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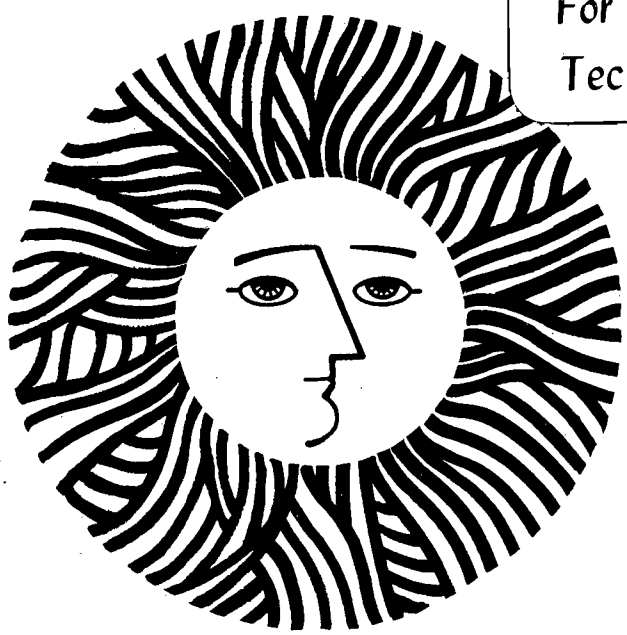

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ACCURACY OF THE EDGEWORTH EXPANSION FOR LOLP CALCULATIONS IN SMALL POWER SYSTEMS*

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Abstract - Because they speed numerical calculations and exhibit functional dependences, Edgeworth-type series are increasingly used to approximate and calculate LOLP's. They are usually sufficiently accurate for large power systems (>15 000 MW), but can be very inaccurate for small systems or those with low forced outage rates. This is because the approximating Edgeworth-type series are appropriate only for continuous probability densities, while discrete lattice-type density functions describe a typical power system's probability properties. This paper investigates these inaccuracies for small systems (≤5000 MW), working out specific examples in a numerical approach and examining underlying functional dependences in an analytic approach.

INTRODUCTION

Numerical methods have historically been used to calculate reliability indices like loss-of-load probability (LOLP) for electric power systems [1]. For many purposes, however, numerical methods are a cumbersome means of computation. In capacity expansion models, for example, 20-year planning horizons are evaluated over a wide range of input assumptions. To reduce computational complexity and add conceptual perspective, several authors have studied analytic methods for approximating the numerical calculations. The most successful of these methods is an expansion based on the normal probability distribution, its moments and derivatives (see [2] to [6]). There are several variations on this approach, depending on the exact form of expansion used. Individual forms of the series are associated with Edgeworth and Gram-Charlier [7].

The purpose of this paper is to make a systematic investigation of the accuracy of these methods with particular attention to power systems of 5000 MW and less. Two approaches will be described: (1) a numerical approach where specific examples are worked out, and (2) an analytic approach where underlying functional dependences are examined.

The most successful applications of the Edgeworth expansion and related approaches has been to large systems, typically in excess of 15 000 MW capacity. Even for systems of this size, if the average forced outage rate (FOR) is very low, the series can have disturbing properties and limited accuracy [8]. Since the method relies in part on central-limit theorem arguments, it can be expected to work well only when the number of random variables, i.e., generators, is large. Hence, there is reason to believe that the Edgeworth expansion will be less useful to approximate small systems than large ones.

Edgeworth-type expansions are usually appropriate when the underlying random variables take on continuous values [9]. This condition is violated in the case of most LOLP calculations where the generators' output characteristics are represented by probability distributions that limit the variables to discrete values only. Usually units are assumed to be either on or off in the two-state model. Where partial outages are considered, the standard model is again discrete rather than continuous. The underlying discrete probability distributions used to describe generators give rise to an aggregate probability distribution that is not continuous and that is referred to as a lattice-type probability distribution. In the limit of an infinite number of generators ($N \to \infty$), lattice as well as continuous probability distributions can be accurately described with the central-limit theorem [9,10]. In practice, the number of generators (N), is finite so corrections to the central-limit theorem's normal distribution need to be considered. To calculate these corrections, most authors [2-6] have used an Edgeworth series even though, as we have remarked, such expansions are inappropriate for lattice distributions and simple criteria for their accuracy are lacking.

There are several reasons for focusing attention on the problems analytic methods pose for small power systems. Recent investigations of the accuracy question have found good results for systems of 10 000 MW and more [11]. Schenk, however, found very disturbing results for some forms of the Gram-Charlier series applied to a small (1800 MW) system with an average forced-outage rate of around 3% [12]. A more practical reason lies in the near-term market for solar electric technologies. This market is likely to be largest in the southwestern part of the United States because of climate (the solar resources are good) and fuel dependence (there is much oil to be displaced). Excess capacity is less likely to be a market restraint in this region, and most utilities in the southwest are relatively small. Moreover, there are few institutionalized pooling arrangements to perform an aggregating function for reliability planning in this region compared to the eastern part of the country. In addition, adequate modeling of solar electric technologies requires representing the available capacity distributions in much detail [13]. Finally, the Edgeworth series is more computationally efficient than numerical techniques where multi-state generator models are used [5]. With these constraints and opportunitites in mind, we conducted a study of the Edgeworth series as an approximation of the LOLP of certain utilities in Texas, Arizona and Oklahoma.

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FORMULATION

To describe the case studies in detail and to lay the foundation for deeper analysis, it is necessary to describe the particular version of the Edgeworth expansion that we used. Our notation primarily follows Abramovitz and Stegun [14].

