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AEC Contract No. W-7405-eng-48

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PARTICLE-WAVE AMBIGUITIES IN THE INTERPRETATION OF HEAVY-ION REACTIONS†

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October 1972

Abstract

The correlation between the partial wave scattering amplitudes in ℓ-space and the reaction cross section angular distributions are derived in the limits of classical and diffractive scattering. Several ambiguities are noted and illustrated with DWBA calculations for "quasi-elastic" heavy-ion reactions.

I. Introduction

Most direct heavy-ion reactions are characterized by a simple angular distribution which is peaked about an angle θ_{gr} [1]. The observed dependence of θ_{gr} on the energy and charge of the incident ion leads to the interpretation of these reactions as 'grazing reactions', in which the projectile and target move on classical trajectories such that the ions' surfaces just touch [2]. The reaction is limited to such trajectories by two effects: if the ions pass

†Work performed under the auspices of the U. S. Atomic Energy Commission.

farther from each other, the interactions between them are weaker and less likely to induce the reaction, while if the ions approach each other too closely, competition from other processes such as compound-nucleus formation, reduces the direct cross section. Alternatively, the 'grazing reactions' may be thought of as being limited to a narrow range of angular momentum of the projectile and target [3].

One finds, however, that the parameters obtained using various models often differ substantially and cannot be simply related. This makes the physical meaning of such parameters very uncertain. In this paper we present an explanation for some of these apparent ambiguities and give an illustration using DWBA.

II. Theory

For simplicity we shall consider a "quasi-elastic" reaction, i.e., one in which the angular momentum-, energy-, and mass-transfers are negligible. The DWBA reaction amplitudes [4] may then be conveniently represented by a decomposition into partial-wave amplitudes f_ℓ, which represent the overlap of distorted waves $\chi_\ell^{(-)}(r)$, $\chi_\ell^{(+)}(r)$ with a form factor

$$f_\ell = \int dr\ \chi_\ell^{(-)*}(r)\ \mathcal{F}(r)\ \chi_\ell^{(+)}(r) \tag{1}$$

$$= e^{2i\delta_\ell}\ F((\ell - \ell_{gr})/\Delta) \tag{1a}$$

In eq. (1a), a separation has been made between the rapidly-varying phase of f_ℓ, and an amplitude factor F which is peaked smoothly within a width Δ of an ℓ-value, ℓ_{gr}. The amplitude $f(\theta)$ is given by

$$f(\theta) = \frac{1}{2ik} \sum_{\ell} (2\ell + 1)\, f_\ell\, P_\ell(\cos\theta) \tag{2}$$

The largest contributions to $f(\theta)$ for heavy ion reactions come from partial-wave amplitudes $\ell \gg 1$. Therefore, we can approximate the sum over ℓ in eq. (2) by an integral and use the asymptotic expression for $P_\ell(\cos\theta)$ to obtain

$$f(\theta) = (-2\pi i k^2 \sin\theta)^{-1/2} \int d\ell (\ell + \tfrac{1}{2}) F((\ell - \ell_{gr})/\Delta)$$
$$e^{2i\delta_\ell} (e^{i(\ell + 1/2)\theta} + i e^{-i(\ell + 1/2)\theta}) \tag{2a}$$

The integral in (2a) can be evaluated in two simple limiting cases. One is the classical limit, in which F varies sufficiently slowly that the integral is dominated by angular momenta near the classical value $\ell_{cl}(\theta)$ corresponding to scattering through an angle θ,

$$d\delta_\ell/d\ell = -\theta/2 \quad \text{for} \quad \ell = \ell_{cl}(\theta) \quad . \tag{3}$$

This limit is obtained when Δ is large:

$$\Delta^2 \gg 2/\delta_{min}{}^2$$

where

$$\delta^2_{min} = -\frac{d\theta}{d\ell_{cl}} \tag{4}$$

The cross section is then given by the classical formula,

$$\sigma_{cl}(\theta) = |f(\theta)|^2 = \left[\frac{\ell_{cl} + 1/2}{k^2 \sin\theta} \frac{d\ell_{cl}}{d\theta} \right] |F((\ell_{cl}(\theta) - \ell_{gr})/\Delta)|^2 \qquad (5)$$

The expression in square brackets is just the classical cross section determined by the relation (3) between ℓ_{cl} and θ, so that $|F|^2$ can be interpreted as the probability of the reaction occuring when the ions collide on a classical path leading to the angle θ. The width of the peak in the angular distribution is directly proportional to Δ.

