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THE MINKOWSKI METRIC FOR A MULTIDIMENSIONAL GEOMETRY*

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In a recent note we presented a new geometrical description in terms of a multidimensional space (1). The dimensions of this manifold are expressed in terms of a generalized set of physical variables termed quantal units (1,2). These variables can be uniquely expressed as unit dimensions in terms of universal constants; Wheeler's "quantum of length" is one such dimension (3,4). Each quantal unit, in its dimensional representation, is associated with a quantized variable. The quantal units are one-component, single-dimensional quantities. Each quantized variable has an operator representation (1). We denote the quantal unit dimensions of the multidimensional space or Descartes space as $\{x_j\}$ and the associated quantized variables as ν_j, where the index j runs over the dimensionality of the space.

The quantal units represent the geometrical structure of the manifold and can be expressed in metrical form as a generalized Minkowski Metric, which is an extension of Minkowki's four dimensional geometry (5). In the present work, we construct the generalized Minkowski metric for a multidimensional geometry in terms of the quantal units, and present new light-cone relations implied by this metric. See Table I for the quantal units used in this paper.

Invariant relations hold for $(\underline{x}, t)$ and $(\underline{p}, E)$ in terms of the constancy of the universal constant, c (6,7). We have the usual special relativistic expression

$$s_1^2 = x_1^2 + x_2^2 + x_3^2 - c^2t^2, \tag{1}$$

where s_1^2 is the four-vector invariant for $(\underline{x}, t) = (\underline{x}_i, t_i)$ for the vector length $\underline{x} = (x_1, x_2, x_3)$ and the scalar time $t = (t, 0, 0)$, where the index i runs 1 to 3. The usual four-vector invariant for (p_i, E) is given as,

$$s_2^2 = p_1^2 + p_2^2 + p_3^2 - \frac{1}{c^2} E^2 \tag{2}$$

for the momentum vector $\underline{p} = (p_1, p_2, p_3)$ and the scalar energy $E = (E, 0, 0)$, where we use the Minkowski notation (5, 7).

The concept of invariance is extended to include invariance in terms of other universal constants (including c), or quantities expressed in terms of universal constants, such as the quantal units. We form a space $\{x_j\}$ for $1 \leq j \leq n$ for n orthogonal dimensions, where n is the number of physical-variable dimensions considered for a particular space. Each dimension such as space, time, momentum, or energy is considered on an "equal footing," forming a multidimentional Descartes space (1).

From any two physical-variable dimensions, μ and η, of the set $\{x_j\}$, we can form an invariant line-element s_{jk}^2 in terms of the matrix element m_{jk} of the Minkowski metrical matrix $\mathfrak{M}$. We have the general expression,

$$s_{jk}^2 = \mu_{ij}^2 + m_{jk}\,\eta_{ij}^2 . \tag{3}$$

The index i runs from 1 to 3, as before, and the indices j and k run from 1 to n, where n is the dimensionality of the space considered (8).

We introduce a "superspace" (9) of eight components, $n = 8$, which is constructed in terms of the three components of x, the three components of p, and the two scalars, t and E. This superspace

covers a four-space, $\mathcal{N} = 4$, with dimensions $\{x_j\}$, where a single scalar component of x and p is considered so that each dimension of the space is expressed in terms of a single physical variable $\{x_j\} = \{x, t, p, E\}$. The metrical elements m_{jk} of the generalized Minkowski metric $\mathcal{M}$ can be written as

$$m_{jk} = \begin{pmatrix} 1 & c^2 & \frac{c^2}{F} & \frac{1}{F^2} \\ \frac{1}{c^2} & 1 & \frac{1}{F^2} & c^2F \\ \frac{F}{c^2} & F^2 & 1 & c^2 \\ F^2 & \frac{1}{c^2F} & \frac{1}{c^2} & 1 \end{pmatrix} \tag{4}$$

constructed explicitly in terms of the scalar dimensions $\{x_j\}$. A more expanded form of $\mathcal{M}$ for a larger dimensional space is given in Table II. The elements of $\mathcal{M}$ in Eq. (4) are expressible explicitly in terms of the universal constant, c and the universal quantal force, F (Table I). We shall examine the manner in which this metric relates to the Einstein metric $g_{\alpha\beta}$, where α and β run 1 to 4. (In general, Latin indices run from 1 to 3, Minkowski notation (7), and Greek indices run from 1 to 4.)

