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FLUCTUATIONS ANALYSIS

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FLUCTUATIONS ANALYSIS

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I. Introduction

This paper briefly reviews sources of noise in Josephson junctions, and the limits they impose on the sensitivity of dc and rf SQUIDS. The results are strictly valid only for a resistively shunted junction (RSJ) with zero capacitance, but should be applicable to point contact junctions and microbridges in so far as these devices can be approximated by the RSJ model. We first discuss fluctuations arising from Nyquist noise in the resistive shunt of a single junction in the limit $eI_oR/k_BT \ll 1$ in which a classical treatment is appropriate, and then extend the treatment to the limit $eI_oR/k_BT \gtrsim 1$ in which quantum effects become important.

The Nyquist limit theory is used to calculate the noise in a dc SQUID, and the results are compared with a number of practical devices. The quantum limit is briefly considered. Results for the predicted sensitivity of rf SQUIDS are presented, and also compared with a number of practical devices. Finally, the importance of 1/f noise (f is the frequency) in limiting the low frequency performance of SQUIDS is discussed.

II. Noise in the Resistively Shunted Junction

We consider a current-biased RSJ[1] in which current fluctuations are produced by the equilibrium noise in the shunt resistor.

In the classical limit $eI_oR/k_BT<<1$ this problem was solved by Likharev and Semenov[2] and Vystavkin et al.[3] We consider only the limit $V>>2\pi k_BTR/\phi_o$ (R is the shunt resistance and ϕ_o is the flux quantum) in which noise rounding of the I-V characteristic is negligible[4], and

$$V = R(I^2-I_o^2)^{1/2}, \tag{1}$$

where I_o is the critical current. We assume $V<2\Delta/e$ (Δ is the energy gap) so that we may neglect the Ridiel singularity[5], and $T<<T_c$ (the transition temperature) so that the quasiparticle tunneling current is negligible compared with the current in the shunt resistance. The phase difference, $\delta(t)$, across the junction evolves with time as

$$(\hbar/2eR)\dot{\delta}(t) = I-I_o\sin\delta(t) + I_N(t), \tag{2}$$

where $I_N(t)$ is the fluctuating current produced by the shunt resistor. We now consider solutions for the spectral density of the voltage noise.

A. Classical limit

In the classical limit, the spectral density of the current fluctuations in the shunt resistor at angular frequency ω is

$$S_I(\omega) = 2k_BT/\pi R. \tag{3}$$

Ambegoakar and Halperin[4] computed the average voltage from Eq. (2) using Eq. (3) as the spectral density of $I_N(t)$, and found that the I-V characteristic was rounded at low voltages. This rounding arises from the thermal activation of the junction into a non-zero voltage state at currents below I_o. In the limit in which departures from the noise-free solution are small, Eq. (2) has been solved[2,3] using Eq. (3) as the spectral density of $I_N(t)$. At angular frequencies $\Omega<<\omega = 2eV/\hbar$ the spectral density of the voltage fluctuations is white and has magnitude

$$S_v(\Omega) = \left[1 + \frac{1}{2}\left(\frac{I_o}{I}\right)^2\right] \frac{2k_BTR_D^2}{\pi R}, \tag{4}$$

where $R_D = (\partial V/\partial I)_{I_o}$ is the dynamic resistance of the junction. The first term, $2k_BTR_D^2/\pi R$, represents the spectral density of the voltage fluctuations across the resistance R_D produced by a Johnson noise current at angular frequency Ω, while the second term, $(I_o/I)^2\ k_BTR_D^2/\pi R$, represents noise at higher frequencies mixed down by the non-linear device with a "mixing coefficient" $(I_o/I)^2/2$. The second contribution vanishes in the limit $I>>I_o$, and one recovers the Nyquist result.

