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Paul Langacker and Mahiko Suzuki

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THE πN TOTAL CROSS SECTION, NEUTRINO REACTIONS, AND ELECTROPRODUCTION IN THE REGGE ASYMPTOTIC REGION*

Paul Langacker† and Mahiko Suzuki††

Lawrence Berkeley Laboratory and Department of Physics
University of California, Berkeley, California 94720

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ABSTRACT

It is shown from the Deser-Gilbert-Sudarshan representation that the Pomeranchukon residue of the W_2 function for the axial vector current is independent of Q^2. This justifies the relation $\sigma^{\pi p}_{tot}(\infty) = \pi f_\pi^{-2} F_2^{(A)}(\infty)$ previously derived from generalized scaling.

In the formal framework of the Deser-Gilbert-Sudarshan (DGS) representation,[1] we study the Q^2 dependence of the Regge residues of the function $W_2^{(A)}(\nu,Q^2)$ for the axial vector current.[2,3] We find that besides the scaling of $\nu W_2^{(A)}(\nu,Q^2)/m_N^2$ and the Regge behavior in ω of $F_2^{(A)}(\omega)$, <u>the Pomeranchukon residue is independent of Q^2 all the way from $Q^2 = 0$ to ∞</u>. The high energy limit of the πN total cross section is thus related unambiguously to $F_2^{(A)}(\infty)$, which provides us with a test of the free quark algebra result $F_2^{(A)}(\omega) = F_2^{(V)}(\omega)$ [4] at least as $\omega\ (= 2\nu/Q^2) \to \infty$. Generalized scaling[5,6] is made plausible for secondary Regge trajectories.

The axial vector structure function $W_2^{(A)\pm}(\nu,Q^2)$ is defined as

$$\sum \int \frac{d^4x}{4\pi} e^{iqx} \langle p|[A_\mu^{\mp}(x),\ A_\nu^{\pm}(0)]|p\rangle$$

$$= \frac{1}{m_N^2} \tilde{p}_\mu \tilde{p}_\nu\ W_2^{(A)\pm}(\nu,Q^2) + \text{other terms}, \tag{1}$$

where $\tilde{p} = p - q(p\cdot q)/q^2$. Through PCAC $W_2^{(A)\pm}(\nu,0)$ is related to πN scattering by[6]

$$\sigma^{\pi^\pm p}_{tot}(\nu) = \pi f_\pi^{-2} \nu W_2^{(A)\pm}(\nu,0)/m_N^2\ , \tag{2}$$

where $\sigma^{\pi^\pm p}_{tot}(\nu)$ is the $\pi^\pm p$ total cross section for a massless external pion at laboratory energy ν/m_N, and $\sqrt{2}\, f_\pi = 0.96\ m_\pi$ is the pion decay constant.

We now write a DGS representation[1] for $W_2^{(A)\pm}(\nu,Q^2)$; this representation incorporates causality and the mass spectral conditions,

and it is useful for relating different kinematical regions in Q^2. It is written as

$$W_2^{(A)\pm}(\nu,Q^2) = \int_0^\infty d\mu \int_{-1}^{1} db \;\; \sigma^\pm(\mu,b)\; \delta(-Q^2 + 2b\nu - \mu)\; \epsilon(\nu + bm_N^{\,2}) \tag{3}$$

The support of $\sigma^\pm(\mu,b)$ is determined by the mass spectral conditions.[1] For positive Q^2 and ν, we obtain, by integrating Eq. (3) over b,

$$\frac{\nu W_2^{(A)\pm}(\nu,Q^2)}{m_N^{\,2}} = \frac{1}{2m_N^{\,2}} \int_0^\infty d\mu \; \sigma^\pm\left(\mu, \frac{Q^2+\mu}{2\nu}\right) \quad . \tag{4}$$

As was pointed out by many people, the right-hand side of Eq. (4) becomes independent of Q^2 in the Bjorken scaling limit ($Q^2 \to \infty$ with $\omega = 2\nu/Q^2$ fixed) if $\sigma^\pm(\mu,b)$ decreases fast enough in μ to permit interchange of the order of $Q^2 \to \infty$ and $\int d\mu$. If $\sigma^\pm(\mu,b)$ is truncated effectively at $\mu = \mu_{max}$, scaling sets in for $Q^2 \gg \mu_{max}$.

In the Regge limit ($\nu \to \infty$ with Q^2 fixed)

$$\nu W_2^{(A)\pm}(\nu,Q^2)/m_N^{\,2} \xrightarrow[\text{Regge}]{} \sum_i \beta_i^{\pm}(Q^2)\nu^{\alpha_i - 1} \quad . \tag{5}$$

This asymptotic behavior is realized if $\sigma^\pm(\mu,b)$ behaves near $b = 0$ as

$$\sigma^\pm(\mu,b) \xrightarrow[b\to 0]{} \sum_i \gamma_i^{\pm}(\mu)\; b^{1-\alpha_i} \quad . \tag{6}$$

Combining (4), (5), and (6), we obtain a relation

$$\beta_i^{\pm}(Q^2) = \frac{1}{2m_N^{\,2}} \int_0^\infty d\mu \; \gamma_i^{\pm}(\mu) \left(\frac{2}{Q^2+\mu}\right)^{\alpha_i - 1} , \tag{7}$$

which leads to the well known Q^2 dependence of the Regge residues $\beta_i^{\pm}(Q^2) \sim (Q^2)^{1-\alpha_i}$ as $Q^2 \to \infty$.[7] We stress here that Eq. (7) predicts that the Pomeranchukon residue is absolutely independent of Q^2;

$$\beta_P^{\pm}(Q^2) = \frac{1}{2m_N^{\,2}} \int_0^\infty d\mu \; \gamma_P^{\pm}(\mu) \; . \tag{8}$$

The same argument does not go through for the vector current:[2] To take account properly of the gauge condition $W_2^{(V)}(\nu,0) = 0$ for $|\nu| > Q^2/2$, we must write a DGS representation for $W_2^{(V)}(\nu,Q^2)$ with an explicit factor of Q^2, which changes the form of the residue considerably.