We are looking for a way to compute the instantaneous probability that available capacity on a power system is less than the load it must meet. This is the loss-of-load probability (LOLP).

We define the random variable, S, as the total available capacity for the system of interest.

In terms of each generator's output x_i,

$$S = \sum_{i=1}^{N} x_i \quad , \tag{1}$$

where N is number of generators.

We denote the probability density function for S as P(S), where

$$P(S) = \int \delta\left(S - \sum_{i=1}^{N} x_i\right) \prod_{i=1}^{N} P_i(x_i)\,dx_i \quad , \tag{2}$$

and the $P_i(x_i)$ are the probability density functions for each generator. If W denotes the load, then:

$$\text{LOLP} = \int_0^W P(S)\,dS \tag{3}$$

is the required distribution function. We write the LOLP as a function of a standard variable:

$$x = \frac{S - \bar{S}}{\sigma^2} \quad , \tag{4}$$

with

$$\bar{S} = \int_0^\infty S P(S)\,dS \quad ,$$

and

$$\sigma^2 = \int_0^\infty (S - \bar{S})^2 P(S)\,dS \quad . \tag{5}$$

The Edgeworth approximation may be written as [14]:

$$\begin{aligned}\text{LOLP}(x) = P(x) &- \overset{1}{\left[\frac{\gamma_1}{6} Z^2(x)\right]} + \overset{2}{\left[\frac{\gamma_2}{24} Z^3(x) + \frac{\gamma_1^2}{72} Z^5(x)\right]} \\ &+ \overset{3}{\left[\frac{\gamma_3}{120} Z^4(x) + \frac{\gamma_1\gamma_2}{144} Z^6(x) + \frac{\gamma_1^3}{1296} Z^8(x)\right]} \\ &+ \overset{4}{\left[\frac{\gamma_4}{720} Z^5(x) + \frac{\gamma_2^2}{1152} Z^2(x) + \frac{\gamma_1\gamma_3}{720} Z^7(x)\right.} \\ &+ \left.\frac{\gamma_1\gamma_2}{1728} Z^9(x) + \frac{\gamma_1^4}{31104} Z^{11}(x)\right] \quad ,\end{aligned} \tag{6}$$

where

$$P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-(w^2/2)}\,dw \quad ,$$

$$Z^n(x) = \frac{e^{(-x^2)/2}}{\sqrt{2\pi}} (-1)^n He_n(x) \quad ,$$

$$\gamma_{r-2} = \frac{\sum_{i=1}^{N} K_{r,i}}{\left[\sum_{i=1}^{N} \sigma_i^2\right]^{r/2}} \quad . \tag{7}$$

Here, $He_n(x)$ are the standard Hermite polynomials [14], $K_{r,i}$ is the r'th order cumulants for the i'th generator, and σ_i^2 is the variance of the i'th generator with the relation

$$\sigma^2 = \sum_{i=1}^{N} \sigma_i^2 \quad . \tag{8}$$

In (6), P(x) is the term which would remain in the limit of an infinite number of generators. It is the central limit theorem's approximation term and is simply the complementary error function erfc(x).

Each bracketed correction term in (6) includes all terms which appear to a given order in the parameter $1/\sigma^2$. In particular, we refer to each bracketed group of terms as one term and note that each term is of order $[1/(\sigma^2)]^{n/2}$, where n = 1,2,3,4. If all N generators were identical, then each bracketed term would be of order $[1/(N\sigma_0^2)]^{n/2}$, with σ_0^2 the variance of one generator. In this case, terms go to zero as $N \to \infty$.

CASE STUDIES

For our case studies, we take generators represented by a two-state probability density. They have capacity x_i and a forced outage rate L_i. Their probability density is

$$P_i(x_i) = L_i\,\delta(x_i) + (1 - L_i)\,\delta(x_i - \alpha_i). \tag{9}$$

Table 1 gives the relevant means μ_i, variances σ_i^2, and cumulants $K_{r,i}$ for these density functions. For our numerical analysis we compare the exact LOLP calculated by computer to the approximation formula (6), in each case comparing the effects of including more of the bracketed terms in the Edgeworth expansion.

To lend concreteness to these calculations, we examine the generator mix of particular utilities, using data obtained from Department of Energy compilations [15]. Since forced-outage rate data for the systems studied are not readily available, we relied upon the EEI averages listed in Table 2. In Table 3 we list the units, by size, used in calculations for each company.

Figure 1 shows a typical result for the Arizona Public Service generator mix. The two-term Edgeworth expansion gives a very good approximation down to the region of roughly 5×10^{-4}. At this level of risk, the corresponding load is roughly 1700 MW. In terms of reserve margin such a peak load on a 2600 MW system would imply a reserve margin of more than 50%. Such a level is impractical.

TABLE 1. Formulas for relevant means μ_i, variances σ_i^2, and cumulants $K_{r,i}$ for density functions of (9).

$$\mu_i = \alpha_i(1 - L_i)$$

$$\sigma_i^2 = \alpha_i^2 L_i(1 - L_i)$$

$$\mu_{3,i} = \alpha_i^3 L_i(1-L_i) \{L_i^2 - (1-L_i)^2\}$$

$$\mu_{4,i} = \alpha_i^4 L_i(1-L_i) \{L_i^3 + (1-L_i)^3\}$$

$$\mu_{5,i} = \alpha_i^5 L_i(1-L_i) \{L_i^4 - (1-L_i)^4\}$$

$$\mu_{6,i} = \alpha_i^6 L_i(1-L_i) \{L_i^5 + (1-L_i)^5\}$$

$$K_{3,i} = \mu_{3,i}$$

$$K_{4,i} = \mu_{4,i} - 3\sigma_i^2 \cdot \sigma_i^2$$

$$K_{5,i} = \mu_{5,i} - 10\, \mu_{3,i} \cdot \sigma_i^2$$

$$K_{6,i} = \mu_{6,i} - 15\, \mu_{4,i} \cdot \sigma_i^2 - 10\mu_{3,i} \cdot \mu_{3,i} + 30\, \sigma_i^2 \cdot \sigma_i^2 \cdot \sigma_i^2$$

TABLE 2. Average full forced outage rates [15].