B. Quantum limit

In general one should use the full expression for the spectral density of the Johnson noise

$$S_I(\omega) = (\hbar\omega/\pi R)\ \coth\ (\hbar\omega/2k_BT). \tag{5}$$

In the limit $k_BT>>\hbar\omega$, Eq. (5) reduces to Eq. (3), while in the limit $k_BT<<\hbar\omega$, $S_I(\omega)\rightarrow\hbar\omega/\pi R$, the spectral density of the zero point fluctuations in the resistor. Koch et al.[6] have used the general approach of Likharev and Semenov[2] to solve Eq. (2) using Eq. (5) as the spectral density of $I_N(t)$, and find

$$S_v(\Omega) = \left[\frac{2k_BT}{\pi R} + \left(\frac{I_o}{I}\right)^2 \frac{eV}{\pi R}\ \coth\ \left(\frac{eV}{k_BT}\right)\right] R_D^2. \tag{6}$$

The first term is the same as in Eq. (4), but the second contains quantum corrections. It is instructive to consider three limiting cases, all with $\Omega<<2eV/h$:

(i) $eV<<k_BT$: We recover Eq. (4).

(ii) $2k_BT(I/I_o)^2>>eV>>k_BT$ or $eV>>(eI_oR)^2/2k_BT$: We recover the usual Nyquist result $S_v(\Omega) = 2k_BT/\pi R$.

(iii) $eV>>k_BT$, $2(I/I_o)^2k_BT$, or $1<<eV/k_BT<<(eI_oR/k_BT)^2/2$: We obtain the quantum noise limit

$$S_V(\Omega) = eV(I_o/I)^2R_D^2/\pi R = eI_o{}^2R^2/\pi(I^2-I_o{}^2)^{1/2}. \quad (7)$$

Note that this limit can be obtained only if

$$\kappa \equiv eI_oR/k_BT>>1. \quad (8)$$

Although Eq. (7) is true only for[1] $\beta_c = 2\pi I_oR^2C/\phi_o = 0$ (C is the junction capacitance), it is necessary to satisfy Eq. (8) to obtain the quantum limit in the more general non-hysteric case $\beta_c \lesssim 1$. Thus, Eq. (8) can be rewritten as

$$\kappa \equiv (e/k_BT)(\beta_c\phi_oj_1/2\pi c)^{1/2}>>1, \quad (9)$$

where j_1 is the critical current density and c is the capacitance per unit area. Thus, one obtains the quantum limit by working with sufficiently high critical current densities and/or low temperatures.

At the characteristic voltage $V=I_oR$ in the limit $\kappa>>1$, we obtain

$$S_V(\Omega)|_{V=I_oR} = \frac{eI_oR^2}{\pi}. \quad (10)$$

The quantity eI_o/π is just the spectral density of the shot noise in a current I_o. However, it should be apparent that the noise arises from the zero point fluctuations in the shunt resistor. There is no intrinsic shot noise in the zero voltage pair current.

III. Noise in the dc SQUID

A. Model calculation

Tesche and Clarke[7,8] studied the effects of Nyquist noise in the resistive shunts of the two junctions of a dc SQUID, and computed the I-V characteristics and noise spectral densities for $\beta_c=0$. To compare their results with real SQUIDS with non-zero capacitance they used the maximum value of resistance for non-hysteric behavior, $R = (\phi_o/2\pi I_o C)^{1/2}$.' The introduction of a non-zero capacitance modifies both the I-V characteristic and the level of the voltage noise, but the overall error is believed to be no more than a factor of 2. Optimum performance is obtained for $\beta = 2LI_o/\phi_o = 1$, where L is the SQUID inductance. Most of the calculations were for a SQUID in the He^4 range with L=1nH and I_o=1μA. These choices fix the noise parameter $\Gamma = 2\pi k_B T/I_o\phi_o$ to be about 0.05 at 1.2K. The parameter Γ determines the noise rounding of the I-V characteristic as well as the magnitude of the noise, and its value can drastically affect the value of the transfer function $V_\phi \equiv (\partial V/\partial\phi)_I$. Thus, predictions for a given value of Γ are not necessarily immediately applicable to a device with a very different value of Γ.

