

MULTIPLE WELL VARIABLE RATE WELL TEST ANALYSIS OF DATA
FROM THE AUBURN UNIVERSITY THERMAL ENERGY STORAGE PROGRAM

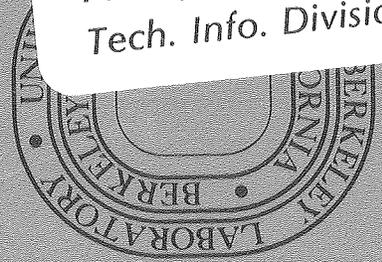
Christine Doughty, Donald McEdwards, Chin Fu Tsang

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MULTIPLE WELL VARIABLE RATE WELL TEST ANALYSIS OF DATA
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ABSTRACT

The computer program ANALYZE is used to determine reservoir parameters and the unique location of a hydrologic boundary for the aquifer used by Auburn University in a series of aquifer thermal energy storage field experiments. ANALYZE can deal with several observation wells and account for variable flow rates in multiple production wells. Two injection tests and one production test were analyzed. All yielded similar values of reservoir parameters and barrier location.

I. INTRODUCTION

The Water Resources Research Institute of Auburn University has conducted two series of field tests (one in 1976¹ and the other in 1978²) involving the feasibility of thermal energy storage on a shallow aquifer in Mobile County, Alabama. One of the goals of these tests was to obtain temperature and pressure data that may be used to check the validity of mathematical models that numerically simulate heat and fluid flow. To this end, information concerning the physical properties of the aquifer is required. The aquifer characteristics that need to be determined from pressure and flow data are the transmissivity - kh/μ - and the storativity - ϕch - of the aquifer. If a hydrologic boundary influences the data, its location and type (no-flow or constant potential) must be determined also.

Part of the first series of tests was a 36-hour constant flow transient pressure measurement. This has been analyzed by the USGS³ using conventional type-curve analysis techniques. Values of kh/μ and ϕch were obtained, and a barrier boundary approximately 300-500 meters from the production well was indicated.

The Lawrence Berkeley Laboratory has recently developed a computer-assisted well test analysis method, program ANALYZE^{4,5}, that rigorously accounts for the variable flow rates of several production or injection wells; the method can also treat simultaneous pressure data from several observation wells. Hence the technique is not restricted to constant flow rate tests as are type-curve analyses. Further, simultaneous measurements from several observation wells can give an unambiguous location for a hydrologic boundary. This paper describes the application of this technique to the Auburn data with the intent to confirm

the USGS transmissivity and storativity values, and to determine a unique location for the boundary.

In the next section, the program ANALYZE will be described. Following this, the results of three well test analyses using data from the 1976 and 1978 tests will be presented.

II. PROGRAM ANALYZE

Program ANALYZE treats well test data for the reservoir characteristics transmissivity - kh/μ , storativity - ϕch , the location and type (barrier or leaky) of a linear hydrologic boundary. The program rigorously handles variable production rates from up to 20 production wells. Pressure data from up to 20 observation wells may be analyzed simultaneously. The program models production wells as fully penetrating line sources or sinks and the reservoirs as a constant thickness, laterally infinite, isotropic porous medium.

The computation basis of ANALYZE is a least squares minimization routine that uses parameters kh/μ , ϕch , and the angle and distance to a hydrologic boundary to calculate the pressure change at locations and times corresponding to observed pressure data. It then adjusts the values of the parameters (collectively called X_j) such that the sum of the normalized squared difference between calculated pressure changes and observed pressure changes is a minimum. The set of parameters associated with the minimum is then accepted as representative of the reservoir and well system. Written mathematically, we minimize

$$\chi^2 = \frac{1}{I} \sum_{i=1}^I \left[\frac{\Delta P_{\text{calc}}^{(i)}(X_j) - \Delta P_{\text{obs}}^{(i)}}{\Delta P_{\text{obs}}^{(i)}} \right]^2$$

in which the summation is taken over all the times and locations at which the I observed pressure changes are specified. To enable the user to judge the reasonableness of a minimum and its associated parameters, the observed and calculated pressure changes, the differences and ratios between them, and times of observation are listed for each observation well. Following this a log-log plot of pressure vs. time comparing the observed and calculated pressure changes is printed.

For a given set of data, program ANALYZE is most effectively used when repeated analyses are made in a prescribed order using certain portions of the pressure data and solving for certain parameters. The analysis procedure given below is based on experience gained in analyzing synthetic and various field test data.

Analysis Procedure

Step One

Purpose: 1) To determine if a boundary is present, and if so the type of boundary and approximate values of kh/μ and ϕch , 2) to determine values of kh/μ and ϕch if a boundary is not indicated.

Procedure: Perform sequential 2-parameter (kh/μ , ϕch) analyses for which later data is progressively deleted. For example, in a series of three analyses, the first analysis will consider all the pressure data, the second analysis will consider the earliest half of the data from each observation well, and the third analysis will consider the earliest quarter of the data from each observation well. If, as the later portion of the data is progressively deleted, the value of kh/μ becomes progressively larger (smaller) coincident with a progressive decrease in the χ^2 value, a barrier (leaky) boundary is indicated. If the kh/μ value and the χ^2 value do not change appreciably, then the data does not contain boundary information and the values of kh/μ and ϕch are accepted as the final results. If a boundary is indicated,

the values of kh/μ and ϕch of the earliest time data analysis are close approximations to the final values because the boundary's influence is small for early times. These values are used as initial guesses for Step Two.

Step Two

Purpose: To determine simultaneous values of kh/μ , ϕch and the angle and distance to a boundary.

Procedure: Perform a four parameter (kh/μ , ϕch , angle, and distance) analysis using all the pressure data and using as initial values of kh/μ and ϕch the values found for the earliest data fit in Step One. With the initial values of angle and distance set to zero, implement the SEARCH option. The SEARCH option is a preminimization grid search for the location of a minimum in angle-distance coordinates holding the initial values of kh/μ and ϕch constant. The final preliminary grid search values of angle and distance are used as initial guesses for these parameters in the four parameter analysis. The returned values of the four parameter analysis are accepted as final results.

III. WELL TESTS

The aquifer used by the Auburn Experiment is about 21m thick and lies between 40 and 61m below the surface. It is primarily sand confined above and below by clay layers. Two production/injection wells and fourteen observation wells penetrate the aquifer. Figure 1 shows the well field layout.

1. Old Injection Test (June-September 1976)

Warm water (35°C) was injected into the central injection/production well, I_1 , at a variable rate for 74 hours. This injection period was followed by a quiescent period of 93 hours after which injection was restarted. The injection data was reported as cumulative flow only, so flow rate was represented by step changes calculated from this data. Wells 1 through 10 served as observation wells.

2. 36-Hour Pumping Test (May 1976)

Well I₁ was pumped at a constant rate of 521 gpm for 36 hours. Wells 7, 8, 9 served as observation wells in the LBL analysis. The USGS analyzed this test, reporting results of analysis of data from wells 7 and 10.

3. New Injection Test (March - June 1978)

Hot water (50-65°C) was injected into well I₂ at a variable rate for 1900 hours. Flow rate data was again calculated using the cumulative flow rate data reported as in part 1. Wells 4 through 11 served as observation wells.

IV. WELL TEST ANALYSES

The old injection test data, the first data considered, was analyzed in great detail with very satisfactory results. The 36-hour pumping test data and new injection test data were analyzed to confirm and refine these results rather than determine them from scratch. Hence, these analyses are not as extensive as the first injection test analyses.

1. Old Injection Test

Two-Parameter Analysis

Individual Well Analyses: Sequential two-parameter individual well analyses were done for data from wells 1 through 10 with latest pressure data considered ranging from 10 to 167 hours ($10 \leq \tau_{\max} \leq 167$). Because the injection well only partially penetrates the aquifer, predicted pressures close to the injection well won't be as comparable with observed pressures as those further away. Figure 2 shows the variation of kh/μ and ϕ_{ch} with τ_{\max} for wells 7, 9, and 10. The similarity of the results from different wells exhibits the overall homogeneity of the reservoir.

Multiwell Analysis: Since the individual well analyses give similar values for reservoir parameters, it is reasonable to do sequential two-parameter multiwell analyses to yield spatially averaged values of kh/μ and ϕ_{ch} . The results of multiwell analyses using data from wells 7, 9 and 10 are shown in Figure 3. The decrease in kh/μ correspondent to an increase in χ^2 for $\tau_{max} \leq 20$ indicates the presence of a barrier boundary.

