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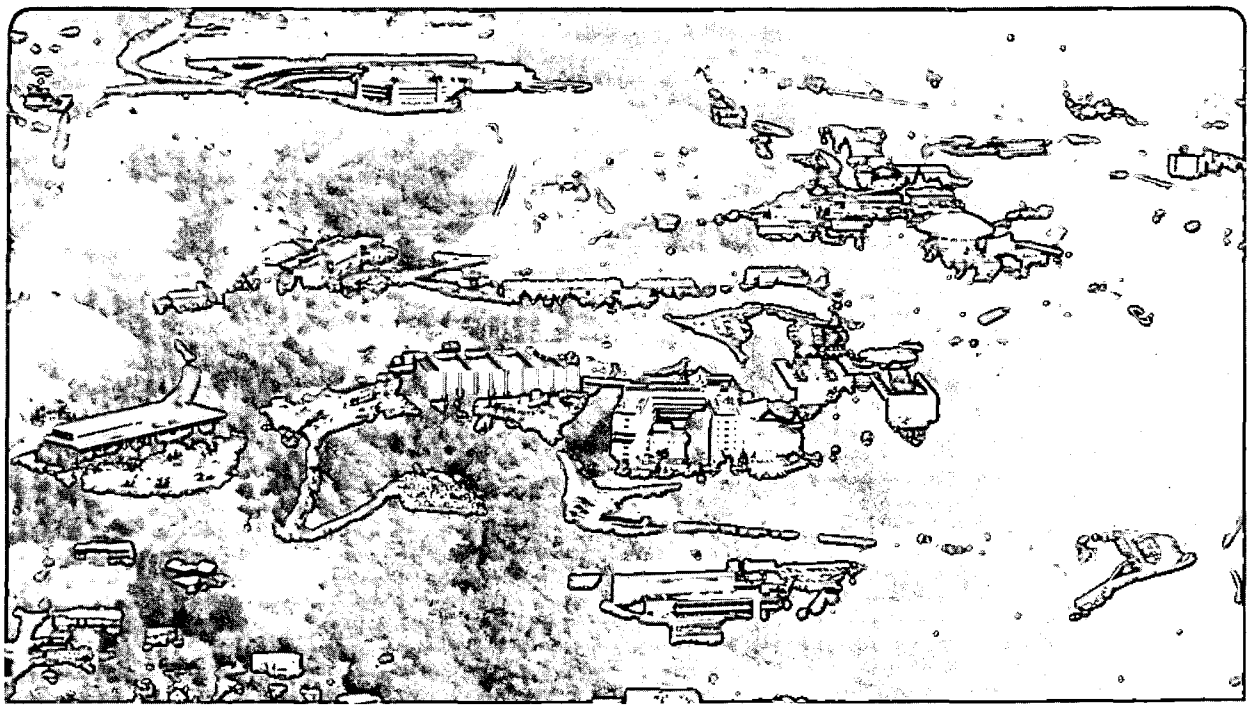

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ENGINEERING NOTE

CODE	SERIAL	PAGE
P41000	M5875	1 of 18

AUTHOR	DEPARTMENT	LOCATION	DATE
M. A. Green	MECHANICAL ENGINEERING	Berkeley	15 March 1982

PROGRAM — PROJECT — JOB

PEP-4 DETECTOR

POLE-TIP CALORIMETERS

TITLE

A CHECK OF THE UCLA CALCULATIONS OF DEFLECTION IN THE CALORIMETER

This ENGINEERING NOTE presents a summary of the calculations done to check the UCLA calculations of deflection of various components of the TPC pole-tip calorimeters. UCLA calculated and measured deflections of various components under gravitational force, earthquake accelerations and quench forces which arise during the collapse of the TPC magnetic field.

THE UCLA Report HEE-129[1] by H. K. Ticho provided the basic information about the calorimeter. In addition information was obtained from UCLA Physics Department drawing PTC-1175-GN[2] which was given to me by Gene Miner.[3] Many of the deflection calculations were done by the UCLA Mechanical Engineering Department using a computer. My calculations for checking the "order of magnitude" of the deflections were made using equations given in the 5th edition of Roark.[4] My calculations are made in SI units. The electrical calculations are rationalized with the permeability of air $\mu_0 = 4\pi \times 10^{-7}$.

The deflection calculations are made for the following cases: 1) Deflection of the posts which support the strong back, lead plates and the aluminum pressure plate; 2) Deflection of the strong back plate and aluminum pressure plate under gravitational loading (i.e., earthquake forces); and 3) Deflection of the lead laminants under gravitational loading perpendicular to their faces (i.e., earthquake loading and z direction loading due to magnetic field collapse). All deflection calculations were made under acceleration of gravity. Magnetic forces and earthquake loading can be expressed in terms of gravity loading.

