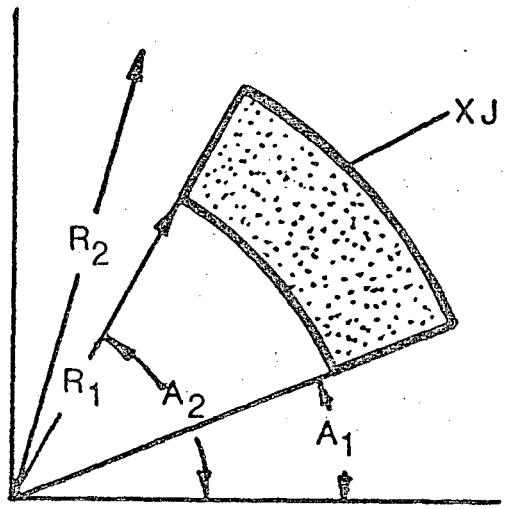
| NT | R1 | R2 | A1 | A2 | XLEN | XJ | A3 | MK |
|---|---|---|---|---|---|---|---|---|
| 2 | 4.6610 | 5.4590 | 22.0236 | 19.8122 | 590.0000 | 35813. | .1500 | 1 |
| 2 | 4.6610 | 5.4590 | 59.0047 | 7.7526 | 582.9321 | 35813. | .1500 | 2 |
| 1 | 4.6610 | 5.4590 | 0. | 2.0000 | 590.0000 | 30889. | .1500 | 3 |
| -0 | -0. | -0. | 1.0000 | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | 1.0000 | -0. | 2.0000 | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |

CHANGED BY PROGRAM

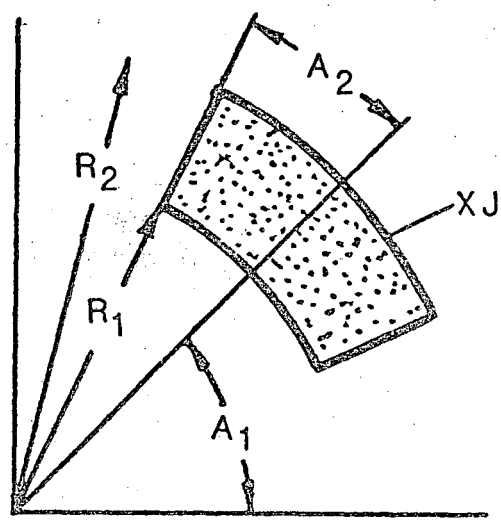
of data is the output magnet parameters. This gives the new angles and lengths needed to calculate the multipole coefficients in the final solution. Note all multipole coefficients give at RO the useful aperture radius (for all cases, RO = 2.54 cm). If there is no change in angle or coil length (no optimization is done) the output magnet parameters are eliminated. (The integrated values of multipole coefficients are also eliminated.)

All of the cases have NT = 1 or NT = 2. Case 0 optimizes the integrated field only. Case 1, case 13, and case 52 optimize both the two-dimensional and integrated field. Case 16, case 33, case 45, case 58, case 61, and case 62 have no optimization in them. The two-dimensional and integrated fields are calculated for the given input parameters.

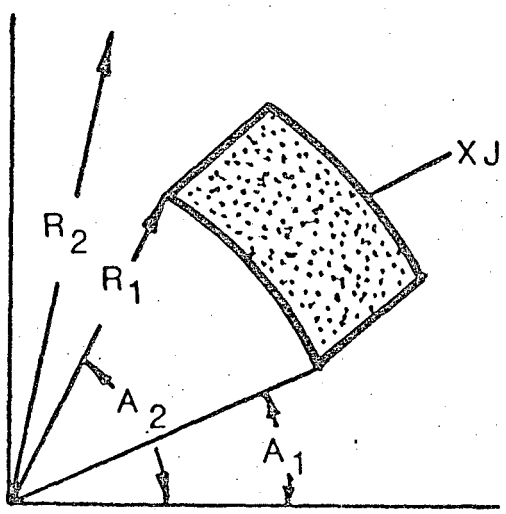
Figure 4 - Various dipole coil configurations case calculated by the programs. (Note: for quadrupoles these configurations would be confined to an angle between 0 and 45 degrees. Symmetry is used in these codes.)