Boundary Analysis

Multiwell Analysis: Three observation wells is the optimal number to consider when searching for a boundary. Data from fewer than three wells yields a mathematically ambiguous solution, while using data from a large number of wells makes fitting overly expensive. As all observation wells exhibited pressure changes due to injection, any boundaries are assumed to lie beyond the observation well field. A four-parameter analysis was used to locate the barrier indicated by the two parameter analysis sequence. Since more distant boundaries may also exist, a sequence of four parameter analyses was done with τ_{max} ranging from 4.6 to 167 hours (Figure 4 - distance is measured from the origin in Figure 1, angle is measured clockwise from the +y axis). Trends in χ^2 and kh/μ values indicate the presence and type of the next-encountered boundary.

To interpret these results, note that the χ^2 values for all times are much less than the corresponding χ^2 values for the two-parameter no-barrier analysis (Figure 3). Thus, for all times, the assumption of one barrier fits the data better than no barrier. The increase in χ^2 for $12 \leq \tau_{max} \leq 20$ indicates that the influence of one barrier fits the data less well for these times. This increase in χ^2 for $12 \leq \tau_{max} \leq 20$, coupled with the decrease in kh/μ for the same time period suggests that a second barrier influences pressure data for $t > 12$ hours. Thus, the shortest time data, $4.6 \leq \tau_{max} \leq 12$, determines the reservoir parameters and the location of the first boundary, while subsequent variation in kh/μ and χ^2 indicates the presence, but not the location, of a more distant boundary. The scatter of data and unaccounted for variations may be due

in part to the variable flow rate and manner in which it was modeled. Also, very few data points are available for small values of τ_{\max} , so the results of small τ_{\max} analyses may not be as meaningful as results of large τ_{\max} analyses. Although the influence of a second barrier is indicated after 12 hours, analysis of data for large values of τ_{\max} may still be used to determine the location of the first barrier, if its effect is dominant. Angle and distance values are less scattered for this range of τ_{\max} than for smaller values of τ_{\max} , so the barrier location given by the analyses for which $\tau_{\max} > 12$ is thought to be the most realistic result.

In light of the above discussion, the set of parameters chosen to represent the aquifer are:

$$\begin{aligned} kh/\mu &= .115 \times 10^7 \text{ mdm/cp} \\ \phi_{ch} &= .464 \times 10^{-3} \\ \text{Angle} &= 315^\circ \\ \text{Distance} &= 330 \text{ m} \end{aligned}$$

These values agree quite well with the USGS results which made use of the 36-hour constant flow rate test (Case 2).

$$\begin{aligned} kh/\mu &= .113 \times 10^7 \text{ mdm/cp} \\ \phi_{ch} &= .34 \times 10^{-3} - .5 \times 10^{-3} \\ \text{DISTANCE} &= 306 - 496 \text{ m} \end{aligned}$$

2. 36-Hour Pumping Test

Two Parameter Analyses

Individual Well Analyses: Sequential, two-parameter individual well analyses were done for wells 7 and 9 for τ_{\max} ranging from .5 to 36 hours (Figure 5). As in the old injection test, the individual

well analyses yield similar results, however in this test there appears to be a greater variation in kh/μ between wells (compare Figure 2).

Multiwell Analysis: Figure 6 shows the results of a sequence of two-parameter multi-well analyses using data from wells 7, 8 and 9 for τ_{max} from .5 to 36 hours. The rapid decrease with τ_{max} in kh/μ and the increase in χ^2 for $2.5 \leq \tau_{max} \leq 8$ indicates a barrier boundary. Although the increase in χ^2 with τ_{max} in this test is similar to that in the old injection test, two parameter multiwell analysis (Figure 3), in this test the values of χ^2 are much larger than the old injection test values of χ^2 for corresponding values of τ_{max} . These large χ^2 values may be due to the greater variation in kh/μ between wells noted in the individual well analyses.

Boundary Analyses

Individual Well Analyses: Because of the large χ^2 values associated with the two-parameter multiwell analysis, sequences of four-parameter individual well analyses were done for wells 7, 8, and 9; selected results are shown in Figures 7 and 8.

Results of analysis of data from Well 7 is scattered for $\tau_{max} < 3$ hours (Figure 7), probably because of the small number of data points available for analysis at these early times. Constant values of parameters are returned for $3 \leq \tau_{max} \leq 13$ hours, after which kh/μ decreases and χ^2 increases, indicating a second barrier is affecting the data. For $3 \leq \tau_{max} \leq 13$ hours, then, we select these values:

$$\begin{aligned} kh/\mu &= .118 \times 10^7 \text{ mdm/cp} \\ \phi_{ch} &= .459 \times 10^{-3} \\ \text{Distance} &= 305 \text{ m} \end{aligned}$$

(angle is not determined in an individual well analysis). These results agree quite well with the old injection test and USGS results. Analysis of data from wells 8 and 9 yields results similar to each other but quite different from those of well 7 (Figure 8). kh/μ is rather stable for $\tau_{max} < 1$ hour then decreases rapidly until 2 hours after which it decreases gradually. The best χ^2 value corresponds to $\tau_{max} \approx 1$ hours, implying the first barrier is felt before one hour and the second barrier after that time. For $\tau_{max} \approx 1$, then we average results for wells 8 and 9 to obtain these values:

$$\begin{aligned} kh/\mu &= .17 \times 10^7 \text{ mdm/cp} \\ \phi ch &= .26 \times 10^{-3} \\ \text{Distance} &= 250 \text{ m} \end{aligned}$$

These results are qualitatively different than previous results.

Multiwell Analysis: Figure 9 shows the results of a sequence of four-parameter multiwell analyses using data from observation wells 7, 8 and 9. Unlike any previous analysis sequence, χ^2 smoothly decreases as τ_{max} increases from 0 to 20 hours, after which it increases through the end of the test - 36 hours. All values of χ^2 are larger than the corresponding old injection test values (Figure 4). This odd χ^2 behavior is not entirely unexpected in light of the disparity between individual well analysis results. For the region around the minimum χ^2 , $10 \leq \tau_{max} \leq 30$ hours, we select the values:

$$\begin{aligned} kh/\mu &= .138 \times 10^7 \text{ mdm/cp} \\ \phi ch &= .328 \times 10^{-3} \\ \text{Angle} &= 325^\circ \\ \text{Distance} &= 275 \text{ m} \end{aligned}$$

However, these values should not be considered very reliable.

Of all the 36-hour test data, only well 7 yields results that agree with the old injection test and USGS results. It will be shown in Part V that well 7 parameters do, in fact, model the aquifer better than the other sets of parameters found.

3. New Injection Test

Two Parameter Analyses

Individual Well Analyses: Sequential two-parameter individual well analyses were done for data from wells 4 through 11 for τ_{max} ranging from 1 to 7 hours. All results show similar behavior: kh/μ decreases as τ_{max} increases, as expected (Figure 10). However, χ^2 doesn't monotonically increase, but increases dramatically between τ_{max} values of 2 and 4 hours, then decreases till 7 hours. This is probably due to the errors made in measuring the large change in injection rate that occurred at 3.3 hours.

Boundary Analyses

Individual Well Analysis: Sequential four-parameter individual well analyses also give similar results for all wells. For $\tau_{max} < 2.5$ hours, depending on the values of the initial guesses used either 1) kh/μ and χ^2 match the two parameter analysis values for these times and the barrier distances are quite large (no barrier effective), or 2) kh/μ is quite large and the barrier distance is small (20-250m) (Figure 11). This small barrier distance is physically unreasonable since the real barrier must lie beyond the well field. These conflicting results are believed to be caused by the paucity of data available for analysis for $\tau_{max} < 2.5$ hours. For $\tau_{max} > 3$ hours, the four-parameter analyses gives much lower χ^2 values than do the two parameter analyses. Hence it is inferred that after 3 hours the barrier is felt. The results are independent of the initial guess of parameters for τ_{max} values greater than 3 hours. The results for all the wells are summarized in Table 1 for $\tau_{max} = 7$ hours, the value of τ_{max} that corresponds to the minimum χ^2 .

