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IMAGE COMPENSATION

Andris Faltens

August 1981

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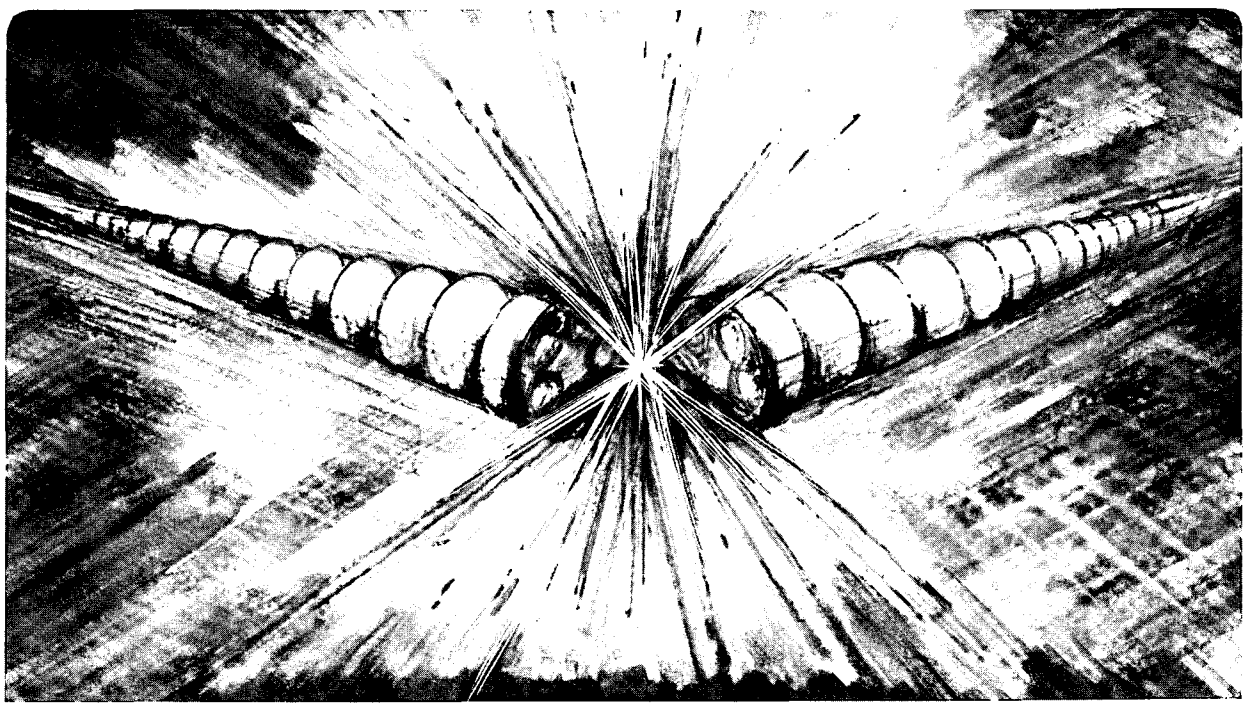

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Lawrence Berkeley Laboratory

Technical Report of the Betatron Design Study

IMAGE COMPENSATION*

Andris Faltens

August 1981

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The electrical and magnetic image problem is discussed for a high current betatron for both the quasi-static and the rf fields, and a wall geometry is prescribed which satisfies the electrostatic and ac image requirements. The quasi-static magnetic images may be sustained by supplementary conductors carrying externally controlled current.