For this particular Minkowski four-space, six invariant line-elements, s^2_{jk}, can be constructed for any two physical variables, μ_{ij} and η_{ij} of $\{x_j\}$, in terms of the constancy of the quantal units c and F (10). The symmetry properties for each element m_{jk} of $\mathcal{M}$ are $m_{ik} = \tilde{m}_{jk}^{-1} = m_{kj}^{-1}$ and $\mathrm{tr}\,\mathcal{M} = \mathcal{N}$.

Returning to Eqs. (1) and (2) and using the signature (+,+,+,-), we have

$$s_3^2 = x_1^2 + x_2^2 + x_3^2 - \frac{1}{F^2} E^2 \tag{5}$$

for $\mu_{i1} = \underline{x} = (x_1, x_2, x_3)$ and $\eta_{i3} = (E_1, 0, 0) = E$ and $m_{13} = 1/F^2$. The invariant for the variable pair $\mu_{i4} = \underline{p}$ and $\mu_{i2} = t$ is given by

$$s_4^2 = p_1^2 + p_2^2 + p_3^2 - (F^2)t^2; \tag{6}$$

and for the pair $\mu_{i1} = \underline{x}$ and $\eta_{i3} = \underline{p}$, we have

$$s_5^2 = x_1^2 + x_2^2 + x_3^2 - \frac{c^2}{F^2}(p_1^2 + p_2^2 + p_3^2). \tag{7}$$

The last expression that comprises the invariant relations of this eight-component superspace is for the pair $\mu_{i2} = t$ and $\eta_{i4} = E$:

$$s_6^2 = t^2 - (c^2F)E^2. \tag{8}$$

The four-vector invariants in Eqs. (1) and (2) are subspaces of the $\mathcal{N} = 8$ superspace; also, the pairs (x, E) and (p, t) forming the respective invariants s_3^2 in Eq. (5) and s_4^2 in Eq. (6) represent four-vector spaces, whereas the pairs $(\underline{x}, p)$ forming the invariant s_5^2 in Eq. (7) represent a six-subspace and the pair (t, E) forming the invariant s_6^2 in Eq. (8) represents a two-subspace. (Note that the subspaces refer to subspaces of the superspace $\mathcal{N} = 8$ but are termed partial spaces when related to the Minkowski metrical Descartes space of $\mathcal{N} = 4$. We usually term the space for which we write the Minkowski metric the Descartes space.)

The metrical elements for each subspace can be related to the Einstein metric, $g_{\alpha\beta}$. For example, for s_1^2 and s_2^2 (Eqs. (1) and (2)), we can write the Minkowski form of the invariant $s_1^2 = x_1^2 + x_2^2 + x_3^2 - c^2t^2$ as

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta, \tag{9}$$

where the indices α and β run from 1 to 4 so that $\mu_{ik=1} = \underline{x}$ for i = 1 to 3 and $\eta_{ik=2} = t$ for i = 1; then $m_{jk} = g_{\alpha\beta}$ for $x_4 = -ict$. In the usual notation,

$$g_{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{10}$$

where $c = 1$ "natural" units are chosen. Equation (1) can also be written $s_1^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$ in Einstein's special relativistic notation (7); and Eq. (9) is the general relativistic form of this equation.