Figure 12 shows the results of Table 1 displayed on a plan view of the well field. Note the general trends in kh/μ and ϕ_{ch} : kh/μ decreases south to north; ϕ_{ch} decreases SE to NW. This variation in ϕ is consistent with the geological information that the aquifer is a buried river channel and that the barrier to the NW corresponds to the edge of this buried river channel. Thus our data indicates a porosity decreasing from channel center to edge, that is, from SE to NW.

V. SUMMARY

Table 2 lists values of χ^2 that correspond to a comparison of pressures calculated using various sets of parameters and observed pressures. It's clear that the set of parameters from the 36-hour test, wells 8 and 9, yield significantly larger χ^2 values than do the other analysis results. Further the χ^2 values from the other analyses are very similar for corresponding values of τ_{max} . Thus, the range of reliable reservoir parameters is taken from well test analyses whose χ^2 values are similar. These parameter ranges are:

$$\begin{aligned} kh/\mu &= .109 \times 10^7 - .118 \times 10^7 \text{ mdm/cp} \\ \phi_{ch} &= .464 \times 10^{-3} - .484 \times 10^{-3} \\ \text{Angle} &= 315^\circ \\ \text{Distance} &= 305 - 337 \text{ m} \end{aligned}$$

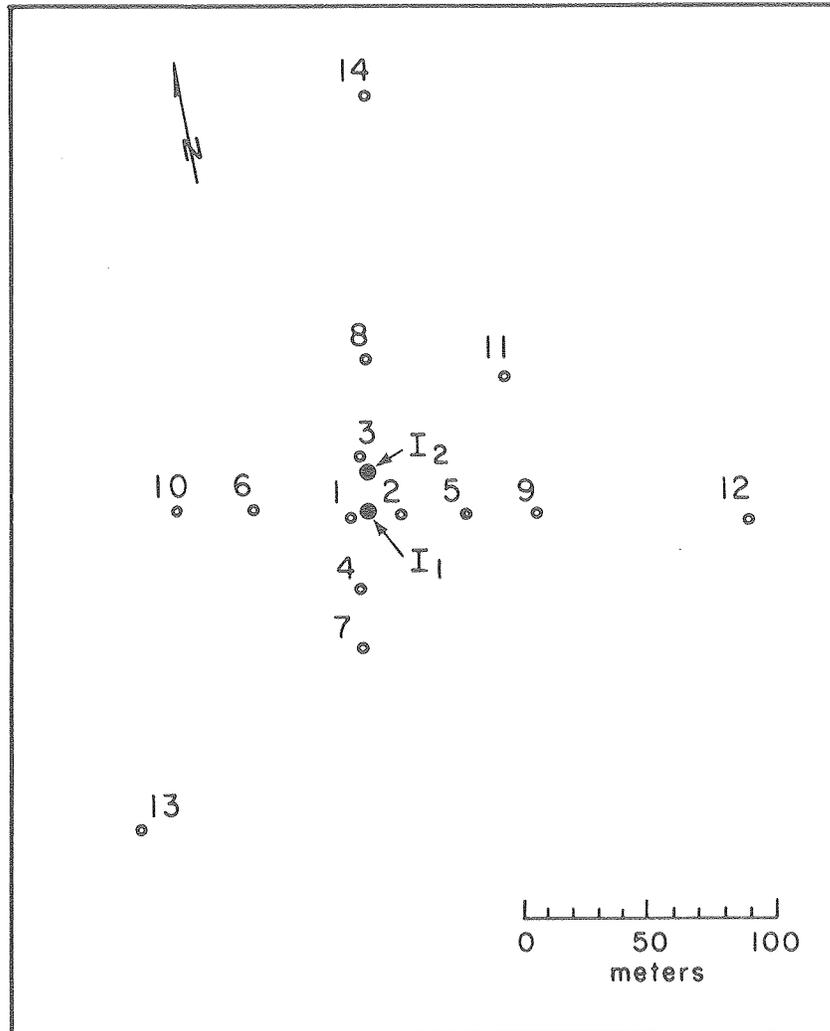
Also, in our analysis, there is an indication of decreasing permeability and porosity from SE to NW.

VI. ACKNOWLEDGEMENT

We gratefully acknowledge the cooperation of Fred Molz and Auburn University in supplying the data for this analysis. This work was supported by the U. S. Department of Energy under contract W-7405-ENG-48 through Oak Ridge National Laboratory.

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2. Molz, F. J. et al., 1979. Thermal energy storage in confined aquifers. WRRRI Preprint.
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XBL 795 - 7445

Figure 1. Top view of the Auburn well field.