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IMAGE COMPENSATION

Andris Faltens

The design of the betatrons has been based on the recognition that at high energies the magnitude and location of the beam image charges and currents in the chamber walls enter into determining the maximum current in the ring. Both DC and AC images are important: the former from their effect on incoherent or single-particle focussing and the latter from their effect on instabilities. Throughout the design process various simplifying approximations have been used for the images, which would be replaced by calculations and measurements of the specific geometries selected for a real machine. It is convenient to discuss electric, magnetic, and AC images separately. The relation between the current in a ring and the incoherent betatron tune shift in a static situation is given by

$$I_{incoh} = \frac{8.5\ kA\ b(a+b)\ \beta^3\gamma^3\nu\Delta\nu}{R^2F} \tag{1}$$

where

$$F = 1 + \varepsilon_1\gamma^2\ \frac{b(a+b)}{h^2} + \varepsilon^2(\gamma^2-1)\ \frac{b(a+b)}{g^2}\ ,$$

R is the beam major radius,

a and b are the beam half height and half width,

h and g are the chamber half height and half width,

β and γ are the relativistic factors,

ν and $\Delta\nu$ the betatron tune and tune shift, and

ε_1 and ε_2 are the Laslett image coefficients.

Electrostatic Images

The effect of electrostatic images is included in Eq. (1) in the image coefficient, ε_1, which has a value near 0.2 for a line beam situated in a metallic vacuum chamber that is wide compared to its height, and accordingly this value has been used throughout the design process. This coefficient decreases or becomes more favorable when the side walls are included and when the beam width becomes comparable to the chamber height. The chamber wall has to have enough conductivity to form an electrostatic shield around the beam, a requirement easily met even in the presence of the pulsed magnetic fields of the betatron.

Magnetic Quasi-Static Images

The effect of image currents or magnetic images is included in the coefficient ε_2 in Eq. (1), but the application of this relation to a betatron directly is inappropriate. The coefficients ε_1 and ε_2 are ordinarily calculated for static fields; in a betatron the magnetic images fields may not reach their static values. Temporally, the image currents may be imagined as being present at injection time on the surfaces of the nearest conductors, then redistributing with time into other conductors according to their circuit resistances and inductances, and only decaying after a considerably longer time. The redistribution and decay of the image current is accompanied by some magnetic field leaking out of the beam enclosing conductors, at which time the effect of any external magnetic materials becomes evident. In the present designs, an iron flux core encloses the beam, making

in effect a tightly coupled transformer between the beam and the drive coils. An equivalent circuit for the betatron is:

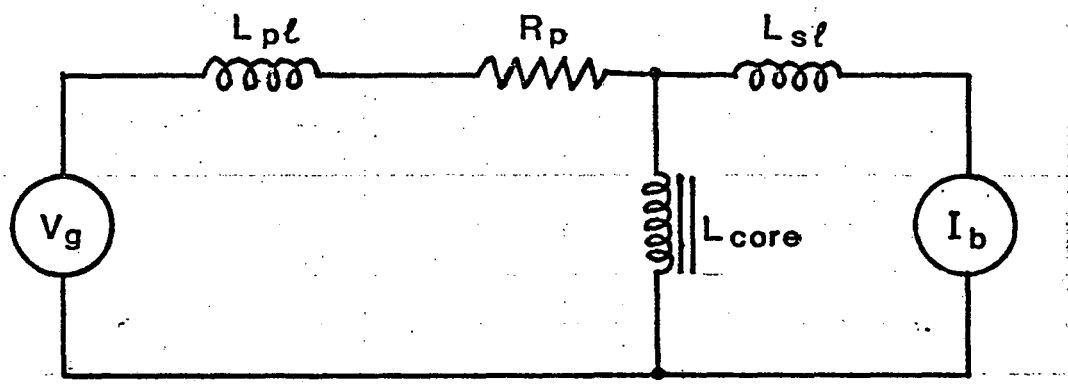

Fig. 1 Equivalent Circuit for the Betatron

where: V_g is the drive generator or a large, low impedance capacitor bank; and the current source, I_b, is the beam, which may be depicted as driving the circuit directly, as shown, or though a 1:1 transformer.

$L_{p\ell}$ is the primary circuit leakage inductance;

R_p is the primary circuit resistance;

L_A is the betatron flux core inductance;

$L_{s\ell}$ is the secondary circuit leakage inductance; and

I_b is the beam, approximated as a current source.