α_i	L_i
Nuclear	0.110
Fossil: 50 MW	0.023
200 MW	0.053
400 MW	0.095
600 MW	0.160
800+ MW	0.180
Combustion turbine	0.100

This illustrates a generic problem of using LOLP indices on smaller power systems. Many large pools set a risk criterion of "one day in ten years" for LOLP. Where this is an annual index, the corresponding peak risk will be typically on the order of 10^{-2}. Some versions of the "one day in ten years" criterion refer to a peak risk that is much lower, i.e., about 4×10^{-4}. Usually these criteria will yield reserve requirements in the 20-25% range for large systems. For smaller systems analyzed in isolation, very much higher reserves are needed at the LOLP levels usually adopted by the large systems.

In practical applications of LOLP methods to smaller systems, it is often assumed that interconnection support is available to reduce risk, i.e., LOLP, to appropriate or acceptable levels at reserve margins of 20-25% on the isolated small systems. One example of this approach is given in a Power Technologies Inc. study of the Northern States Power System [17]. Here a system is expanded from about 6600 MW to over 13 500 MW at an isolated risk criterion of six days per year, or about 2×10^{-2}. Thus acceptable accuracy for reserve margin analysis might focus on the LOLP region of 10^{-2} to 1. For other purposes, however, it might be desirable to have accuracy at lower levels of probability. In the analysis which follows we will focus on the region from LOLP = 10^{-3} to LOLP = 1.

TABLE 3. Power ratings (MW) of generators in four utility systems.

Arizona Public Service	Texas Electric	PS Oklahoma	El Paso
26	123	120	15
27	188	85	23
33	396	85	25
119	248	2	34
120	387	4	37
116	44	170	50
263	75	473	50
263	405	450	82
24	455	450	82
51	455	473	85
24	18	473	85
51	18	3	103
24	46	84	120
51	75	84	166
33	170	315	957
12	518	6	
63	20	33	
56	15	170	
56	81	95	
115	13	170	
115	13	8	
56	13	25	
56	13	25	
115	115	30	
99	536	67	
55	13	67	
55	13	67	
25	4466	4034	
22			
22			
57			
57			
21			
4			
1			
105			
105			
105			
2602			

Low FOR

Not every case turns out to be as well behaved as Fig. 1. Indeed, several different kinds of pathologies can appear using the Edgeworth expansion. Let us consider the effects of low forced outage rates. We illustrate the kind of outcome Schenk [11] depicts for very reliable systems in Figs. 2-4. These are based on the generator mix of Texas Electric (4466 MW total capacity). In these calculations, we fix all forced outage rates at 10%, 5%, and 2%. Figure 2, with FOR = 0.10, is quite similar to Fig. 1. The two-term approximation is good down below 5×10^{-4} and higher order terms add very little. In Fig. 3, with average FOR = 0.05, we see oscillations and pathologies with the fourth term in the series, while fairly reasonable accuracy is obtained for the two- and three-term expansions. But the series gets less stable and indeed the distribution has negative values in a small region around 3200 MW.

The possibility that the Edgeworth series can go negative has been noted in the literature [18]. This becomes a substantial problem in Fig. 4. Here, the

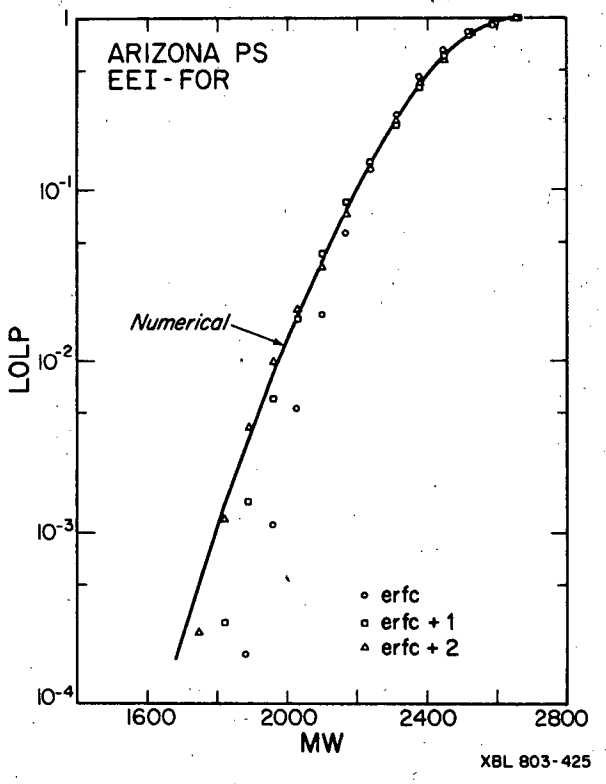

Fig. 1. LOLP plotted against generating capacity of the Arizona Public Service System (Edison Electric Institute forced outage rate data): Edgeworth expansion error functions compared to numerical results.