INDIVIDUAL WELLS - 2 PARAMETER ANALYSIS

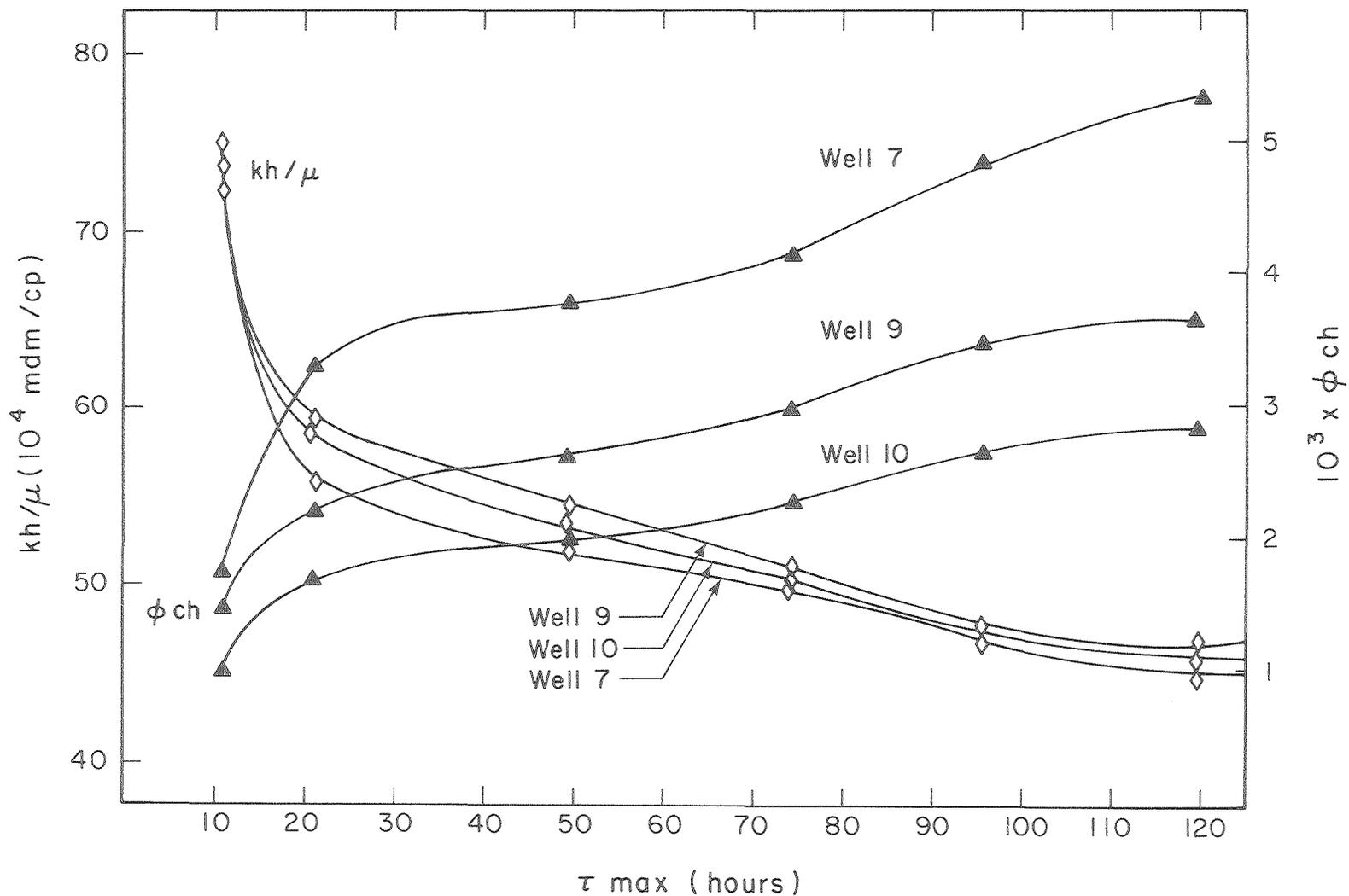
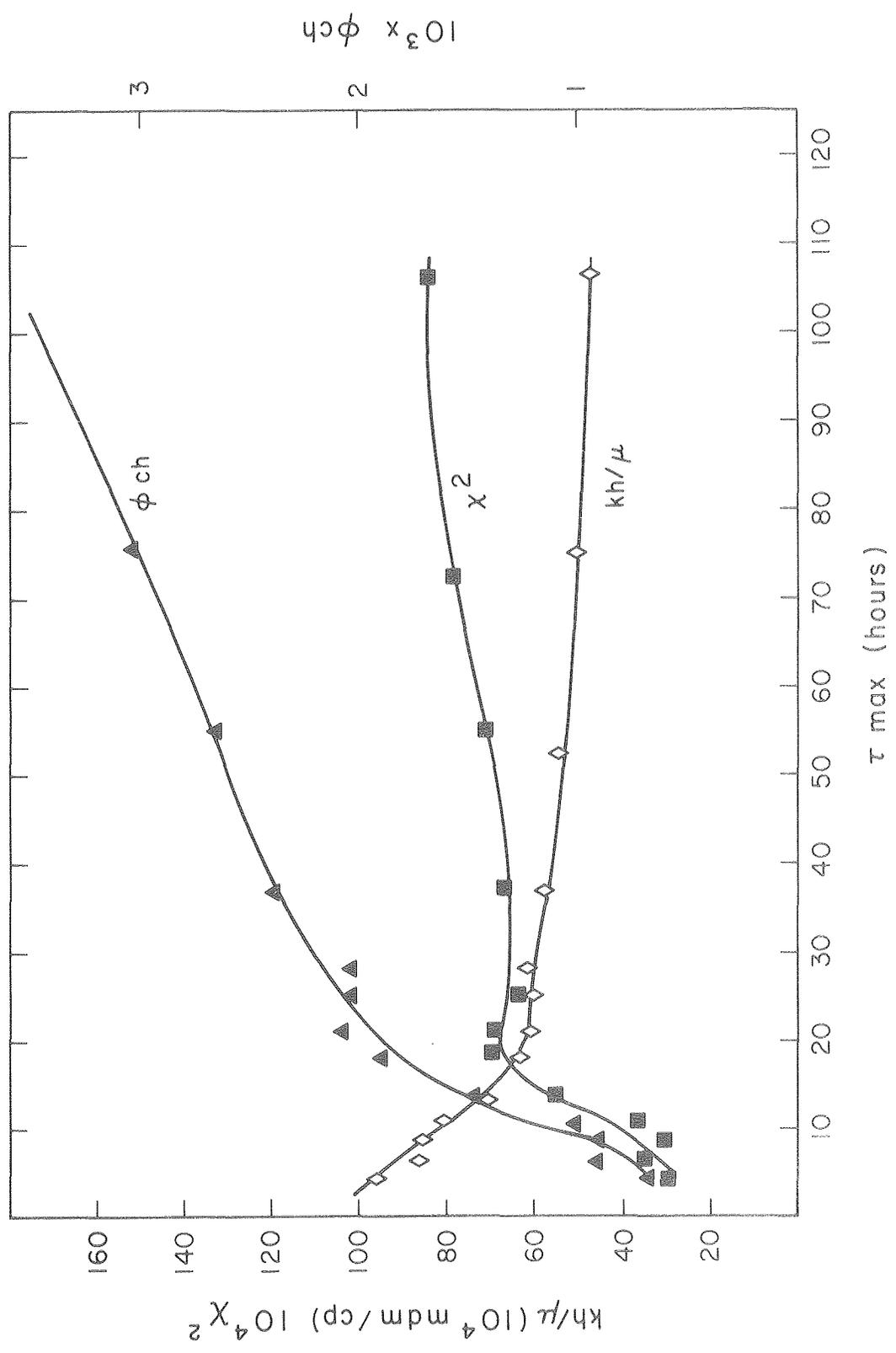


Figure 2.

XBL 795-7497

MULTIWELL - 2 PARAMETER ANALYSIS



XBL 795-7471

Figure 3.

MULTIWELL - 4 PARAMETER ANALYSIS

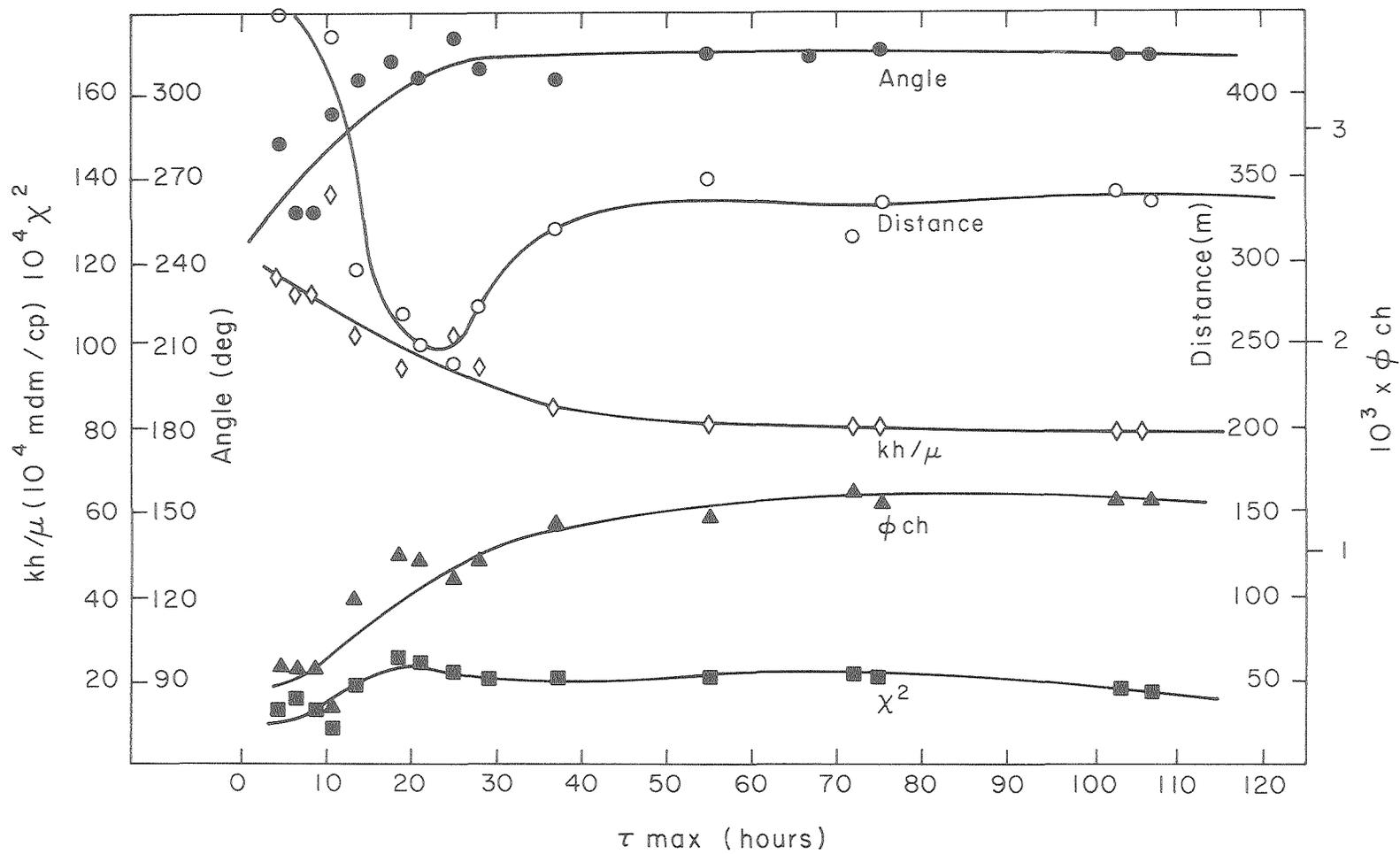


Figure 4.