A check of the magnetic force loading during a collapse of the TPC magnet field was made. A calculation of the field decay of the new TPC magnet was made and it was compared to the worst-case calculations made for the old TPC magnet. These worst-case calculations were the bases for the magnetic force calculations made by M. L. Stevenson[5] and the UCLA group.[1] The UCLA force and hence the deflection calculations are conservative. The rate of field collapse for the conventional magnet is estimated. The resulting magnetic forces are compared to the UCLA calculations.

After checking the UCLA calculations for force and deflection, the following conclusions can be drawn. The deflection calculations appear to be correctly made. The calculations for magnetic force and deflection appear to be conservative by nearly a factor of two for the superconducting TPC magnet. The forces during a collapse of the conventional magnet field appear to be about 25% of those encountered during a collapse of the superconducting magnet field.

1. Deflection calculations

The UCLA report measures and/or calculates the deflections of several elements of the lead argon calorimeters. The calculations include: a) post deflection, b) deflection of the lead plates under gravitational loading, c) deflection of the stack of plates in the direction of the 310 Stainless Steel strongback, and d) deflection of the stack of plates in the direction of the aluminum pressure plate. These calculations were checked using equations from the 5th edition of Roark. The UCLA calculations were made by Professor S. Dong of the UCLA School of Engineering. Many of these calculations were made using the computer.

a) Deflection of the posts

The support posts (six of them) extend from the magnet pole tip some 370mm into the calorimeter. On these posts are hung the stainless steel strongback, 26 half-washers with 0.090-inch-thick lead plates, 76 half-washers with 0.045-inch-thick lead plates, and the 0.5-inch-thick aluminum pressure plates. Using some formulae given in Roark, a deflection 0.42mm was calculated.(See Appendix A.) The UCLA calculations and measurements show a deflection of 0.66mm. The primary difference between the calculations given in this report and the UCLA calculations is that our calculations do not consider the fact that the edge of the post loses contact with the support surface.

b) Deflection of the lead plates

The UCLA calculation of the lead plate deflection was made using the EASE-2 computer program (See Figures 11a and 11c of the UCLA report). The LBL model assumed that the lead plate is a 30-degree sector clamped at the radial edges. The maximum deflection is assumed to occur at a radius of 750mm.(This corresponds to the maximum deflection calculated by the EASE-2 program.) The UCLA calculations show a deflection of 0.56mm for the 0.045-inch lead plates.

Our calculations showed a deflection of 0.27mm. The UCLA calculation of deflection of the 0.090-inch lead plates shows a maximum deflection of 0.32mm. Our calculations show a deflection of 0.16mm. (See Appendix B for the calculations of the lead plate deflection under a gravity load.)

c) Deflection of the 310 Stainless Steel strongback

The stainless steel strongback is supported simply along the inner radius and it is supported simply at the six points around the outer radius of the plate (the six support posts). The strongback supports forces in the Z direction toward the iron pole. The strongback is not supported at all in the direction away from the iron pole. The LBL calculations assumed that the strongback plate is simply supported at the inner and outer edges. The force on the plate was assumed to be uniform across the plate surface. The UCLA calculations used the actual support conditions in the EASE-2 computer code. The LBL calculation of deflection showed a deflection of 0.18mm at the radius between the inner and outer radii of the plates. The UCLA calculation showed a deflection of 0.25mm (between the posts) from a point midway between the two radii and the outer radius of the plate.

d) Deflection of the aluminum pressure plate

The method of calculation of deflection in the aluminum pressure plate is the same as for the strongback. The combined force on the plate surface is the total weight of the lead, stainless steel and the aluminum. The plate is assumed to be simply supported on both the inner and outer radii (See Appendix C for the method of calculation). The calculated deflection was 1.90mm. This is due solely to the fact that the aluminum plate stiffness factor is one order of magnitude lower than the stiffness factor of the stainless steel plate. UCLA calculated a deflection of 2.5mm using the EASE-2 computer code. I agree that the deflection along the Z axis may be excessive during an earthquake. (The deflection is excessive only when one considers the impairment of clearance with the TPC amplifiers.) One could reduce the deflection of the pressure plate by a factor of three by replacing the aluminum pressure plate with a 310 Stainless Steel plate of the same thickness.

e) Deflection of the whole structure under earthquake loading

When one calculates earthquake deflections one assumes a

maximum acceleration of 1g (9.8 ms^{-2}) in any direction. Deflection in the radial direction is dominated by the deflection of the six stainless steel posts. Therefore it is correct to assume that maximum deflection in this direction is of the order of the 0.66mm calculated by UCLA. Z direction deflection depends on the direction of the acceleration. If it is toward the pole, the deflection of the strongback controls. The Z deflection toward the pole is 0.25mm. If the acceleration is away from the pole the 2.5mm deflection of the aluminum pressure plate dominates. The laminations themselves should not deflect more than 0.56mm calculated for the plates with 0.045 in. of lead on them.