Figure 1(a) shows V_ϕ vs. I/I_o for three values of average flux in the SQUID. The SQUID must be operated at or near the peak in V_ϕ to obtain optimum performance. For L=1nH, I_o=1μA, and Γ=0.05, to a good approximation the optimum value of V_ϕ at $\phi \approx (2n+1)\phi_o/4$ is[7]

$$V_\phi \approx \frac{R}{L} . \tag{11}$$

Figure 1(b) shows the spectral density, $S_v(0)$, of the voltage noise across the SQUID at low frequencies (<<2eV/h); S_v is white, proportional to Γ, and modulated by the average flux. S_v has peaks at the same bias current as the peaks in V_ϕ. For the values of L, I_o, Γ, and ϕ given above, the peak has the value

$$S_v \approx 16k_B TR. \tag{12}$$

Figure 1(c) shows the equivalent flux noise $S_\phi^{1/2} = S_v^{1/2}/V_\phi$. For the same values of L, I_o, Γ, and ϕ,

$$S_\phi \approx 16k_B TL^2/R. \tag{13}$$

To compare the intrinsic sensitivities of different SQUIDS, we define a noise energy

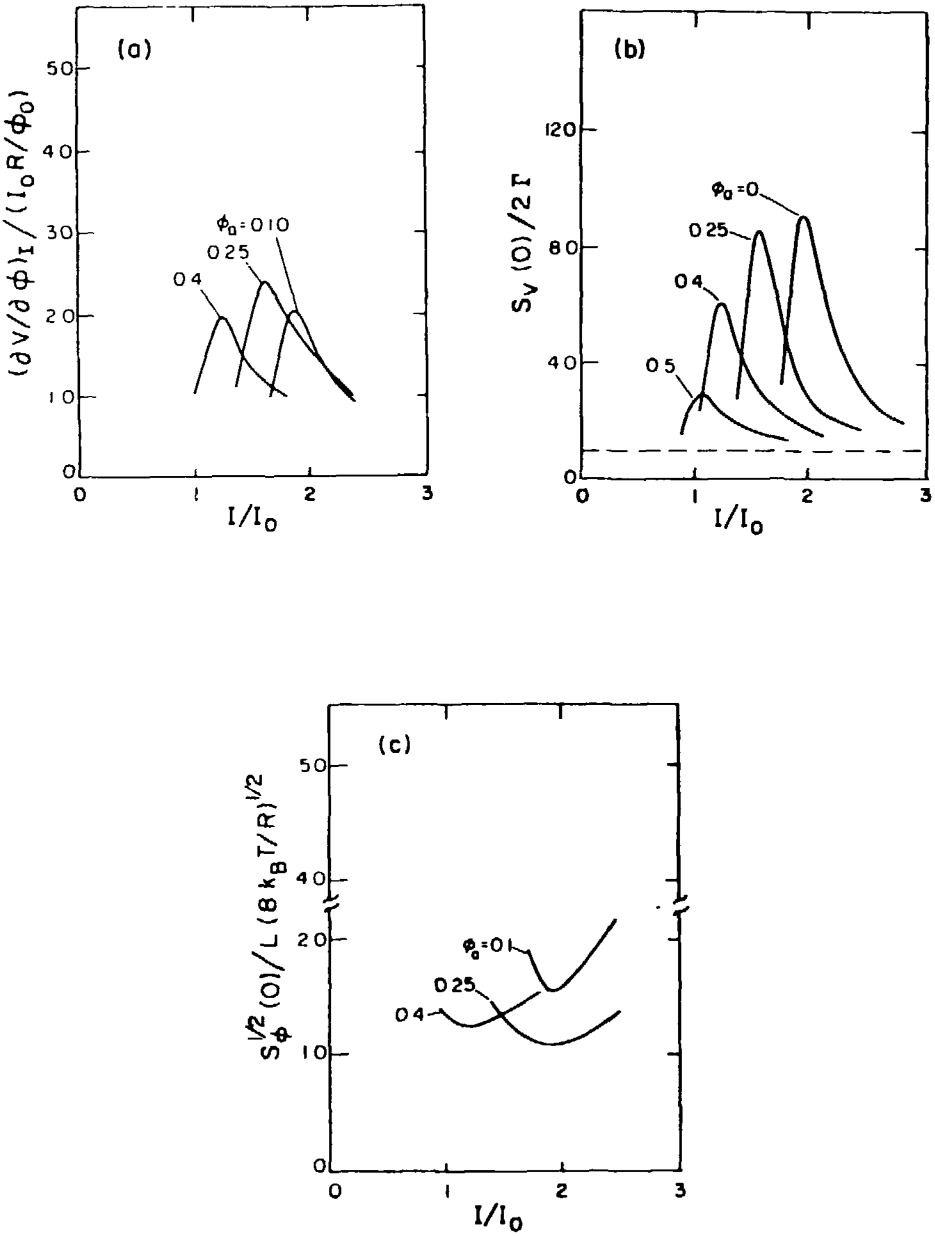

Fig. 1. (a) Computed transfer functions, (b) spectral density of voltage noise, and (c) rms flux noise per $Hz^{1/2}$ vs. I/I_o for a dc SQUID with $2LI_o/\Phi_o = 1$ and $\Gamma = 2\pi k_B T/I_o\Phi_o = 0.05$.

$$\frac{\varepsilon}{1\text{Hz}} = \frac{S_\Phi}{2L} \approx \frac{8k_BT}{R/L} \approx 8k_BT(\pi LC)^{1/2}, \tag{14}$$

where we have used $\beta=\beta_c=1$ to obtain the last expression. Equation (14) demonstrates that one can improve the sensitivity of the dc SQUID by reducing L, C, and/or T.

It is important to realize that when the SQUID is coupled to an input circuit to make a voltmeter or magnetometer, the equivalent flux noise may not be sufficient to characterize the sensitivity. There is a second noise source, namely the circulating noise current in the SQUID[8] that produces a real flux noise in the device. This flux noise produces, in turn, a real voltage noise in the input circuit. Thus, the input circuit should be characterized by two noise sources, a current noise that arises from the voltage noise across the SQUID, and a voltage noise that arises from the current noise in the SQUID loop. The circulating current noise in the SQUID produces a component of the voltage noise across it provided $V_\Phi \neq 0$; this contribution is already contained in S_V. Thus, the current and noise sources in the input circuit are partially correlated. A detailed account has been given of the optimization of various input circuits taking these effects into account.[9]

