LBL-12623

CONF-8105104--7

Lawrence Berkeley Laboratory
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Presented at the Fifth High Energy Heavy Ion Study, Lawrence Berkeley Laboratory, University of California, Berkeley, CA, May 18-22, 1981

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July 1981

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Prepared for the U.S. Department of Energy under Contract W-7405-ENG-48

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THEORY OF COULOMB DISTORTION OF PION SPECTRA BY HEAVY-ION SPECTATOR FRAGMENTS*

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1. Introduction

It has been known for over two years now that the π^- spectrum from heavy ion collisions peaks near beam velocity and that the π^+ spectrum has a corresponding depression. We have guessed that these features are Coulomb effects of projectile fragments. In papers just submitted for publication[1,2] the experimental data and the theoretical interpretation are presented. We aim in the present paper to give a complementary report with somewhat more mathematical detail than in ref. 2. We also highlight and discuss implications of the theoretical parameters deduced from the pion spectra.

2. Outline of the theory

2.1. Construction of Coulomb Green's Function

We start by considering a fragment of charge Z_F and mass A_F and a pion of mass m_π at a distance r from the center of this fragment under consideration. For production of slow pions near the beam velocity, we seek a solution of a Schrodinger equation describing the relative motion (nonrelativistic) of the two particles of the form

*This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract W-7405-ENG-48.
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$$\left\{-\frac{\hbar^2}{2\mu}\nabla^2 + V(r) + W(r) - E\right\}\psi(\vec{r}) = 0 \quad . \tag{1}$$

It is possible to attempt solving this equation via Green's Function if one writes as

$$\{E - H(\vec{r})\}\psi(\vec{r}) = \rho(\vec{r}) \quad , \tag{2}$$

where

$$H(\vec{r}) = -\frac{\hbar^2}{2\mu}\nabla^2 + V(r) \qquad \text{with } V(r) = \frac{Z_\pi Z_F e^2}{r} \tag{3}$$

In Eq. (2) E stands for the kinetic energy of the pion when it is far from the fragment under consideration. We may write the solution of Eq. (2) as

$$\psi(\vec{r}) = \int d^3r' \; G(\vec{r},\vec{r}')\rho(\vec{r}') \quad , \tag{4}$$

where $G(\vec{r},\vec{r}')$ is the three-dimensional Green's Function and satisfies the differential equation

$$\{E - H(\vec{r})\}\; G(\vec{r},\vec{r}') = \delta(\vec{r} - \vec{r}') \quad . \tag{5}$$

G satisfies the homogeneous differential equation for $\vec{r} \neq \vec{r}'$ but is singular at $\vec{r} = \vec{r}'$. It can be shown that the three-dimensional Green's Function has a Legendre expansion of the form[3,4]

$$G(\vec{r},\vec{r}') = \frac{1}{4\pi}\;\frac{2\mu}{\hbar^2}\;\frac{1}{rr'}\sum_{\ell=0}^{\infty}(2\ell + 1)\; P_\ell(\hat{r}\cdot\hat{r}')g_\ell(r,r') \quad , \tag{6}$$

where the radial Green's Function $g_\ell(r,r')$ satisfies the differential equation

$$\left\{\frac{d^2}{dr^2} + k_\pi^2 - \frac{\ell(\ell+1)}{r^2} - \frac{Z_\pi Z_F e^2}{r}\right\} g_\ell(r,r') = \delta(r - r') \text{ with } k_\pi = \frac{P_\pi}{h} \quad . \tag{7}$$

2.2. The delta source at a distance

We may solve Eq. (2) with the aid of the delta-function source at $\vec{r}' = \vec{r}_s$. In this case $\rho(\vec{r}_s) = \lambda\delta(\vec{r} - \vec{r}_s)$ and the solution to Eq. (1),

which we seek, becomes related to the three-dimensional Green's Function by

$$\psi(\vec{r}) = \lambda G(\vec{r},\vec{r}_s) \tag{8}$$

One has to solve Eq. (7) for $g_\ell(r,r_s)$ for $r > r_s$ by imposing the boundary condition of an outgoing wave on the radial wave function. This could be done by changing the variable r to the dimensionless variable $\rho = kr$ and introducing the Sommerfeld parameter $\eta = Z_\pi Z_F e^2/(\hbar c|\vec{\beta}_\pi - \vec{\beta}_F|)$, where $|\vec{\beta}_\pi - \vec{\beta}_F|$ is the velocity (in units of c) of the produced pion relative to the assumed spectator fragment. We see immediately that

$$g_\ell(r,r_s) = -k^{-1}\, F_\ell(\eta,\rho_s)[G_\ell(\eta,\rho) + iF_\ell(\eta,\rho)] \quad . \tag{9}$$

F and G stand for the regular and irregular Coulomb wave functions, respectively.

