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MULTI-DIMENSIONAL CANONICAL/SYMPLECTIC MAPS FOR GYRO-RESONANCE CROSSINGS

Celso Grebogi and Allan N. Kaufman

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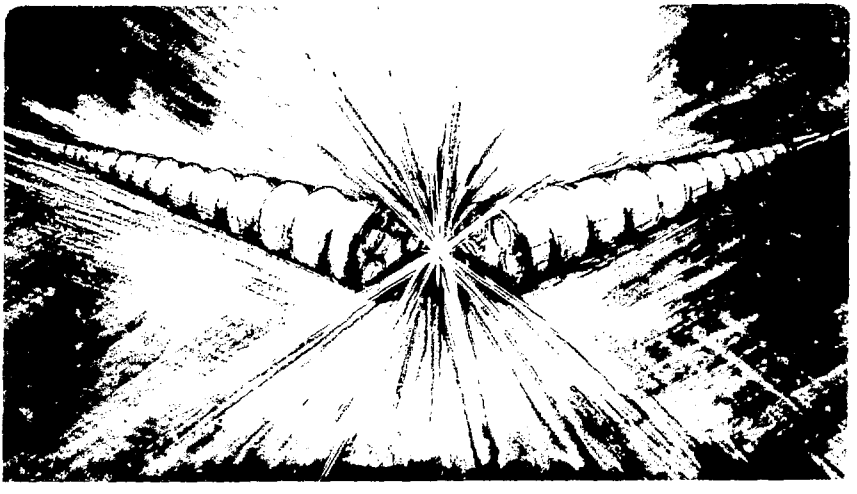

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Multi-Dimensional Canonical/Symplectic Maps for Gyro-resonance Crossings*

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ABSTRACT

The locally resonant interaction of a magnetically confined charged particle with a high-frequency electromagnetic wave can be modeled by a mapping of the particle's phase space. This mapping represents the jump in the dynamical variables caused by each passage through the resonance zone. Two formalisms are presented for mappings which preserve the phase-space structure: the Poincaré transformation, appropriate for canonical variables; and the Lie map, valid for any coordinate system on phase space. In both cases, the generating function for the map is the action integral, across the resonance, of the interaction Lagrangian.

In the non-resonant region of phase space, the interaction is transformed to higher order by standard Lie methods. Particle motion in this (adiabatic) region can then be represented also by a mapping. Consequently, the Hamiltonian flow in phase space can be replaced by iterates of a composite map, the product of the resonance and adiabatic maps.

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The key words for this conference are conservative and long-time prediction. In this talk, we interpret "conservative" to refer to the phase space of a Hamiltonian system (not to its energy), and we concentrate on developing a method, for evolution by means of iterated maps, which preserves the structure of that space. For "prediction," the requirement is reliability, and we maintain that any approximation method which destroys the structure of phase space cannot be reliable.

Our studies are motivated by a specific problem, of some importance in plasma physics [1]. Let there be a magnetostatic field, in which charged particles move with a complete set of 3 adiabatic invariants. Such fields characterize plasma confinement devices, such as tokamaks and mirror machines. One of the methods for heating the plasma is to apply an electromagnetic wave, of definite frequency ω, which resonates with particle motion in a restricted region, called the "resonance zone." As a particle crosses this zone, its adiabatic invariants and its energy suffer jumps, whose values depend on the relative phase of the particle motion and the wave. Outside the resonance zone, the particle oscillates non-resonantly in the field of the wave; there the invariants and Hamiltonian of the unperturbed particle motion are replaced by those of its oscillation center [2]. We call this the "adiabatic region".

The basic idea of our study is to replace the Hamiltonian differential equations, for continuous flow in phase space, by an iterated sequence of point mappings, which successively move a particle from one resonance crossing to the next. Our formalism for the mapping guarantees the preservation of phase-space structure. The generating

function for the resonance-crossing map is obtained by physically motivated perturbation methods from the action integral representing the effect of the wave.

There are two reasons for replacing a set of differential equations by an iterated mapping. From the analytic point of view, a mapping is often more amenable to further theoretical development than its equivalent set of differential equations. This has become especially apparent in the last decade in respect to studies of intrinsic stochasticity. From the computational point of view, a mapping is far more efficient, since it represents a large finite time interval.