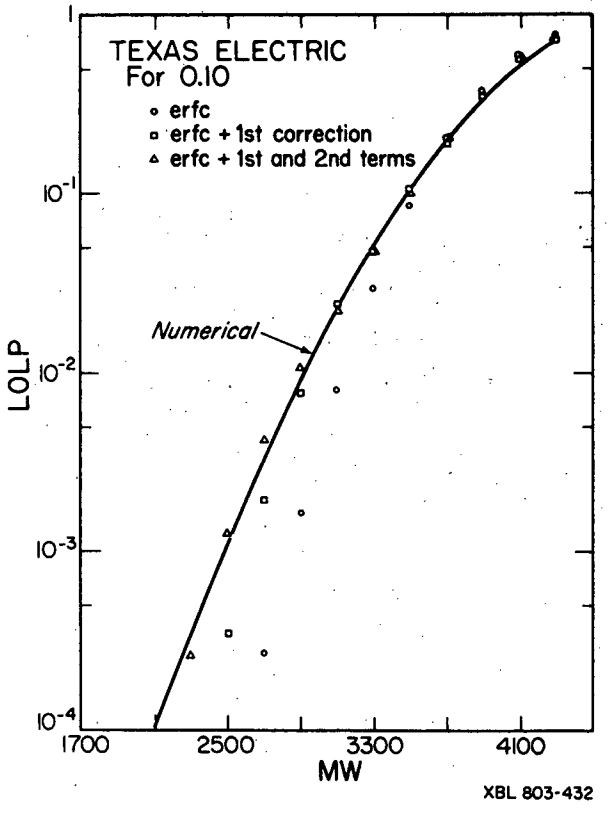

Fig. 2. LOLP plotted against generating capacity of Texas Electric System at FOR = 0.10. Edgeworth expansion error functions compared to numerical results.

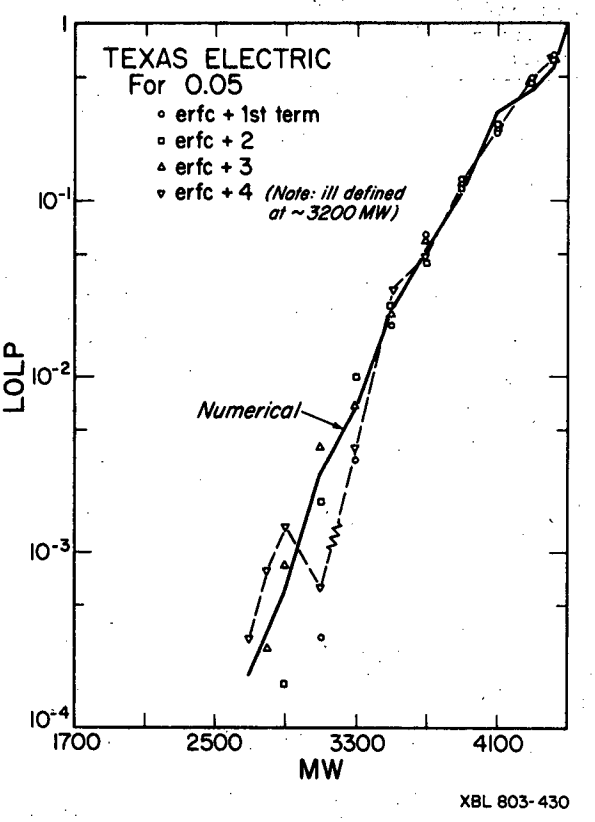

Fig. 3. Same as Fig. 2, except that FOR = 0.05.

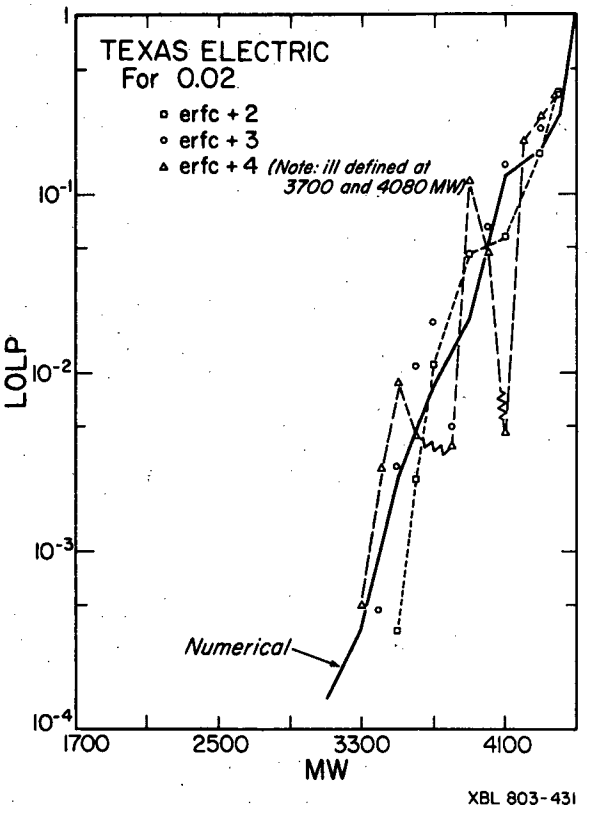

Fig. 4. Same as Fig. 2, except that FOR = 0.02.

average FOR = 0.02. The oscillations of the four-term series are more pronounced. The accuracy of the two- and three-term series is poorer than in Fig. 3. There is also a small region, not shown in Fig. 4, where the series goes negative with the third term. The diminishing accuracy in Figs. 2 - 4 is not simply a feature of the generator mix assumed. The same qualitative results obtained in sensitivity studies of the Arizona Public Service system (2602 MW total capacity).

