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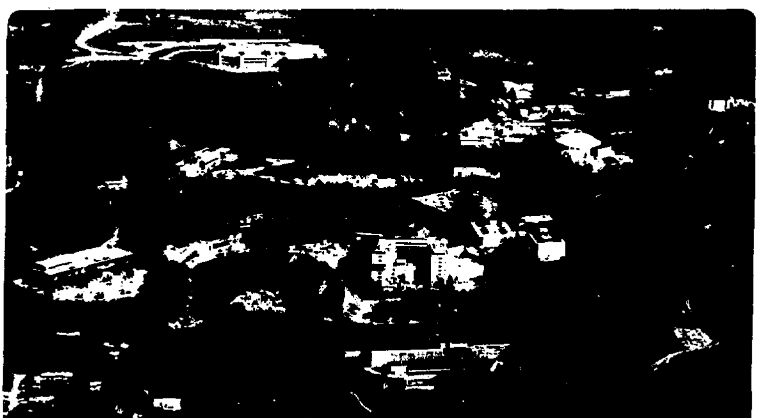

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THE MEASUREMENT AND THEORETICAL CALCULATION OF QUENCH VELOCITIES WITHIN LARGE FULLY EPOXY IMPREGNATED SUPERCONDUCTING COILS

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Abstract - The velocity of normal region propagation was measured in a 2-m diameter superconducting coil. The measured data made with small sense coils, did not agree very well with theories which have been used for the last 15 years. An adiabatic quench propagation theory, which takes into account the properties of both the superconductor and the matrix material and which assumes there is no heat transfer out of the wire, was found to agree with the experimental measurements. Both the theory and experimental measurements are given in this paper.

INTRODUCTION

An analytic, although approximate, expression for quench velocities is derived in this paper. It is used to predict some quench velocities, which are then compared with experimental data [1-4].

The theory applies to insulated wires for which the amount of heat conducted radially away from the wire can be neglected [2,4]. The effect of current sharing between the matrix material (copper) and superconductor is included [3-4]. The theory also takes into account the difference in specific heat between superconducting and normal materials [3,4]. The result is a condition, derived from the heat balance equation, that the quench velocity, v, has to satisfy. Therefore, the equation for v is implicit. However, it has been possible to write the equation in such a way that v can be solved by computing successive iterations. This method of computation is described here.

The quench velocities were measured in a coil 2 m in diameter and 0.7 m long [5]. The wire was 1.5 mm in diameter, insulated by Formvar and completely cast in epoxy by vacuum impregnation. The quenches were induced at a point the winding, using a pulsed 1-cm diameter coil. Propagation times were measured by the signal induced in a similar 1.3-cm diameter coil some distance from the pulsed coil as the superconducting to normal transition passed by. The measured quench velocities fit the theory described in this paper better than other expressions to be found in the literature.

BASIC THEORY

The quench process in a superconductor can be characterized by the following one dimensional equation [1]:

$$C_p \frac{\partial T}{\partial t} = \rho_{wire}\epsilon(T)\, J^2_{wire} + \frac{\partial}{\partial x}\left[k_{wire}\frac{\partial T}{\partial x}\right] \qquad (1)$$

where C_p is the specific heat per unit volume; T is temperature; t is time; ρ_{wire} is the resistivity of the wire when it is in the normal state; $\epsilon(T)$ is a function which depends on the state of the conductor; k_{wire} is the thermal conductivity of the wire. C_p, ρ_{wire}, $\epsilon(T)$, and k_{wire}, which are functions of temperature, are defined later.

The preceding equation is a heat balance equation between Joule heating P_{joule}, heat absorption P_{abs}, and heat conduction P_{cond}. The basic equation (1) can be restated:

$$P_{abs} = P_{joule} + P_{cond} \qquad (2)$$

The following assumptions are used to develop (2) into a viable theory for normal region propagation: (a) the electric field is uniform over the wire cross section and is directed along the wire axis, (b) the temperature is assumed to be uniform across the wire cross section, and (c) the wire is assumed to be insulated electrically and thermally, which makes the problem one dimensional.

Joule heating

The amount of heat released per unit volume at a point x of the wire by Joule heating can be stated as follows:

$$P_{joule} = \rho_{wire}\,\epsilon(T) J^2_{wire} \qquad (3a)$$

where J_{wire} is the current density in the wire cross section defined as follows:

$$J_{wire} = \frac{i}{A_{wire}} \qquad (3b)$$

i is the current carried by the wire, and A_{wire} is the cross-sectional area of the wire.