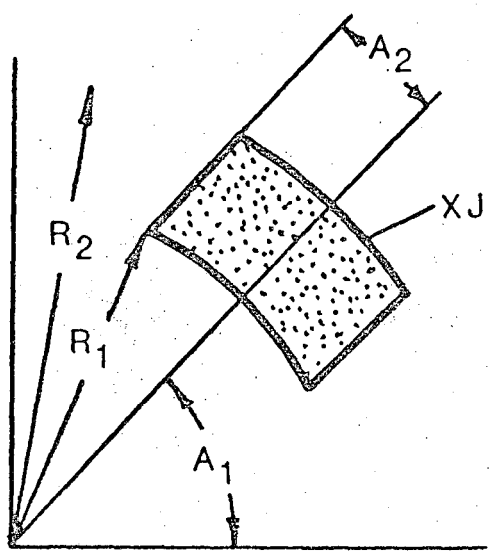

Case 1 - Sector Coils




Case 2 - Sector Coils




Case 3 - Modified Sector Coils




Case 4 - Modified Sector Coils

Table 6. Input data which describes the magnet coils in the SCMAG programs. The dimension of each of the parameters is shown.

| | Coil Type | Coil R Parameters | | Coil A Parameters | | Coil Straight Sect. Leng. | Coil Current Density | Cond. spacing in A direct. or cond. cur. | Assembly Number |
|---|---|---|---|---|---|---|---|---|---|
| | NT | $R_1$ | $R_2$ | $A_1$ | $A_2$ | XLEN | XJ | $A_3$ | MK |
| Case 1 | 1 | cm | cm | deg | deg | cm | A $cm^{-2}$ | cm | N.D. |
| Case 2 | 2 | cm | cm | deg | deg | cm | A $cm^{-2}$ | cm | N.D. |
| Case 3 | 3 | cm | cm | deg | deg | cm | A $cm^{-2}$ | cm | N.D. |
| Case 4 | 4 | cm | cm | deg | cm | cm | A $cm^{-2}$ | cm | N.D. |
| Case 5 | 5 | cm | cm | cm | cm | cm | A $cm^{-2}$ | cm | N.D. |
| Case 6 | 6 | cm | cm | cm | cm | cm | A $cm^{-2}$ | cm | N.D. |
| Case 7 | 7 | cm | cm | cm | cm | cm | A $cm^{-2}$ | A | N.D. |
| Case 8 | 8 | cm | cm | deg | deg | cm | A $cm^{-2}$ | A | N.D. |

cm = Dimensions are given in centimeters.

deg = Angles are given in degrees.

A $cm^{-2}$ = Coil current densities are given in amps per square centimeter.

A = Conductor currents given in amperes.

N.D. = Not dimensionalized (this is an integer number).

TEST CASE 0, FERMI LAB DIPOLE AS BUILT

INPUT PARAMETERS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 20 | 4 | 2.5400 | 9.5630 | -0. | -0. | |
| 1 | -0. | -0 | -0. | -0 | -0. | -0 | -0. | |
| 1 | -0. | -0. | -0. | -0. | -0. | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3.8100 | 4.6080 | .1700 | 72.5400 | 590.0000 | 35918. | .1500 | 1 |
| 1 | 4.6610 | 5.4590 | 2.0000 | 36.5000 | 590.0000 | 35813. | .1500 | 2 |
| 1 | 4.6610 | 5.4590 | 0. | 2.0000 | 590.0000 | 30889. | .1500 | 3 |
| -0 | -0. | -0. | -0. | -0. | 2.0000 | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |

TWO DIMENSIONAL AND INTEGRATED FIELD MULTIPOLE SOLUTIONS

| N | 2D FIELD | INTG. FIELD |
|---|---|---|
| 1 | -4.33173 | -26.34717 |
| 3 | -.00811 | .00000 |
| 5 | -.00002 | .00354 |
| 7 | -.00316 | -.01887 |
| 9 | .00534 | .03247 |
| 11 | -.00162 | -.00979 |
| 13 | .00033 | .00200 |
| 15 | -.00001 | -.00008 |
| 17 | -.00002 | -.00013 |
| 19 | .00002 | .00013 |