A Very Small System

To verify the problems that can come up with very small systems, we examined El Paso Electric (957 MW capacity). Figure 5, based on EEI forced outage rates, shows the results. In this case, there is reasonable accuracy using the two-term series, but the higher order terms make matters worse instead of better. The three-term series shows a region of insensitivity at about 600-750 MW of load. In this region the approximation provides essentially no information. A negative distribution occurs in the four-term series at about 660 MW (not shown in Fig. 5).

Large-Unit Additions

The last series of studies performed focuses on the large-unit phenomenon. We consider Public Service Company of Oklahoma (4034 MW capacity). This company is planning a two-unit nuclear project, called Black Fox, which will add 2300 MW to its system, a 57% increase. Figure 6 examines the accuracy of the Edgeworth expansion before adding the large units. The story is the same as in Figs. 1 and 2. The two-term series does well, and nothing is gained or lost with more terms. Figure 7 shows the outcome with the addition of the first 1150 MW unit. Here the result resembles Fig. 5.

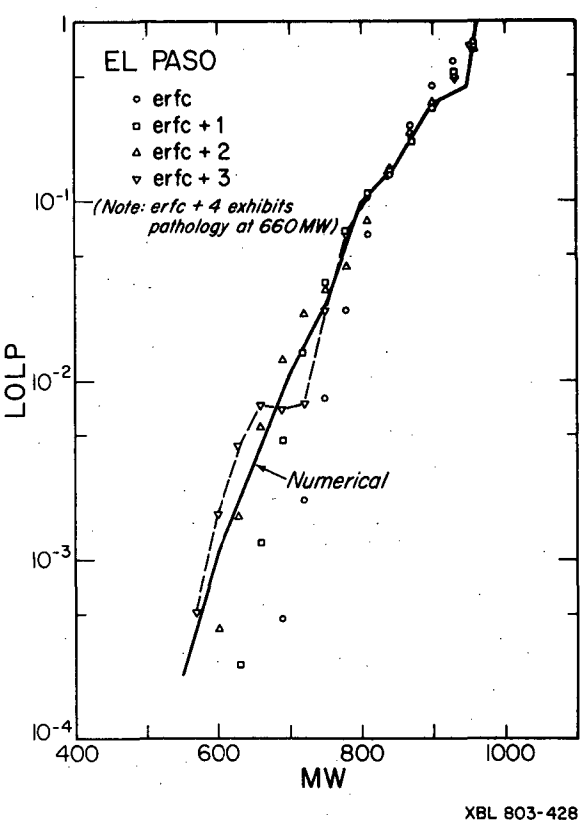

Fig. 5. LOLP plotted against capacity of El Paso Electric system (based on EEI forced outage rates); Edgeworth expansion error functions compared to numerical results.

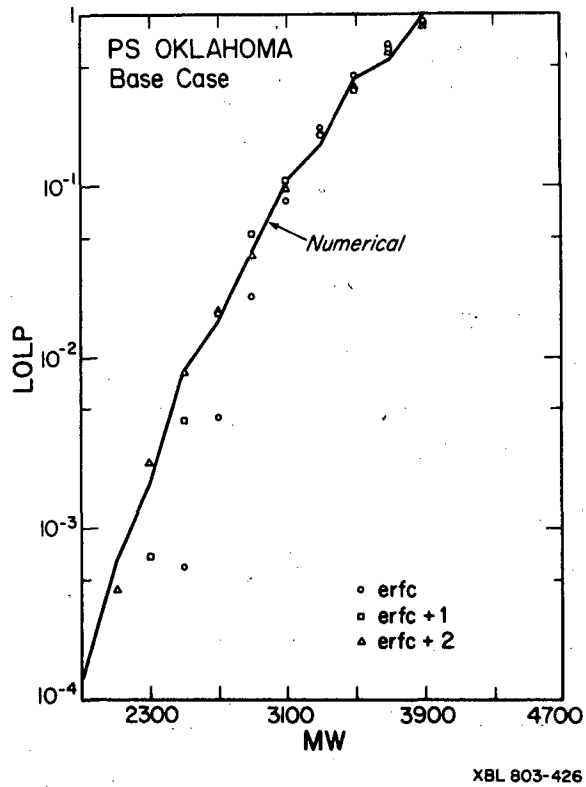

Fig. 6. LOLP vs. capacity, Public Service Company of Oklahoma (before addition of nuclear units); Edgeworth expansion error functions.

The two-term series works with tolerable accuracy, although less well than in Figs. 1, 2 and 6. The third term shows a substantial region of insensitivity, 600 MW or more at LOLP of 5×10^{-3}. The fourth term exhibits a negative distribution at loads that correspond to LOLP of less than 10^{-3}.

Finally, Fig. 8 shows the results with both large units added. Here we begin with LOLP = 5×10^{-3}. This corresponds to an 80% reserve margin at the corresponding load of about 3500 MW. The two- and three-term series perform acceptably over this region. The four-term series exhibits a negative value at loads that correspond to 100% reserve margin. In general, Fig. 8 shows better results than Fig. 7. The large units in this case are each about 18% of the total capacity. That is roughly the fraction represented by the largest unit in the El Paso Electric case. In Fig.7, the large unit represents about 22% of the total capacity. This probably accounts for the poorer performance of the Edgeworth series, since the underlying distribution is more "lumpy," i.e., less easily approximated continuously.