The SQUID is usually operated with an ac flux of (say) 100kHz, and the resulting ac voltage is amplified by a cooled tank circuit or transformer. When the circuitry is optimized, the noise temperature of the room temperature preamplifier can be less than 1K, so that the noise contribution of the preamplifier is negligible for a SQUID operated in the He^4 range.

B. Performance of five dc SQUIDS

Table I lists the sensitivity of five thin film dc SQUIDS with which the author is familiar. This list is not meant to be exhaustive, but is intended to show improvements that have been made over the 1974 cylindrical dc SQUID. The first device was flux-modulated and operated in a feedback loop, while the others were not modulated, and the noise was measured open loop at the optimum flux and current bias. The noise energy of the more sensitive devices is also given in units of Planck's constant, h.

The agreement between the measured and predicted performance is generally quite good. SQUIDS (i)-(iii) illustrate how the sensitivity improves approximately as $C^{1/2}$. The temperature dependence of $S_\phi/2L$ for (iii) scales approximately as T. The fourth device illustrates that a two-order of magnitude reduction in the inductance produces approximately the predicted improvement in $S_\phi/2L$. However, the sensitivity did not improve further as the temperature was lowered. The last device, fabricated from microbridges, has the highest performance reported so far, again in reasonable agreement with the predicted value.

We conclude that Eq. (14) adequately predicts the sensitivity over a wide range of parameters. Detailed deviations from the model certainly occur, for example, due to self-resonant modes in the junctions or in the SQUID loop. However, although at biases near a self-induced step the dynamic resistance, R_D, may be substantially higher than the RSJ model would estimate, since both S_v and V_ϕ^2 tend to scale as R_D^2, S_ϕ is still in reasonable agreement with the model.

C. Quantum limit

Koch *et al.*[6] have performed preliminary numerical calculations of the limiting sensitivity set by zero point fluctuations under optimum bias conditions, and find $\varepsilon/1\mathrm{Hz} \approx \hbar$. Further work is in progress.