Previous work along these lines has usually led to maps of a two-dimensional phase plane [3], which is required to be area-preserving, or, more generally, measure-preserving. For the six-dimensional phase space of a particle, the corresponding requirement is that the map be symplectic [4], i.e., that it preserve the "fundamental 2-form" of phase space, which reads (in canonical variables)

$$\Omega \equiv \sum_{i=1}^{3} dq_i \wedge dp_i \tag{1}$$

(The wedge $\wedge$ may be omitted by the reader unfamiliar with exterior algebra.) The use of canonical variables is sometimes inconvenient, and for a general phase-space coordinate system, Eq. (1) becomes

$$\Omega = \sum_{i,j=1}^{6} \Omega_{ij}\,(z)dz^i \wedge dz^j \tag{2}$$

To be specific, we consider first the motion of a particle in the unperturbed field $\underline{B}_0(\underline{x})$, $\phi_0(\underline{x})$. It has long been customary [5] to describe this motion in terms of gyration about a guiding center (g.c.), which tends to follow magnetic field lines, but slowly drifts across them. The standard g.c. equations do not reflect the underlying Hamiltonian structure of the exact system. Only recently has Littlejohn succeeded in developing a Hamiltonian theory for g.c. motion [6]; here the symplectic structure is precisely preserved, even though the Hamiltonian function itself is approximate. A further advantage of Littlejohn's approach is that the dynamical variables are physical, although non-canonical. Finally, by being a Hamiltonian theory, this approach is able to utilize the powerful Lie-transform methods [7] developed over the last few years.

The g.c. variables of Littlejohn's formalism are $\underline{X}$ (g.c. position in physical space, in any three-dimensional coordinate system) and P (g.c. parallel momentum). These four are non-canonical, with the Poisson brackets:

$$[\underline{X},\underline{X}] = (B^*)^{-1}\,\hat{b} \times \underset{\sim}{I}\ ,$$

$$[\underline{X},P] = (B^*)^{-1}\,\underline{B}^*; \tag{3}$$

$\hat{b}(\underline{X})$ is the unit vector along $\underline{B}_0(\underline{X})$,

$$\underline{B}^*(\underline{X},P) = \underline{B}_0(\underline{X}) + P\nabla \times \hat{b}(\underline{X}),$$

$$B^*(\underline{X},P) = \hat{b}(\underline{X}) \cdot \underline{B}^*(\underline{X},P).$$

These brackets (which are exact, not approximations) satisfy the Jacobi condition, and thus are a legal "symplectic" structure. For applications, they must be supplemented by a Hamiltonian function $H(\underline{X},P;t)$ and by a relation to the particle variables ($\underline{r}$, $\underline{v}$).

To illustrate the use of (3), consider the Hamiltonian $H(P) = \frac{1}{2}P^2$, representing the parallel kinetic energy alone. The equation of evolution under <u>any</u> H, for any function g(z) on phase space, is $\dot{g} = [g,H]$. For this Hamiltonian, we obtain $\dot{P} = 0$ and $\dot{\underline{X}} = (B^*)^{-1}\underline{B}^*P$. Thus, $\hat{b}\cdot\dot{\underline{X}} = P$; the parallel component of g.c. velocity equals the (invariant) g.c. momentum, for this Hamiltonian. The perpendicular part of $\dot{\underline{X}}$ is the centrifugal drift.

As another example, take as the Hamiltonian $H(\underline{X}) = \mu B_0(\underline{X})$, the gyration kinetic energy alone. (We discuss μ, the magnetic moment, shortly.) Now we obtain $\dot{P} = -(B^*)^{-1}\,\underline{B}^* \cdot \nabla B_0$, the mirror force; and $\dot{\underline{X}} = \mu(\hat{b} \times \nabla B_0)(B^*)^{-1}$, the ∇B drift.

As the final example, we take as our Hamiltonian the electrostatic energy $\phi_0(\underline{X})$. (Note that we set e=m=c=1 wherever convenient.) We now find $\dot{P} = -(B^*)^{-1}\,\underline{B}^* \cdot \nabla\phi_0$, and $\dot{\underline{X}} = (B^*)^{-1}\,\hat{b} \times \nabla\phi_0$, the parallel electric force and the electric drift.

In addition to the g.c. variables ($\underline{X}$, P), Littlejohn's formalism includes the canonical pair (μ,θ), the gyromomentum (magnetic moment) and the gyrophase. These are conjugate: $[\theta,\mu] = 1$, and commute with ($\underline{X}$,P). Let us examine their evolution under each of the above Hamiltonians. Since none of them depend on θ, the magnetic moment μ is invariant, as expected. However, for $H = \mu B_0(\underline{X})$, we obtain $\dot{\theta} = B_0(\underline{X})$; the gyrofrequency is given by the local magnetic field, as expected.