Analytic Results

We illuminate the above numerical examples with a simplified version of our analytic results. For low forced outage rates and a finite number of generators, we describe: (1) origins of negative LOLP's using Edgeworth-type approximations, (2) problems induced by including a generator much larger or smaller than those already present, and (3) non-convergence of Edgeworth series.

We first note that for calculations where the key region of interest is $10^{-4} \leq \text{LOLP} \leq 10^{-2}$, a typical range of the variable x is $-3\sqrt{2} \leq x \leq -2\sqrt{2}$. In this

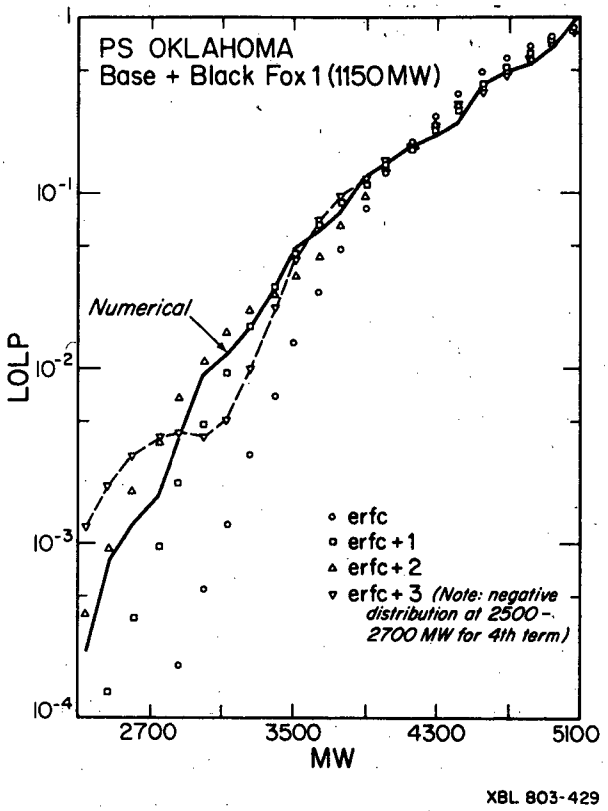

Fig. 7. LOLP plot for PS Oklahoma after addition of 1150 MW nuclear unit.

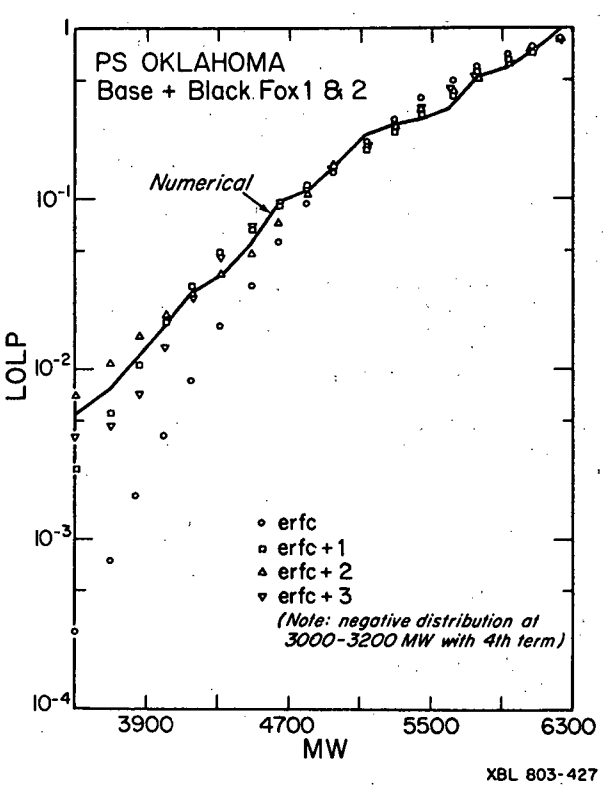

Fig. 8. LOLP plot for PS Oklahoma after addition of two 1150 MW nuclear units.

region the Edgeworth expansion, including the first two bracketed terms of (6) looks like:

$$\text{LOLP} \approx \frac{e^{-x^2/2}}{\sqrt{\pi}} \left\{ \frac{1}{x+\sqrt{x^2+1.64}} - \left[\frac{\gamma_1}{6\sqrt{2}} (2x^2-1) \right] + \left[\frac{\gamma_2}{24} (2x^3-3x) + \frac{\gamma_1^2}{72} (4x^5-20x^3+15x) \right] \right\} . \quad (10)$$

For $x = 3\sqrt{2}$, the above expression becomes

$$\text{LOLP} = 0.7\times 10^{-4} \cdot \Big\{ \underbrace{0.16}_{0} - \underbrace{[\gamma_1 2.12]}_{1} + \underbrace{[\gamma_2 1.88 + \gamma_1^2 6.63]}_{2} \Big\} ,$$

and for $x = -2\sqrt{2}$ it becomes

$$\text{LOLP} = 10^{-2} \cdot \Big\{ 0.23 - [\gamma_1 \cdot 0.83] + [\gamma_2 \cdot 0.42 - \gamma_1^2 \cdot 2.0] \Big\} .$$

Thus, we see that if γ_1 and/or γ_2 becomes too large, this approximation to the LOLP can become negative. By requiring γ_1 and γ_2 to be sufficiently small, we can make the first and second order corrections to the Edgeworth expansion small and unable to cause the LOLP to have negative values. Nonetheless, this does not guarantee that the fraction of the series retained is an accurate approximation to the exact probability distribution.