For an observation point far from the region of interaction, one can find for G the following:

$$G(\vec{r},\vec{r}_s) \underset{r\to\infty}{\sim} F_\delta(\vec{r},\vec{r}_s)\, \frac{e^{i(kr-\eta\, \ln 2\rho)}}{r} \tag{10}$$

The logarithmic term is a direct consequence of the long-range nature of the Coulomb force. This term produces no net current at infinity, so we know that F_δ can be interpreted as a wave amplitude for the production of pions from a δ-source at r_s and with

$$\sigma = \left|f_\delta(\vec{r},\vec{r}_s)\right|^2 \quad , \tag{11}$$

where

$$F_\delta(\vec{r},\vec{r}\,) = -\lambda\, \frac{2u}{\hbar^2} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} (-1)^\ell e^{i\sigma_\ell} y^*_{\ell m}(\theta,\phi)\, y_{\ell m}(\theta_s,\phi_s) \left\{\frac{F_\ell(\eta,\rho_s)}{\rho_s}\right\} \quad , \tag{12}$$

and σ_ℓ is the well-known Coulomb phase shift.

For comparison with zero degree pions, we let $\vec{r} = (\infty,0,0)$ and take the

beam along the z-axis. This will eliminate the m-dependence of Eq. (12). For $\vec{r}_s = (r_s, 0, 0)$ or $\vec{r}_s = (r_s, \frac{\pi}{2}, 0)$ f_δ is proportional to a sum over ℓ of $a_\ell F_\ell(\eta, \rho_s)$ plus $F_0(\eta, \rho_s)$. We found that F_0 is the predominant term in the expansion and it is good approximation to write

$$\left|F_\delta(\vec{r}, \vec{r}_s)\right|^2 \simeq \lambda^2 \left(\frac{2\mu}{\hbar^2}\right)^2 \left|\frac{F_0(\eta, \rho_s)}{\rho_s}\right|^2 \tag{13}$$

Hence

$$\sigma_{\pi^\pm} \propto \left|\frac{F_0(\eta_\pm, \rho_{s_\pm})}{\rho_{s_\pm}}\right|^2 = G_\pm(1 + \rho_{s_\pm}\eta_\pm + \ldots)^2 \tag{14}$$

where

$G_\pm = \dfrac{2\pi\eta_\pm}{[\exp(2\pi\eta_\pm)-1]}$, and the +,- signs stand for π^+ and π^-, respectively.

2.3. Averaging over products and velocity dispersion

For comparison with inclusive pion data, we should calculate an average value of Eq. (14) over all products and relative velocity of pion and fragment. The velocity dispersion formula obtained by Greiner et al.[5] is used and has the form

$$P \propto \exp[-\beta_F^2/(2\alpha_F^2)] \tag{15}$$

with $\alpha_F = \sigma_0/(m_N c)\sqrt{(A_0 - A_F)/[A_F(A_0 - 1)]}$,

and where A_0 is the projectile mass number, σ_0 is a constant. The grand average of the pion cross section is given by

$$\langle\sigma_{\pi^\pm}\rangle = N_\pi \frac{\sum_{A_F, Z_F} N_F \sigma(A_F, Z_F) F(A_0 - A_F) G\,(\langle\eta\rangle_{\beta_F}) \left|1 + \frac{m_\pi r_0 c}{\hbar}\beta_{\pi^\pm}\langle\ldots\rangle_{\beta_F} A_F^{1/3}\right|^2}{\sum_{A_F, Z_F} N_F \sigma(A_F, Z_F) F(A_0 - A_F)} \tag{16}$$

where

$N_F = \sqrt{2\pi}\alpha_F^3$

$\sigma(A_F, Z_F)$ is the cross section for forming a fragment of charge Z_F and mass A_F