The other limit in which the cross section takes a simple form is the diffraction limit, where Δ is small:

$$\Delta^2 << 2/\delta_{min}{}^2 \qquad (6)$$

The contributions to the integral in (2) then come from the peak in F and the cross section is

$$\sigma_{diff}(\theta) = \frac{\Delta^2(\ell_{gr} + 1/2)}{2\pi k^2 \sin\theta} |\tilde{F}(\Delta(\theta - \theta_{gr})|^2 \qquad (7)$$

where $\tilde{F}$ is the fourier transform of F. This is the result obtained by Frahn and Ventner [3]. The width of the peak in the angular distribution is <u>inversely</u> proportional to Δ.

Furthermore, one may show that the width of $\sigma(\theta)$ is a minimum for Δ between the limits given by (4) and (6). The parameter Δ is therefore model dependent and not uniquely determined by the shape of $\sigma(\theta)$ since both the

diffraction and classical models can give similar shapes for $\sigma(\theta)$, but for very different parameters. In DWBA one can satisfy either limit (4) or (6) by suitable adjustment of the form factor, $\mathcal{F}(r)$, or the distorted waves $\chi^{(\pm)}(r)$, the latter by adjustment of the optical potentials.

III. Calculations

We illustrate the ambiguities noted above in figs. 1 and 2.

In fig. 1 we show the results of DWBA calculations for $^{208}Pb(^{16}O,^{16}O)$. We have used a purely absorptive optical potential of Woods-Saxon shape with $W = -15$ MeV, $R_I = 11$ fm, and $a_I = 0.5$ fm. The absence of a real potential allows one to use the classical relations between l, θ, r for Coulomb trajectories, but otherwise does not alter the qualitative results deduced from calculations. The form factor was taken to be a derivative of a Woods-Saxon shape (see fig. 2) with a variable width and peak radius (=R). Plotted in fig. 1 is the FWHM of the calculated $\sigma(\theta)$ vs. the FWHM of the form factor for two values of R. One finds two solutions which give the same width for $\sigma(\theta)$: FWHM $\mathcal{F}(r) \lesssim \lambda$ (classical limit), and FWHM $\mathcal{F}(r) \gg \lambda$ (diffraction limit), where λ is the wave length of the projectile.

In fig. 2 we show DWBA calculations obtained with two very different form factors which, however, give similar shapes for $\sigma(\theta)$. Also shown is the quantity $|\beta_L^M|^2 \propto f_\ell^* f_\ell = F_\ell^* F_\ell$ vs. ℓ, where L is the angular momentum transfer ($L = M = 0$). The decrease in $|\beta_L^M|^2$ for small ℓ values is due to the decrease of $|\beta_L^M|^2$ arising from the absorptive potential, $W(r)$, $r \sim R_I$ ($\ell_{r=R_I} = 30$). Similarly, the shape of $\sigma(\theta)$ at large angles is sensitive to the optical potential (which was not adjusted). At forward angles the calculations are nearly identical, even though the asymptotic parts of the form factors are quite different.

Results similar to those shown in figs. 1 and 2 were obtained for angular momentum (L), energy (Q), and mass transfers typical of many heavy ion reactions.

IV. Conclusions

We conclude from this study that

(i) Classical and diffraction models can give similar shapes for heavy ion reaction angular distributions but often this will require very different parameters.

(ii) DWBA calculations exhibit similar ambiguities in that the shape of $\sigma(\theta)$ does not uniquely determine the shape of the form factor even in the asymptotic region $r \to \infty$.