In a superconducting wire, there are three distinct regions in which current can flow in the wire: the superconducting region where $\epsilon(T) \approx 0$, the current shared region where current is carried in both the superconductor and the matrix metal and $(0 \le \epsilon(T) \le 1)$, and the normal region where current is carried in the normal metal only $(\epsilon(T) = 1)$.

Let $i_c(T)$ be the critical current of the wire at a temperature T and some magnetic induction B, and correspondingly let $T_c(i)$ be the critical temperature of the wire at a current i and some magnetic induction, B. The superconducting critical region is defined as follows for a given magnetic induction B:

$$i_c(T) = i_c(o)\left[1 - \left(\frac{T}{T_c(o)}\right)^2\right] \qquad (4a)$$

and

$$T_c(i) = T_c(o)\left[1 - \frac{i}{i_c(o)}\right]^{1/2} \qquad (4b)$$

where $i_c(o)$ is the critical current at T = 0 and B and $T_c(o)$ is the critical temperature at i = 0 and B.

Using (4a) and 4b), one can define the $\epsilon(T)$ term in (3a) by the following:

$$\epsilon(T) = 0 \text{ when } T < T_c(i) \qquad (5a)$$

$$\epsilon(T) = 1 - \frac{i_c(T,B)}{i} \text{ when } \begin{matrix} T > T_c(i) \\ T < T_c(0) \end{matrix} \qquad (5b)$$

$$\epsilon(T) = 1 \text{ when } T > T_c(0) \qquad (5c)$$

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and because a superconductor in the normal state has a very high resistivity

$$\rho_{wire} = \rho_m \frac{1 + r_{sc}}{r_{sc}} \tag{5d}$$

ρ_m is the resistivity of the matrix metal, and r_{sc} is the matrix material to superconductor ratio by volume.

Heat Absorbtion and Enthalpy Change

As the temperature T rises, a certain quantity of heat is absorbed by the material per unit of volume and time

$$P_{abs} = C_p(T) \frac{\partial T}{\partial t} \tag{6}$$

where Cp(T) is the specific heat of the wire per unit of volume, averaged over the superconductor and the matrix. We define the enthalpy as usual:

$$h(T) = \int_0^T C_p(T)\, dT \tag{6a}$$

Then

$$P_{abs} = \frac{\partial h}{\partial t} \tag{6b}$$

In the normal state, in our range of temperatures, the wire specific heat can be expressed as [6].

$$C_p(T) = \gamma T + \Gamma T^3 \tag{7}$$

$$\gamma = \frac{1}{1 + r_{sc}} \gamma_s + \frac{r_{sc}}{1 + r_{sc}} \gamma_m \tag{7a}$$

$$\Gamma = \frac{1}{1 + r_{sc}} \Gamma_s + \frac{r_{sc}}{1 + r_{sc}} \Gamma_m \tag{7b}$$

where γ_s (γ_m) and Γ_s (Γ_m) are the parameters defined as the parameters for specific heat per unit of volume of the superconductor (matrix). Then, in the normal state, $h(T) = h_n(T)$ where

$$h_n(T) = \frac{\gamma T^2}{2} + \frac{\Gamma T^4}{4} \tag{8}$$

For a type II superconductor, (8) is still valid when the superconductor carries only its critical current in the current sharing region. However, in the superconducting region, there is an additional term Q associated to the magnetization process [3,7]. The amount of enthalpy necessary to bring a unit volume of the wire from the ambient temperature T_0 to the current-dependent critical temperature $T_c(i)$ is

$$\Delta h = \Delta h_n + \Delta Q(i) \tag{9}$$

where $\Delta Q(i)$ is approximated as

$$\Delta Q(i) = \frac{B_{c1}(0)\, B_{c2}(0)}{8\pi \times 10^{-7}(1 + r_{sc})} \; \frac{T_c^2(i) - T_0^2}{T_c^2(0)} \; \frac{T_c^2(i) + 3T_0^2}{T_c^2(0)} \tag{9a}$$