2. Forces and deflections during a magnet quench

The UCLA calculations of magnetic forces during a quench are based on a PHYSICS NOTE by L. Stevenson.[5] In this physics note, the effect of field collapse in the coil and field collapse in the iron are considered. In simple terms, the collapsing coil field provides the $d\phi/dt$ to generate azimuthal currents in the support plates and lead half disks. The flux decay in the iron is much slower (about 10^4 sec according to Klaus Halbach).[6] Magnetic flux lines coming from the iron are the source of radial and longitudinal fields which interact with the azimuthal currents to produce longitudinal and radial force components in the strongback, lead half disks and the aluminum pressure plates. The radial forces act radially outward; the longitudinal forces act in a direction toward the iron pole.

The calculations of TPC magnet field collapse time constant in the UCLA note came from an ENGINEERING NOTE I wrote in 1978.[7] This note presents a worst calse calculation field decay in the old TPC magnet. Field decay data for the new magnet will be presented in a latter section of this report along with the calculated field decay time constant for the conventional TPC magnet. The calculated maximum field collapse time constant for the new TPC magnet is 5.4 sec as compared to a "worst case" field collapse time constant of 3 sec used in the UCLA force calculations. The conventional magnet field decay time constant is about 1.4 seconds.

The M. L. Stevenson[5] and UCLA analysis[1] of the magnetic force problem separates the magnetic induction and vector potential into two components. The first component is generated by the coil package (designated by a subscript zero); the second component is generated by eddy currents in the iron which refuse to allow the magnetic flux

in the iron to die. In vector terms the magnetic induction and vector potential are:

$$\vec{B} = \vec{B}_0 e^{-t/\tau_0} + B_1 (1 - e^{-t/\tau_0}) e^{-t/\tau_1} \qquad \text{-1-}$$

and

$$\vec{A} = A_0 e^{-t/\tau_0} + A_1 (1 - e^{-t/\tau_0}) e^{-t/\tau_1} \qquad \text{-2-}$$

where B_0 and A_0 are generated by the coil package and B_1 and A_1 are generated by the iron. The inductions B_0 and B_1 have only radial and longitudinal terms (B_0 is predominantly longitudinal except very close to the pole). The vector potentials A_0 and A_1 are only azimuthal. τ_0 is the coil decay time constant (τ_0 = 3 seconds in the UCLA Note); and τ_1 is the iron decay time constant (τ_1 is from 10^3 to 10^4 seconds depending on how one looks at it.[6] It is assumed that $\tau_1 >> \tau_0$.

The magnetic force vector

$$\vec{F} = \vec{J} \times \vec{B} = \frac{1}{\rho} \vec{E} \times B = \frac{1}{\rho} \frac{\partial \vec{A}}{\partial t} \times B \qquad \text{-3-}$$

where $\vec{F}$ is magnetic force; $\vec{J}$ is current density; $\vec{B}$ is induction; $\vec{E}$ is electric field; ρ is electrical resistivity; and $\vec{A}$ is vector potential. If one sets

$$\eta_0 = e^{-t/\tau_0}, \quad \eta_1 = e^{-t/\tau_1} \qquad \text{-4a-}$$

$$\delta\vec{A} = \vec{A}_1 - A_0, \quad \text{and} \quad \vec{B} = \vec{B}_1 - \vec{B}_0 \qquad \text{-4b-}$$

Then one finds that when $\eta_1 \approx 1$

$$\vec{F} = \frac{\eta_o(1-\eta_o)}{\rho\tau_o}\,\vec{\delta B} \times \vec{\delta A} + \frac{\eta_o}{\rho\tau_o}\,\vec{B_o} \times \vec{\delta A} \qquad \text{-5-}$$

The equation above has two terms. The first peaks at $\eta_o=0.5$ and the other peaks at $\eta_o=1$. In order to get a worst case calculation, UCLA assumed that the force was the sum of the two terms at their highest values. Thus, the r and z components of the force can be given as follows:

$$F_{r\,max} = \frac{1}{\rho\tau_o}\left[B_{oz}\,\delta A + \frac{1}{4}\,\delta B_z\,\delta A\right] \qquad \text{-5a-}$$

and

$$F_{z\,max} = \frac{1}{\rho\tau_o}\left[B_{or}\,\delta A + \frac{1}{4}\,\delta B_r\,\delta A\right] \qquad \text{-5b-}$$

The r-component of the force will apply hoop stresses to the strongback and pressure plates which can be ignored because the plates have a large radial extent. The r-component can be ignored on the lead half disks as well because they alternate azimuthally on the support posts. Only the z-component of force is of interest in the calculation of deflection.