By examining γ_1, we can see how adding a generator whose output is much larger or smaller than the other units already present would make an Edgeworth-type expansion less accurate. From (7), we have [in the case of small forced outage (L_i), and a finite number of generators (N)]:

$$\gamma_1 = \sum_{i=1}^{N} \frac{\alpha_i^3 L_i}{\left[\sum_{i=1}^{N} \alpha_i^2 L_i \right]^{3/2}} . \quad (11)$$

This expression is minimized when the added generator size (α_{N+1}) is

$$\alpha_{N+1} = \frac{\sum_{i=1}^{N} \alpha_i^3 L_i}{\sum_{i=1}^{N} \alpha_i^2 L_i} . \quad (12)$$

In the case where the originally present N generators are of equal size (α), the minimizing added generator size is $\alpha_{N+1} = \alpha$. In this case

$$\gamma_1 = \left[\sum_{i=1}^{N+1} L_i \right]^{-1/2} ;$$

whereas if $\alpha_{N+1} \gg \alpha$, γ_1 would become $[L_{N+1}]^{-1/2}$; or approximately $\sqrt{N+1}$ times larger than the minimized γ_1 with $\alpha_{N+1} = \alpha$.

As indicated in (6), the higher-order cumulants and the associated products of lower-order cumulants determine whether higher-order terms in the Edgeworth expansion are accentuated or not. With this in mind, we can easily demonstrate that for low forced outage rates (L_i) and finite numbers of generators (N), higher order terms may contribute significantly to the Edgeworth expansion. To see this, we consider a typical form that contains the above mentioned cumulants and which multiplies

directly the various Hermite polynomials in Edgeworth-type expansions. In particular, we consider the term γ_{r-2} to lowest order in L_i; it may be written to good approximation as

$$\gamma_{r-2} \sim \frac{1}{\left[\sum_{i=1}^{N} L_i\right]^{\frac{r}{2}-1}}$$

$$\cdot \left[1 + \frac{r(r-2)}{2} \cdot \frac{\sum_i \left\{\alpha_i[\sum_j L_j] - [\sum \alpha_j L_j]\right\}^2 L_i}{[\sum \alpha_j L_j]^2}\right] \tag{13}$$

From expression (13), we see that if $\Sigma L_i < 1$, ever higher order terms of the Edgeworth expansion will be increasingly weighted. In short, there is no convergence of the series.

CONCLUSION

In summary, for the case of a finite number of generators N and low forced outage rates L_i, we have shown why adding a generator much larger than those already present makes the Edgeworth-type expansion more inaccurate than adding one of the optimal size, where:

$$\alpha_{optimal} = \sum_{i=1}^{N} \alpha_i^3 L_i \Big/ \sum_{i=1}^{N} \alpha_i^2 L_i \quad .$$

We have also shown that, when the forced outage rates are too low, i.e.,

$$\sum_{i=1}^{N} L_i < 1 \quad ,$$

that the Edgeworth-type series are very poor approximations to the LOLP. We have also illuminated the origins of negative LOLP's when Edgeworth-type approximations are used, using the above two criteria.

In the Appendix, a more detailed look at the underlying lattice characteristics of power system distribution functions and their impact on approximation methods are given.

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APPENDIX

(1) In most LOLP calculations, the power spectra for the individual generators are described by discrete probability densities (see (9)). This leads to aggregate power system probability densities that are also discrete. These densities and their associated probability distributions are referred to as lattice distributions (densities). In the limit of an infinite number of generators, both lattice as well as continuous probability distributions can be accurately described using the central-limit theorem.

(2) Edgeworth-type approximations are appropriate for continuous distributions, not lattice distributions, so criteria for their accuracy are of an ad-hoc nature. We use some formalism to demonstrate the point. We note that quite generally an aggregate probability density for N generators can be written as a Fourier transform:

$$P(S) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega(S-\bar{S})}\left(G(\omega)\right)d\omega$$

$$G(\omega) = \prod_{i=1}^{N} f_i(\omega) \tag{14}$$

$$f_i(\omega) = \int dx_i \; p_i(x_i)\, e^{i\omega(\mu_i - x_i)}$$

where $p_i(x_i)$ is the i'th generator's power spectrum. Edgeworth approximations are appropriate when $G(\omega)$ satisfies the following conditions [9]:

$$G(\omega) = 1 \quad , \qquad \omega = 0$$

$$|G(\omega)| < 1 \quad , \qquad \omega \neq 0 \tag{15}$$

$$\lim_{\omega \to \pm\infty} |G(\omega)| \to 0 \quad .$$

When these conditions hold, $G(\omega)$ is expanded in a series of terms about the point $\omega = 0$ with the following form

$$G(\omega) = e^{-\frac{\omega^2\sigma^2}{2}} \left\{ \sum_{m=0}^{\infty} c_m \omega^m \right\} \tag{16}$$

where the c_m are evaluated in terms of the standard cumulants and moments of the exact distribution function. Performing the indicated Fourier transform (10) and gathering terms of the same order in $(1/\sigma^2)^{n/2}$, we arrive at the standard Edgeworth approximation for $P(S) = P[(S-\bar{S})/\sqrt{\sigma}] = P(x)$. When the above appropriate conditions are met, the size of the terms $(1/\sigma^2)^{n/2}$ gives an estimate of the accuracy of including terms up to that order.

$$\begin{aligned} P(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}} \Bigg\{ & 1 + \left[\frac{\gamma_1}{6} He_3(x)\right] + \left[\frac{\gamma_2}{24} He_4(x) + \frac{\gamma_1^2}{72} He_6(x)\right] \\ & + \left[\frac{\gamma_3}{120} He_5(x) + \frac{\gamma_1\gamma_2}{144} He_7(x) + \frac{\gamma_1^3}{1296} He_9(x)\right] \\ & + \left[\frac{\gamma_4}{720} He_6(x) + \frac{\gamma_2^2}{1152} He_8(x) + \frac{\gamma_1\gamma_3}{720} He_8(x)\right. \\ & \left. + \frac{\gamma_1^2\gamma_2}{1728} He_{10}(x) + \frac{\gamma_1^4}{31\ 104} He_{11}(x)\right] \Bigg\} \quad . \end{aligned} \tag{17}$$

See (6) for the Edgeworth approximation of the LOLP probability distribution function.