The total unperturbed Hamiltonian is (to lowest order) the sum of the three examples discussed above; it was instructive to examine them individually. Now we add the perturbation of an electromagnetic wave: $\delta\underline{E}(\underline{x},t) = \underline{\tilde{E}}(\underline{x}) \exp i[\psi(\underline{x}) - \omega t] + c.c.$, which we express in eikonal form. (In general, we expect a sum of such terms, but for clarity we consider only one term here.) The local wave-vector is $\underline{k}(\underline{x}) = \nabla\psi(\underline{x})$. The perturbation Lagrangian is

$$\delta L(\underline{r},\underline{v};t) = \underline{v}\cdot\delta\underline{A}(\underline{r},t) = (i\omega)^{-1}\ \underline{v}\cdot\underline{\tilde{E}}(\underline{r}) \exp i[\psi(\underline{r}) - \omega t] + c.c.$$

At this point it is necessary to know the relation between the (non-canonical) particle variables $(\underline{r},\underline{v})$ and the (non-canonical) g.c. variables $(\underline{X},P,\mu,\theta)$. This is expressed by Littlejohn [6a,6c] as a power series in the small parameter ε, representing the ratio of gyroradius to magnetic scale length. (We do not quote this relation here.) Upon substituting it into δL, we find (after some algebra) $\delta L(\underline{X},P,\mu,\theta,t) = \Sigma_m L_m(\underline{X},P,\mu) \exp i\psi_m(\underline{X},\theta,t) + c.c$, where the Fourier coefficient L_m is given by

$$L_m = J_m i^m\ \omega^{-1}\ \underline{\tilde{E}}(\underline{X})\cdot[P\hat{b} + (mB_0/k_\perp)\hat{k}_\perp + (2iB_0/k_\perp)(\partial \ell n\ J_m/\partial \ell n\mu)\hat{b} \times \hat{k}_\perp], \tag{4}$$

and the phase is

$$\psi_m(\underline{X},\theta,t) = \psi(\underline{X}) + m\theta - \omega t. \tag{5}$$

Here J_m is the Bessel function $J_m(k_\perp r_g)$ with gyroradius $r_g = [2\mu/B_0(\underline{X})]^{1/2}$, and $\hat{k}_\perp$ is the unit vector along $\underline{k}_\perp$, the projection of $\underline{k}(\underline{X})$ on the plane perpendicular to $\underline{B}_0(\underline{X})$.

Let us examine a <u>single</u> term of the perturbation Lagrangian :

$$L_m(\underline{X},P,\mu) \exp i\, \psi_m(\underline{X},\theta,t),$$

The crucial question is whether the phase is <u>rapidly varying</u> along an orbit or is <u>nearly stationary</u>. From (5), we have

$$\dot{\psi}_m = \underline{k}(\underline{X})\cdot\dot{\underline{X}} + m\dot{\theta} - \omega.$$

Using the <u>unperturbed</u> Hamiltonian for $\dot{\theta}$ and $\dot{\underline{X}}$, we see that the phase is stationary where

$$\omega - \underline{k}(\underline{X})\cdot\dot{\underline{X}}(\underline{X},P,\mu) = mB_0(\underline{X}); \qquad (6)$$

i.e., where the wave frequency in the local g.c. frame is an integer multiple of the local gyrofrequency. The gyro-resonance condition (6) is a 5-dimensional surface in the 6-dimensional g.c. phase space ($\underline{X}$, P, μ,θ). It is convenient to think of the unperturbed particle moving in that space, and repeatedly (and rapidly) crossing the gyro-resonance surface (6).

Where the phase ψ_m is rapidly varying, i.e., for the adiabatic regions of phase space, not near the resonance zone, the interaction can be transformed away [8]. This means that the g.c. motion is replaced by

oscillation center (o.c.) motion: the wave field produces rapid oscillations about the o.c., which moves (relatively) slowly under a new Hamiltonian, wherein the wave field appears quadratically, producing ponderomotive effects. The quadratic, or ponderomotive, term is proportional to $|L_m|^2 \, |\dot{\psi}_m|^{-2}$; thus it can be considered a weak perturbation (order $|\delta E|^2$), except in the resonance zone, where $\dot{\psi}_m \to 0$. We now examine the resonance zone, where the ponderomotive term diverges. We return to the non-resonant region later.