OUTPUT MAGNET PARAMETERS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3.8100 | 4.6080 | .1700 | 72.5400 | 603.5816 | 35918. | .1500 | 1 |
| 1 | 4.6610 | 5.4590 | 2.0000 | 36.5000 | 590.0000 | 35813. | .1500 | 2 |
| 1 | 4.6610 | 5.4590 | 0. | 2.0000 | 590.0000 | 30889. | .1500 | 3 |
| -0 | -0. | -0. | -0. | -0. | 2.0000 | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |

TEST CASE 1, FERMI LAB DIPOLE

INPUT PARAMETERS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 20 | 4 | 2.5400 | 9.5630 | -0. | -0. | |
| 1 | -0. | -0 | -0. | -0 | -0. | -0 | -0. | |
| 1 | -0. | -0. | -0. | -0. | -0. | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3.8100 | 4.6080 | .1700 | 72.5400 | 590.0000 | 35918. | .1500 | 1 |
| 1 | 4.6610 | 5.4590 | 2.0000 | 36.5000 | 590.0000 | 35813. | .1500 | 2 |
| 1 | 4.6610 | 5.4590 | 0. | 2.0000 | 590.0000 | 30889. | .1500 | 3 |
| -0 | -0. | -0. | -0. | 1.0000 | 2.0000 | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | 1.0000 | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |

ITERATED VALUES OF THE MULTIPOLE COEFFICIENTS

| | |
|---|---|
| .008106 | .000016 |
| -.000165 | .000004 |
| -.000000 | .000000 |
| N=3 | N=5 |

TWO DIMENSIONAL AND INTEGRATED FIELD MULTIPOLE SOLUTIONS

| N | 2D FIELD | INTG. FIELD |
|---|---|---|
| 1 | -4.37385 | -25.88557 |
| 3 | .00000 | .00000 |
| 5 | -.00000 | .00455 |
| 7 | -.00231 | -.01157 |
| 9 | .00507 | .02958 |
| 11 | -.00157 | -.00957 |
| 13 | .00037 | .00213 |
| 15 | -.00003 | -.00017 |
| 17 | -.00001 | -.00008 |
| 19 | .00002 | .00011 |

OUTPUT MAGNET PARAMETERS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3.8100 | 4.6080 | .1700 | 72.9565 | 576.2217 | 35918. | .1500 | 1 |
| 1 | 4.6610 | 5.4590 | 2.0000 | 37.3869 | 590.0000 | 35813. | .1500 | 2 |
| 1 | 4.6610 | 5.4590 | 0. | 2.0000 | 590.0000 | 30889. | .1500 | 3 |
| -0 | -0. | -0. | -0. | 1.0000 | 2.0000 | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | 1.0000 | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |

TEST CASE 13, MODIFICATION 9 OF FERMI DIPOLE

INPUT PARAMETERS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 20 | 4 | 2.5400 | 9.5630 | -0. | -0. | | |
| 1 | -0. | -0 | -0. | -0 | -0. | -0 | -0. | |
| 1 | -0. | -0. | -0. | -0. | -0. | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3.8100 | 4.6080 | .1700 | 65.6700 | 590.0000 | 35918. | .1500 | 1 |
| 1 | 3.8100 | 4.6080 | 70.5300 | 77.9100 | 590.0000 | 35918. | .1500 | 2 |
| 1 | 4.6610 | 5.4590 | 2.0000 | 25.5000 | 590.0000 | 35813. | .1500 | 3 |
| 1 | 4.6610 | 5.4590 | 29.3500 | 41.2500 | 590.0000 | 35813. | .1500 | 4 |
| 1 | 4.6610 | 5.4590 | 0. | 2.0000 | 590.0000 | 30889. | .1500 | 5 |
| -0 | -0. | -0. | -0. | 1.0000 | 2.0000 | -0. | -0. | -0 |
| -0 | -0. | -0. | 1.0000 | 1.0000 | 2.0000 | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | 1.0000 | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | 1.0000 | 1.0000 | 2.0000 | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |

ITERATED VALUES OF THE MULTIPOLE COEFFICIENTS

| | | | | | |
|---|---|---|---|---|---|
| -.009148 | -.001464 | -.000354 | -.000225 | -.000080 | -.000016 |
| -.002276 | -.000746 | -.000291 | -.000076 | .000014 | .000023 |
| .000066 | .000031 | .000001 | -.000004 | .000000 | -.000001 |
| -.000000 | -.000000 | -.000000 | -.000000 | .000000 | .000000 |
| N=3 | N=5 | N=7 | N=9 | N=11 | N=13 |

TWO DIMENSIONAL AND INTEGRATED FIELD MULTIPOLE SOLUTIONS

| N | 2D FIELD | INTG. FIELD |
|---|---|---|
| 1 | -4.15731 | -24.53127 |
| 3 | .00000 | -.00000 |
| 5 | .00000 | .00000 |
| 7 | .00000 | -.00000 |
| 9 | .00000 | .00024 |
| 11 | -.00000 | .00008 |
| 13 | -.00000 | .00001 |
| 15 | -.00007 | -.00041 |
| 17 | .00010 | .00056 |
| 19 | -.00004 | -.00024 |

OUTPUT MAGNET PARAMETERS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3.8100 | 4.6080 | .1700 | 64.6964 | 577.2557 | 35918. | .1500 | 1 |
| 1 | 3.8100 | 4.6080 | 69.7939 | 77.4461 | 576.7550 | 35918. | .1500 | 2 |
| 1 | 4.6610 | 5.4590 | 2.0000 | 20.3544 | 590.0000 | 35813. | .1500 | 3 |
| 1 | 4.6610 | 5.4590 | 25.1634 | 38.6138 | 575.2013 | 35813. | .1500 | 4 |
| 1 | 4.6610 | 5.4590 | 0. | 2.0000 | 590.0000 | 30889. | .1500 | 5 |
| -0 | -0. | -0. | -0. | 1.0000 | 2.0000 | -0. | -0. | -0 |
| -0 | -0. | -0. | 1.0000 | 1.0000 | 2.0000 | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | 1.0000 | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | 1.0000 | 1.0000 | 2.0000 | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |

TEST CASE 16, THEORETICAL SECTOR BLOCK 1, CASE 14 PLS CASE 15

INPUT PARAMETERS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 20 | 4 | 2.5400 | 9.5630 | -0. | -0. | |
| 1 | -0. | -0 | -0. | -0 | -0. | -0 | -0. | |
| 1 | -0. | -0. | -0. | -0. | -0. | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3.8100 | 4.6080 | .1700 | 32.3743 | 590.0000 | 35918. | .1500 | 1 |
| 1 | 3.8100 | 4.6080 | 36.3041 | 52.6675 | 584.2200 | 35918. | .1500 | 2 |
| 1 | 3.8100 | 4.6080 | 63.0682 | 71.6283 | 587.7600 | 35918. | .1500 | 3 |
| 1 | 4.6610 | 5.4590 | 2.0000 | 42.3799 | 590.0000 | 35813. | .1500 | 4 |
| 1 | 4.6610 | 5.4590 | 51.5902 | 66.9627 | 582.8200 | 35813. | .1500 | 5 |
| 1 | 4.6610 | 5.4590 | 0. | 2.0000 | 590.0000 | 30889. | .1500 | 6 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |

TWO DIMENSIONAL AND INTEGRATED FIELD MULTIPOLE SOLUTIONS

| N | 2D FIELD | INTG. FIELD |
|---|---|---|
| 1 | -4.53126 | -27.12436 |
| 3 | -.00000 | -.00001 |
| 5 | .00000 | .00274 |
| 7 | -.00000 | .00234 |
| 9 | .00169 | .01042 |
| 11 | -.00039 | -.00228 |
| 13 | .00106 | .00624 |
| 15 | -.00039 | -.00228 |
| 17 | -.00010 | -.00059 |
| 19 | .00007 | .00042 |