The pair $(\underline{x}, E)$ invariant, s_3^2, in Eq. (5) and the pair $(\underline{p}, t)$ invariant, s_4^2, in Eq. (6) are expressed in terms of the constancy of the quantal force F; and to relate m_{jk} to the metrical form $g_{\alpha\beta}$, one must assume units in which $F = 1$ for a fourth component $x_4 = -iFE$ (Eq. (5)) and $p_4 = -i\,1/F\,t$ (Eq. (6)) Einstein notation (7). See Fig. 1 for light-cone relations for these subspaces. The pair $(\underline{x}, \underline{p})$ represents an invariant six-subspace, and the pair (t, E) represents an invariant two-subspace. Some properties of six-spaces, for example invariance properties and their relationship to isospin space, are given by E. Y. Hsu(11).

The Minkowski metric, in the representation in Eq. (4) and Table II, is a rank 3 matrix. Alternate forms and symmetry properties are given in Ref. (14). A contracted form and diagonal form are given in this reference.

In Fig. (1a) and (1b), we can define the space-like and time-like events in terms of greater than or less than the velocity of light signal propagation. In Fig. (1b) we could speak of momentum-like (for the three spatial components) or energy-like (for the fourth temperal component) events for $v > c$ or $v < c$ signal propagation. In Fig. (1c) and (1d), we can define "space-like" and "time-like" quantities in terms of forces in the manifold greater than or less than F, the quantal force. Just as the velocity of light, c, is considered an upper bound on velocity F can be considered an upper bound on force (13). For a force $f < F$, we have an energy-like "event" in Fig. (1c) and a time-like "event" in Fig.(1d); and for $f > F$, we have a space-like "event" in Fig. (1c) and a momentum-like "event" in Fig.(1d). Events represented

in Fig. (1e) and (1f), relate to both c and F bounds. We usually speak of a light signal as the propagator of information content in the manifold; it is proposed that force stresses in the manifold also represents information (6).

Let us look at another example Minkowski space. If the Minkowski five space ($\nu = 5$), $\{x_{j \to n}\} = \{x, t, p, E, m\}$, is considered, then we can specify ten line invariants in terms of elements of the generalized Minkowski metric expressed in terms of the three quantal units c, F and t (see Table II) which, in turn, can be expressed in terms of the three constants c, G and $\hbar$, (see Table I). Note that this Minkowski space is covered by the superspace of $\eta = 9$ dimensional components.

Most quantal units are expressed in terms of c (associated with micro- and macro-phenomenon), G (associated with macro-phenomena) and $\hbar$ (associated with micro-phenomenon). Elements of the generalized Minkowski space of $\nu = 10$ for $\{x_j\} = \{x, t, p, E, m, F, c, a, \mathfrak{p}, L\}$ are expressed in terms of c, F, t and m in quantal unit form which can be expressed in terms of c, G and $\hbar$ by use of the expression in Table I. The expressions of the metric elements in Table II in terms of c, F, t and m instead of c, G and $\hbar$ result in simpler and more convenient terms, some of which appear in Einstein's field Eqs. (2). For more details see Ref. (14). The formulation of the generalized Minkowski metric in terms of the micro-parameter $\hbar$ is an aspect of the tie-in of quantum phenomena and the geometry of our multidimentional space which is is manifest as a set of constraints in cosmology (2) and quantum mechanics (12). The quantum properties of the quantal unit terminology are discussed in Ref. (12). The specific relationship of the generalized Heisenberg relations (12) and the metrical elements in Table II are given in Ref. (14). Also, simultaneous observability of the quantized

variables is discussed in this paper. The general concept of four-vectors and Lorentz invariance is presented in terms of these quantities.

We have presented a metric, termed the generalized Minkowski metric, for the multidimensional space, termed the Descartes space (1), in terms of physical variables. We have discussed in detail the Minkowski metric for space $\{x_{j \to n=4}\}$ which is covered by $\eta = 8$ superspace. Higher order, higher dimensional spaces are presented and discussed in detail in Refs. (13, 14).