Representative values of these quantities are:

$$L_p \simeq 1\mu Hy$$

$$R_p \simeq 5m\Omega$$

$$L_{core} \simeq 5\ mHy$$

$$L_{s\ell} \simeq 1\mu Hy$$

The core inductance is not a linear component because of the presence of the magnetic material, but its exact value, as well as the effect of eddy currents within it, are unimportant as long as the core impedance is high compared to the other circuit impedances. For the above values the "DC" conditions would be approached with a time constant of 1 sec whereas the acceleration time is near 1 msec. Well before the DC conditions could be reached, a beam would significantly decelerate itself from the back-emf, and the flux core would saturate. For our purposes we therefore may assume that the full beam image current flows during the acceleration cycle.

By judicious placement of the flux-core-drive conductors the effect of the image currents on the beam may be minimized. With multiple conductors, the equivalent circuit becomes

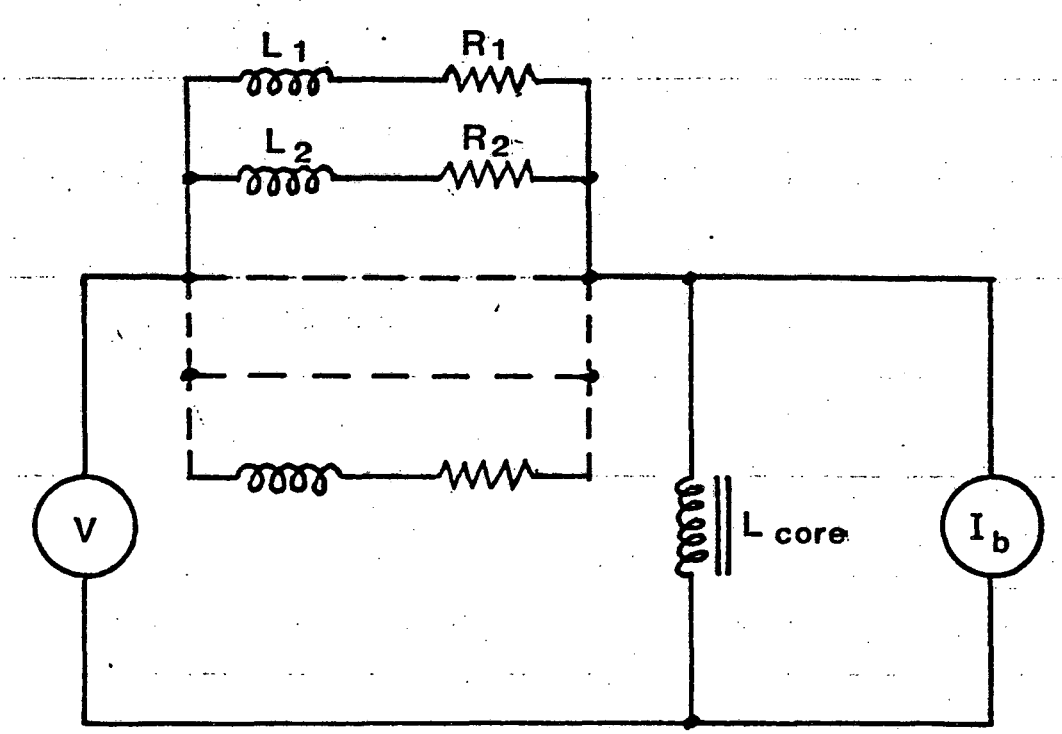

Fig. 2 Equivalent Circuit of Image Conductors

The n conductors are coupled to each other and the beam with mutual inductances. The initial current distribution at injection is determined by these inductances, while the redistribution of the image currents is due to the resistances. The time scale for this redistribution of the image current is of the order $L_n/R_n = 10\mu Hy/10\ m\Omega = 1$ msec. The final distribution of image currents could of course be forced, by suitable choices of the R_n's, to be the same as the initial distribution, if desired.

In the betatron the beam radial width is determined by the momentum spread and emittance, both of which damp during acceleration. The damping varies as $\gamma^{-1/2}$, or a factor of 3.2, in the booster, and as γ, or a factor of 2, in the main rings. This change in the beam profile will tend to induce the correct image currents in the walls, towards a distribution more peaked above and below the beam than the initial distribution. This distribution may be modified by the R_ns, or if desired, by small, externally applied voltages in order to make corrections on a 100 μsec time scale during acceleration.