TEST CASE 33, THEORETICAL SECTOR BLOCK 2, CASE 14 PLS CASE 32

INPUT PARAMETERS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 20 | 4 | 2.5400 | 9.5630 | -0. | -0. | |
| 1 | -0. | -0 | -0. | -0 | -0. | -0 | -0. | |
| 1 | -0. | -0. | -0. | -0. | -0. | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3.8100 | 4.6080 | .1700 | 32.3743 | 590.0000 | 35918. | .1500 | 1 |
| 1 | 3.8100 | 4.6080 | 36.3041 | 52.6675 | 584.2200 | 35918. | .1500 | 2 |
| 1 | 3.8100 | 4.6080 | 63.0682 | 71.6283 | 587.7600 | 35918. | .1500 | 3 |
| 1 | 4.6610 | 5.4590 | 2.0000 | 31.8597 | 590.0000 | 35813. | .1500 | 4 |
| 1 | 4.6610 | 5.4590 | 35.8965 | 52.4172 | 583.0900 | 35813. | .1500 | 5 |
| 1 | 4.6610 | 5.4590 | 62.9207 | 71.5183 | 586.8500 | 35813. | .1500 | 6 |
| 1 | 4.6610 | 5.4590 | 0. | 2.0000 | 590.0000 | 30889. | .1500 | 7 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |

TWO DIMENSIONAL AND INTEGRATED FIELD MULTIPOLE SOLUTIONS

| N | 2D FIELD | INTG. FIELD |
|---|---|---|
| 1 | -4.47660 | -26.77811 |
| 3 | -.00000 | -.00001 |
| 5 | .00000 | .00000 |
| 7 | .00000 | .00219 |
| 9 | .00000 | .00067 |
| 11 | -.00000 | .00003 |
| 13 | .00124 | .00726 |
| 15 | -.00045 | -.00264 |
| 17 | -.00010 | -.00061 |
| 19 | .00008 | .00044 |

TEST CASE 45, BLOCK DIPOLE WITH FERMI BORE, CASE 37 PLS CASE 41

INPUT PARAMETERS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 20 | 4 | 2.5400 | 9.5630 | -0. | -0. | |
| 1 | -0. | -0 | -0. | -0 | -0. | -0 | -0. | |
| 1 | -0. | -0. | -0. | -0. | -0. | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 3.8100 | 4.6080 | 16.0380 | 15.5333 | 590.0000 | 35918. | .1500 | 1 |
| 2 | 3.8100 | 4.6080 | 44.3352 | 8.2844 | 584.3700 | 35918. | .1500 | 2 |
| 2 | 3.8100 | 4.6080 | 67.5853 | 4.1422 | 587.6800 | 35918. | .1500 | 3 |
| 2 | 4.6610 | 5.4590 | 22.0236 | 19.8122 | 590.0000 | 35813. | .1500 | 4 |
| 2 | 4.6610 | 5.4590 | 59.0047 | 7.7526 | 582.9300 | 35813. | .1500 | 5 |
| 1 | 4.6610 | 5.4590 | 0. | 2.0000 | 590.0000 | 30889. | .1500 | 6 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |

TWO DIMENSIONAL AND INTEGRATED FIELD MULTIPOLE SOLUTIONS

| N | 2D FIELD | INTG. FIELD |
|---|---|---|
| 1 | -4.45974 | -26.69557 |
| 3 | .00000 | .00001 |
| 5 | .00000 | .00266 |
| 7 | .00002 | .00244 |
| 9 | .00181 | .01115 |
| 11 | -.00001 | .00001 |
| 13 | .00116 | .00683 |
| 15 | -.00039 | -.00232 |
| 17 | -.00010 | -.00058 |
| 19 | .00007 | .00041 |