XBL 795-7476

INDIVIDUAL WELLS -2 PARAMETER ANALYSIS

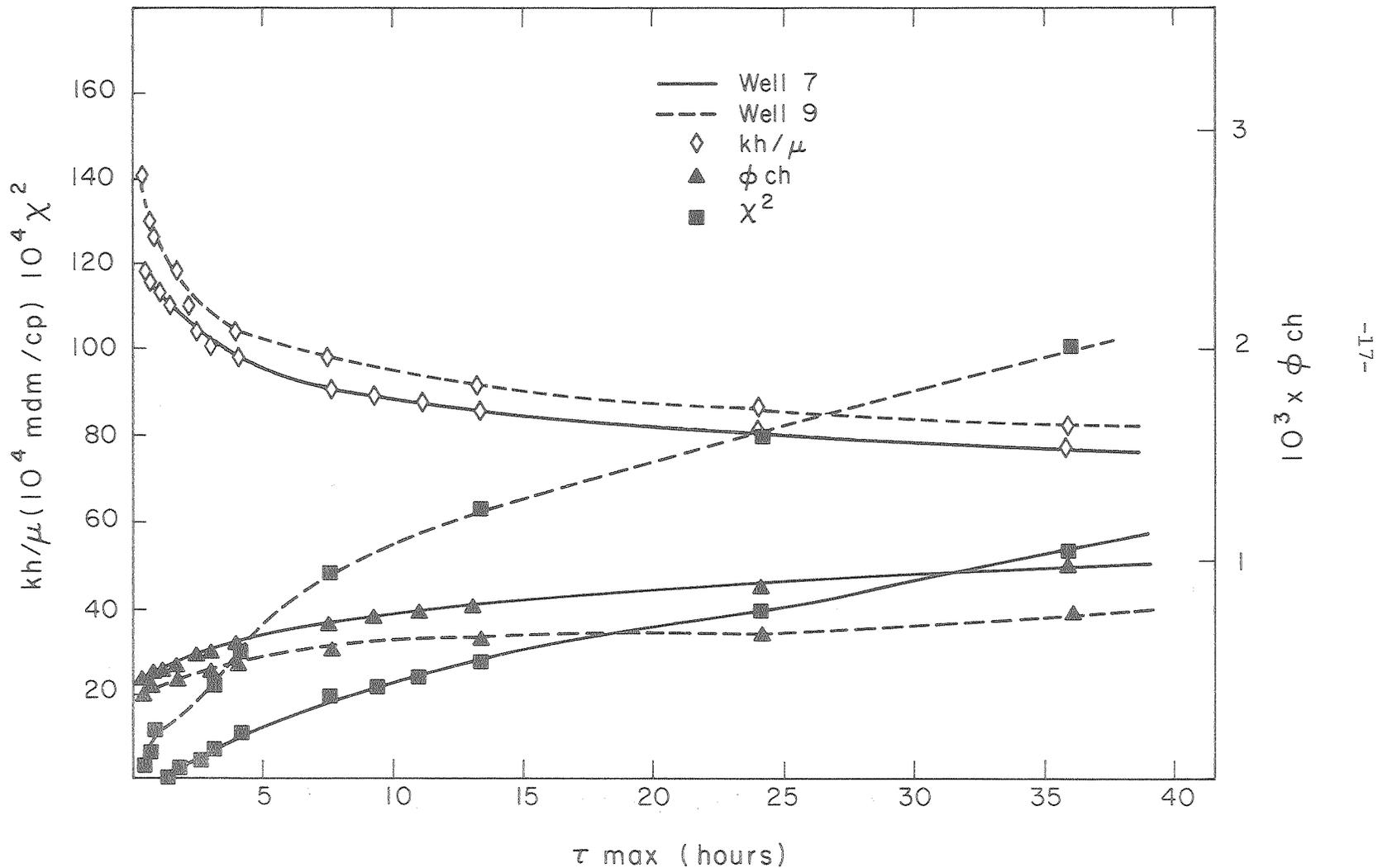


Figure 5.

MULTIWELL - 2 PARAMETER ANALYSIS

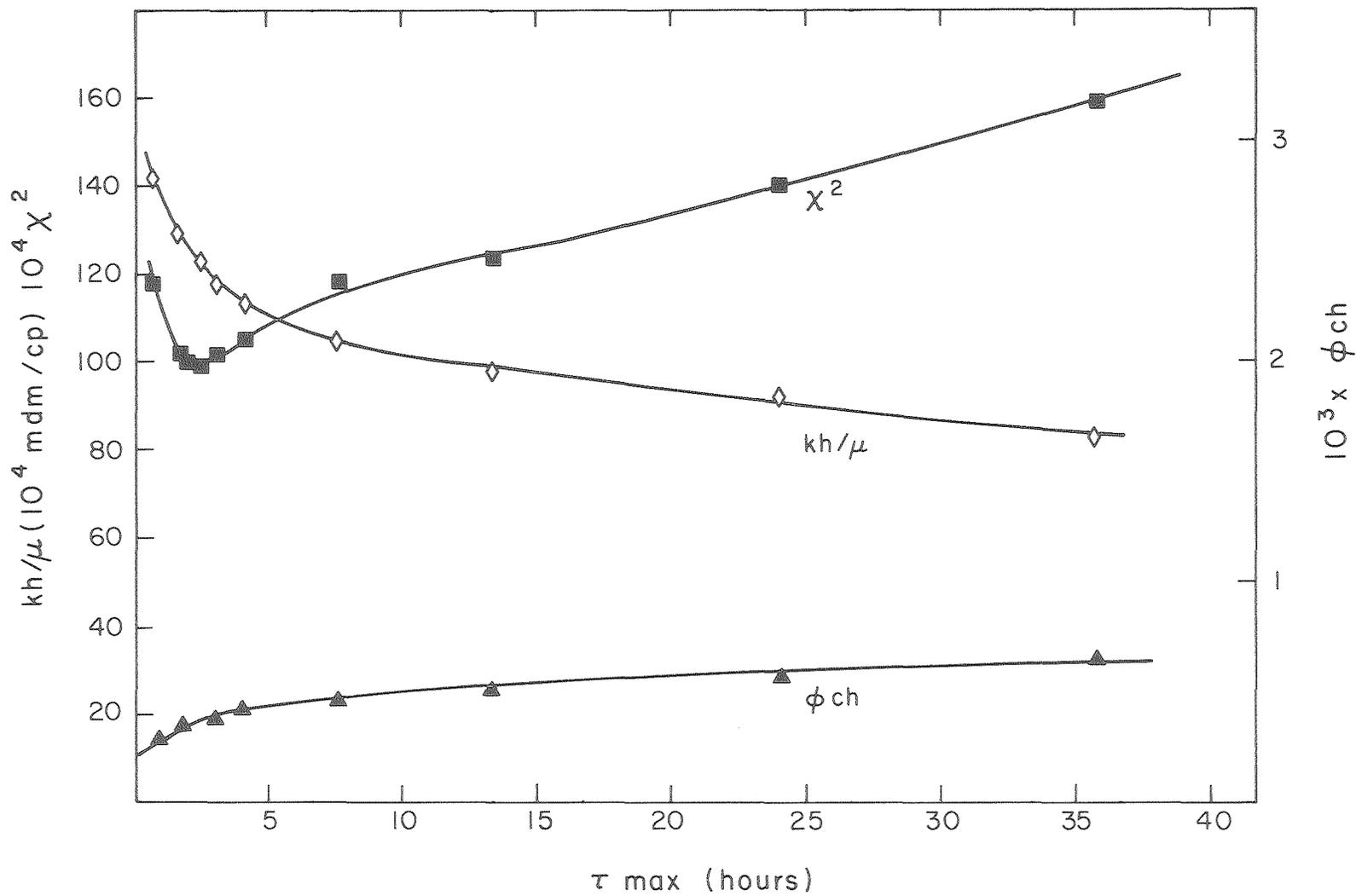


Figure 6.