Our result Eq. (8) can be combined with Eq. (2) to give

$$\sigma_{tot}^{\pi^\pm p}(\infty) = \pi\, f_\pi^{-2}\, F_2^{(A)\pm}(\infty) \; . \tag{9}$$

This relation may be tested in high energy neutrino reactions when a relation between the vector and axial vector structure functions

$$F_2^{(V)}(\omega) = F_2^{(A)}(\omega) \tag{10}$$

is supplemented. Eq. (10) results in many models, for instance, in the free quark algebra model of light-cone singularities. Eq. (9) may then be written in the form appropriate to the neutrino reactions;

$$\sigma_{tot}^{\pi^{\pm}p}(\infty) = \frac{1}{2}\pi f_{\pi}^{-2}(\cos\theta_C)^{-2} F_2^{\nu,\bar{\nu}}(\infty)\Big|_{\Delta S=0} , \tag{11}$$

where $F_2^{\nu,\bar{\nu}}(\omega)\Big|_{\Delta S=0}$ is the strangeness conserving part of the neutrino (antineutrino) structure function. The free quark algebra result (10) leads us also to a similar relation with the $F_2(\omega)$ of electroproduction

$$\sigma_{tot}^{\pi^{\pm}p}(\infty) = \pi f_{\pi}^{-2} F_2^{\gamma_V}(\infty) , \tag{12}$$

where γ_V is the isovector part of the photon. If we further assume SU(3) symmetry,[8] these relations become

$$\sigma_{tot}^{\pi^{\pm}p}(\ ) = \frac{1}{2}\pi f_{\pi}^{-2} F_2^{\nu,\bar{\nu}}(\infty)\Big|_{\Delta S=0 \text{ and } 1} , \tag{11'}$$

and

$$\sigma_{tot}^{\pi^{\pm}p}(\infty) = \frac{3}{4}\pi f_{\pi}^{-2} F_2^{\gamma}(\infty) , \tag{12'}$$

where $F_2^{\gamma}(\omega)$ and $F_2^{\nu,\bar{\nu}}(\omega)$ are the full structure functions for electro- and neutrino production. The same relations were previously obtained from a more phenomenological argument based on generalized scaling. We have so far used (1) the DGS representation, (2) PCAC and a massless pion extrapolation, and (3) the free quark algebra result $F_2^{(A)}(\omega) = F_2^{(V)}(\omega)$. If we trust the formal manipulation of the DGS representation, Eq. (11) may be regarded as a test of $F_2^{(A)}(\omega) = F_2^{(V)}(\omega)$ at $\omega \to \infty$.

Since the residues $\beta_s^{\pm}(Q^2)$ of the secondary Regge trajectories involve a nontrivial Q^2 dependence, there is no simple relation between the residues in the Regge expansion in ν for $\sigma_{tot}^{\pi^{\pm}p}(\nu)$ and those in ω for $F_2^{(A)\pm}(\omega)$. However, if we assume that $\gamma_s^{\pm}(\mu)$ in Eq. (7) has a peak around $\mu = \mu_s$ or in the extreme case $\gamma_s^{\pm}(\mu) = 2m_N^2\, c_s\, \delta(\mu - \mu_s)$, then

$$\beta_s^{\pm}(Q^2) = c_s\left(\frac{Q^2 + \mu_s}{2}\right)^{1-\alpha_s} . \tag{13}$$

This allows us to interpolate smoothly between $\sigma_{tot}^{\pi^{\pm}p}(\nu)$ and $F_2^{(A)\pm}(\omega)$ using a generalized scaling variable. Eq. (13) may be obtained from the generalized scaling behavior

$$\nu\, W_2^{(A)\pm}(\nu,Q^2)/m_N^2 = F_2^{(A)\pm}\left(\frac{2\nu + \kappa}{Q^2 + \mu_s}\right) , \tag{14}$$

where the constant κ is related to satellite terms in the Regge expansion. Hence, generalized scaling and saturation with a single value of μ of the DGS spectral function are identical in the Regge asymptotic region.

FOOTNOTES AND REFERENCES

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† Address after September 15, 1972: The Rockefeller University, New York, New York 10021.

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3. We follow the conventions of C. H. Llewellyn Smith, Phys. Reports 4C, 262 (1971). In particular, $Q^2 = -q^2 > 0$ (q spacelike), $\nu = p \cdot q$, and $\omega = 2\nu/Q^2$.

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8. We have some reason to expect that the Pomeranchukon will act as a better SU(3) singlet in the current amplitudes than in the pseudoscalar meson amplitudes since SU(3) mass breaking will be insignificant at $Q^2 \to \infty$.

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