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POLYNOMIAL CHROMODYNAMICS

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P O L Y N O M I A L   C H R O M O D Y N A M I C S *

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ABSTRACT

A model of classical field theory with internal symmetry Z(3) is partially solved in 1+1 dimensions. The dynamical variables are two pairs of scalar and spinor fields Φ_1,Φ_2 and ψ_1,ψ_2. The Φ and ψ fields form composite finite-size particles. We study the mass spectrum and the interactions of these particles.

1. INTRODUCTION: The action of the model is [1]:

$$S(\Phi_1,\Phi_2,\psi_1,\psi_2) = \int dtdx \,[\, \tfrac{1}{2}(\partial_\mu\Phi_1)^2 + \tfrac{1}{2}(\partial_\mu\Phi_2)^2 \tag{1}$$
$$- \lambda(\Phi_1^2+\Phi_2^2)^2 + \alpha(\Phi_1^3-3\Phi_1\Phi_2^2) + \beta(\Phi_1^2+\Phi_2^2) + \delta + i\overline{\psi}_1\gamma_\mu\overleftrightarrow{\partial}_\mu\psi_1$$
$$+ i\overline{\psi}_2\gamma_\mu\overleftrightarrow{\partial}_\mu\psi_2 + g(\overline{\psi}_1\psi_1-\overline{\psi}_2\psi_2)\Phi_1 - g(\overline{\psi}_1\psi_2+\overline{\psi}_2\psi_1)\Phi_2 \,].$$

With $\alpha=2\lambda\Phi_v/3$, $\beta=\lambda\Phi_v^2$, $\delta=-\beta\Phi_v^3$; $\partial_0=\partial/\partial t$, $\partial_1=\partial/\partial x$; $\gamma_0=-\sigma_2$, $\gamma_1=i\sigma_1$ and $\overline{\psi}=\psi^+\sigma_2$. $\sigma_1,\sigma_2,\sigma_3$ are the Pauli matrices. The coupling constants λ, g and the vacuum field Φ_v are free positive parameters.

The symmetry Z(3) -- a rotation of the Φ and ψ fields by 120° -- leaves the action invariant.

"Toy" models of nuclear physics in 1+1 dimensions have been studied at the level of mesons and nucleons [2], e.g. the σ-model. The aim of the present work is to start at a deeper level, and derive first the structure of the nucleons and from thereon their properties.

Using the vocabulary of QCD we consider the Φ-fields to play the role of scalar gluons [3], due to their cubic and quartic selfinteraction. The spinor fields ψ and the Z(3) group are assumed to play the role of quarks [4] and the color group respectively [5].

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2. STRUCTURE OF COMPOSITE PARTICLES: Particular solutions of the field equations are obtained in a closed form when the effect of the quarks upon the gluons is neglected-- an approximation also made in the MIT-bag model. We find soliton solutions for the Φ-fields [6], and localized solutions for the ψ-fields:

$$\Phi_1=\Phi_v(1+3\tanh(ax))/4, \quad \Phi_2=\sqrt{3}\Phi_v(1-\tanh(ax))/4; \qquad (2)$$

$$\psi_1=NU\exp(iEt-H(x)) \quad , \quad \psi_2=i\sigma_3\psi_1. \qquad (3)$$

With $E=-\frac{1}{2}g\Phi_v$, $H(x)=(2\lambda)^{-\frac{1}{2}}\,g\,\ln(\cosh(ax))$, $a=\sqrt{3\lambda/2}$. U is the spinor part and N the normalization constant.

The total field energy generated by these solutions defines the mass of the ground-state of a finite-size composite particle:

$$M = a\Phi_v^3 - g\Phi_v \ . \qquad (4)$$

A perturbational calculation of the time dependent oscillations of the Φ-fields gives two excited states with the masses $M^* = M+\sqrt{3}a$, and $M^{**} = M+2a$.

We can think of removing one quark ($\psi_1=0$ or $\psi_2=0$). This process breaks the symmetry of the field equations and thus costs energy. The mass of this "colored" state is larger: $M^c=M+7g\Phi_v/8$. Removing finally both quarks ($\psi_1=\psi_2=0$) we obtain a "glueball" with the mass $M^g=M+g\Phi_v$.