Of course, if one calculates the form factor from some nuclear model and obtains the distorting potentials from other sources (e.g. an optical model analysis of elastic scattering) then one apriori determines which behaviour, classical or diffractive (particle or wave), will dominate (if either). Lacking such a prescription, however, can result in an ambiguous and unphysical determination of parameters.

References

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Figure Captions

Fig. 1. The variation of the width (FWHM) of the DWBA angular distribution *vs*. the width (FWHM) and peak ($r = R$) of the form factor (see fig. 2). λ is the projectile wavelength ($r = \infty$).

Fig. 2. DWBA calculations for a "quasi-elastic" reaction for two different form factors (a,b). A Woods-Saxon optical potential was used with $V = 0$, $W = -15$ MeV, $R_I = 11$ fm, and $a_I = 0.5$ fm. The quantity $|\beta_L^M|^2$ is proportional to $f_\ell^* f_\ell$ (see text).

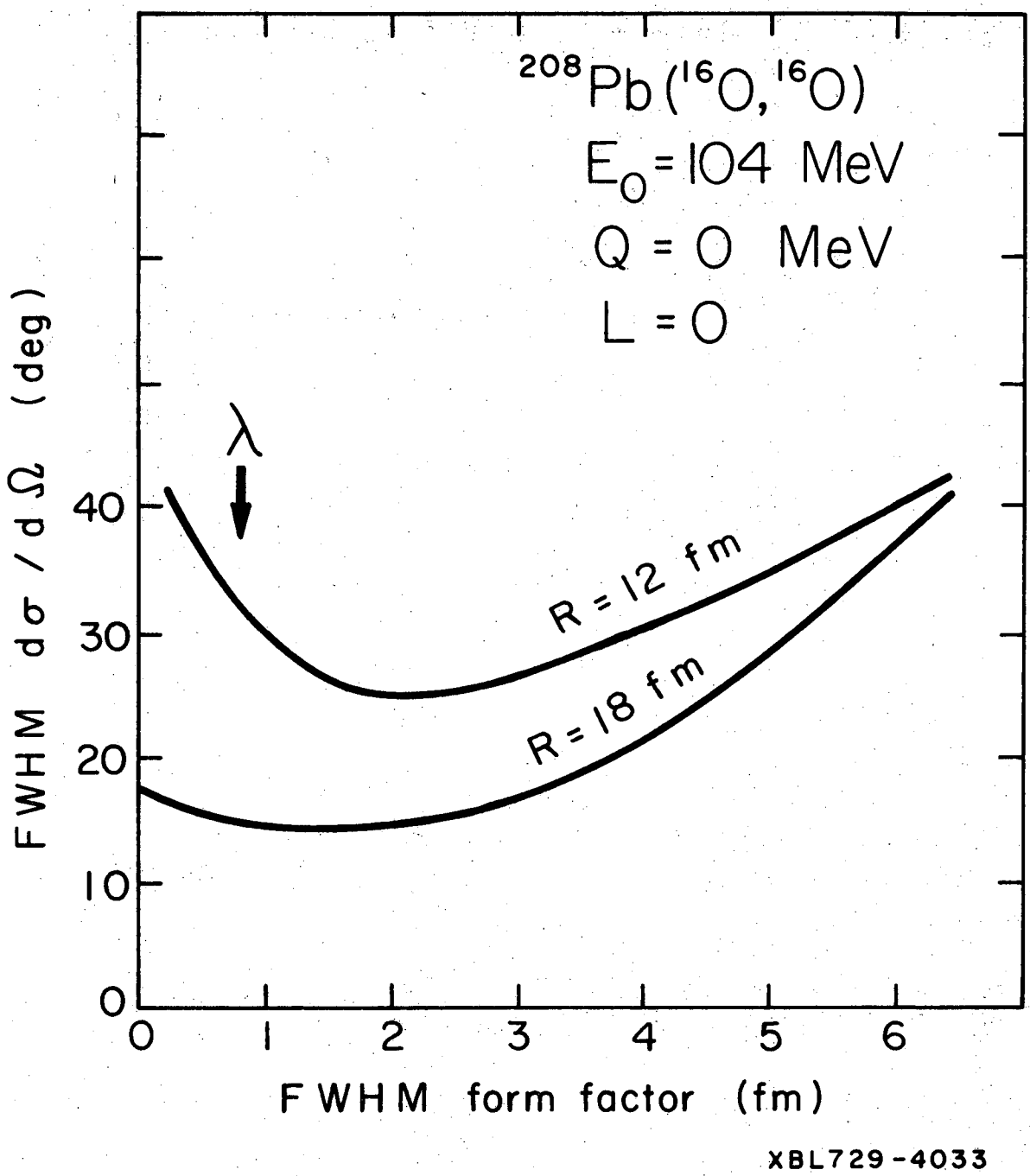
$^{208}Pb(^{16}O,^{16}O)$
$E_0 = 104$ MeV
Q = 0 MeV
L = 0
λ
R = 12 fm
R = 18 fm
FWHM dσ/dΩ (deg)
0
10
20
30
40
0
1
2
3
4
5
6
FWHM form factor (fm)
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Fig. 1

