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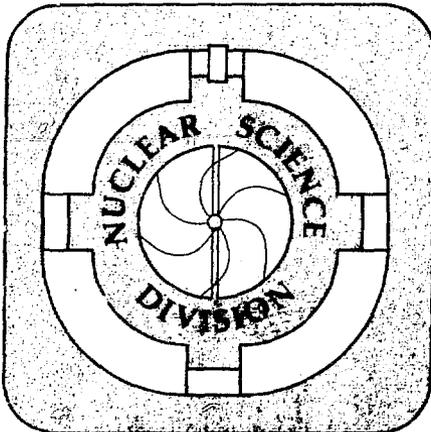
MASTER

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FUSION CROSS^gSECTIONS AND THE NEW DYNAMICS

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Abstract

The prediction of the need for an extra push over the interaction barrier in order to make the heavier nuclei fuse is made the basis of a simple algebraic theory for the energy-dependence of the fusion cross-section. A comparison with recent experiments promises to provide a quantitative test of the "New Dynamics" (the theory of macroscopic nuclear shape evolutions based on the one-body dissipation concept).

$$(Z^2/A)_{\text{eff}} \equiv 4Z_1 Z_2 / A_1^{1/3} A_2^{1/3} (A_1^{1/3} + A_2^{1/3}) \quad (2)$$

and $(Z^2/A)_{\text{eff}}^{\text{thr}}$ is a pure number, specifying the threshold value of the effective fissility, beyond which an extra push is needed. The quantity (L/L_{ch}) is the angular momentum of the system, L , in units of a characteristic angular momentum, given by

$$L_{\text{ch}} = \frac{e\sqrt{m}r_0}{2f} \frac{A_1^{2/3} A_2^{2/3} (A_1^{1/3} + A_2^{1/3})}{\sqrt{A_1 + A_2}} \quad (3)$$

1. Introduction

For relatively light nuclear systems one would expect the interaction-barrier configuration of two tangent nuclei to be driven automatically toward fusion by the cohesive nuclear forces. For sufficiently heavy systems, or for systems with sufficient angular momentum, however, the electric repulsion as well as the centrifugal force would be expected to prevent automatic fusion. An extra bombarding energy over the interaction barrier--an extra push--would then be required to overcome the relevant saddle point in configuration space and achieve fusion.

2. Nuclear Fusion according to the New Dynamics

These qualitative expectations were analyzed^{1,2,4)} using a schematic model based on the "New Dynamics" obtained by combining the electrostatic and surface tension forces with the "one-body" nuclear dissipation (a type of viscosity appropriate, under certain assumptions, for an assembly of particles whose mean free paths are long, rather than short, compared to the size of the system^{2,3)}). In the schematic model the nuclear shapes were parametrized as two spheres connected by a portion of a cone.

A result of those studies, which follows largely on dimensional grounds (given the structure of the New Dynamics together with an approximation exploiting the relative smallness of the neck between the two nuclei), is that the kinetic energy in the radial (or approach) degree of freedom, i.e., the radial injection energy E_r over the interaction barrier, necessary to overcome the saddle point for fusion, should have the following approximate appearance:

$$E_r = \left\{ \begin{array}{l} 0, \text{ for } (Z^2/A)_{\text{eff}} + (L/L_{\text{ch}})^2 < (Z^2/A)_{\text{eff}}^{\text{thr}} \\ K \left[(Z^2/A)_{\text{eff}} + (L/L_{\text{ch}})^2 - (Z^2/A)_{\text{eff}}^{\text{thr}} \right]^2 + \\ \text{higher powers of the square bracket,} \\ \text{for } (Z^2/A)_{\text{eff}} + (L/L_{\text{ch}})^2 > (Z^2/A)_{\text{eff}}^{\text{thr}} \end{array} \right\} \quad (1)$$

In the above, $(Z^2/A)_{\text{eff}}$ is the effective fissility of the colliding nuclear system, defined by

In the above equations, Z_1, Z_2, A_1, A_2 are the atomic and mass numbers of the two colliding nuclei, m is the nuclear mass unit (taken as 931 MeV/c²), r_0 is the nuclear radius constant (taken as 1.2249 fm), e is the proton charge, and f is the effective "angular momentum fraction", i.e., that fraction of the total angular momentum which is responsible for the centrifugal force in the separation degree of freedom^{1,2,4,5)}. This force, as represented by the term $(L/L_{\text{ch}})^2$, along with the electric repulsion, proportional to $(Z^2/A)_{\text{eff}}$, opposes capture inside the fusion saddle point and calls for an increased injection energy according to eq. (1). (For approaching nuclei $f = 1$; for two spheres rolling on each other without sliding $f = 5/7$. The use of a fixed effective value of f represents a rough attempt to handle the actually intricate problems associated with the presence of angular momentum.)

