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The Application of the Shunted Junction Model to Point Contact Josephson Junctions*

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We have studied the dc I-V characteristics of oxidized Nb-Nb point contacts[1] as a function of temperature, contact pressure, and rf power at 36 GHz. Results for junctions which do not show hysteresis have been compared in detail with calculations from the resistively shunted junction (RSJ) model first proposed by McCumber.[2] The agreement between the data and the RSJ model, with a constant shunt resistance and including the effects of noise, is significantly better than has been reported for Dayem bridges.[3,4]

The point contacts used in our experiments were essentially similar to those described by Buhrman, et al.[1] The point was typically oxidized for several days in air and then adjusted at room temperature to a resistance of ~ 0.5 ohm and sealed in He gas. The junction resistance usually decreased by ~ 30 percent on cooling to LHe temperature.

The points were placed along the E-field direction in a standard Q-band microwave waveguide.

In the absence of a matching circuit, the rf source impedance is high compared with the junction impedance so it acts like a constant current source. If a constant current dc bias source is used, and if the junction's capacitance can be neglected, the time dependence of the junction phase for the RSJ model can be written[5] as

$$\frac{d\theta}{d\tau} + \sin\theta = \alpha_{dc} + \alpha_{rf} \sin \Omega\tau, \tag{1}$$

in terms of the normalized currents $\alpha_{dc} = I_{dc}/I_c$, $\alpha_{rf} = I_{rf}/I_c$ and time $\tau = (2eRI_c/\hbar)t$. The normalized frequency or junction parameter $\Omega = \hbar\omega/2eRI_c$ determines the important features of the rf response of the junction.

Without an rf signal, (1) can be solved exactly to give a hyperbolic I-V curve

$$I_{dc}^2 = I_c^2 + (V_{dc}/R)^2, \tag{2}$$

where $V_{dc} = (\hbar/2e)\langle d\theta/dt\rangle$. If we compare this form with the data of Fig. 1 we find a good fit only in the low voltage region. The value of R required for the fit is ~ 2/3 the value of R_N, the asymptotic resistance at high voltages. If we draw a hyperbola with the experimental value $R = R_N$ it falls below the entire experimental curve. Either heating or phase slip effects might explain this discrepancy.

Static I-V curves measured with a constant voltage bias source are similar to those with a constant current source. The equivalent circuit should thus contain a series inductance in the case of voltage bias to account for the absence of negative resistance.

When an rf current with frequency ω is driven through the junction, constant voltage steps appear on the I-V curve at voltages given by the Josephson relation $V = n\hbar\omega/2e$. Equation (1) has no analytic solutions for finite rf current. Russer[5] has used an analog computer to show that the dependence of step height on rf current in the RSJ model differs significantly from the Bessel's function dependence obtained from Josephson's theory with constant voltage sources. We have used a digital computer to calculate a series of I-V curves from (1) for different values of the normalized rf current and the normalized frequency Ω.

In Fig. 2(a) we show a series of experimental I-V curves measured for different values of α_{rf} using a point contact at 4.2K with $\Omega = 0.16$. It is a common practice to estimate the height of steps from data which are rounded by noise by fitting the region between steps to a straight line of finite slope. When this is done we obtain the comparison between the estimated heights of the zeroth and first steps which are compared with the noise-free RSJ model in Fig. 2(b). The value of the shunt resistance R which was used in the fit is the value required to fit the static I-V curve in this low voltage region. The fit is very good despite the crude method used to obtain the points from the measured I-V curves. Similar fits[3,4] for thin film bridges have shown less good agreement. In particular, there is a characteristic drop in the height of the zeroth step as α_{rf} approaches zero known as the Dayem effect, which does not appear in the point contact data.