XBL 795-7473

WELL 7 - 4 PARAMETER ANALYSIS

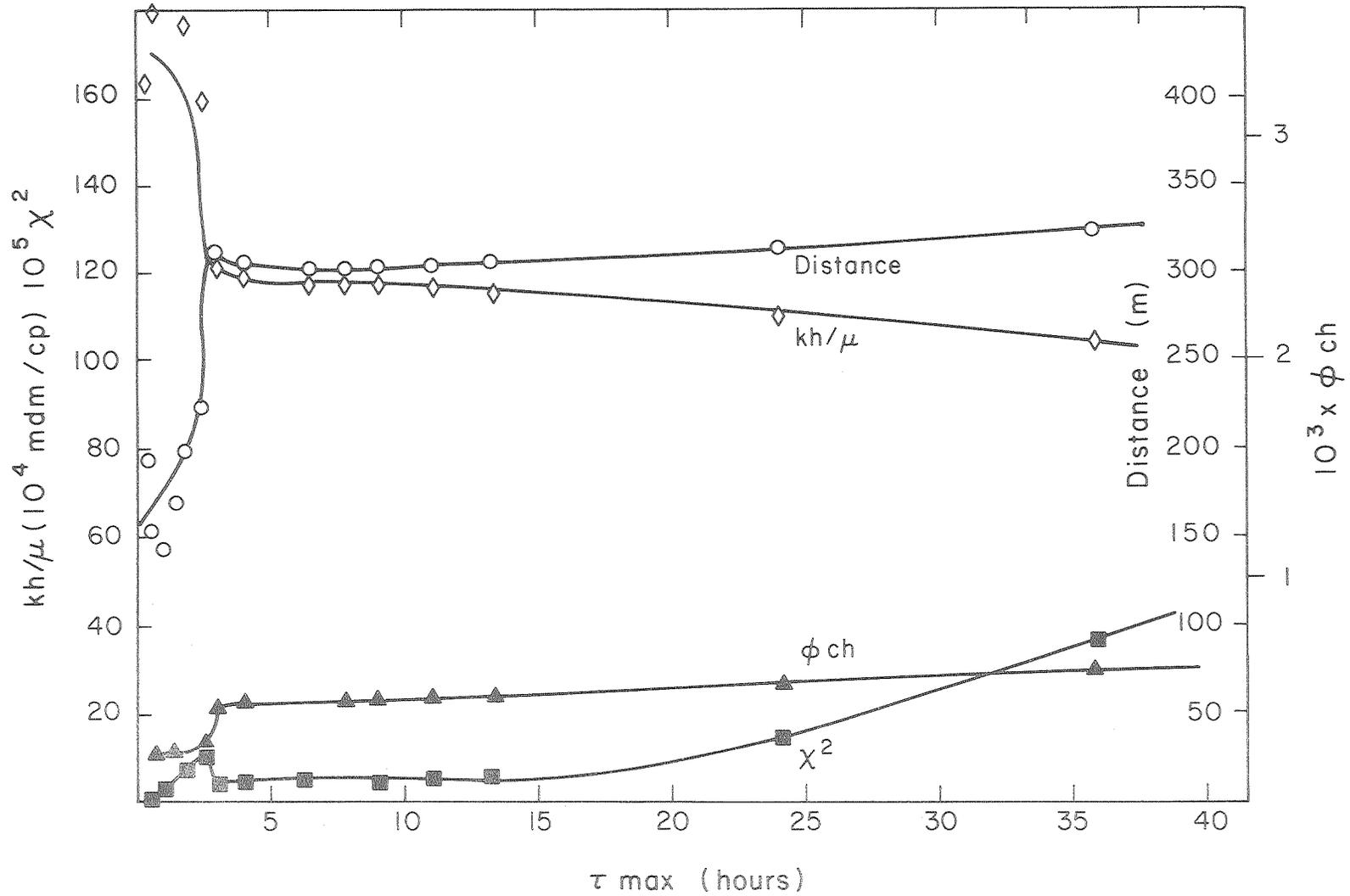


Figure 7.

XBL 795-7478

WELL 8 - 4 PARAMETER ANALYSIS

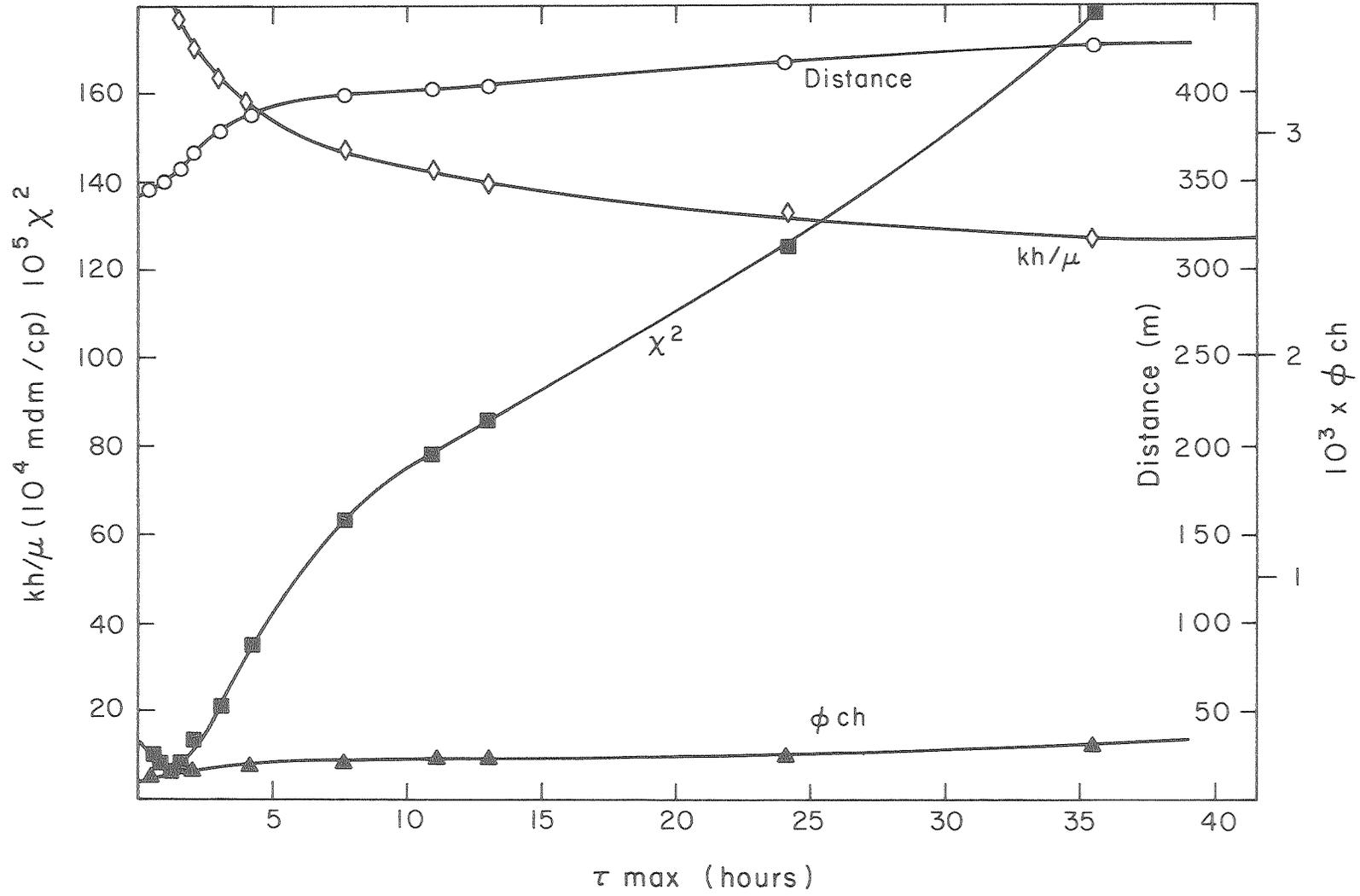


Figure 8.