IV. Noise in the rf SQUID

A. Model calculation

Kurkijärvi[15] and Kurkijärvi and Webb[16] first calculated the intrinsic noise in the rf SQUID, which arises from the thermally induced uncertainty in the value of flux at which the SQUID makes transitions between adjacent quantum states. They found the spectral density of the intrinsic equivalent flux noise to be

$$S_\phi^{(i)} \approx 1.3 \frac{(LI_o)^2}{\omega_{rf}} \left(\frac{2\pi k_B T}{I_o \Phi_o} \right)^{4/3} , \tag{15}$$

provided the rf frequency, $\omega_{rf}/2\pi$, is less than $10^7(R/1\Omega)$Hz. The noise also tilts the steps in the rf current-voltage characteristic measured across the tank circuit coupled to the SQUID. If one defines α as the ratio of the voltage increase along a step to the voltage separation between successive steps, it can be shown that[17-19]

$$\alpha^2 = \omega_{rf} S_\phi^{(i)} / \pi \phi_o^2 . \tag{16}$$

Although $S_\phi^{(i)}$ may be substantially higher[17] than the value predicted by Eq. (15), if one <u>measures</u> α, $S_\phi^{(i)}$ is accurately predicted by Eq. (16). In addition, the noise of the preamplifier and tank circuit are usually by no means negligible, particularly if the preamplifier is at room temperature so that the resonant circuit includes dissipative elements well above the bath temperature. The contribution to the noise energy is[7,18,20,21]

$$\varepsilon^{(a)}/1\mathrm{Hz} = 2\pi\alpha k_B T_a^{(eff)}/\omega_{rf}, \tag{17}$$

where $T_a^{(eff)}$ is the effective noise temperature of the preamplifier and tank circuit. Adding $\varepsilon^{(i)}/1\mathrm{Hz} = S_\phi^{(i)}/2L$ to $\varepsilon^{(a)}/1\mathrm{Hz}$ we find

$$\frac{\varepsilon}{1\mathrm{Hz}} = \frac{1}{\omega_{rf}} \left(\frac{\pi\alpha^2 \phi_o^2}{2L} + 2\pi\alpha k_B T_a^{(eff)} \right) . \tag{18}$$

Equation (18) demonstrates that for fixed α and $T_a^{(eff)}$ the noise energy scales as $1/\omega_{rf}$. However, in practice, $T_a^{(eff)}$ tends to increase as ω_{rf} is increased. In general, the relative importance of the two terms depends largely on whether the preamplifier is at room temperature or the bath temperature. Not only does the preamplifier noise temperature tend to decrease when the preamplifier is cooled, but, in addition, the contribution of the tank circuit becomes insignificant.

As in the case of the dc SQUID, when an input coil is coupled to the rf SQUID, a more detailed noise analysis is necessary to characterize the circuit.[18,19,22,23,24] Amplifier and tank circuit current noise are downconverted by the SQUID to induce a real flux noise in the SQUID which in turn produces a voltage noise source in the input circuit. Thus, there are current and voltage noise sources in the input circuit, just as for the dc SQUID, although they are usually assumed to be uncorrelated.

B. Performance of rf SQUIDS

Table II lists the performance of six rf SQUIDS; again this list is intended to be representative rather than exhaustive. It should be noted that the noise energy has been divided by κ^2, where κ is the coupling coefficient between the SQUID and the input circuit. The noise energy is thus referred to the input circuit, and is a much more useful characterization of the noise for most practical purposes.

The noise energy of SQUIDS (i), (iii) and (iv) is in good agreement with the predictions of the model. The advantage of using a cooled preamplifier is clearly demonstrated. When (iii) was cooled to $\sim$0.1K, the intrinsic noise became negligible, and the device was limited by preamplifier noise. The sensitivity of SQUID (v), on the other hand, seems to be substantially higher than predicted. It should be noted, however, that the value of α listed was not given in ref. 27, but was taken from an earlier paper on the same SQUID by the same group. The reasons for this discrepancy are not clear. Finally, devices (ii) and (vi) were operated at X-band. At these frequencies, the model is certainly invalid, and one cannot make any comparisons between theory and experiment, but it seems unlikely that one can improve significantly on the performance of the X-band device of Hollenhorst and Giffard.[28]