The I-V curves predicted from the noise-free RSJ model are compared with the experimental curves in Fig. 2(a). For such small values of Ω, the theoretical I-V curves are essentially straight horizontal lines between steps. We also calculated the I-V characteristics in the presence of Gaussian white noise corresponding to a temperature T_N = 100K by using a random number generator. These calculations show that the I-V curves predicted by the RSJ model with noise are, for small values of Ω, essentially straight lines with finite slope over most of the region between steps. The slope has been calculated for selected I-V curves and is in good agreement with the experimental curves. The rounding of the corners of a step due to noise has been investigated previously by Henkels and Webb[6] and has shown to be in agreement with Stephen's theory.[7]

Our noise calculations show that the method used to estimate experimental step heights for Fig. 2(b) underestimates the height of the noise free step. If there actually were a systematic step height error, there would be a cumulative error in Fig. 2(a) in the current at higher order steps. The agreement is in fact still very good for the 4th order step. In general, the fit does become less good for higher order steps.

Comparisons similar to those shown in Fig. 2 are given in Fig. 3 for a higher pressure Nb-Nb point contact operated at 8.7K with a normalized frequency value $\Omega = \hbar\omega/2eRI_c = 1.67$. Both theory and experiment give a step height dependence which is closer to Bessel's functions for larger Ω. The agreement between theory and experiment, however, is much less good than for small Ω. The fit is obtained only over a smaller range of experimental parameters. Even after careful corrections are made for noise the height of the first step relative to I_c is less than the predicted value.

A very important application of point contact Josephson junctions is for non-linear devices to be operated at high frequencies. In this application it is desirable to use small values of Ω so that the non-linear effects are not shorted out by a small shunt resistance. The performance of many of these devices depends on the height of induced steps and on the differential resistance of the I-V curve between steps. We have found that for $\Omega \lesssim 0.3$ there is excellent agreement for these quantities between the RSJ model including noise and experimental results for oxidized Nb-Nb point contacts. Consequently, this model should be very useful in predicting the detailed performance of high frequency devices. The fit between theory and experiment becomes significantly worse for larger values of Ω which are obtained by operating with higher contact pressure or closer to T_c.

There is no evidence for a temperature dependence of the shunt resistance or a voltage dependence in the region of the energy gap which would be expected from a significant tunneling contribution to the junction current. The shunt resistance is also independent of rf power. Our oxidized point contacts can be best thought of as nearly ideal microbridges which can be closely represented by the resistively shunted jucntion (RSJ) model including thermal noise.