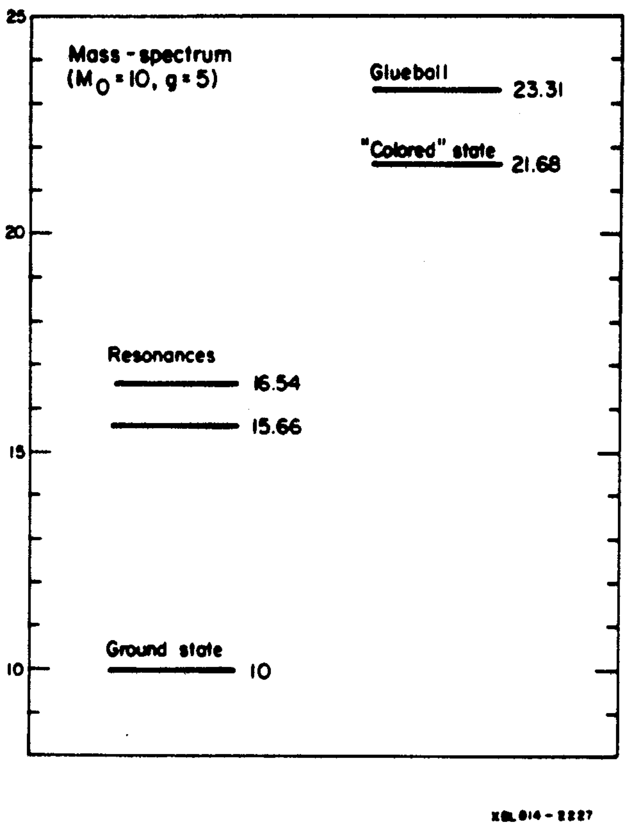

Fig. 1. Example of mass spectrum. The ground-state is chosen to have mass ten to give an easy reference point for the rest of the spectrum. The parameters are $\lambda = 1$, $g = 5$ and $\Phi_v = 2.6717$. The mass is measured in units of $\sqrt{\lambda}$.

3. TWO PARTICLE INTERACTIONS: Two-center Φ-fields are constructed by combining two soliton solutions with the centers localized at x_1 and x_2 ($x_2 \le x_1$) as follows:

$$\Phi_1(x,x_1,x_2)=\Phi_v(1+3\tanh(ax-ax_1))(-1+3\tanh(ax-ax_2))/8, \quad (5)$$

$$\Phi_2(x,x_1,x_2)=\sqrt{3}\Phi_v(1-\tanh(ax-ax_1))(1+\tanh(ax-ax_2))/8. \quad (6)$$

The ψ-fields are superposed according to the Pauli exclusion principle [7] (applied to color):

$$\psi_1(x,x_1,x_2) = \psi_1(x-x_1) - \psi_1(x-x_2), \quad (7)$$

$$\psi_2(x,x_1,x_2) = \psi_2(x-x_1) - \psi_2(x-x_2). \quad (8)$$

The interaction potential between two particles at a distance $d=x_1-x_2$ is defined as the difference between the field energy generated by the two-center fields and their total mass at large distance of separation.

The potential for two ground-state particles shows a shallow attraction where the particles begin to touch, and a strong repulsion at complete overlap. The particles deficient in quarks have strong attractive potentials.

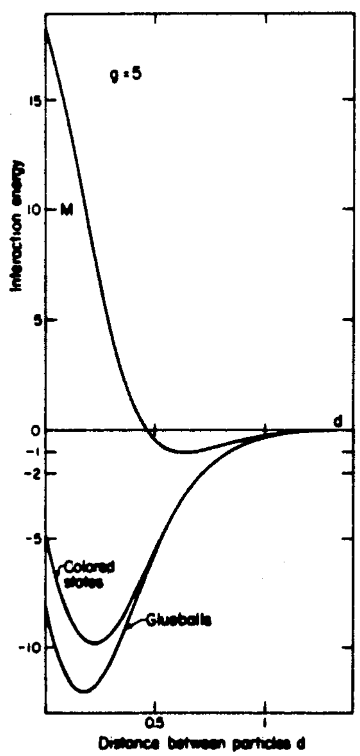

Fig. 2. Static potential of two composite particles in their ground-state with M=10 (upper curve). The attraction is due to the gluons, whereas the repulsion is due to the quarks. The extension of such particles calculated as an average value is 0.387 in units of $1/\sqrt{\lambda}$. The potential of colored-states and glueballs are purely attractive (lower curves). The parameters are the same as in Fig.1.

<u>4. MANY PARTICLE INTERACTIONS:</u> The ansatz in Eqs. (5-8) is generalized to systems with 2P particles, with the centers of the particles located at x_m, m=1,2P. The centers have to be ordered: $x_{2P} \le x_{2P-1} \le \ldots \le x_2 \le x_1$. The Φ-fields are:

$$\Phi_1 = 8^{-P}\Phi_v \prod_{m=1}^{P} (1+3\tanh(ax-ax_{2m-1}))(-1+3\tanh(ax-ax_{2m})), \quad (9)$$

$$\Phi_2 = 8^{-1}\sqrt{3}\Phi_v \sum_{m=1}^{P} (1-\tanh(ax-ax_{2m-1}))(1+\tanh(ax-ax_{2m})), \quad (10)$$

$$\psi_1 = \sum_{k=1}^{2P} (-1)^{k+1}\psi_1(x,x_k), \quad \psi_2 = \sum_{k=1}^{2P} (-1)^{k+1}\psi_2(x,x_k). \quad (11)$$

Calculations for a string of equidistant particles , $d=x_m-x_{m-1}$, show a marked dependence of the many-body