For design purposes at the present study we have assumed that 75% of the image current is in its optimum location and the remaining 25% is absent completely. This means that in the booster and in the top and bottom rings of the main ring stack a quarter of the beam current would be imaged in the iron yoke above and below the beam, and consequently we have made a first order correction by increasing the distance to the poles by an additional $(\sqrt{2} - 1)h$. In the actual machine nearly the full image current is expected to persist within the yoke throughout acceleration, but its distribution may be imperfect. The detailed studies of what constitutes tolerable imperfections must be left for the actual design of a real machine, at which time the

effects of various current distributions could be calculated and the fields measured and corrected or compensated as necessary.

One of the questions which may be clarified at this time concerns the coarseness of an acceptable current distribution such as would result from the use of discrete conductors. Because of the large required momentum spread, the beams have more of a ribbon-like than a line appearance. For a wide ribbon the desired wall current distribution is

$$K = \frac{I_{beam}}{4a} \tag{2}$$

where a is the half width of the beam. Letting s be the separation between evenly space image conductors, each conductor carries a current

$$i = \frac{I_{beam}\, s}{4a} \tag{3}$$

which may be Fourier analyzed as a wall current distribution

$$K(r) = \frac{I_{beam}}{4a}\left[1 + \sum_{\ell} 2 \cos \frac{2\pi \ell r}{s}\right] \tag{4}$$

This current, in front of an iron pole face, generates a field

$$B_{\ell} = \frac{\mu_o\, I_{beam}}{4a}\left[\sin \frac{2\pi \ell r}{s}\, i_z + \cos \frac{2\pi \ell r}{s}\, i_r\right] e^{-\frac{2\pi \ell z}{s}} \tag{5}$$

for the ℓ'th component in a rectangular geometry approximation. In the present designs the distance from the beam center to the chamber wall, h, has been about three times the beam vertical height, resulting in a distance from the wall to the edge of the beam of (2/3)h. The field gradient due to the image current therefore is

$$\frac{dB_z}{dr} = \frac{\pi u_o I_{beam}}{2as} e^{-\frac{4\pi h}{3s}} \qquad (6)$$

for the largest harmonic, $\ell = 1$.

To keep this gradient at the 1% level of the guide field gradients, nB/r = 2 gauss/cm and 10 gauss/cm for the booster and main rings respectively, requires $s \leqslant 7.5$ cm and $s \leqslant 4$ cm for the two machines. For engineering purposes, a 2" conductor spacing for the booster and 1" for the main rings are satisfactory.

In the preceding, the guide field has been omitted from consideration in order to simplify the discussion. The guide field has an average value which provides the bending of the beam and a gradient which provides focussing. Because the guide field must increase in step with the particle energy, and hence the core flux, both may be driven by a common voltage source through a suitable transformer. The preceding circuit must be slightly embellished to prevent the changing guide field from inducing circulating currents in the image windings. Each image conductor must be connected to the voltage source through a small transformer which just bucks out the voltage induced by the changing guide field, as shown:

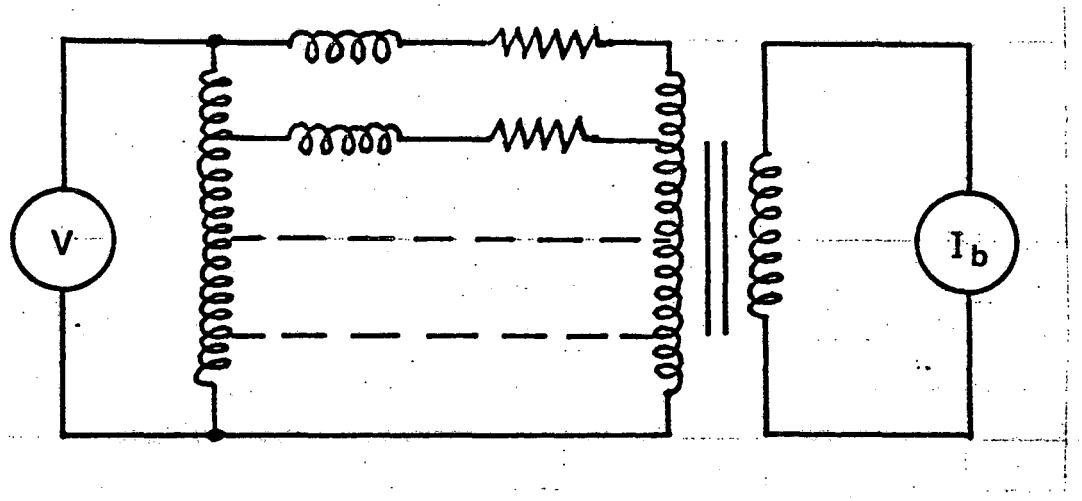

Fig. 3 Voltage Compensation in Image-Conductor Circuits

The guide field induces a voltage of 70 V/cm of radial extent in a loop of the type formed by the image conductors, for an acceleration rate of 20 kV/turn. The balancing of the voltages would be most easily accomplished by nulling out the current flowing in each image conductor in the absence of beam. Errors in voltage balancing result in currents which increase with time, eventually leading to a constant field distortion; errors caused by image currents would at injection be small, and depending on current redistribution times could have their largest effect during the acceleration or at its end.

AC Images

The AC image currents are helpful for transverse and longitudinal beam stability and for control of the coherent tune shifts. Unlike the slowly-varying DC or quasi-static current required for the incoherent focussing problem, the AC image currents do not necessarily have to pass through the

power supply of the accelerator, although that to the extent that they are permitted to flow by the high-frequency internal impedance of the supply connecting leads these currents are beneficial. Instead, most of the AC image currents may be bypassed capacitively within the ring(s).

The lowest apparent frequencies at which oscillating currents appear in the wall conductors are

$$(1-\nu_z)\omega_0 \cong \frac{\omega_0}{4} = \frac{c}{4R} = 2.5 \times 10^7 \text{ sec}^{-1} \tag{7}$$

$$(1-\nu_r)\omega_0 \cong \frac{\omega_0}{3} = \frac{c}{4R} = 3.3 \times 10^7 \text{ sec}^{-1} \tag{8}$$

The bypass capacitors must be essentially short circuits at these frequencies. The value of capacitance that constitutes an AC short circuit is based on keeping its impedance small compared to the allowable AC resistance, which is related to the transverse resistive wall stability requirement by

$$\frac{\Delta p}{p} > \frac{8}{\gamma}\left[1 + i\left\{\begin{matrix} 20\ R_s & \text{for booster} \\ 160\ R_s & \text{for main rings}\end{matrix}\right\}\right] \tag{9}$$

The surface resistivity of a thick copper sheet at the required frequencies is $R_s = \rho/\delta = 0.6\ \Omega/\text{square}$ where ρ is the resistivity and δ is the skin depth.. Therefore, with the momentum spreads required for longitudinal stability, the threshold values are sixty times greater than this value in the booster and ten times greater in the main rings. At the threshold values, the total ring resistance may be approximated by

$$R = \frac{2\pi R_o R_s}{2\pi h} = 0.6\Omega \text{ and } 0.2\Omega \text{ respectively} \tag{10}$$

therefore a value of $-j(.06)\Omega$, obtained with a $0.67\mu Fd$, capacitance is satisfactory for the booster, and $-j(.02)\Omega$ obtained with $2\mu Fd$ in the main rings.

The capacitors are essentially open circuits for the acceleration fields, which appear across them to accelerate the beam.

The low harmonic longitudinal image currents are satisfied by the above configuration; the tolerable longitudinal coupling impedance is in the 10-20 ohm range, whereas the capacitively coupled walls can contribute only a few milliohms beyond the impedance which would result from perfectly conducting walls. At about the 50th harmonic of the revolution frequency, that is, for frequencies near 1 Ghz, undesirable cavity-like resonances of the entire chamber are possible, and for these frequencies an rf absorber is required.