It is apparent that by using higher rf frequencies and/or cooled amplifiers one can improve substantially on the noise energy of the 20MHz toroidal SQUID (i). At frequencies around 400MHz, the use of cooled GaAs-FETS is relatively straightforward and inexpensive. One does gain sensitivity by going to 10GHz, but the system becomes considerably more complicated and expensive. It is possible that one could improve on the sensitivityof the 400MHz SQUIDS without a great increase in cost by working at (say) 3GHz, at which frequency one could still hope to use a cooled GaAs-FET preamplifier. However, to remain in the range of validity of the model, one would require a junction resistance >300Ω, implying that it would probably be necessary to use a tunnel junction.

V. 1/f Noise

Relatively little attention has been paid to 1/f noise in the devices developed recently. In fact, 1/f noise may become a serious limitation on the low frequency performance of SQUIDS.

Clarke and Hawkins[29] measured the voltage noise in current-biased resistively shunted tunnel junctions and found a 1/f spectral density at frequencies between about 0.1Hz and a few 10's of Hz. The noise at lower frequencies was not measured. The spectral density was proportional to $(\partial V/\partial I_o)_I^2$, indicating that the noise involved 1/f fluctuations in I_o, and to $(dI_o/dT)^2$, suggesting that the critical current fluctuations arose from temperature fluctuations. With these assumptions, one finds the spectral densities for the voltage fluctuations to be given by[29]

$$S_V^{(1/f)} \sim \frac{k_B T^2}{3Ac_v f}\left(\frac{dI_o}{dT}\right)^2 \left(\frac{\partial V}{\partial I_o}\right)_I^2 , \tag{19}$$

where A is the junction area, and c_v is the heat capacity of a unit area of junction with a thickness equal to the sum of the coherence lengths of the two superconductors. Equation (19) is not likely to be valid down to very low frequencies because the spectral density of the temperature fluctuations must flatten out as the frequency is lowered. However, this expression should give at least a rough lower bound of the noise at frequencies above (say) 0.1Hz. In a dc SQUID, this noise source will produce both a voltage and a circulating current noise; however, the contribution of the latter (with $V_\phi \neq 0$) to the overall voltage noise is relatively small, and we shall neglect it. We can adapt Eq. (19) for a dc SQUID by dividing by 2 (for the two junctions), dividing by V_ϕ^2 to convert to an equivalent flux noise spectral density, and by noting that $(\partial V/\partial I_o)/V_\phi \approx 2L$. Setting $2LI_o = \Phi_o$, we obtain

$$\frac{\varepsilon^{1/f}}{1\text{Hz}} \sim \frac{k_B T^2 \Phi_o^2}{12 w^2 c_v L f}\left(\frac{dI_o}{I_o dT}\right)^2 , \tag{20}$$

where w is the width of the junction, which is assumed to be square. At 4.2K, taking $dI_o/I_o dT \sim 0.3\text{K}^{-1}$ and $c_v \sim 8\times10^{-16}\text{JK}^{-1}\mu\text{m}^{-2}$ as reasonable estimates, we find

$$\frac{\varepsilon^{1/f}}{1\text{Hz}} \sim \frac{10^{-29}\text{JHz}^{-1}}{\left(\frac{w}{\mu\text{m}}\right)^2\left(\frac{L}{\text{nH}}\right)\left(\frac{f}{1\text{Hz}}\right)} . \tag{21}$$

From Eq. (14), with $c = 0.04\text{pF}\mu\text{m}^{-2}$, the white noise energy level at 4.2K is

$$\frac{\varepsilon^{w}}{1\text{Hz}} \approx 6\times10^{-33}\left(\frac{w}{\mu\text{m}}\right)\left(\frac{L}{1\text{nH}}\right)^{1/2}\text{JHz}^{-1} . \tag{22}$$

