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ANOMALIES AND THE PARTICLE CONTENT OF GRAND UNIFIED THEORIES*

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ABSTRACT

For the largest class of grand unified models based on simple gauge groups, the presence of anomalies not only results in unrenormalizability, but in addition means that there are charged, massless fermions in the theory. Conversely, the absence of charged, massless particles in such models guarantees the absence of anomalies. For models outside this class, if there are anomalies but no charged massless fermions, the anomalies occur only for rather exotic currents.

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It has long been known that the renormalizability of spontaneously broken gauge theories is destroyed by the presence of anomalies.[1] Accordingly, many model builders have required the absence of anomalies as a fundamental principle.[1,2] Thus, in the prescient paper of Georgi and Glashow on SU(5), the absence of anomalies was regarded as something of a surprise.[3] (Of course, as their future work showed, the absence of anomalies in the model was easy to understand in terms of the SO(10) model). Similarly, in the SU(7) technicolor model of Farhi and Susskind, the fermion representation was required to be anomaly free so that the reducible representation carried by the fermions was $\underset{\sim}{7}^* + \underset{\sim}{35}^* + \underset{\sim}{21}$ (This again is easily understood in terms of the 64 dimensional representation of SO(14)).[4]

The constraint that models have no anomalies significantly reduces the possibilities. This was exploited by Frampton who showed that insisting on many of the desiderata proposed by Georgi resulted in the absence of any solutions at all.[5]

It might be objected that these requirements of Georgi[2] and Frampton[5] are too restrictive and in particular that the absence of anomalies is somehow a "formal" requirement that ought to be ignored since anomalies represent an extreme short-distance pathology which might somehow be cured by new physics (gravity?) which is not relevant at low energies (like the grand unification mass!).

This very short note is intended to demonstrate that the requirement that anomalies be absent has a much less formal significance: theories with anomalies have particle content not in accord with what we see in nature. In particular, for a large

class of models, if there are anomalies there are (electromagnetically) charged particles with only one chiral state, that is, charged massless fermions.

The demonstration of these facts is entirely straight-forward. The works of Georgi and Glashow[6] and of Okubo[7] show that for semi-simple gauge groups (thus excluding $SU(2) \times U(1)$), only $SU(n)$ factors can result in anomalies. We henceforth confine ourselves to the consideration of simple groups with anomalies (i.e. $SU(n)$). Banks and Georgi,[8] and Okubo[7] have shown that there is really only one anomaly for such a model in the following sense. We take as our fundamental fermion fields all the left-handed fields including both fermions and antifermions. The generators of the (reducible) representation are indicated by T_a, T_b, etc. The anomaly of the currents J_a, J_b, and J_c is proportional to $\mathrm{Tr}\left(\{T_a,T_b\}\,T_c\right)$ so we write

$$A_{abc}(R) = \mathrm{Tr}\left(\{T_a,T_b\}\,T_c\right) . \tag{1}$$

We define the symmetric tensor d_{abc} by

$$d_{abc} = \mathrm{Tr}\left(\{t_a,t_b\}\,t_c\right) , \tag{2}$$

where t_a, t_b, and t_c are the particular matrices for the fundamental representation of $SU(n)$. As mentioned above, Banks and Georgi,[8] and Okubo[7] have shown that

$$A_{abc}(R) = A(R)\,d_{abc} . \tag{3}$$

Okubo's proof may be paraphrased as follows. Let us write the $SU(n)$ commutation relations as $[t^a,t^b] = i\,f_{abc}t^c$ so for any representation, R, with matrices $T^a{}_{ij}$ we have as well

$[T^a,T^b] = i\,f_{abc}T^c$. A vector operator belonging to the representation R is a set of matrices $V^a{}_{ij}$ (where i and j run over the same values as in $T^a{}_{ij}$) satisfying the commutation relation $[T^a,V^b] = i\,f_{abc}V^c$. Consider the matrix elements of V: $(Rj \mid V^a \mid Ri)$. These are determined up to an overall factor (the reduced matrix elements) by the Wigner-Eckart theorem to be equal to the Clebsch-Gordan coefficients, except that there may be more than one way of making the final representation R, out of the product of the initial representation R and the adjoint representation (carried by the vector operator V). Thus the number of independent vector operators in the representation R is simply the number of times R occurs in the product of R and the adjoint representation.