In the lattice case, $G(\omega)$ violates conditions (Eq. (11)). To see this, consider $G(\omega)$ for our case where the generators' power spectra are given by Eq. (9).

$$\begin{aligned} G(\omega) &= \prod_{i=1}^{N} e^{i\omega\bar{S}} \left[L_i + (1-L_i)\, e^{-i\omega\alpha_i}\right] \\ &= \sum_{\substack{m = \text{sum of all} \\ \text{possible states}}} a_m\, e^{i\omega\Delta_m} \end{aligned} \tag{18}$$

and

$$\Sigma a_m = 1 \quad .$$

Here

$$G(\omega) = 1 \quad , \qquad \omega = 0$$

$$|G(\omega)| \leq 1 \quad , \qquad \omega \neq 0$$

$$\lim_{\omega \to \pm\infty} |G(\omega)| \Rightarrow 1 \quad . \tag{19}$$

As we can see, contributions for appropriately applying an Edgeworth-type expansion are not met. Furthermore, the point $\omega = 0$ does not leap out as an obvious point to center an expansion. Hence, if we use an Edgeworth expansion to describe such a lattice distribution, establishing conditions for its range of validity will be possible only on an ad-hoc basis.

(3) The smoother or smaller the steps in the exact lattice distribution, the better the continuous functions of the Edgeworth expansion will describe the exact distribution.

(a) Large N means a smoother distribution.

(b) Small forced outage rates (L_i), mean less smoothness. The nearer in value to 1 that the expression

$$\prod_{i=1}^{N} (1 - L_i)$$

approaches, the less smooth the exact distribution.

In Figs. 2, 3 and 4, we compare LOLP curves for power systems, each with the same generator mix. Each system's forced outage rates are equal for each generator, but different between systems. In particular, one system has $L_i = 0.1$ (Fig. 2); another, $L_i = 0.05$ (Fig. 3); and a third, $L_i = 0.02$ (Fig. 4). We see clearly that smaller L_i gives rise to a more lumpy distribution.

(4) The Edgeworth expansion uses as its zero'th order term a symmetric Gaussian centered at $\bar{S}$ with variance σ^2. Thus, the more skewed the exact distribution, the less accurate is the approximation. The parameter γ_1 is a measure of relative skewness and in the limit of small forced outage rates (L_i small) becomes

$$\gamma_1 = \frac{\Sigma\, \alpha_i^3 L_i}{[\Sigma\, \alpha_i^2 L_i]^{3/2}} \quad . \tag{11}$$

This parameter clearly demonstrates two properties already discussed in the preceding text.

(a) As the L_i decrease, γ_1 increases (see (13)). For example, if all the α_i are equal,

$$\gamma_1 \to \frac{1}{[\Sigma\, L_i]^{\frac{1}{2}}} \tag{20}$$

and for nonequal α_i, γ_1 becomes (see (13)):

$$\gamma_1 \to \frac{1}{[\Sigma\, L_i]^{\frac{1}{2}}} \cdot \left[1 + \frac{3}{2} \sum_{i=1}^{N} \frac{[\alpha_i(\sum_j L_j) - (\sum_j \alpha_j L_j)]^2\, L_i}{[\sum_j \alpha_j L_j]} \right],$$

which also increases as $1/[\Sigma\, L_i]^{\frac{1}{2}}$, when the L_i become small.

(b) Adding a unit much larger or smaller than the already existing generators increases γ_1.

(5) As is clear from Eq. (17), the Edgeworth expansion is based on the zero'th order term being a Gaussian centered at $\bar{S}$ with variance σ^2. In this limit, N goes to infinity. This is the only term that remains (central limit theorem).

The Edgeworth corrections to this Gaussian term are a series of Hermite polynomials that are weighted by various products of the standardized cumulants γ_i (see (13)). As the variable $x = (S-\bar{S})/\sqrt{\sigma^2}$ leaves the origin ($x = 0$) and increases in absolute values ($|x| > 0$), the Hermite polynomials approach their large x limit of $He_\omega(x) \Rightarrow x^n$. Depending on the size of the cumulants multiplying them, these polynomials can dominate the expressions for $P(S)$ and $LOLP(\omega)$. Since they can and do go negative in certain regions of (x), they can cause overall negative probability densities and/or distributions.

One ad-hoc criterion used by several authors is to use only the first two bracketed terms in the Edgeworth expansion and restrict the values of γ_1 and γ_2 such that the probability density does not go negative in the region of interest for the variable x [12]. See text.

Another ad-hoc criterion is to fix the range of x and then require that the cumulants be such that the maximum values of the first two bracketed terms be small compared to the zero'th order term in this range of x. Neither of these two requirements is sufficient to prove that contributions from higher-order corrections aren't larger or more negative than the first two corrections plus the zero'th order term. This is adequately demonstrated in our computer runs for certain cases.

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