Thus, we see that as we reduce LC, ε^{w} scales as $(LC)^{1/2}$ while $\varepsilon^{1/f}$ scales as $(LC)^{-1}$, so that, according to this model, 1/f noise will dominate out to progressively higher frequencies. This result is illustrated in Fig. 2 where the predictions of Eqs. (21) and (22) are plotted for L=1nH and w=100, 10 and 1μm. It should be noted that the model seriously underestimates the 1/f noise in the cylindrical dc SQUID[10] (indicated by the dashed lines) to which the parameters L=1nH and w=100μm correspond. It has been speculated[10] that the motion of flux trapped in the SQUID might contribute to 1/f noise. However, preliminary measurements by Koch *et al.*[11] on SQUIDS with L≈1nH and 100μm^2 junctions, and by Ketchen and Tsuei[30] on a SQUID with L≈10pH

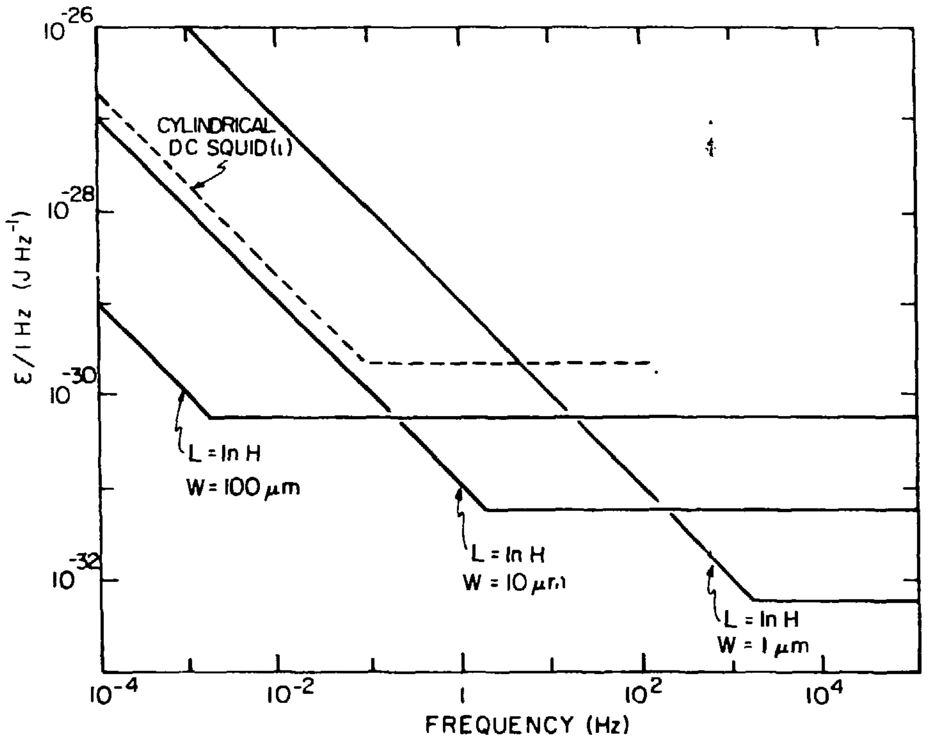

Fig. 2. Spectral densities of 1/f and white noise according to Eqs. (21) and (22) for dc SQUIDS with L=1nH and junction widths w=100, 10 and 1μm (solid lines). The dashed line shows the measured noise in a cylindrical dc SQUID (i).

and $10\mu m^2$ junctions are in qualitative agreement with the general predictions of Fig. 2.

Thus, it seems likely that as the white noise levels of dc SQUIDS are improved by reducing L and/or C, the 1/f noise will inevitably be increased. If so, the optimum sensitivity at, for example, 1Hz may not necessarily be obtained by minimizing LC, but rather by adjusting LC to trade off the white and 1/f noise contributions optimally. One should also bear in mind that the 1/f noise will be reduced if one can operate the SQUID at a low enough temperature that dI_o/dT becomes vanishingly small; however, it should also be remembered that $c_v \propto T^3$. To my knowledge, there have been no measurements of 1/f noise in rf SQUIDS as a function of LC, but 1/f noise in the critical current of the junction will certainly lead to 1/f noise in the equivalent flux noise. Furthermore, it is not known whether or not 1/f noise in point contacts or microbridges is due to temperature fluctuations. Further work is urgently needed to resolve these questions.