Investigation of equation 5b yields the following observations. The maximum force (hence deflection) is inversely proportional to the peak decay time constant and material resistivity. The peak force is directly proportional to the starting induction squared. (B_{or}, δA and δB_r are directly proportional to the magnet starting central induction.) While the conventional magnet has a shorter decay time constant than the superconducting magnet, the magnetic forces are lower because the starting central induction is 0.4 Tesla instead of 1.5 Tesla.

I am convinced that the UCLA calculations of magnetic deflection are indeed conservative. The value of τ_o is the lowest "worst case" value based upon the bore tube time constant. Equation 5b

assumes that the peak force maximum occurs at the same point in time. In addition, a quench is a variable time constant phenomena with the shorter time constants occurring at lower fields. The basic assumptions of the calculation will always tend to a conservative "worst case" value.

3. <u>The rate of flux change of the central field of the conventional and the new superconducting TPC magnet</u>

In both the conventional and superconducting TPC magnets, the time constant for current decay is the longer time constant which comes from a set of coupled equations. The conventional magnet has two time constants (τ_1 the coil time constant and τ_2 the winding mandral time constant) which make up the time constant for the collapse of the field τ_L. The superconducting coil has an additional time constant (τ_3 the time constant for the pure copper circuit which comes into play. In both the conventional and superconducting magnets, the coupling between elements is reasonably good (say better than 90 per cent). As a result τ_o is the so-called longer time constant.

a) the conventional magnet

For the conventional magnet, the value of τ_o is:

$$\tau_o \approx \tau_1 + \tau_2 \qquad \text{-6-}$$

where

$$\tau_1 = \frac{L_1}{R_1} \quad \text{and} \quad \tau_2 = \frac{L_2}{R_2}$$

where L_1 is the self inductance of the coil; L_2 is the self inductance of the winding mandrel; R_1 is the resistance of the coil; and R_2 is the resistance of the winding mandrel. In the conventional magnet, the following values apply: $L_1 \simeq 0.32$1H, $L_2 \simeq 1.32 \times 10^{-6}$H, $R_1 \simeq 0.23\Omega$, and $R_2 \simeq 2.19 \times 10^{-6}\Omega$. The values of the time constants are $\tau_1 = 1.43$s, $\tau_2 = 0.61$s, and $\tau_o = 2.04$s. (Note: τ_o is the time constant for magnetic field decay, not current decay. The current will drop sharply at first with a much shorter time constant. When the current reaches sixty per cent of its starting value it decays exponentially with a time constant τ_o.) The measured τ_o is around 1.4s for the conventional magnet.[8]

b) the superconducting coil

The superconducting coil has no set values of τ_1, τ_2 and τ_3. The time constants are much longer at the start of a quench when both temperature and resistance are low. The resistance of the coil grows rapidly as the quench propagates. Before the coil stored energy has begun to dissipate, the whole coil has become normal through quench-back (this is true when the TPC magnet is at full design current). The average temperature of the coil package is 50K or below. At low temperature the longer time constant τ_L is lomger than it is at high temperatures. In order to use an equation like 5b, one must have an effective time constant τ_e. The minimum value of τ_e is τ_o in equation 5b. The following expression can be used to estimate τ_e:

$$\tau_e = \tau_L \left[\frac{E_o - E_D}{E_o}\right]^{1/2} \qquad \text{-7-}$$

where

$$\tau_L = \tau_1 + \tau_2 + \tau_3$$

and

$$\tau_1 = \frac{L_1}{R_1}, \quad \tau_2 = \frac{L_2}{R_2}, \quad \tau_3 = \frac{L_3}{R_3}$$

and L_1 is the coil self inductance; L_2 is the bore tube self inductance, L_3 is pure copper curcuit self inductance; R_2 is the coil resistance; R_3 is the copper circuit resistance. (Note: L_1, L_2 and L_3 are independent of temperature but R_1, R_2 and R_3 are very much dependent on temperature.) E_o is the starting stored energy of the magnet. E_D is the energy dissipated in resistive heating in the three circuits.