TEST CASE 52, PALMER TYPE MODIFICATION OF FERMI DIPOLE, FINITE WIRE, SEE CASE 13

INPUT PARAMETERS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 20 | 4 | 2.5400 | 9.5630 | -0. | -0. | |
| 1 | | -0. | -0 | -0. | -0 | -0. | -0 | -0. |
| 1 | | -0. | -0. | -0. | -0. | -0. | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 3.8100 | 4.6080 | 32.2721 | 32.1021 | 590.0000 | 35918. | .1500 | 1 |
| 2 | 3.8100 | 4.6080 | 73.6200 | 4.1422 | 590.0000 | 35918. | .1500 | 2 |
| 2 | 4.6610 | 5.4590 | 10.6140 | 8.6140 | 590.0000 | 35813. | .1500 | 3 |
| 2 | 4.6610 | 5.4590 | 31.8886 | 6.8912 | 590.0000 | 35813. | .1500 | 4 |
| 1 | 4.6610 | 5.4590 | 0. | 2.0000 | 590.0000 | 30889. | .1500 | 5 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | 1.0000 | -0. | 2.0000 | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | 1.0000 | -0. | 2.0000 | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |

ITERATED VALUES OF THE MULTIPOLE COEFFICIENTS

| | |
|---|---|
| -.008539 | .001606 |
| .000747 | -.001005 |
| .000036 | -.000041 |
| .000000 | -.000000 |
| N=3 | N=5 |

TWO DIMENSIONAL AND INTEGRATED FIELD MULTIPOLE SOLUTIONS

| N | 2D FIELD | INTG. FIELD |
|---|---|---|
| 1 | -4.10685 | -24.38555 |
| 3 | -.00000 | .00000 |
| 5 | .00000 | .00000 |
| 7 | .00176 | .00535 |
| 9 | -.00195 | -.01021 |
| 11 | .00085 | .00509 |
| 13 | -.00033 | -.00194 |
| 15 | .00000 | .00007 |
| 17 | .00010 | .00057 |
| 19 | -.00005 | -.00031 |

OUTPUT MAGNET PARAMETERS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 3.8100 | 4.6080 | 32.2721 | 32.1021 | 590.0000 | 35918. | .1500 | 1 |
| 2 | 3.8100 | 4.6080 | 75.5193 | 4.1422 | 570.1722 | 35918. | .1500 | 2 |
| 2 | 4.6610 | 5.4590 | 10.6140 | 8.6140 | 590.0000 | 35813. | .1500 | 3 |
| 2 | 4.6610 | 5.4590 | 31.8644 | 6.8912 | 550.2615 | 35813. | .1500 | 4 |
| 1 | 4.6610 | 5.4590 | 0. | 2.0000 | 590.0000 | 30889. | .1500 | 5 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | 1.0000 | -0. | 2.0000 | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | 1.0000 | -0. | 2.0000 | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |

TEST CASE 58, BLOCK DIPOLE WITH FERMI BORE, CASE 37 PLS CASE 57

INPUT PARAMETERS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 20 | 4 | 2.5400 | 9.5630 | -0. | -0. | |
| 1 | -0. | -0 | -0. | -0 | -0. | -0 | -0. | |
| 1 | -0. | -0. | -0. | -0. | -0. | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 3.8100 | 4.6080 | 16.0380 | 15.5333 | 590.0000 | 35918. | .1500 | 1 |
| 2 | 3.8100 | 4.6080 | 44.3352 | 8.2844 | 584.3700 | 35918. | .1500 | 2 |
| 2 | 3.8100 | 4.6080 | 67.5853 | 4.1422 | 587.6800 | 35918. | .1500 | 3 |
| 2 | 4.6610 | 5.4590 | 16.6438 | 14.6438 | 590.0000 | 35813. | .1500 | 4 |
| 2 | 4.6610 | 5.4590 | 43.7469 | 8.6140 | 583.2800 | 35813. | .1500 | 5 |
| 2 | 4.6610 | 5.4590 | 67.9895 | 4.3070 | 586.7200 | 35813. | .1500 | 6 |
| 1 | 4.6610 | 5.4590 | 0. | 2.0000 | 590.0000 | 30889. | .1500 | 7 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |

TWO DIMENSIONAL AND INTEGRATED FIELD MULTIPOLE SOLUTIONS

| N | 2D FIELD | INTG. FIELD |
|---|---|---|
| 1 | -4.42981 | -26.49702 |
| 3 | .00000 | .00001 |
| 5 | .00000 | -.00000 |
| 7 | .00041 | .00451 |
| 9 | -.00024 | -.00070 |
| 11 | .00037 | .00226 |
| 13 | .00136 | .00797 |
| 15 | -.00046 | -.00270 |
| 17 | -.00010 | -.00061 |
| 19 | .00007 | .00043 |

TEST CASE 61, TEST CASE 0 WITH E = 1000000 PSI

INPUT PARAMETERS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 20 | 4 | 2.5400 | 9.5630 | -0. | -0. | |
| 1 | -0. | -0 | -0. | -0 | -0. | -0 | -0. | |
| 1 | -0. | -0. | -0. | -0. | -0. | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3.8100 | 4.6080 | .1700 | 72.1740 | 603.5800 | 35918. | .1500 | 1 |
| 1 | 4.6610 | 5.4590 | 2.0000 | 36.4550 | 590.0000 | 35813. | .1500 | 2 |
| 1 | 4.6610 | 5.4590 | 0. | 2.0000 | 590.0000 | 30889. | .1500 | 3 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |

TWO DIMENSIONAL AND INTEGRATED FIELD MULTIPOLE SOLUTIONS

| N | 2D FIELD | INTG. FIELD |
|---|---|---|
| 1 | -4.32458 | -26.30397 |
| 3 | -.01259 | -.02720 |
| 5 | .00187 | .01499 |
| 7 | -.00379 | -.02273 |
| 9 | .00545 | .03314 |
| 11 | -.00159 | -.00962 |
| 13 | .00029 | .00180 |
| 15 | .00000 | .00002 |
| 17 | -.00003 | -.00016 |
| 19 | .00002 | .00013 |

TEST CASE 62, TEST CASE 45 WITH E = 1000000 PSI

INPUT PARAMETERS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 20 | 4 | 2.5400 | 9.5630 | -0. | -0. | |
| 1 | | -0. | -0 | -0. | -0 | -0. | -0 | -0. |
| 1 | | -0. | -0. | -0. | -0. | -0. | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 3.8100 | 4.6080 | 15.9640 | 15.5333 | 590.0000 | 35918. | .1500 | 1 |
| 2 | 3.8100 | 4.6080 | 44.1620 | 8.2844 | 584.3700 | 35918. | .1500 | 2 |
| 2 | 3.8100 | 4.6080 | 67.3820 | 4.1422 | 587.6800 | 35918. | .1500 | 3 |
| 2 | 4.6610 | 5.4590 | 21.9300 | 19.8122 | 590.0000 | 35813. | .1500 | 4 |
| 2 | 4.6610 | 5.4590 | 58.8200 | 7.7526 | 582.9300 | 35813. | .1500 | 5 |
| 1 | 4.6610 | 5.4590 | 0. | 2.0000 | 590.0000 | 30889. | .1500 | 6 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |
| -0 | -0. | -0. | -0. | -0. | -0. | -0. | -0. | -0 |

TWO DIMENSIONAL AND INTEGRATED FIELD MULTIPOLE SOLUTIONS

| N | 2D FIELD | INTG. FIELD |
|---|---|---|
| 1 | -4.46662 | -26.73713 |
| 3 | -.00388 | -.02298 |
| 5 | .00044 | .00518 |
| 7 | -.00035 | .00022 |
| 9 | .00177 | .01093 |
| 11 | -.00011 | -.00061 |
| 13 | .00116 | .00682 |
| 15 | -.00037 | -.00219 |
| 17 | -.00011 | -.00065 |
| 19 | .00007 | .00040 |

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*TECHNICAL INFORMATION DEPARTMENT*
*LAWRENCE BERKELEY LABORATORY*
*UNIVERSITY OF CALIFORNIA*
*BERKELEY, CALIFORNIA 94720*