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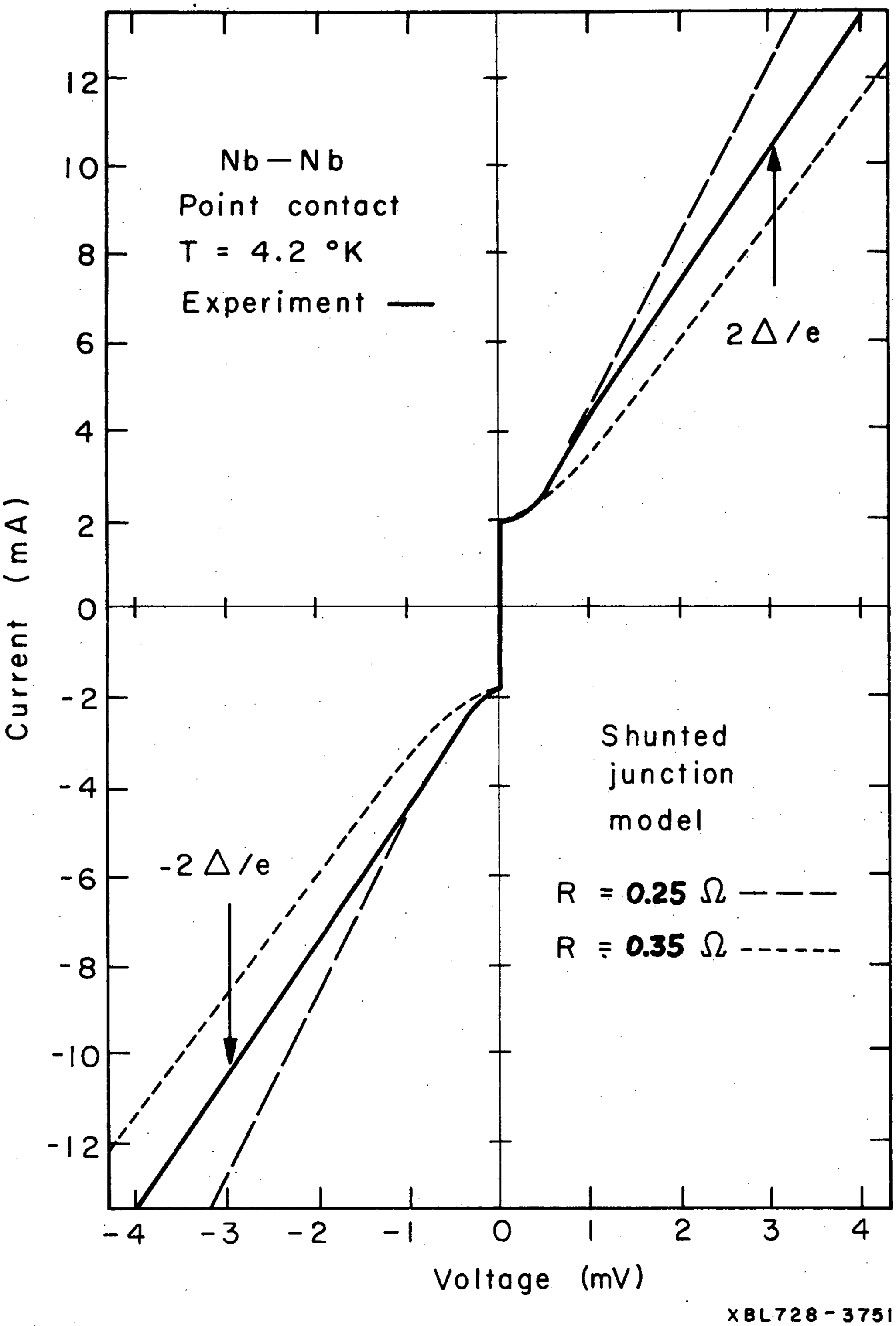
FIGURE CAPTIONS

Fig. 1. An experimental static I-V curve for a Nb-Nb point contact at 4.2K, compared with two hyperbolas. The hyperbola with R = 0.25 ohms, which fits the data in the low voltage region has too small a value of differential resistance at high voltage. The hyperbola with the differential resistance of 0.35 ohms which is correct at high voltage lies below the experimental curve.

Fig. 2(a). Experimental and theoretical I-V curves for a Nb point contact with T = 4.2K and Ω = 0.16 at 36 GHz. The thin solid line is the prediction of the RSJ model without noise and the thin dashed line is the prediction of the RSJ model with 100K thermal noise.

(b). Experimental and theoretical dependence of the zeroth and first step heights on rf current. The theory is fitted to the data at the two marked calibration points. This two-point calibration is used to fit all curves in Fig. 2(a)-(b). The resistance value deduced is the same as that required to fit the static I-V curve in the low voltage region. The experimental step heights were crudely estimated from the experimental I-V curves of Fig. 2(a) as discussed in the text. The agreement at the subsidiary maxima is improved if noise is considered more carefully.

Fig. 3(a)-(b). Results similar to Fig. 2(a)-(b) for a Nb-Nb point contact at 8.7K for which Ω = 1.67 at 36 GHz. The agreement between theory and experiment is less good for such large values of Ω. In particular, the height of the first step in Fig. 3(b) is appreciably less than the predicted value. This discrepancy cannot be explained by noise.