When a quench propagates at a uniform velocity, v, the functions T and h are a function of x − vt. Therefore, (6b) takes the following form:

$$P_{abs} = -v \frac{\partial h}{\partial x} \tag{10}$$

Heat Conduction

Heat conduction drives heat along the wire. If one assumes that the heat conductivity in the superconductor is negligible with respect to the heat conductivity K_m of the matrix

$$k_{wire} = \frac{r_{sc}}{1 + r_{sc}} K_m \tag{11}$$

If the matrix material has the standard ratio between electrical and thermal conductivity determined by the Lorentz Number L ($L = 2.45 \times 10^{-8}\ W\Omega K^{-2}$),

$$k_m \rho_m = LT \tag{12}$$

It follows from (11), (12) and (5d) that

$$k_{wire} \rho_{wire} = LT \tag{12a}$$

The heat conductance term can be derived from (1), (2), and (12a) to yield

$$P_{cond} = \frac{d}{dx}\left[\frac{LT}{\rho_{wire}} \frac{dT}{dx}\right] \tag{13}$$

Heat Balance

Using (5), (10), and (13) one can rewrite the heat balance equations (1) and (2) to yield the following form:

$$-v \frac{dh}{dx} = \rho_{wire}\, j_{wire}^2\, \varepsilon(T) + \frac{d}{dx}\left(\frac{LT}{\rho_{wire}} \frac{dT}{dx}\right) \tag{14}$$

where h, ρ_{wire}, j_{wire}, $\varepsilon(T)$, and L are previously defined by (9), (5d), (3a), (5a) to (5c), and (12), respectively.

SOLUTION

Equation (14) can be solved using reasonable approximations in each of the three regions of the superconductor to normal transition. For example, the temperature over which (14) is valid is from the operating temperature (about 4 K) to 20 K. Above 20 K, (1) is dominated by the heat absorption and joule heating term. If the residual resistivity ratio of the conductor is less than 200, ρ_{wire} is a constant, and a constant value for the Lorentz number L can be reasonably assumed. Using the preceding approximations, one ends up with an implicit expression for v. The mathematical details of this calculation are given elsewhere [8].

To compute the quench velocity, one needs to know: i, the current in the wire; A_{wire}, the wire cross section area; R_{sc}, the ratio of matrix to superconductor by volume; T_0, the operating or "bath" temperature; $T_c(o)$, the critical temperature for zero current in the wire (for Nb-Ti, $T_c(o) = 9.4K$); $i_c(o)$, the wire critical current for T = 0 K and the local magnetic induction, B (for the calculations presented here $j_c(o) = 4.5 \times 10^9 Am^{-2}$); $B_{c1}(0)$ $B_{c2}(0)$, the product of the two critical fields at T = 0 K (we used $B_{c1} = 0.014$ T and $B_{c2} = 14$ T for Nb-Ti) [9]; and $\mu = 2.045$ (see [8] for the derivation of this parameter). In addition, one needs L, γ_s, γ_m, Γ_s, Γ_m and A_c. For Nb-Ti, $\gamma_s = 789\ Jm^{-3}k^{-2}$ and $\Gamma_s = 9.27\ Jm^{-3}k^{-4}$; for Cu, $\gamma_m = 98.1\ Jm^{-3}k^{-2}$ and $\Gamma_m = 6.80\ Jm^{-3}k^{-4}$ [6]. $L = 2.45 \times 10^{-8}\ W\ k^{-2}$, and $A_c = 1.77 \times 10^{-6}\ m^2$.

From the numbers in the previous paragraph, one computes Υ (7b), γ (7a),

$$h^* = \frac{\Gamma T_c^4(o)}{4}, \tag{15a}$$

$$J_{wire} = \frac{i}{A_{wire}}, \tag{15b}$$

$$V = \sqrt{\frac{L}{2}}\left(\frac{J_{wire}\ T_c(o)}{h^*}\right), \tag{15c}$$

$$\beta = \frac{B_{c1}(o)\ B_{c2}(o)}{8\pi 10^{-7}(1 + r_{sc})\ h^*}, \tag{15d}$$

$$\eta = \frac{\gamma}{T_c(o)^2}, \tag{15e}$$

$$\psi = (1 - q_o)\ (1 + q_o + 2E) + \beta(q_c - q_o)\ (q_c + 3q_o) \tag{15f}$$

$$q_c = \frac{i}{i_c(o)} \tag{15g}$$

and

$$q_o = \left(\frac{T_o}{T_c(o)}\right)^2 \tag{15h}$$

Then one can define the functions

$$\chi(R_v) = \frac{1-e^{-\mu(1 + \eta)R_v^{2/3}}}{1 + \eta} \tag{16a}$$

and

$$R_v(\chi) = \frac{\chi}{\sqrt{2(\psi\chi - 1 + q_c)}} \tag{16b}$$

(16a) and (16b) can be solved by iteration stanting with (16a) and $R_v \to \infty$. Once convergence has been achieved, the quench velocity v can be calculated using