To first order in the wave amplitude, the perturbation Hamiltonian is the negative of the perturbation Lagrangian:

$$\delta H_m(\underline{X},P,\mu,\theta,t) = -L_m(\underline{X},P,\mu) \exp i\psi_m(\underline{X},\theta,t).$$

Each dynamical variable is affected by δH_m; let us examine μ, which is invariant under H_0. We have $\dot{\mu} = -\partial\delta H_m/\partial\theta = -\, im\delta H_m$; thus the invariance of μ is broken by the wave.

We wish to determine, for each dynamical variable z^i, its jump $\Delta z^i = z^{i\prime} - z^i$ to a new value $z^{i\prime}$, as a result of the resonance crossing. We shall work to lowest order in wave amplitude, but stress preservation of phase-space structure. Denote the value <u>at</u> the crossing by $\bar{z}^i = \frac{1}{2}(z^i + z^{i\prime})$, and the time dependence relative to that value by $\delta z^i(t) = z^i(t) - \bar{z}^i$.

Returning to μ, we see that the jump is $\Delta\mu = \int dt\, \dot{\mu} = -\int dt\, \partial\delta H_m(z,t)/\partial\theta = -\int dt\, \partial\delta H_m(\bar{z} + \delta z(t), t)/\partial\theta = -(\partial/\partial\bar{\theta}) \int dt\, \delta H_m(z,t) = +(\partial/\partial\bar{\theta}) \int \delta L\, dt$. We introduce the perturbation action integral:

$$S(\bar{z};\bar{t}) = \int \delta L\, dt,$$

and note that $\Delta\mu = \partial S/\partial\bar{\theta}$. Analogously, we calculate the resonance jump in gyrophase as $\Delta\theta = -\partial S/\partial\bar{\mu}$.

We evaluate S by the method of stationary phase, expanding the phase $\psi_m(t)$ about the stationary time $\bar{t}$, where $\dot{\psi}_m = 0$, i.e., at the resonance surface. With $\psi_m(t) = \psi_m(\bar{t}) + \frac{1}{2}(t-\bar{t})^2\,\ddot{\psi}_m(\bar{t}) + \ldots$, we obtain

$$S(\bar{z};\bar{t}) = (-2\pi/\ddot{\psi}_m)^{1/2}\, L_m(\bar{z})\, \exp i\, \psi_m(\bar{\underline{X}},\bar{\theta},\bar{t}). \tag{7}$$

(Validity conditions for the approximations made are discussed below.)

Regardless of the evaluation of S, or even of its expression in terms of the perturbation Lagrangian, the formulation of the jumps $\Delta\mu$, $\Delta\theta$ in terms of the derivatives of S guarantees the preservation of area in the μ,θ plane: $d\mu'\wedge d\theta' = d\mu\wedge d\theta$. (This can be verified with a bit of straightforward exterior algebra.) We now need to extend this preservation to the full 6-dimensional phase space, since each of the g.c. variables experiences a jump Δz^i.

For a <u>canonical</u> coordinate system (q,p), the simplest generalization is the <u>Poincaré transformation</u> [9]. For <u>any</u> function $S(\bar{q},\bar{p})$, it is again straightforward to verify that the jump relations:

$$\Delta p_i = \partial S/\partial\bar{q}_i \quad , \quad \Delta q_i = -\,\partial S/\partial\bar{p}_i \ , \tag{8}$$

together with $q_i = \bar{q}_i - \frac{1}{2}\Delta q_i$, $q_i' = \bar{q}_i + \frac{1}{2}\Delta q_i$ (and similarly for p_i), lead to preservation of the 2-form:

$$\sum dq_i' \wedge dp_i' = \sum dq_i \wedge dp_i \ .$$

To apply these ideas in our 3-degree-of-freedom case, two alternative approaches are possible. One approach is to impose the requirement of using canonical variables, since the Poincaré method is simply invalid for our non-canonical set $(\underline{X},P)$. Any canonical set will do, so one is guided by the physics of the particular problem being studied. A natural choice is the set of action-angle variables for the unperturbed g.c Hamiltonian $H_0(\underline{X}, P)$. Another possibility (unnatural, in our opinion), is the standard set of canonical particle coordinates $(\underline{r}, \underline{p})$. One must now express S, given explicitly in (7), in terms of the canonical set chosen. This would produce such a mess, that the beauty of the Littlejohn formalism is lost. For that reason we have turned to a second approach, which works for a non-canonical coordinate system as well.