A possible configuration of the chamber conductors is shown in Fig. 4:

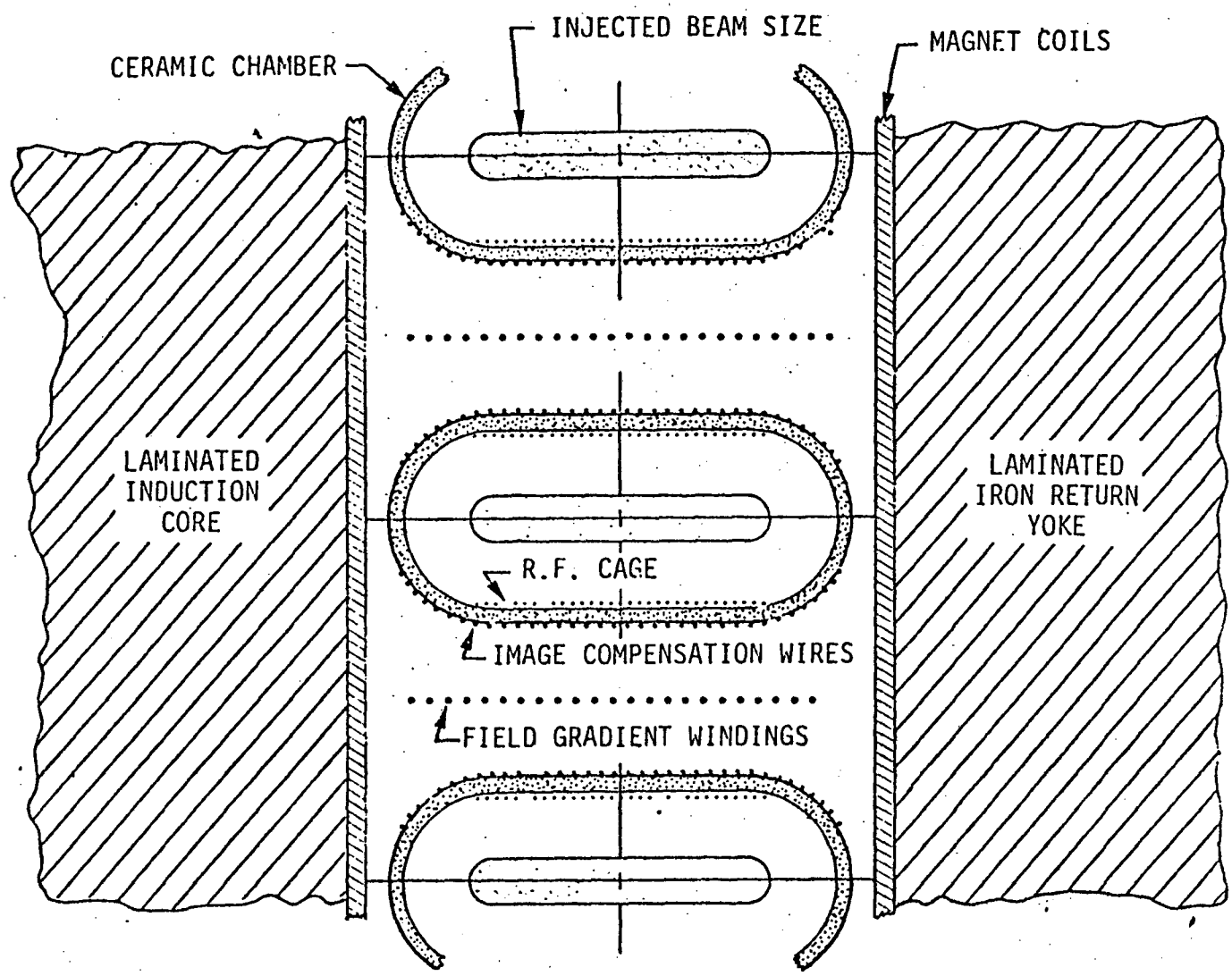

Fig. 4 Possible Image Conductor Configuration

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