Now the adjoint representation is given by $[T^a{}_{adj}]_{bc} = -\,i\,f_{abc}$ where the structure constants have been arranged to be completely anti-symmetric. Let T^a be the matrices of any representation, R, and define

$$D^c_{ab} = \mathrm{Tr}\left(\{T^a,T^b\}\,T^c\right) .$$

This is a vector operator belonging to the adjoint representation. To prove this, conjugate all the T's by $\exp(-i\alpha T^e) \approx I - i\alpha T^e$. This leaves the trace invariant so we have

$$i\,f_{ead}\,D^c_{db} + i\,f_{ebd}\,D^c_{ad} = -\,i\,f_{ecd}\,D^d_{ab} ,$$

or,

$$[\,T^e_{adj},D^c\,] = i\,f_{ecd}\,D^d .$$

Thus both $T^a{}_{adj}$ and D^a are vector operators belonging to the adjoint representation. But there are only two independent vector operators

belonging to the adjoint representation because the adjoint occurs only twice in the product of the adjoint with itself for SU(n), $n > 2$. This last fact is familiar for SU(3) where $8 \times 8 = 27 + \overline{10} + 10 + 8 + 8 + 1$. It is easy to show that it is true for higher n simply by examining the Young tableaux and seeing that there are precisely two ways of forming the adjoint (one column of n-1 boxes, and one column of one box) from the product of two adjoints.

Since $T^a{}_{\mathrm{adj}_{bc}} = -i f_{abc}$ is antisymmetric in b and c, while $D^a{}_{bc}$ is symmetric, it is clear that the $D^a{}_{bc}$ for different representations must all be proportional to each other. In particular, they are proportional to the $D^a{}_{bc}$ obtained from the fundamental representation. Thus there is only one anomaly, A, to calculate.

In particular, we can calculate A by determining the trace of $\left(\left\{T_a, T_b\right\} T_c\right)$ for any choice of the T's as long as d_{abc} is non-vanishing. Suppose then that we take for each T the matrix Q representing the electromagnetic charge (which is a generator for grand unified theories). We then have

$$\mathrm{Tr}\, Q^3 = 2A\, d_{QQQ} \tag{4}$$

Thus if $A \neq 0$ and $d_{QQQ} \neq 0$, then $\mathrm{Tr}\, Q^3 \neq 0$. But we can calculate $\mathrm{Tr}\, Q^3$ directly. The u_L contributes $(2/3)^3$ for each color etc. while the $(\bar{u})_L$ contrubutes $(-2/3)^3$, etc. Thus we see that $\mathrm{Tr}\, Q^3 = 0$ unless there is a charged fermion lacking its chiral partner. Such a fermion is destined to remain massless since it cannot become a full Dirac particle.

This result can equally well be stated in the contrapositive form: if $d_{QQQ} \neq 0$ and every charged fermion has both chiral pieces, the representation is anomaly free. Thus in the SU(5) Georgi-Glashow model once it is seen that the u, d, e and ν_e fit into the $\underset{\sim}{\bar{5}} + \underset{\sim}{10}$ it is unnecessary to check that the anomalies cancel. To see that $d_{QQQ} \neq 0$ it suffices to find any representation for which $\mathrm{Tr}\, Q^3 \neq 0$. In particular, for the $\underset{\sim}{5}$, $\mathrm{Tr}\, Q^3 = 8/9$. Similarly, in the SU(7) technicolor model, that it is possible to assign the electric charges consistently establishes that there is no anomaly.

If there is an anomaly, but no massless charged fermion we must have $d_{QQQ} = 0$. This can certainly not occur if SU(5) is simply extended in the usual technicolor fashion. On the other hand, it is easy to construct models with this property by putting all the fermions into the fundamental representation of SU(n), which always has an anomaly. Thus we can take SU(15) and put the usual fifteen fermion fields associated with u, d, e and ν_e into the fundamental representation. Clearly $\mathrm{Tr}\, Q^3 = 0$ for this representation. Moreover, with this embedding of Q in SU(15) we will have $\mathrm{Tr}\, Q^3 = 0$ for every representation by virture of Eq. (3).

This crazy SU(15) model does illuminate the peculiarity of having $d_{QQQ} = 0$. The generator Q is still embedded within SU(5). We can of course regard the $\underset{\sim}{15}$ as $\underset{\sim}{5} + \underset{\sim}{10}$ under this SU(5). Thus by our previous analysis we can see that there are no SU(5) anomalies, that is, for the SU(15) $d_{abc} = 0$ not just when $T_a = T_b = T_c = Q$, but whenever all three lie within the usual SU(5).

We can state the result more formally as follows. Suppose that for some simple group $d_{QQQ} = 0$ and suppose that Q is one of the generators of a simple subgroup with respect to which $d_{QQQ} \neq 0$ (here the trace is over the fundamental representation of the subgroup). Then all the anomalies of the generators of the subgroup vanish.

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