In general terms, we ask what transformation $z \to z'$ preserves the fundamental 2-form. In (2), $\Omega_{ij}(z)$ is the (antisymmetric) Lagrange matrix, the reciprocal of the Poisson matrix $\sigma^{ij}(z) = [z^i, z^j]$. The Jacobi condition on Poisson brackets is equivalent to the requirement that Ω is "closed": $\partial\Omega_{ij}/\partial z^k + \partial\Omega_{jk}/\partial z^i + \partial\Omega_{ki}/\partial z^j = 0$ for all z and all (ijk). This is of course satisfied by Littlejohn's variables, for which (2) reads $\Omega = \underline{B}^*(\underline{X},P)\cdot d\underline{X} \times d\underline{X} - \hat{b}(\underline{X})\cdot d\underline{X}dP + d\mu d\theta$.

To answer the question above, we turn to the Lie map [10]. Again, let S(z) be <u>any</u> function on phase space; no particular coordinate system need be specified as yet. Now define the corresponding Lie transformation operator as $R = \exp[S,\cdot]$. Let g(z) be any "observable,"

i.e., some function (on phase space) of physical interest. Then g transforms under S as $g' = Rg = g + [S,g] + \frac{1}{2}[S,[S,g]] + \ldots$ The fundamental property of R is that Poisson bracket relations are preserved, i.e., if three functions g_1, g_2, g_3, satisfy $[g_1,g_2] = g_3$ for all z, then $[g_1', g_2'] = g_3'$. Let us now apply R to each coordinate z^i. Then $\Delta z = z^{i\prime} - z^i = [S(z), z^i] + \text{order } S^2$. Expressing z in terms of $\bar{z}$ and Δz, we obtain $\Delta z^i = [S(\bar{z}), \bar{z}^i] + \text{order } S^2$. For <u>canonical</u> variables, this reduces to (8) to lowest order; thus we see the relation of the two methods.

The Lie method has the great advantage of being coordinate-free, and of allowing the use of operational methods when one proceeds to iterate. At least two disadvantages are apparent to us: it is asymmetric with respect to the resonance surface, at least in its present form; for numerical computation, an infinite series is called for.

Before proceeding to examine the adiabatic region, we summarize the treatment of the resonance zone. For any observable, and in particular for the dynamical variables, the jump across the resonance is given by $g' = Rg$ with $R = \exp [S,\cdot]$ and S given by Eq. (7). In that equation, the phase ψ_m is given by Eq. (5), and the amplitude L_m by Eq. (4).

In the adiabatic region, we shall use the unperturbed Hamiltonian (in the g.c. variables) $H_0(\underline{X},P,\mu) = \frac{1}{2}P^2 + \mu B_0(\underline{X}) + \phi_0(\underline{X})$, each term of which was discussed above. (The ponderomotive correction, quadratic in the amplitude, will be neglected here, but must be considered eventually). We look for a complete set of 3 invariants. As discussed earlier, μ is one invariant, and H_0 (being time-independent) is a second. The existence of a third invariant $I(\underline{X},P,\mu)$ is guaranteed

(at least locally, i.e., between resonances) by the Darboux theorem [4,6], which also presents an algorithm for its construction. For an axisymmetric system like a tokamak, it can be identified with the angular momentum (and then is μ-independent). In a mirror machine, the bounce action or drift flux will serve as the third invariant.

In the 6-dimensional phase space $(\underline{X}, P, \mu, \theta)$, the oscillation center orbit lies on each of the three (5-dimensional) invariant surfaces μ, $H_0(\underline{X}, P, \mu)$, $I(\underline{X}, P, \mu)$. It thus lies on the intersection $\mu \cap H_0 \cap I$, which is three-dimensional.

To help us visualize the orbit, let us first project out the (μ, θ) variables, reducing the phase-space to four dimensions. Then let us postulate axial symmetry (for simplicity) and project out the toroidal angle ϕ, leaving three dimensions: P and the poloidal coordinates, which we denote α, β. In this space (α, β, P), the two invariant surfaces are $H_0(\alpha, \beta, P) = E$ and $I(\alpha, \beta, P) = p_\phi$. These two-dimensional surfaces intersect in a curve $E \cap p_\phi$, along which the oscillation center moves until it reaches the resonance surface $\dot{\psi}_m(\alpha, \beta, P) = 0$. (In practice, the latter surface usually depends also on ϕ, so one must do the analysis in a higher-dimensional space, where one's geometric intuition may fail.)

For each of the three invariants of the adiabatic region, one can introduce a conjugate variable whose time-derivative is constant along the orbit. One then constructs a relatively trivial symplectic map, denoted A, for the adiabatic motion. (Its explicit form should be tailored to the problem at hand.) This map then takes a point in phase space from one resonance crossing to the next. The map A depends

parametrically on the three invariants of the adiabatic motion, and advances the conjugate variables and time.