VI. Summary

The intrinsic sensitivity of dc SQUIDS has improved from $\sim 2\times10^{-30}$ JHz^{-1} for the cylindrical tunnel junction device (i) to $\sim 2\times10^{-33}$ JHz^{-1} for the planar microbridge device (v). Generally speaking, the performance is in reasonable agreement with model predictions. A SQUID with a sensitivity at or close to the quantum limit ($\varepsilon/1Hz \sim \hbar$) may well emerge within a year. However, the SQUIDS must be efficiently coupled to an input coil to take advantage of the great improvements of sensitivity, and, for most purposes, also operated in a feedback loop. Both requirements require substantial further work.

The sensitivity of rf SQUIDS has also improved over the past several years, by two orders of magnitude compared with the 20MHz toroidal device (i) but by only a factor of about 5 compared with the 1974 X-band SQUID (ii). These improvements have been achieved by using higher frequencies and/or cooled preamplifiers. Further improvements may be possible by cooling the SQUID below 1K or by choosing frequencies around (say) 3GHz. Nevertheless, my general impression is that substantial further improvements will be both difficult and expensive.

The question of 1/f noise has been largely overlooked in recent work, but it appears almost inevitable that as the white noise level of SQUIDS is reduced, 1/f noise will dominate up to higher frequencies. Further study of 1/f noise in tunnel junctions, microbridges and point contacts, as well as in SQUIDS, is urgently needed.

Finally, to keep matters in perspective, one should realize that the highest possible sensitivity is necessary only for a very few exotic applications, for example, gravity wave antennas. For virtually all practical applications a device with a noise energy of (say) $10^{-31} JHz^{-1}$, efficiently coupled to an input circuit and operated in a flux-locked loop, is likely to be more than adequate. Questions of dynamic range, slewing rate, and stability of the flux-locked loop, which are discussed by Professor Giffard elsewhere in these proceedings, as well as of long term reliability and straightforward operations of the device, may then be of greater concern than higher sensitivity.

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Table I. Performance of five thin film dc SQUIDS

Type	Junction Area (μm^2)	R (Ω)	L (nH)	T (K)	$S_\phi/2L$ Calculated ($10^{-30} JHz^{-1}$)	$S_\phi/2L$ Measured ($10^{-30} JHz^{-1}$)
(i) tunnel junction cylindrical[10]	10^4	0.8	1	4.2	0.6	2
(ii) tunnel junction planar[11]	10^2	7	1.2	4.2	0.08	0.18
(iii) tunnel junction planar[12]	1	30	1	4.2	0.015	0.025(37h)
				1.6	0.006	0.011(17h)
(iv) tunnel junction planar[13]	10	2	0.0115	4.2	0.0027	0.003(5h)
				1.8	0.0011	0.003(5h)
(v) micro-bridge planar[14]	-	40	0.1	4.2	0.0012	0.002(3h)

Table II. Performance of six rf SQUIDS

Type	Amplifier	$T_a^{(eff)}$ (K)	Freq. (MHz)	L (nH)	κ^2	α	$S_\phi/2L$ Calculated (10^{-30} JHz^{-1})	$S_\phi/2L$ Measured (10^{-30} JHz^{-1})
(i) point contact toroidal[25]	room temp. FET	60	20	0.8	0.3	0.33	70	50
(ii) thin-film microbridge cylindrical[26]	room temp. parametric	200	10^4					2
(iii) thin-film tunnel junction planar[24]	cooled GaAs-FET	30	60	0.25	0.09	0.2	50	50
(iv) point contact toroidal[24]	cooled GaAs-FET	40	200	0.4	0.25	0.3	8	6
		70	400	0.4	0.25	0.3	5	5
(v) point contact 2-Hole[27]	cooled GaAs-varactor diode	5.5	430	0.5	0.16	0.5	8	0.4
(vi) point contact re-entrant toroidal[28]	room temp. heterodyne	500	9000	0.3	0.5			0.7