XBL 795-7474

MULTIWELL - 4 PARAMETER ANALYSIS

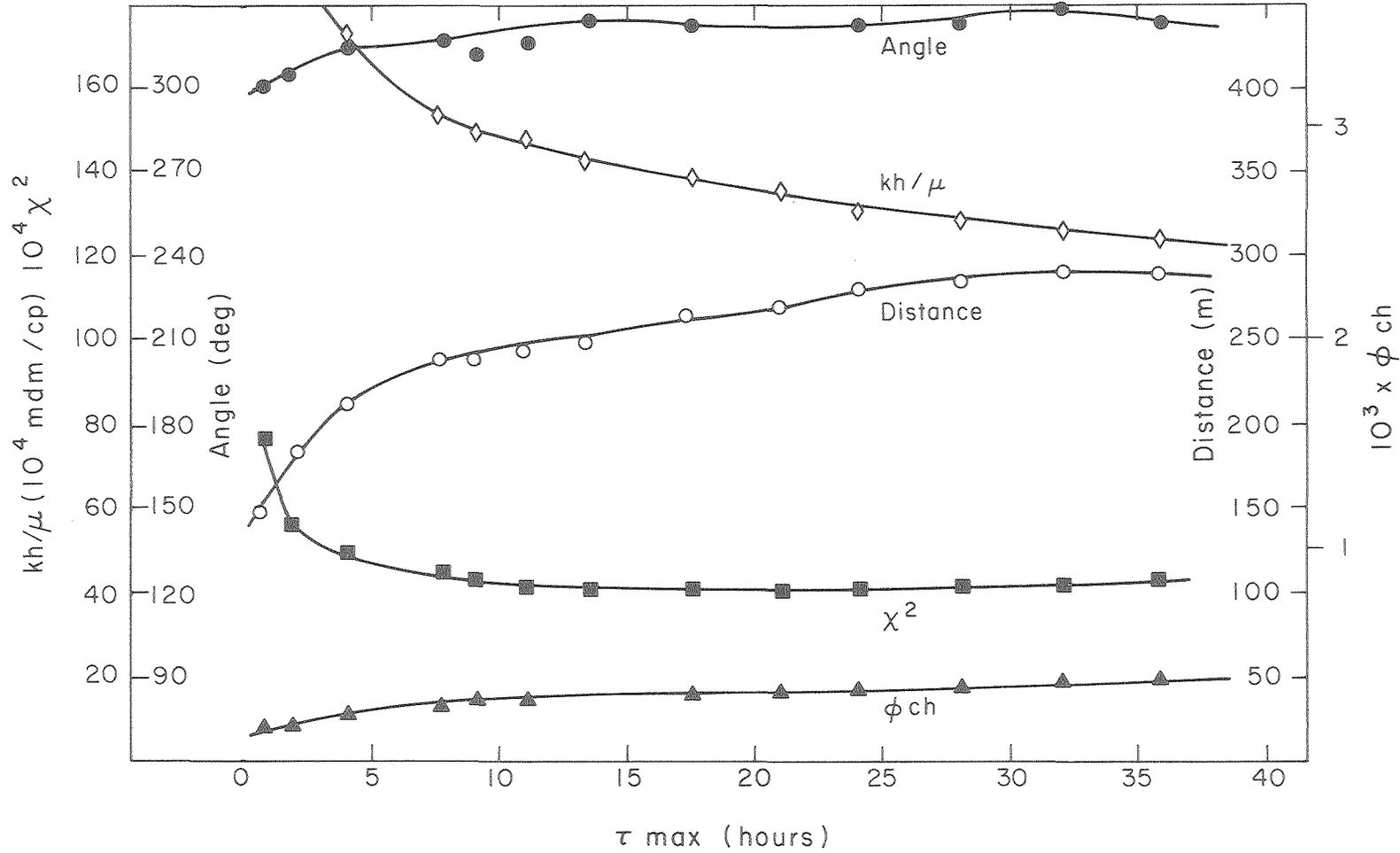


Figure 9.

XBL 795-7475

WELL 10 - 2 PARAMETER ANALYSIS

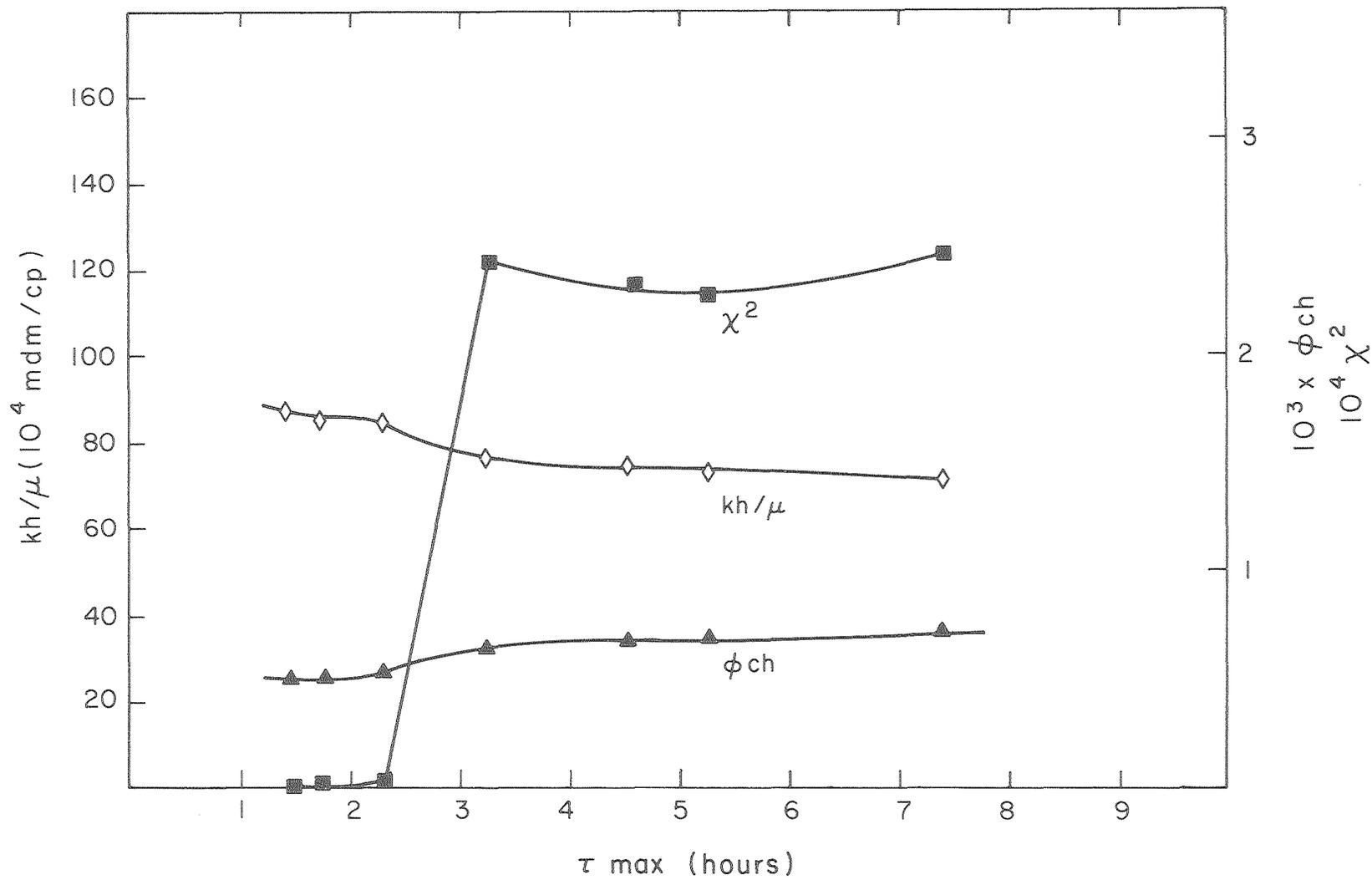


Figure 10.