$F(A_0 - A_F)$ is the factor giving pion production dependence on mass loss $A_0 - A_F$

$r_s = r_0 A_F^{1/3}$ is assumed to be the most probable distance where the pion produced

$$\langle n_\pm \rangle_{\beta_F} = \pm\left(\frac{Z_F e^2}{\hbar c}\right) \frac{\mathrm{erf}\left[\beta_{\pi^\pm}/(\sqrt{2}\alpha_F)\right]}{\beta_{\pi^\pm}} .$$

3. Comparison with Data and Discussion

Figures 1 and 2 show comparisons with zero degree π^- of Sullivan et al.[1] The dashed curve is the direct result of eq. (16), and the solid curve is the direct result after folding in the experimental resolution. For the calculations, the fragment cross sections $\sigma(A_F, Z_F)$ were taken from the fast stage, before particle evaporation, of an abrasion-ablation firestreak code of Patrick McGaughey.[6] It was necessary for best agreement to use a fragment velocity dispersion parameter σ_0 smaller than the experimental value from inclusive fragmentation experiments.[5,7] That is, we use $\sigma_0 = 60$ MeV/c rather than the 86 MeV/c of refs. 5 and 7. Of course, the pion experimental data are sensing a different fragmentation velocity dispersion, that of the fast cascade (abrasion) product compound nuclei rather than the final products measured in the inclusive experiments. It may well be that the heavy ion abrasion stage knocks out correlated chunks of nucleons, leaving the primary fragments with narrow velocity dispersions. These narrow distributions are then broadened in the particle evaporation stage.

We found also that the pion source function $F(A_0 - A_F)$ needs to weight the heavier fragments (for the Ne data only) more than the first guess of simple linear dependence on fragment mass loss. That is, we used for the Ne data of Fig. 1 the function $F(A_0 - A_F) = (A_0 - A_F) - 0.06(A_0 - A_F)^2$. This quadratic term may be due to shadowing effects in pion production. That is, the observed pions must be formed near the surfaces of the nuclei, whereas fragment mass loss may be more nearly proportional to the number of nucleon-nucleon collisions throughout the nuclear volume.

We look forward to the next generation experiments in HISS, where the correlations of pions with specific heavy fragments will be measured.

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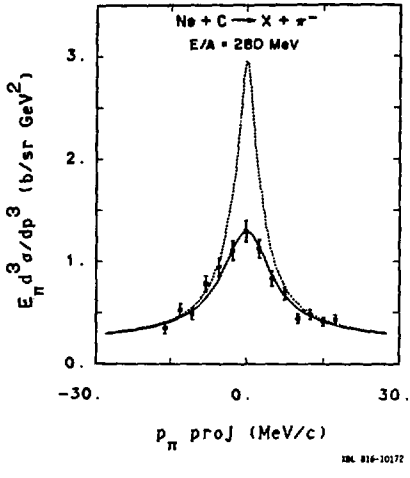

Fig. 1. Lorentz invariant π^- cross sections at zero degrees vs. pion momentume in the projectile frame for $^{20}Ne + {}^{12}C$. Points are data of ref. 1. The dashed (solid) curves are theoretical calculations with eq. (16) before (after) folding in experimental resolution.

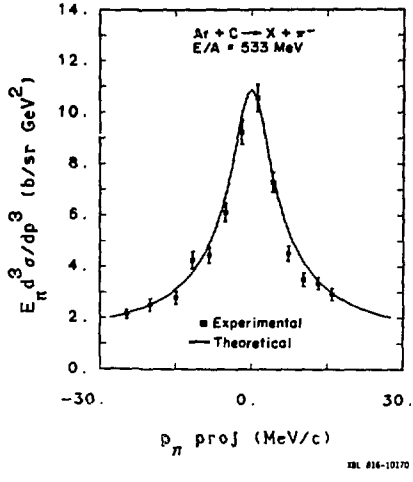

Fig. 2. Same as Fig. 1 except for $^{40}Ar + {}^{12}C$.