Nb—Nb
Point contact
T = 4.2 °K
Experiment
2Δ/e
-2Δ/e
Shunted junction model
R = 0.25 Ω
R = 0.35 Ω
Current (mA)
Voltage (mV)
XBL728-3751

FIG. 1

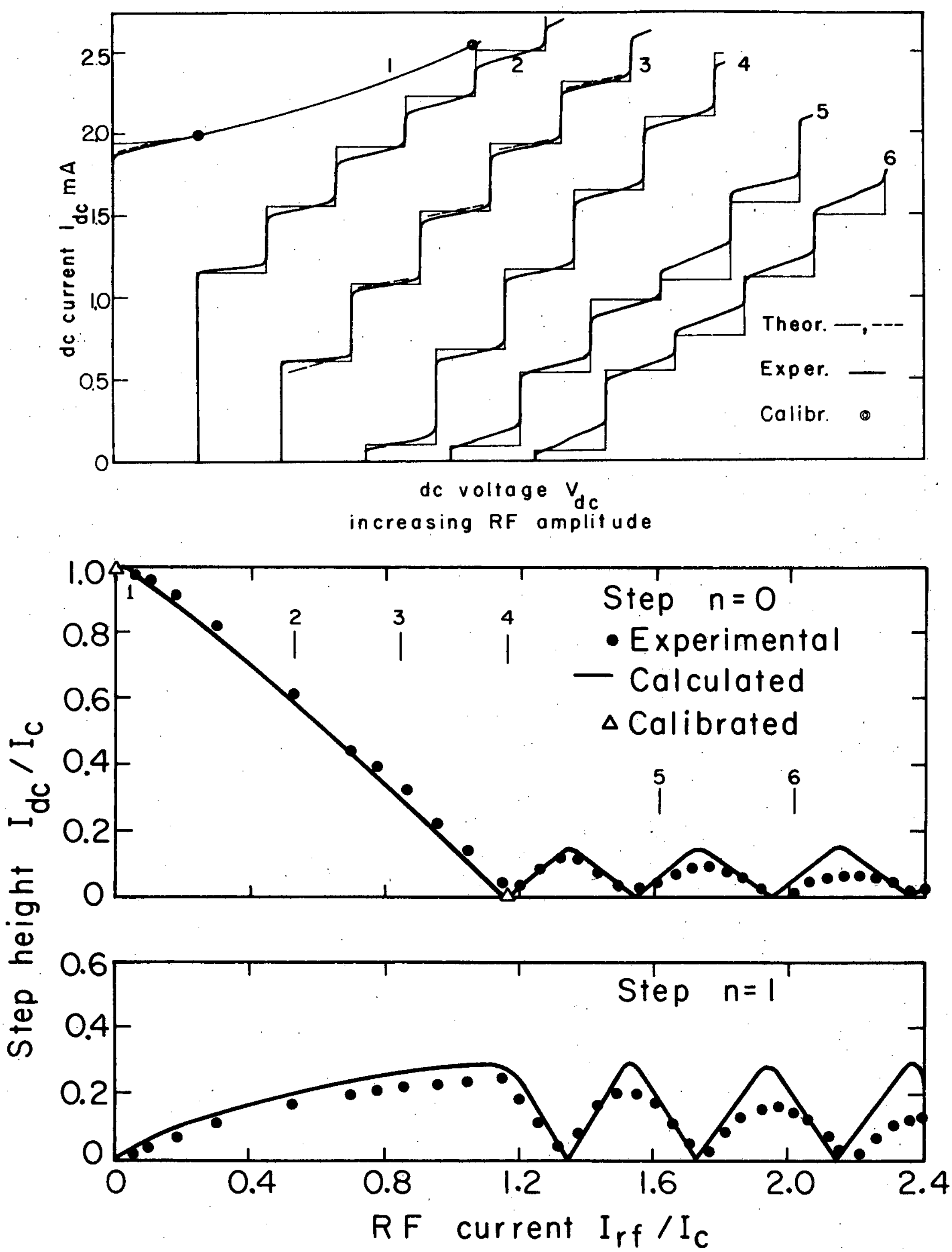