One can estimate τ_L and E_D as a function of temperature if one knows how material resistivity and enthalpy vary with temperature. Use the copper and aluminum tables in *References 9, 10* and *11*. The superconductor is about 64 per cent RRR = 140 copper. The copper circuit itself is assumed to be RRR = 200 copper. The aluminum bore tube is made from 1100-0 aluminum RRR = 25. The results of the time constant calculations are given in TABLE 1 (Note: magnetoresistance was included in these calculations). The major assumption made in

calculating τ_e was that the temperature is the same in the copper, superconductor and bore tube as the quench proceeds. (I don't believe this results in serious error.)

TABLE 1. Various superconducting TPC magnet time constants as a function of temperature

Average T (K)	time constants (s)* τ_1	τ_2	τ_3	τ_L	$\left(\frac{E_0-E_D}{E_0}\right)^{\frac{1}{2}}$**	τ_e (s)
10	7.8	5.4	12.4	25.3	1.000	25.3
30	5.4	4.9	8.4	18.7	0.987	18.9
40	2.9	4.2	4.5	11.5	0.960	12.0
50	1.7	2.9	2.7	7.3	0.900	8.0
60	0.93	2.43	1.45	4.81	0.800	6.00
65	0.71	2.16	1.11	3.98	0.720	5.54
70	0.57	1.94	0.89	3.40	0.631	5.39#
75	0.48	1.62	0.75	2.85	0.527	5.41
80	0.39	1.42	0.60	2.41	0.348	6.91

* See *Reference 10* for the resistivity data needed to calculate the resistance of the various circuits

** Based on the energy being distributed so that all elements have the same temperature

\# The minimum value of τ_e uses this value for τ_0 in equation 5b. (Note: a minimum value of τ_0 = 5.39 seconds results in a smaller magnetic force and deflection than the τ_0 = 3 seconds assumed in the UCLA study.)

— *References* —

1. H. K. Ticho, *MECHANICAL PROPERTIES OF THE POLE TIP CALORIMETER*, UCLA High-Energy Group Note HEE-129, UCLA Physics Department, October 1979

2. UCLA Physics Department Drawing PTC-1175-GN, THE POLE TIP CALORIMETER ASSEMBLY

3. Private communication with Gene Miner concerning changes on UCLA Drawing PTC-1175-GN

4. R. J. Roark and W. C. Young, *FORMULAS FOR STRESS AND STRAIN*, 5th Edition, McGraw-Hill Book Co., New York, 1975

5. M. L. Stevenson, *FORCES DUE TO EDDY CURRENTS*, LBL Group A Physics Note 867, November 1978

6. Klaus Halbach, LBL, private communication on the effect of eddy currents in the iron poles of a magnet. One pole which is one meter in radius will have an eddy current time constant of about 10,000 s:

$$\tau_1 \simeq 10^4 r^2$$

7. M. A. Green, *PEP-4 EXPERIMENT, LARGE SUPERCONDUCTING SOLENOID MAGNET, THE RATE OF MAGNETIC FLUX CHANGE DURING A QUENCH*, LBL Engineering Note M5147, March 1978

8. Private communication from Martin Berndt of SLAC on his measurements of the conventional TPC magnet time constant, March 1982

9. WADD Technical Report 60-56, *A COMPENDIUM OF THE PROPERTIES OF MATERIALS AT LOW TEMPERATURE*, PROPERTIES OF SOLIDS, July 1960

10. *HANDBOOK ON MATERIALS FOR SUPERCONDUCTING MACHINERY*, Materials and Ceramic Information Center and NBS Boulder, MCIC HB-04, 1974

11. M. A. Green, *PEP-4 EXPERIMENT, LARGE SUPERCONDUCTING SOLENOID MAGNET, VARIATION OF BORE TUBE RESISTIVITY AND LONGITUDINAL FORCES DURING A QUENCH*, LBL Engineering Note M5146, March 1978