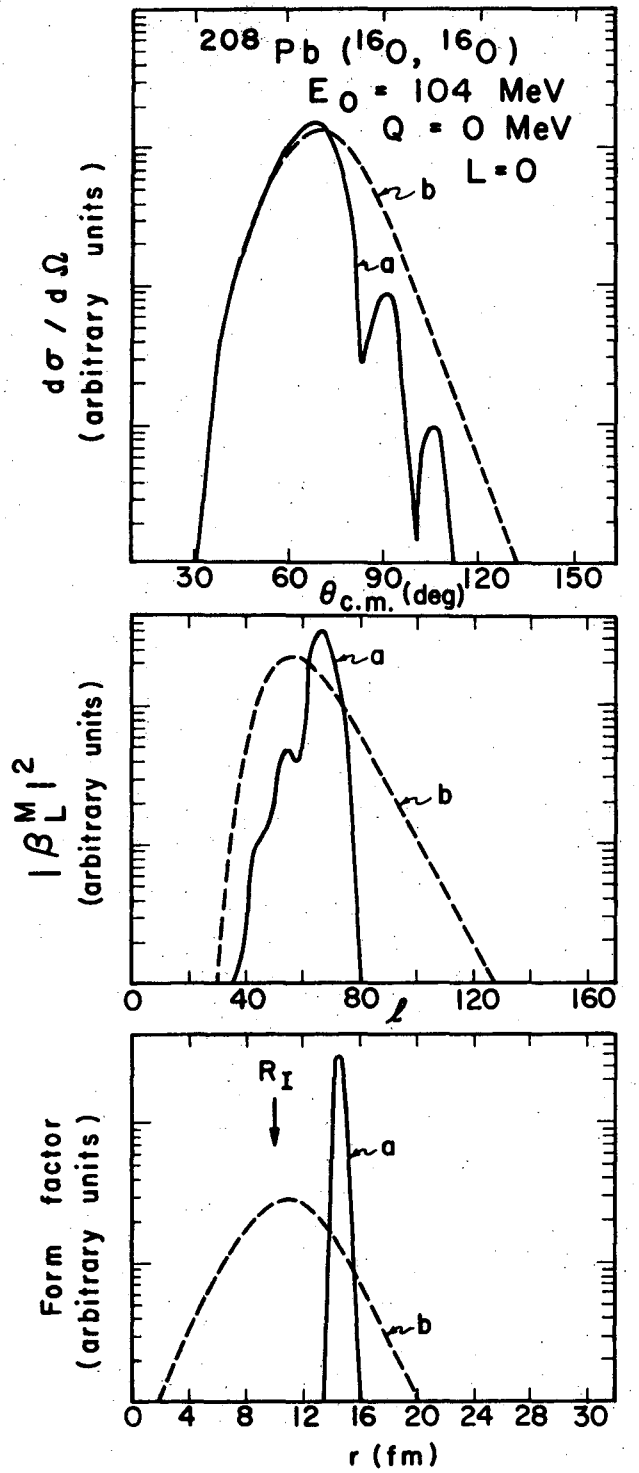

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Fig. 2

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