The link between the quantal units as inert dimensions and as active geometrical constraints on dynamical processes may relate to the suggestions by G. Chew (15), that in order to understand elementary particle phenomena more completely, one may be required to "bootstrap space-time itself." That is, perhaps it is the bootstrapping of the geometrical structure of a multidimensional space that gives rise to the dynamical processes that occur within the "arena" in Wheeler's terminology (3), which we see as particle processes obeying dynamical laws. Actually the arena is the physics (14). The bootstrap concept is applied to a uniform formalism of the four force fields in Ref. (16).

A physical quantity, as a dimension of the manifold, cannot be separated from any other quantity, as Minkowski suggested for x and t ("one cannot imagine a space without a time or a time without a space") when describing his concept of a four-space (5). E. J. Post also states that matter-energy cannot exist without space-time (17).

It is interesting to note the comparison and equivalence of some operator forms in Ref. (1) and Ref. (18). The Hermitian properties and Lorentz covariance of a simular time operator in Ref. (1), constructed from the quantal units, is discussed in Ref. (18).

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Footnotes and References

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7. The Minkowski notation, $s^2 = x^2 + y^2 + z^2 - c^2t^2$ or $s^2 = x_1^2 + x_2^2 + x_3^2 - c^2t^2$ (or as $s^2 = x_i^2 - c^2t^2$ for i running 1 to 3) is used, in which time is explicitly denoted, rather than the Einstein notation, $s^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$, where i runs 1 to 4 and $x_4 = -ict$.

8. Since the variables μ_{ij} and η_{ij} can be either vectors or scalars, the notation to denote vector components is index i running 1 to 3; for scalars, index $i = 1$.

9. In Ref. 4, J. A. Wheeler introduced the general concept of a "superspace"; we use a specific, restricted sense to this terminology.

10. Note that the quantal force F is expressed in terms of c, the velocity of light, and G, the universal gravitational constant; and the

quantal time t can be expressed in terms of c, G, and Planck's constant, $\hbar$; thus the m_{jk} elements can be expressed in terms of the three quantities c, G, and $\hbar$ as well as the three quantities c, F, and t.

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Erratum

Table I. Universal quantal units

Quantal unit	Quantal unit in terms of F, c and $\hbar$[a]	Numerical value of quantal unit[b]
$x = \left(\frac{G\hbar}{c^3}\right)^{\frac{1}{2}}$ length	$\left(\frac{c\hbar}{F}\right)^{\frac{1}{2}}$	1.60×10^{-33} cm
$t = \left(\frac{G\hbar}{c^5}\right)^{\frac{1}{2}}$ time	$\left(\frac{\hbar}{cF}\right)^{\frac{1}{2}}$	5.36×10^{-44} sec
$p = \left(\frac{c^3\hbar}{G}\right)^{\frac{1}{2}}$ momentum	$\left(\frac{\hbar F}{c}\right)^{\frac{1}{2}}$	4.16×10^{10} $\frac{\text{gm-cm}}{\text{sec}}$
$E = \left(\frac{c^5\hbar}{G}\right)^{\frac{1}{2}}$ energy	$\left(c\hbar F\right)^{\frac{1}{2}}$	1.25×10^{16} ergs
$m = \left(\frac{c\hbar}{G}\right)^{\frac{1}{2}}$ mass	$\left(\frac{\hbar F}{c^3}\right)^{\frac{1}{2}}$	2.22×10^{-5} gm
$F = \frac{c^4}{4}$ force	F	1.22×10^{49} dynes
$c = c$ velocity	c	3.00×10^{10} cm/sec
$a = \left(\frac{c^7}{G\hbar}\right)^{\frac{1}{2}}$	$\left(\frac{c^3F}{\hbar}\right)^{\frac{1}{2}}$	5.72×10^{53} cm/sec^2
$\mathcal{P} = \frac{c^5}{G}$ power	cF	3.66×10^{59} dyne $\frac{\text{cm}}{\text{sec}}$
$L = \hbar$ angular momentum	$\hbar$	1.06×10^{-27} erg-sec

[a]The quantal units are expressed in terms of the quantal force F, quantal velocity c, and quantal angular momentum $\hbar$.