APPENDIX A

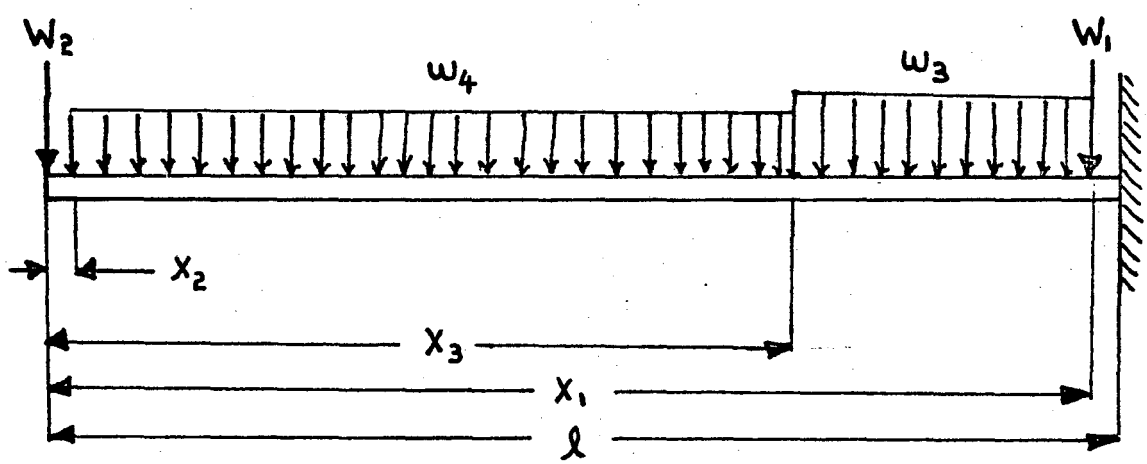
Calculation of the Deflection of the Posts

The six support posts extend from the pole tip 0.37m. These posts are assumed to be made from 310 Stainless Steel which has a modulus of $2.1 \times 10^{11} nm^{-1}$. The posts have an outside diameter of 50.8mm and an inside diameter of 30.8mm. Using

$$I = \frac{\pi}{64}(D_1^4 - D_2^4) \qquad \text{-A1-}$$

where $D_1 = 5.08 \times 10^{-2}m$ and $D_2 = 3.08 \times 10^{-2}m$ one gets an EI product (E is the modulus of the post) of $EI = 5.937 \times 10^4 Nm^2$.

In order to calculate the deflection, one must look at how the forces are applied to the post. Figure A1 shows the weight of the strongback as W_1, the weight of the pressure plate as W_2, the weight of the thick lead plates as a distributed load W_3, and the weight of the thin lead plates as a distributed load W_4. The deflection, which is maximum at the point where W_2 is applied, is simply the sum of the deflections due to each of the forces. The end of the post where W_1 is applied is fixed so no rotation can occur. (This does not quite occur in real life.)



$X_1 = 0.36\ m$
$X_2 = 0.01\ m$
$X_3 = 0.25\ m$
$\ell = 0.37\ m$

$W_1 = 601\ N$
$W_2 = 147\ N$
$w_3 = 17418\ N\,m^{-1}$
$w_4 = 9231\ N\,m^{-1}$

FIGURE A1. Force Diagram for the Calorimeter Support Posts

The deflection at the free end due to force W_1 can be calculated using the following equation (see Figure A1):

$$y_1 = \frac{-W_1}{6EI}(2\ell^3 - 3\ell^2 x_1 + x_1^3) \qquad \text{-A2-}$$

Roark 5th Ed (1975)
Table 3-1a p 96

Using this equation, one gets:

$$y_1 = -1.856 \times 10^{-7}\text{m}$$

The deflection due to force W_2 can be calculated with the equation above except a subscript $_2$ is substituted for subscript $_1$. Using equation A2 one gets the following value for deflection at the free end due to force W_2:

$$y_2 = -4.181 \times 10^{-5}\text{m}$$

The deflection at the free end due to the distributed load W_3 can be calculated using the following equation form:

$$y_3 = \frac{-W_3}{24EI}(\ell - x_3)^3(3\ell + a) \qquad \text{-A3-}$$

Roark 5th Ed (1975)
Table 3-2a p 98

From equation A3 one gets a calculated deflection due to the distributed load W which is as follows:

$$y_3 = 2.873 \times 10^{-5}\text{m}$$

The deflection at the free end due to the distributed load W_4 can be calculated using the following equatiom form:

$$y_4 = \frac{-W_4\,\ell^4}{8EI} + \frac{W_4}{24EI}(\ell - x_3)^3(3\ell + x_3) \qquad \text{-A4-}$$

From equation A4 one gets a calculated deflection due to the distributed load W_4 which is as follows:

$$y_4 = -3.490 \times 10^{-4} m$$

The total free end deflection calculated is

$$y_{max} = y_1 + y_2 + y_3 + y_4 \qquad \text{-A5-}$$

When one solves equation A5 to get the maximum deflection

$$y_{max} = 4.214 \times 10^{-4} m$$

one can compare the results from eq. A5 with a calculation based on uniform loading of the post:

$$y_{max} = \frac{W\ell^4}{8EI} \qquad \text{-A6-}$$

Using $W = 1.063 \times 10\ Nm^{-1}$, one finds that the maximum deflection under uniform loading from equation A6

$$y_{max} = 4.195 \times 10^{-4} m$$

which is very close to that calculated by eq. A5.