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FIG. 2

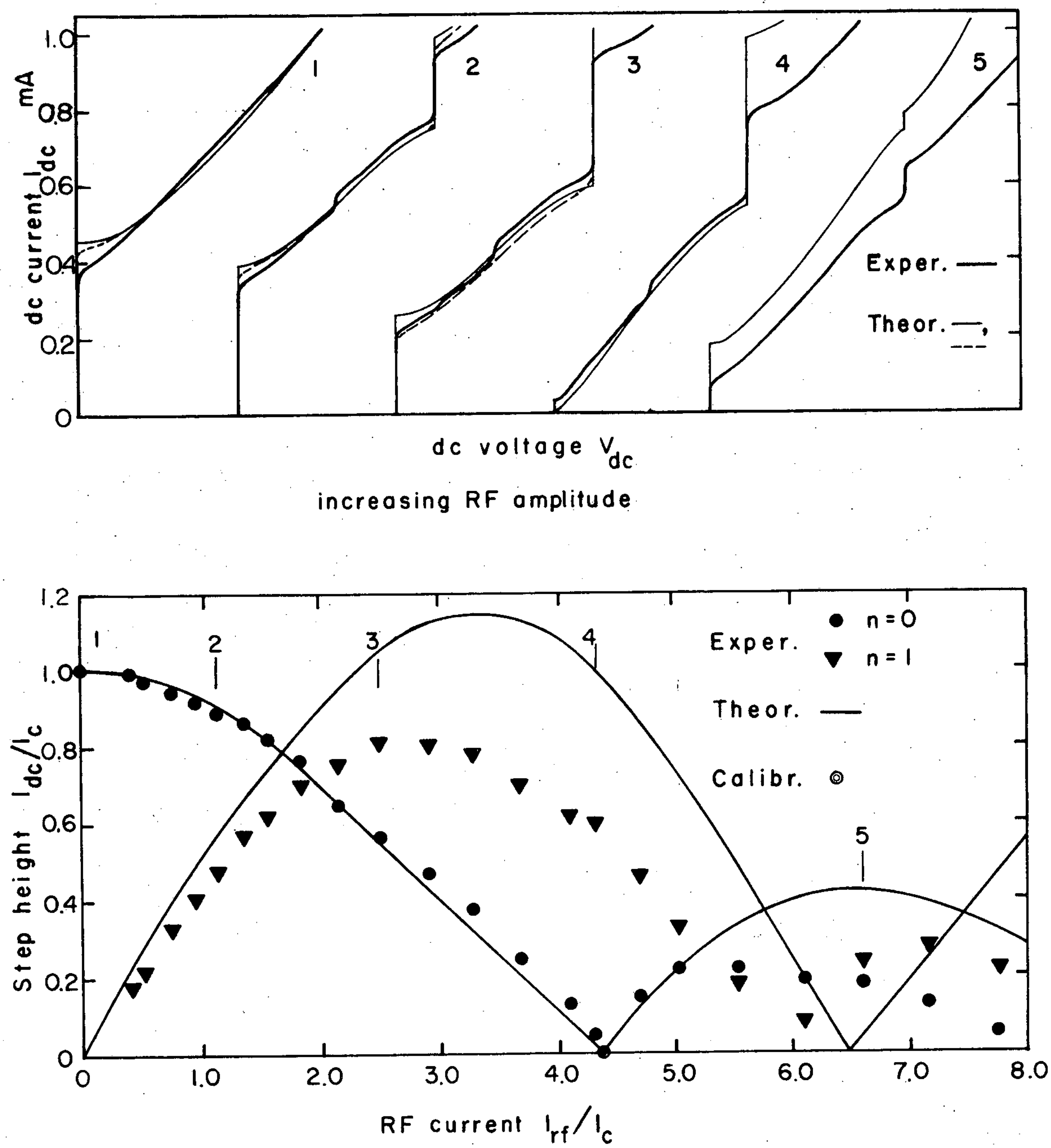

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FIG. 3

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