The constant K , which specifies how rapidly the extra push increases with excess over the threshold condition, follows from the model in ¹⁾ as

$$K = \frac{A_1^{1/3} A_2^{1/3} (A_1^{1/3} + A_2^{1/3})^2}{A_1 + A_2} \frac{32}{2025} \left(\frac{3}{\pi} \right)^{2/3} \left(\frac{e^2}{\hbar c} \right)^2 m c^2 a^2, \quad (4)$$

where a is a pure number (equal to about 5 in the schematic model of ¹⁾). We will refer to K as the "thud wall stiffness coefficient". The reason for the name is that, because of the large absolute magnitude of the one-body dissipation, most of the extra push is dissipated soon after contact in a "thud" and a "clutch". Hence the fusion of systems with effective fissilities exceeding appreciably the threshold value is opposed by a large "thud wall"--see Fig. 13 in ¹⁾. The pure number a , independent of A_1, A_2 , will be called the "thud wall slope coefficient".

Even without describing the workings of the schematic model used to derive the above expressions, I hope that the general idea is clear: when the electric and centrifugal forces exceed a certain threshold value, an extra push is obviously necessary for fusion. This simple physical idea was incorporated in a schematic model based on the New Dynamics, and the structure of the extra push expression, derived to lowest order in the excess over the threshold condition, came out as eq. (1). Since the one-body dissipation theory has no

adjustable parameters (there is no adjustable viscosity coefficient), there are only natural constants and pure numbers in the resulting thr equations. However, the pure numbers $(Z^2/A)_{\text{eff}}$ and a do depend on the approximations of the schematic model, in particular on the parametrization of the nuclear shapes by spheres connected by a conical neck. Also, the factor f is an effective angular momentum fraction, expected to be somewhat less than unity, but not known precisely. So, in addition to taking these numbers from some schematic model, one may also want to treat them as adjustable parameters when comparing the general structure of the theory with experiment.

3. The Extra Push Theorem

In a collision between two spheres the cross-section for just barely making contact (the reaction cross-section) is given by the standard formula

$$\sigma = \pi r_c^2 \left(1 - \frac{E_B}{E}\right), \quad (5)$$

which can also be written as

$$\frac{\sigma E}{\pi r_c^2} = E - E_B, \quad (6)$$

where E_B is the potential energy at contact (the "interaction barrier"), and r_c is the center separation at contact. (For sharp spheres it is just the sum of the radii).

Equations (5,6) follow simply from conservation of energy (and angular momentum). Thus the right-hand side of eq. (6) is the energy excess over the interaction barrier, equal, by conservation of energy, to the tangential (orbital, or rotational) energy at contact (the left-hand side). To verify this, write

$$L^2 = [(mass)(velocity)(moment arm)]^2 = [M\sqrt{2E/Mb}]^2 \\ = 2MEb^2 = 2MEa/\pi, \quad (7)$$

where M is the reduced mass and b the impact parameter, so that $\sigma E/\pi r_c^2$ equals $L^2/2Mr_c^2$, the rotational energy at contact.

Now when one asks for making contact not "just barely", but with a finite radial energy E_r --just sufficient to ensure fusion--the energy-conservation equation (6) is replaced by

$$\frac{\sigma E}{\pi r_c^2} + E_r = E - E_B. \quad (8)$$

Using for E_r eq. (1) (together with eq. (7) to eliminate L^2) one readily verifies that eq. (8) may be rewritten as

$$\frac{\sigma E}{\pi r_c^2} + (c_1 + c_2 \frac{\sigma E}{\pi r_c^2})^2 = E - E_B. \quad (9)$$

where

$$c_1 = \sqrt{K} \left[(Z^2/A)_{\text{eff}} - (Z^2/A)_{\text{thr}} \right], \quad (10)$$

$$c_2 = \frac{\sqrt{K}}{e^2/r_0} \frac{8f^2}{A_1^{1/3} A_2^{1/3}}. \quad (11)$$

Denoting by Σ the energy-weighted reduced cross-section, i.e., the cross-section in units of πr_c^2 , multiplied by the energy E , and calling the deviation of $\sigma E/\pi r_c^2$ from the standard result, $E - E_B$, the "cross-section defect" Δ , where

$$\Delta = E - E_B - \frac{\sigma E}{\pi r_c^2}, \quad (12)$$

we may state the content of the (energy-conservation) equation (9) in the form of the following compact Extra Push Theorem:

"When an extra radial injection energy is needed for fusion, the square root of the cross-section defect should be approximately linear when plotted against the energy-weighted reduced cross-section, viz.:

$$\sqrt{\Delta} = c_1 + c_2 \Sigma + \dots" \quad (13)$$

Thus, by plotting the square root of the experimental values of $E - E_B - \sigma E/\pi r_c^2$ versus $\sigma E/\pi r_c^2$, one should find, approximately, a straight line, with intercept c_1 and slope c_2 . Plotting the ratios c_1/c_2 , multiplied by $Br_0^2 e^2 A_1^{1/3} A_2^{1/3}$, for a series of systems versus the systems' effective fissilities $(Z^2/A)_{\text{eff}}$ should give, according to eqs. (10,11), a straight line with slope $1/f^2$ and intercept $(Z^2/A)_{\text{thr}}/r^2$. Hence follow the three parameters of the theory: the thud wall slope coefficient a , the threshold value of the effective fissility $(Z^2/A)_{\text{eff}}$ and the angular momentum fraction f .

4. Comparison with Experiment

The above analysis was applied to the recent measurements in 4_1 , where a beam of ^{209}Bi was made to interact with seven targets: ^{26}Mg , ^{27}Al , ^{48}Ca , ^{50}Ti , ^{52}Cr , ^{58}Fe and ^{64}Ni . (Fusion in this context means reactions resulting in final fragments with a mass distribution centered around symmetry.)

Figure 1 shows a comparison of the measured fusion cross-sections with theory. The solid curves refer to the standard formula (5) and the dashed curves to the extra push prediction, obtained by solving the quadratic eq. (9) for σ :

$$\sigma = \frac{\pi r_c^2}{E} \left[\sqrt{\left(\frac{c_1 c_2 + 1/2}{c_2^2} \right)^2 - \left(\frac{c_1^2 + E_B - E}{c_2^2} \right)} - \left(\frac{c_1 c_2 + 1/2}{c_2^2} \right) \right]. \quad (14)$$

In constructing the dashed curves in Fig. 1 we took r_c to be the sum of the central radii of the two nuclei, augmented by 1.14 fm to take approximate account of the diffuseness of the nuclear surfaces (this choice reproduces the initial slopes of the reaction cross-sections for ^{26}Mg and ^{27}Al in Fig. 1). Thus:

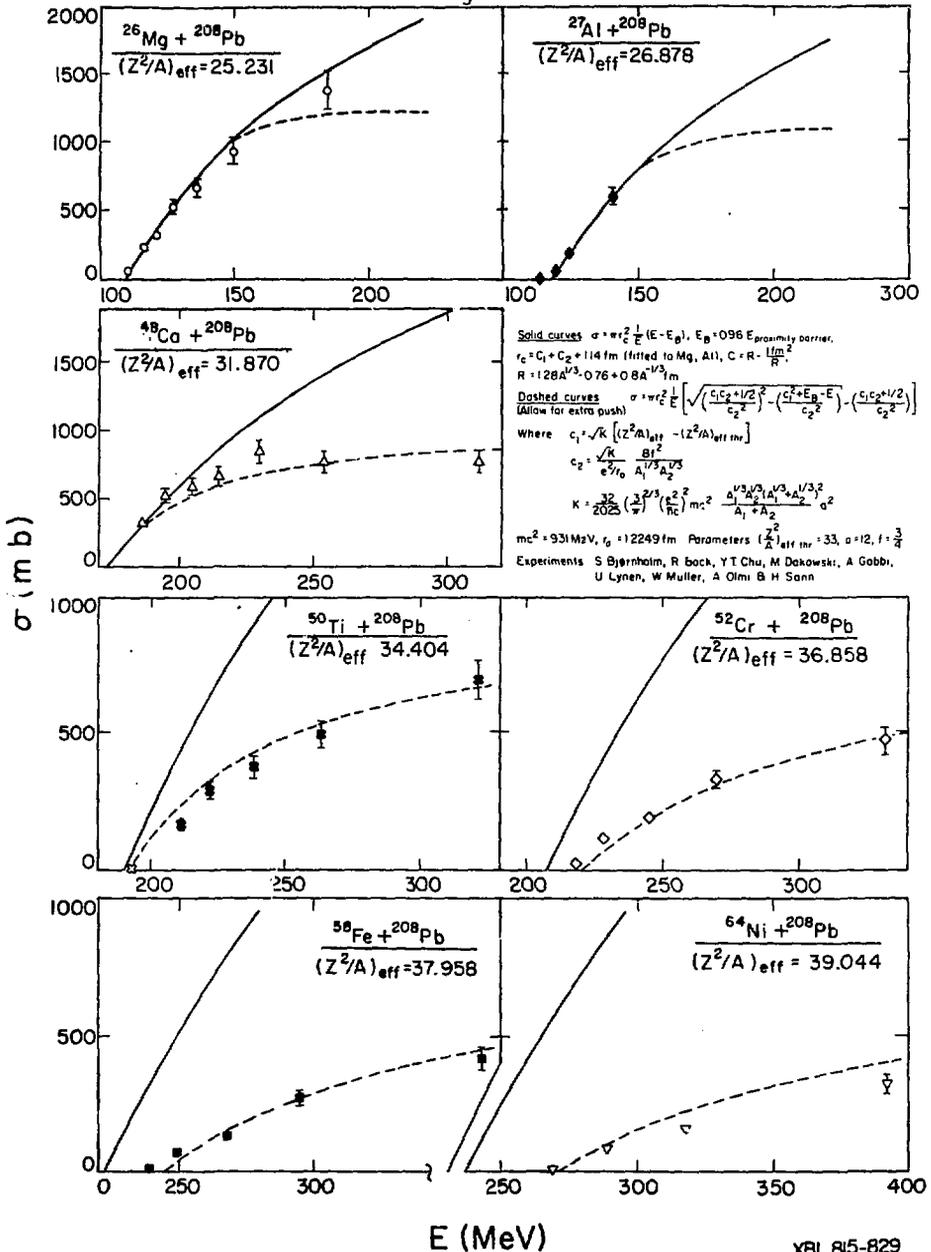


Fig. 1. Comparison of experimental fusion cross-sections (associated with outgoing fragment masses centered around symmetry) with theory. The solid curves are conventional reaction cross-section predictions, and the dashed curves incorporate the requirement of an extra push in the approach degree of freedom. (I deduced the data points from Ref. (4) and added purely nominal 10% error bars.)

$$r_c = C_1 + C_2 + 1.14 \text{ fm} ,$$

$$C = R - 1 \text{ fm}^2/R ,$$

$$R = 1.28 A^{1/3} - 0.76 + 0.8 A^{-1/3} \text{ fm} .$$

For the interaction barrier E_B we used the barrier following from the proximity interaction in B , reduced by 4% γ .

The three parameters of the theory were found to have the following approximate values:

$$(Z^2/A)_{\text{eff}}^{\text{thr}} \approx 33 \pm 1 , \quad (15a)$$

$$a \approx 12 \pm 2 , \quad (15b)$$

$$f \approx (3/4) \pm 10\% . \quad (15c)$$

The deviation of the 33 ± 1 in eq. (15a) from the value 26-27 suggested by the schematic model in $1)$ correlates quantitatively with the deviations of the schematic model's saddle-point shapes from the accurately known macroscopic shapes $8)$. The same is true qualitatively of the deviation of $a \approx 12 \pm 2$ in eq. (15b) from a ≈ 5 , suggested in $1)$. The value of f suggested by eq. (15c) is intermediate between the value appropriate to approaching spheres and to spheres rolling on each other without sliding. (The clutching stage is being investigated within the framework of the one-body dissipation theory by G. Fai, private communication).

5. Conclusions

The degree of correspondence between theory and experiment in Fig. 1 leaves a lot to be desired, and the significance of the correspondence is by no means clear. More work will have to be done on filling in, extending and rechecking the measurements, and on making more nearly quantitative calculations along the lines of $10)$. But it seems that, by measuring the fusion cross-sections, one has available another method of testing quantitatively the predictions of the one-body dissipation theory. When combined with other tests (in particular on evaporation-residue cross-sections, where the theory predicts the need of an "extra-extra push" $1,9)$ and by extending existing tests in the context of deep-inelastic collisions and fission $2,3,10)$, a confrontation of theory and experiment may be achieved which will be sufficiently broad to delineate quantitatively the degree of validity of the New Dynamics.

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