It should be noted that the UCLA calculations considered the fact that the post loses contact with the pole plate. (The fixed boundary condition of the wall is not valid.) The UCLA deflection of $6.6 \times 10^{-4} m$ is higher than y_{max} calculated here as a result. The UCLA paper didn't say what material the post was made of. I assumed it was 310 Stainless Steel (the same as the strongback back plate). If the post were made from 304 Stainless Steel (a likely possibility) the deflection would be 14 per cent more. I am satis- that the UCLA calculations for post deflections are correct. UCLA has experimental data to back up their calculations.

APPENDIX B-

Deflection of the Lead Plates

Calculation of the deflection of the lead plates under gravity loading was done using the assumption of a solid circular sector of infinite radius with a uniformly distributed load q over the entire surface. The straight edges are assumed to be fixed (see Figure B1).

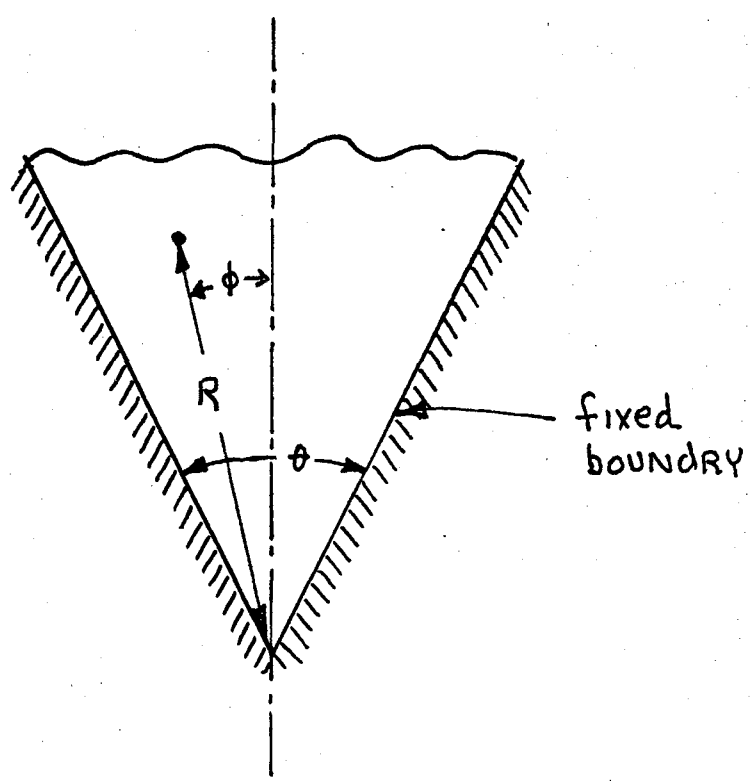


FIGURE B1. Force Diagram for an infinite sector of a circle with a uniform force on it and fixed boundaries at the edges of the sector

Maximum deflection will occur along the center line at $\phi=0$. The radius r corresponds to the point of projected maximum deflection. This point of maximum deflectiom is estimated to lie two thirds of the way out on the lead disks at $R = 0.723$m (the UCLA calculations put the point of maximum deflection at $R = 0.744$m). The equation which calculates the deflection at $\phi=0$ is as follows:

$$y = \frac{qR^4}{32D}\left[\frac{(1-\cos\theta)^2}{1+2\cos\theta}\right] \qquad \text{-B1-}$$

Roark, 5th Ed (1975)
Table 24-29 p 371

where

$$D = \frac{Et^3}{12(1-\nu^2)} \qquad \text{-B1a-}$$

and θ is less than 60°. E is elastic modulus; t is the plate thickness; and ν is Poisson's ratio. R is the radius of the point of projected maximum deflection; θ is the angle between the lines of the sector which are fixed; and q is the pressure force per unit area on the sector.

For the calculation given for 1g loading of the lead plates $\theta = 30^o$ and $R = 0.723$m. TABLE B1 shows the calculated values of D, q and y for both the 45 mil lead laminants and the 90 mil lead laminants.