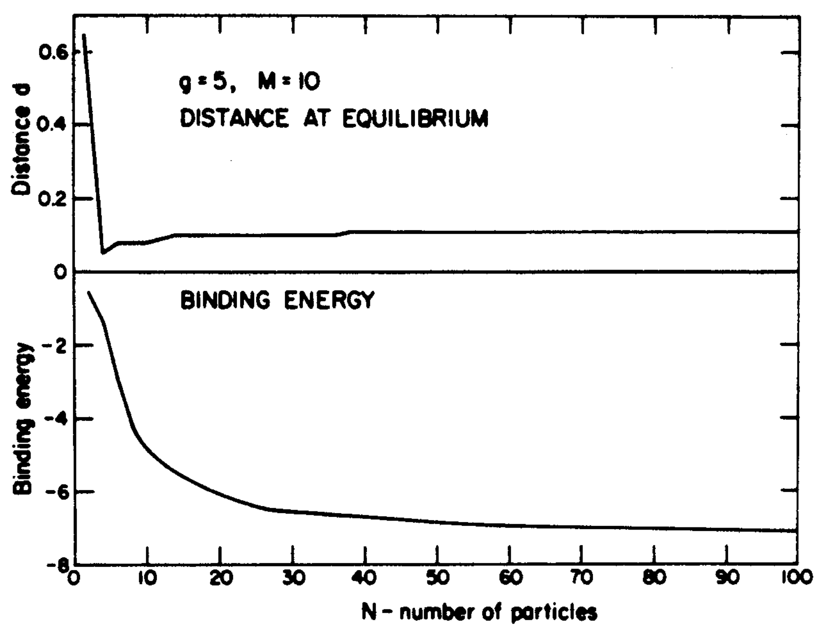

Fig. 3. The interparticle distance and binding energy per particle for a string of N particles at equilibrium.

Saturation of many-body forces is observed [8]. The largest drop in distance and binding energy occurs when passing from two particle systems to four particle systems. The asymtotic value of the binding energy for large numbers of particles is about 70% of the rest mass M = 10 of the particles.

forces on the number of particles. When the number of particles in a system at equilibrium is increased the binding energy per particle increases and the distance between particles decreases, until they reach their saturation limit. The equilibrium point is defined by the minimum of the total energy of the system.

The particle (number) density in one dimensional systems is simply the inverse of the interparticle distance, provided all particles are equally spaced: $\rho=1/d$.

By computing the energy in a system of N particles at a distance d, we can get a representation of the equation of state -- energy as a function of density Fig.4. [9,10]. At the upper end of the curve, where the particles are squeezed into a very small volume, the value of the binding energy depends on the number of particles originally in the system.

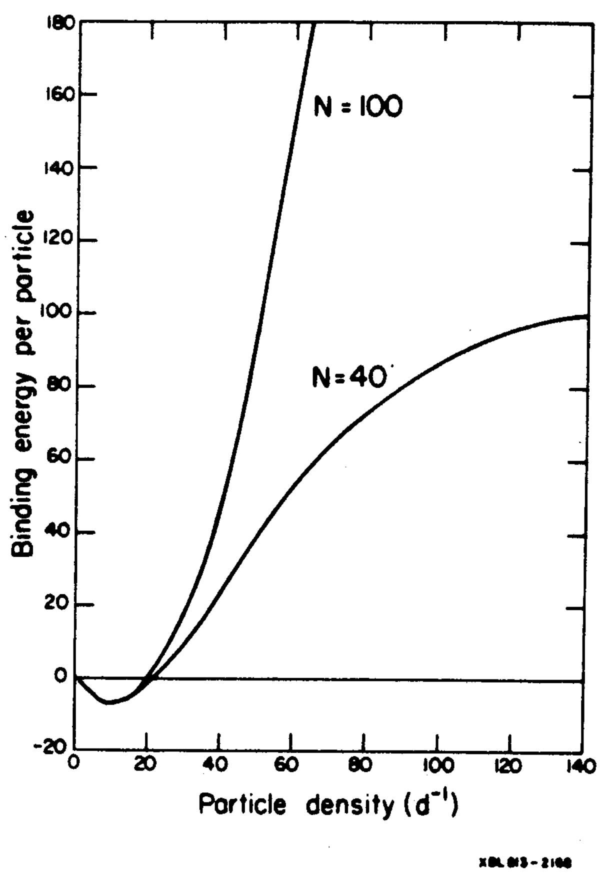

Fig.4. Equation of state for an ensemble of 40 and 100 particles of rest mass M=10.

CONCLUSIONS:

We present a mathematical model in 1+1 dimensions, that mimics i. the quark gluon picture of the structure of nucleons, ii. the two nucleon interaction and iii. some properties of nuclei and nuclear matter.

The calculations are done in a unified way using only the fields of the constituent particles of the "nucleons".

There are some new features as "colored" states, obtained when one fermion (quark) is removed (and transfered to a neighbor particle), and "glueballs" when both quarks are removed.

We calculate the interaction properties of these states (Fig.2), that might turn up in high energy experiments.

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