Upon crossing the next resonance, all the variables jump (both the "adiabatic" invariants and their conjugates) according to the resonance mapping R. The new values of the invariants are then inserted into A for the next adiabatic mapping.

The particle motion is thus represented as a product of alternating symplectic mappings RARA ... = $(RA)^n \equiv T_n$ for n resonance crossings separated by adiabatic motion. The n-fold composite T_n is itself symplectic, since symplectic maps have the group property.

With a formal expression for T_n, one may next inquire into the asymptotic behavior of its orbits, as $n \to \infty$. Since a symplectic map is the multi-dimensional generalization of an area-preserving map, the critical questions are the same, suitably generalized: Is the motion ergodic, chaotic, mixing [11]? What are the Liapunov characteristic exponents [12], and how do they depend on the parameters of the problem, e.g., wave amplitude? What are the diffusion rates for the invariants [13]? Hopefully, operational methods [10] will help provide answers to these important questions.

We now return to the validity criteria for the resonance-crossing maps. In the first place, the wave amplitude must be sufficiently weak that first-order perturbation theory is adequate, since the action integral is evaluated using unperturbed orbits. Physically, this means that the crossing must be sufficiently rapid that the particle cannot get trapped by the resonance. In the second place, the eikonal conditions must allow for the evaluation of the phase integral by the

stationary phase method; this requires $|\dot{\psi}_m|^{1/2} \gg |\ddot{\psi}_m|^{1/3}$. Physically, we require sufficiently slow spatial variation of wave amplitude and wave vector on the gyro-radius scale, as well as rapid passage through resonance. In the third place, the resonances should be sufficiently disjoint in phase space so that the adiabatic regions are well defined. In effect, the crossing time should be short compared to the time between resonances. (Note that Chirikov's "resonance overlap" criterion [3] refers to global resonances, not to the local resonances discussed here. Global resonances can overlap, producing stochasticity, even though the local resonances are disjoint.)

We conclude by outlining elements of a program for future work in this area:

1. A specific model should be chosen, and the map formalism explicitly implemented.
2. The orbits for the iterated map should be compared to those for the exact differential equations of motion [14].

(These first two steps have been carried out by us in earlier work for the case of spatial variation in one dimension [15]. We found quantitative agreement for regular orbits, and qualitative agreement for chaotic orbits, as expected.)

3. Because the irreversible resonant diffusion is responsible for entropy production, it should be possible to extend the quasilinear relation [16] between the diffusion tensor and the dissipative part of the dielectric susceptibility to the nonlinear regime.
4. At some stage, self-consistency for the electromagnetic perturbation should be introduced. The most promising vehicle for this is the Hamiltonian Vlasov-Maxwell theory discovered by Morrison [17].

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(b) G. Benettin, L. Galgani, A. Giorgilli, J. P. Strelcyn, "Tous les Nombres Caracteristiques de Lyapounov sont effectivement calculables," Compt. Rend. Acad. Sci. Paris 286A, 431 (1978);
(c) ——, "Lyapunov Characteristic Exponents for Smooth Dynamical Systems and for Hamiltonian Systems; a Method for Computing all of them," Meccanica, 15, 9, 21 (1980).

13. R. H. Cohen and G. Rowlands, "Calculation of Resonant Transport Coefficients from Mappings," submitted to Phys. Fluids, preprint UCRL-85600 (1981).

14. This comparison should take account of the important studies of:
G. Benettin, M. Casartelli, L. Galgani, A. Giorgilli, and J. Strelcyn, "On the Reliability of Numerical Studies of Stochasticity," Nuovo Cimento 44B, 183 (1978); 50B, 211 (1979).

15. C. Grebogi and A. Kaufman, "Multiple Resonance Crossing in the Eikonal Model of Plasma Turbulence," LBL-12805, in preparation.

16. This relation is explored in:
A. N. Kaufman, "Quasilinear Diffusion of an Axisymmetric Toroidal Plasma," Phys. Fluids 15, 1063 (1972).

17. (a) P. J. Morrison, "Maxwell-Vlasov Equation as a Continuous Hamiltonian System," Phys. Letters 80A, 383 (1980).
The mathematical basis is presented by:
(b) J. Marsden and A. Weinstein, "Hamiltonian Structure of the Maxwell-Vlasov Equation," submitted to Physica D (1981).