TABLE B1 Force per unit area, D modulus and deflection for the lead sheets under gravity loading

	0.045-inch laminant	0.090-inch laminant
D (Nm)*	317.7	1007.5
q (Nm^{-2})	1491	2801
y (m)	2.65×10^{-4}	1.56×10^{-4}

as given by the UCLA report

The deflections calculated using eq. B1 are roughly half of what was calculated by UCLA. The point of maximum deflection for the UCLA case was 0.744m, not 0.723m. The fixed constrained boundary conditions assumed by Roark are not totally indicative of real life. The actual boundary condition lies some where between fixed and free. In the UCLA calculation along line XX in Figure 11a the deflection maximum occurs at $R \approx 0.745$m. If this deflection is added to the LBL calculated deflection value, the deflection given along the YY line is almost achieved. This is the least satisfactory part of the deflection calculation. I feel that the UCLA calculation under gravitational load is realistic.

APPENDIX C

Deflection of the Pressure Plate and Strongback

The 310 Stainless Steel strongback and the 6061 Aluminum pressure plate can be treated as a disk with a hole in it. The boundary conditions at the edge of the disk approximate a simply supported edge on both the inner and outer edges of the disk. See Figure C1. The disk has an inner radius $b = 0.287$m; the outer radius $a = 0.940$m. The loading on the disk due to the gravitational loading of the lead plates is $10813\ Nm^{-2}$.

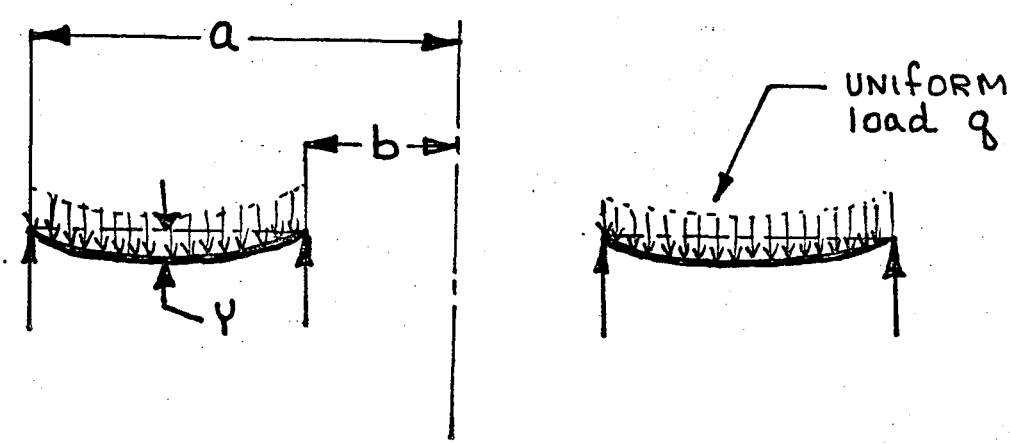


FIGURE C1. Uniform Loading of Disk simply supported on the outer edge and the edge of the central hole

The loaded disk deflection problem is one of the standard problems found in Roark. The equation for maximum deflection of a uniformly loaded circular plate with a hole in it which is simply supported at both the inner and outer edges is given as follows:

$$y = \kappa_y \frac{qa^4}{D}$$

Roark 5th Ed (1975)
Table 24-2c p 339

where

$$D = \frac{Et^3}{12(1 - \nu^2)}$$

and ν Poisson's ratio is 0.3. E is the elastic modulus; t is the plate thickness; a is the outer radius of the disk; b is the inner radius of the disk (see Fig. C1) and κ_y is a coefficient which is a function of b/a. For $b/a = 0.305$, $\kappa_y \simeq 0.0029$. TABLE C1 shows the values of E, t, D and the deflection calculated for the two types of plates.

TABLE C1. The modulus, plate thickness, D, κ_y and the deflection under gravity loading for the strong-back and pressure plates

property		strongback plate 310 St Steel	pressure plate 6061 Aluminum
E	(Nm^{-2})	2.10×10^{11}	0.69×10^{11}
t	(m)	1.91×10^{-2}	1.27×10^{-2}
D	(Nm)	1.34×10^{5}	1.29×10^{4}
b/a		0.305	0.305
κ_y		0.0029	0.0029
y	(m)	1.83×10^{-4}	1.90×10^{-3}

The calculatioms of deflection using the disk which is simply supported at both edges yields a lower deflection than the UCLA calculation using the EASE-2 computer code. The simply supported boundary condition on the outer edge of the disk is not quite accurate. The outer edge of both plates is simply supported at only the six points where the plates are attached to the posts. It is reasonable to expect the disk between the support points to deflect an additional 30 per cent. The UCLA calculations of deflection under gravity loading appear to be correct.

MAG/am

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

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