XBL 795-7472

WELL 10 - 4 PARAMETER ANALYSIS

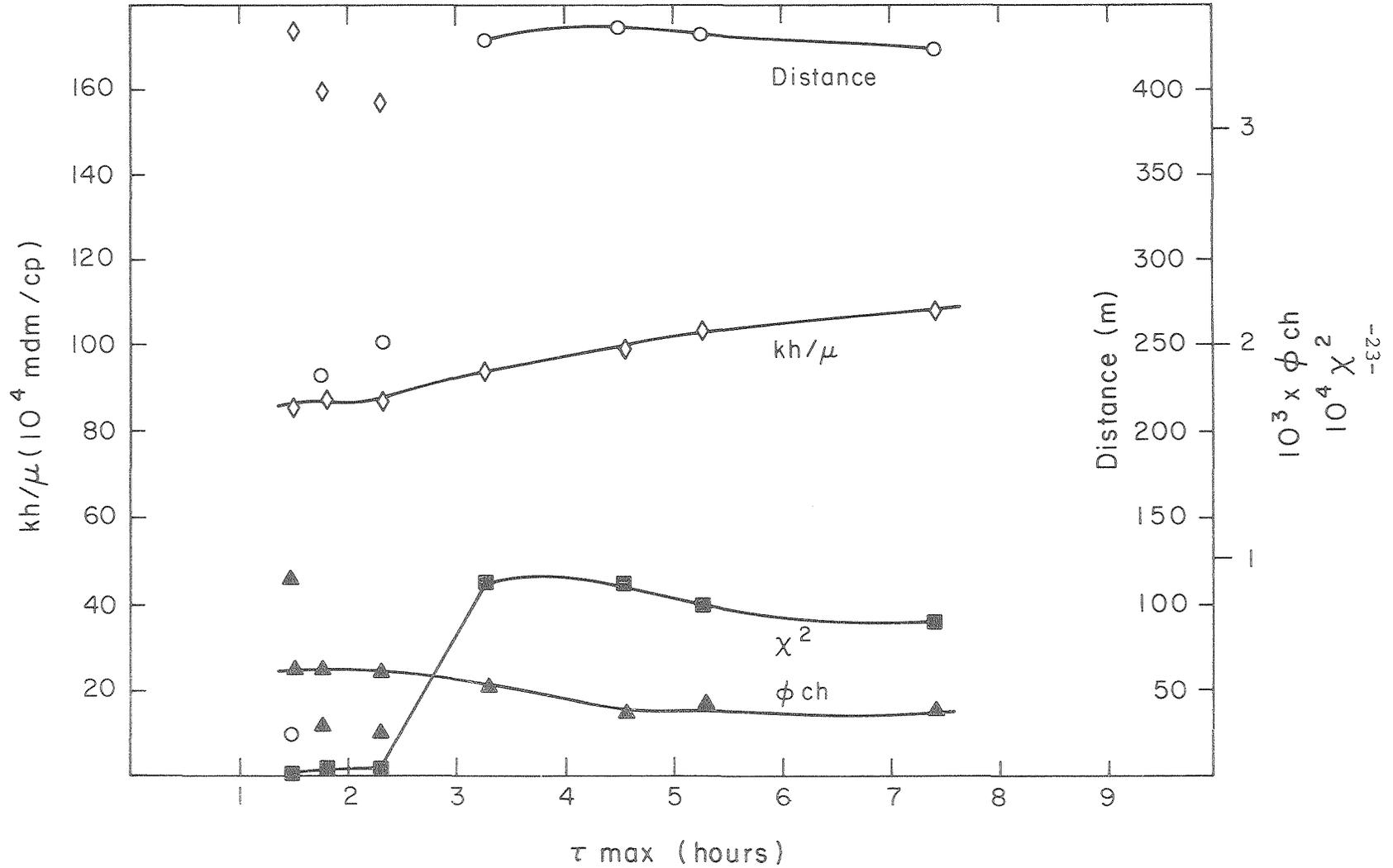
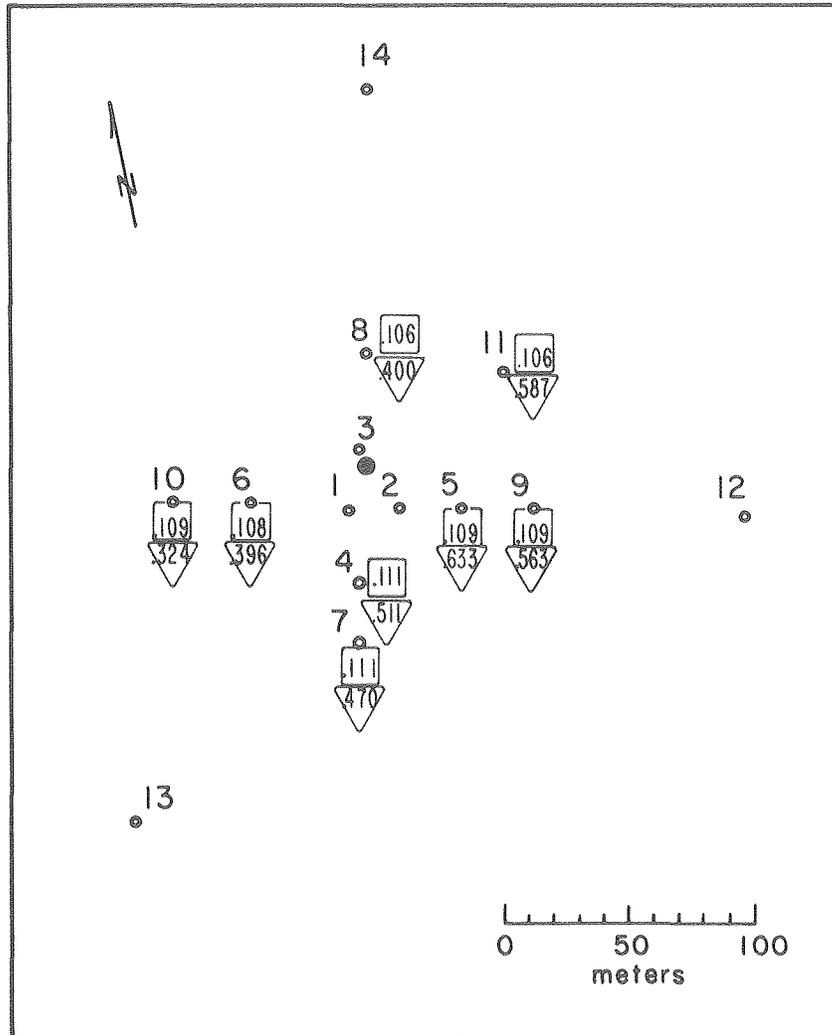


Figure 11.

XBL 795-7477



XBL 795-7443

Figure 12. Top view of the well field showing kh/μ and ϕ_{ch} for wells 4 through 11.

□ $10^{-7} \cdot kh/\mu$ (md m/cp)

▽ $10^2 \cdot \phi_{ch}$

Average values: $kh/\mu = .109 \times 10^7$ md m/cp
 $\phi_{ch} = .484 \times 10^{-3}$

Table 1. New injection test summary,
four-parameter fit, $\tau_{\max} = 7$ hours.

Well	$kh/\mu \left(\frac{\text{md m}}{\text{cp}} \right)$	ϕ_{ch}	Distance (m)	χ^2
4	$.111 \times 10^7$	$.511 \times 10^{-3}$	287.	$.18398 \times 10^{-3}$
5	$.109 \times 10^7$	$.633 \times 10^{-3}$	274.	$.28711 \times 10^{-3}$
6	$.108 \times 10^7$	$.386 \times 10^{-3}$	374.	$.13659 \times 10^{-3}$
7	$.111 \times 10^7$	$.470 \times 10^{-3}$	318.	$.16065 \times 10^{-3}$
8	$.106 \times 10^7$	$.400 \times 10^{-3}$	407.	$.70349 \times 10^{-4}$
9	$.109 \times 10^7$	$.563 \times 10^{-3}$	294.	$.19726 \times 10^{-3}$
10	$.109 \times 10^7$	$.324 \times 10^{-3}$	426.	$.72921 \times 10^{-4}$
11	$.106 \times 10^7$	$.587 \times 10^{-3}$	315.	$.96612 \times 10^{-4}$
Average	$.109 \times 10^7$	$.484 \times 10^{-3}$	337.	

Table 2. χ^2 comparison for various sets of parameters.

Source of Parameters	USGS Well 7	USGS Well 10	Old Injection test, Wells 7, 9, and 10	36-hr test Well 7	36-hr test Wells 8, 9	New injection test Wells 4-11
<u>Parameters</u>						
$kh/\mu \left(\frac{md}{cp} \right)$.113x10 ⁷	.113x10 ⁷	.115x10 ⁷	.118x10 ⁷	.170x10 ⁷	.109x10 ⁷
ϕ_{ch}	.5x10 ⁻³	.34x10 ⁻³	.464x10 ⁻³	.459x10 ⁻³	.26x10 ⁻³	.484x10 ⁻³
Angle (deg)	(315)	(315)	315	(315)	(315)	(315)
Distance (m)	320	470	330	305	250	337
<u>t_{max} (hr)</u>						
2000	.0602	.0600	.0598	.0594	.0881	.0611
300	.0240	.0228	.0251	.0264	.0752	.0189
10	.0069	.0045	.0068	.0072	.0252	.0047

Comments: The USGS did not determine ANGLE and measured distance to the boundary from the observation well rather than the origin, thus yielding a range of values for the DISTANCE parameter. Their value tabulated above was obtained by assuming an ANGLE of 315°.