[b]Values used in the calculation of the universal constants (G, the universal gravitational constant; c, the velocity of light; and $\hbar$, Planck's constant) in terms of which the quantal units are calculated, were taken from B. N. Taylor, W. H. Parker, and D. N. Langerberg, Rev. Mod. Phys. 41, 375 (1969).

Table II. Generalized Minkowski Metric with Elements Expressed in Terms of c, F, t and m.

$\mu_{ij}^2 \downarrow$ \ $1/\eta_{ij}^2 \rightarrow$	$1/x^2$	$1/t^2$	$1/p^2$	$1/E^2$	$1/m^2$	$1/F^2$	$1/c^2$	$1/a^2$	$1/\rho^2$	$1/L^2$
x^2	1	c^2	$\frac{c^2}{F}$	$\frac{1}{F^2}$	$\frac{c^4}{F^2}$	$\frac{t^4}{m^2}$	t^2	t^4	$\frac{t^4}{m^2c^2}$	$\frac{1}{F^2t^2}$
t^2	$\frac{1}{c^2}$	1	$\frac{1}{F^2}$	c^2F	$\frac{c^2}{F^2}$	m^2c^6	F^2m^2	$\frac{t^5}{c}$	m^2c^4	$\frac{c^2F}{t^2}$
p^2	$\frac{F}{c^2}$	F^2	1	c^2	c^2	t^2	m^2	mt	$\frac{t^2}{c^2}$	$\frac{c^2}{t^2}$
E^2	F^2	$\frac{1}{c^2F}$	$\frac{1}{c^2}$	1	$\frac{c^2}{t^2}$	$\frac{1}{c^2t^2}$	F^2t^2	mct	t^2	$\frac{1}{t^2}$
m^2	$\frac{F^2}{c^4}$	$\frac{F^2}{c^2}$	$\frac{1}{c^2}$	$\frac{t^2}{c^2}$	1	$\frac{t^2}{c^2}$	$\frac{F^2t^2}{c^4}$	$\frac{c^4}{F^2t^4}$	$\frac{t^2}{c^4}$	$\frac{1}{c^2}$
F^2	$\frac{m^2}{t^2}$	$\frac{1}{m^2c^6}$	$\frac{1}{t^2}$	c^2t^2	$\frac{c^2}{t^2}$	1	$\frac{m^2}{t^2}$	m^2	$\frac{1}{c^2}$	$\frac{1}{c^2t^4}$
c^2	$\frac{1}{t^2}$	$\frac{1}{F^2m^2}$	$\frac{1}{m^2}$	$\frac{1}{F^2t^2}$	$\frac{c^4}{F^2t^2}$	$\frac{t^2}{m}$	1	t^2	$\frac{t^2}{m^2c^2}$	$\frac{1}{F^2t^4}$
a^2	$\frac{1}{t^4}$	$\frac{c}{t^5}$	$\frac{1}{mt}$	$\frac{1}{mct}$	$\frac{F^2t^4}{c^4}$	$\frac{1}{m^2}$	$\frac{1}{t^2}$	1	$\frac{1}{m^2c^2}$	$\frac{1}{mct^3}$
ρ^2	$\frac{m^2c^2}{t^4}$	$\frac{1}{m^2c^4}$	$\frac{c^2}{t^2}$	$\frac{1}{t^2}$	$\frac{c^4}{t^4}$	c^2	$\frac{m^2c^2}{t^2}$	m^2c^2	1	$\frac{1}{t^4}$
L^2	F^2t^2	$\frac{t^2}{c^2F}$	$\frac{t^2}{c^2}$	t^2	c^2	c^2t^4	F^2c^4	mct^3	t^4	1