$$v = V\ R_v. \tag{17}$$

MEASUREMENTS OF QUENCH VELOCITIES IN A LARGE EPOXY IMPREGNATED COIL

Quench velocities were measured with a solenoid 2 m in diameter and 0.70 m long, wound on an aluminum bore tube 3/8" thick and with two layers of 430 turns each of 1.5 mm diameter insulated wire with copper to superconductor ratio 1.8 to 1. The wire insulation is .05 mm thick. The self-inductance of either layer is 0.46 H and of both layers in series is 1.81 H. The resistance ratio of the wire between 273 K and 10 K was about 120.

Quenches were induced by the technique described in [10]. A capacitor of 1000 μF, charged to about 150 V, was discharged into a small coil 1.3 cm in diameter with an inductance of about 0.5 mH and with an effective area (the average area times the number of turns) of 160 cm^2. The small coil was mounted against the superconducting coil winding and the discharge created enough heat in the superconducting coil wire to induce a quench, probably by $\dot{B}$ effect.

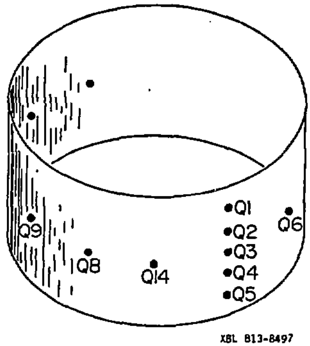

Fig. 1. Location of small coils for initiating and detecting quench propagation.

The method used to measure quench velocities is similar to that described by Goll and Turowski [11]. Small pickup coils were used to detect the passing of the quench along the superconducting wire. Several of these small coils were mounted on the superconducting coil as shown in Fig. 1. The relevant coils, labeled Q3, Q8, Q9, and Q14, were located in the plane of symmetry normal to the axis. Any one of these Q coils could be used for initiating the quench, but quench velocities were measured when the quench was initiated by Q3 only. Soon after the discharge in Q3, signals appeared in other coils located on the same plane, normal to the axis, when these coils were connected directly to the scope with a high impedance [12]. The amplitude of the signals increase with the current in the coil, but the inaccuracy in the measurement makes it difficult to derive a law for that dependence.

Signals were monitored in coils Q_{14}, Q_8, and Q_9. The delay between these signals and the quench trigger in Q_3. This delay is proportional to the distance of each of the coils monitored to Q_3. This phenomenom is expected if the signals in the coils are due to wires turning normal during the time of quench propagation.

CONCLUSIONS

Experimental data and theoretical predictions of quench velocities are plotted in Fig. 2. In most cases, the agreement is better than 10 percent. It is concluded that the model described in this paper is not too far remote from reality when the superconducting wire is deeply imbedded in insulation.

Table 1. Distance, time delay, and quench velocity for various quenches.

Current (A)	Number of layers	Average wire current density ($10^9 A/m^2$)	Small coil used for detection	Distance to Q3 (m)	Time delay for signal (msec)	Quench velocity (m/sec)
500	1	0.28	Q14	0.53	130	4.1
			Q8	0.97	230	4.2
			Q9	1.49	360	4.1
900	1	0.51	Q14	0.53	44	12
			Q8	0.97	83	12
			Q9	1.49	140	11
1300	1	0.74	Q14	0.53	24	22
			Q8	0.97	44	22
			Q9	1.49	69	22
1700	1	0.96	Q14	0.53	15	35
			Q8	0.97	27.5	35
700	2	0.40	Q14	0.53	62	8.6
			Q8	0.97	112	8.7
900	2	0.51	Q14	0.53	40	13.3
			Q8	0.97	71	13.6
1100	2	0.62	Q14	0.53	26.5	20
			Q8	0.97	48.5	20
1300	2	0.74	Q14	0.53	21	25
			Q8	0.97	38	26

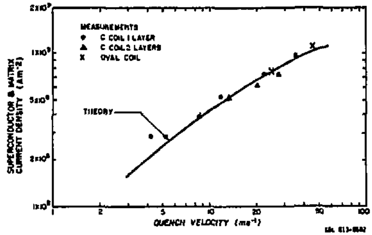

Fig. 2. Predicted and measured quench velocities as a function of the wire current density.

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