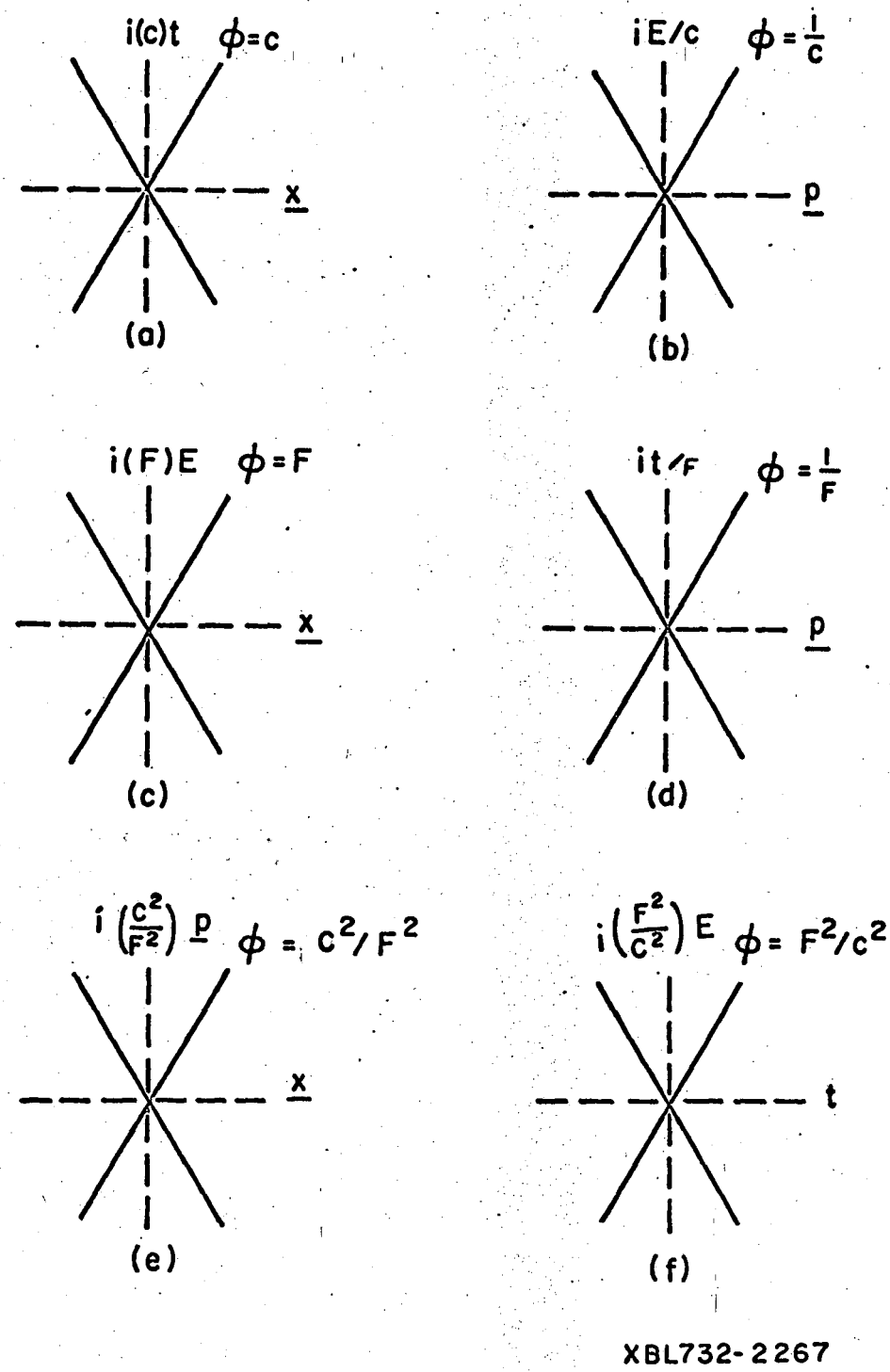

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Fig. 1. The usual light cone relations in Eq. (1) and (2) are represented in Fig. (1a) and (1b). Figure (1c) and (1d) represent the light cone relations for Eq. (5) and (6) and Fig. (1e) and (1f) represent the light cone relations in Eq. (7) and (8) where ϕ is the null vector light cone.