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NEUTRON TRANSFER BETWEEN BCS NUCLEI

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NEUTRON TRANSFER BETWEEN BCS NUCLEI*

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Abstract

Within perturbation theory we present a microscopic approach to calculate neutron transfer in the scattering of two BCS nuclei. The BCS Hamiltonian of the two coupled BCS nuclei is constructed in terms of commuting field operators. Explicit expressions for the transfer cross sections for a single neutron and for two neutrons are obtained. As an example we consider the scattering of two mercury ions below the Coulomb barrier. The limits of perturbation theory are investigated.

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Neutron Transfer in Reactions Between Superfluid Nuclei

1. Introduction

In the near future scattering experiments with heavy ions will be performed in a wide energy range. A large amount of information will be obtained and has already been obtained[1-2]) by varying projectile, target, and scattering conditions. For bombarding energies below the Coulomb barrier the relative motion between a heavy projectile and a heavy target follows[3-5]) a classical Rutherford trajectory except at backward scattering: This case excluded, the de Broglie wave length $\lambda\!\!\!^{-}$ of the relative motion is small compared to the nuclear radii. Furthermore, the gradient of $\lambda\!\!\!^{-}$ is small[6]) compared to unity. For energies above the Coulomb barrier the second statement is not met because of strong nuclear absorption in the overlap region between projectile and target. This leads[7]) to diffraction scattering and the gross structure of the angular distribution shows a diffraction pattern, predominantly of Fresnel type.

A theoretical treatment of the transition region where the incident energy is close to the Coulomb barrier should be accessible from both sides although the approach from the low energy side seems to be easier. Treating the nuclear distortion between the two heavy ions in perturbation theory one still can use the concept of a Rutherford trajectory. Nuclear distortion leads[4,5]) to an optical potential which, of course, depends on the reactions one has in mind. The analysis[8]) of the optical potential is difficult if no data on the elastic scattering are available. In this case one is forced to start from the (model) Hamiltonian and to calculate explicitly the coupling

of the two nuclei. In this paper we investigate the coupling of two superconducting nuclei where an enhanced transfer is expected because of static pair correlations.

When the nuclei approach each other their wave functions begin to overlap and the two sets of their respective field operators commute no longer. One must construct new orthogonal states[5,9]) which depend on the relative distance and on the relative velocity. In section 2 we show how the Hamiltonian is expressed in terms of the new operators up to second order in the overlap of the wave functions. Explicit expressions for the pairing force Hamiltonian are obtained. This model Hamiltonian is used in section 3 to calculate the transfer amplitudes for a single neutron and for two neutrons. We also evaluate the elastic channel, because, if elastic scattering turns out to be very different from the Rutherford cross section then nuclear distortion can no longer be treated by perturbation theory. In section 4 we consider the various approximations for the evaluation of the transfer amplitudes. We also show the results for the scattering of two mercury ions. Section 5 contains a discussion together with some remarks on the kinematics.

An important question is the interference between Coulomb excitation and transfer. In heavy ion reactions the Coulomb field is much too strong[10]) to allow Coulomb excitation to be treated in perturbation theory. There are, however, indications[11]) that Coulomb excitation can be rather small when the transfer takes place. This is possible because of the destructive interference between nuclear forces and the Coulomb force. Nevertheless, the transfer into excited collective states can be calculated only if Coulomb excitation is

taken into account. However, for transfer reactions into ground states or into non-collective excited states Coulomb excitation may not be so important. Having those states in mind we consider in the following the coupling of two almost identical superconducting nuclei and we neglect Coulomb excitation.

2. Orthonormal States for Neutrons

The undisturbed eigenstates $|\Phi_\mu\rangle$ of nucleus A are not orthogonal to the states $|\chi_\nu\rangle$ of nucleus B because the two sets of wave functions overlap. Having transfer reactions in mind we confine ourselves to states $|\Phi_\mu\rangle$ near the fermi levels. If the internucleus distance r (see fig. 1) is not too small we can expand the corresponding orthonormal states $|\bar{\Phi}_\mu\rangle$ and $|\bar{\chi}_\nu\rangle$ in terms of the overlap between the undisturbed states

$$|\bar{\Phi}_\mu\rangle = |\Phi_\mu\rangle - 1/2 \sum_\varkappa \varepsilon^*_{\varkappa\mu}|\chi_\varkappa\rangle + 3/8 \sum_{\varkappa\lambda} \varepsilon_{\varkappa\lambda}\, \varepsilon^*_{\varkappa\mu}\, |\phi_\lambda\rangle - \ldots$$

$$|\bar{\chi}_\nu\rangle = |\chi_\nu\rangle - 1/2 \sum_\lambda \varepsilon_{\nu\lambda}|\phi_\lambda\rangle + 3/8 \sum_{\varkappa\lambda} \varepsilon^*_{\varkappa\lambda}\, \varepsilon_{\nu\lambda}|\chi_\varkappa\rangle - \ldots , \tag{1}$$

where

$$\varepsilon_{\nu\mu} = \langle \Phi_\mu|\chi_\nu\rangle \quad .$$

In the appendix we show that for identical nuclei the symmetric expansion (1) is equivalent to the introduction of orthonormal states which have a maximal localization. Furthermore, the symmetric expansion has the advantage of minimizing the the renormalization in the elastic channel (see below). The summation indices $\varkappa$ and λ should be restricted to the valence states in the neighborhood of the fermi levels. An unrestricted summation would lead to overcounting because the two sets $\{|\Phi\rangle\}$ and $\{|\chi\rangle\}$ are both complete sets. The nucleon annihilation operator $\psi(\underset{\sim}{r})$ is expanded as

$$\psi(\underset{\sim}{r}) = \sum a_\mu\, \bar{\Phi}_\mu(\underset{\sim}{r}) + \sum b_\nu\, \bar{\chi}_\nu(\underset{\sim}{r}) \quad , \tag{2}$$

where the annihilation operators a_μ and b_ν now commute. We insert this expression in the Hamiltonian

$$H = \int d^3r\, \psi^+(\underset{\sim}{r})\, (K + U_A + U_B)\, \psi(\underset{\sim}{r})$$
$$+ 1/2 \int d^3r\, d^3r'\, \psi^+(\underset{\sim}{r})\, \psi^+(\underset{\sim}{r}')\, W(\underset{\sim}{r}-\underset{\sim}{r}')\, \psi(\underset{\sim}{r}')\, \psi(\underset{\sim}{r}) \tag{3}$$

with K, U_A and U_B denoting the operator for the kinetic energy and the potential wells of the two nuclei, respectively. W is the residual nuclear interaction. Assuming that $|\Phi_\mu\rangle$ and $|\chi_\nu\rangle$ are eigenfunctions of the potential wells

$$(K + U_A)\, |\Phi_\mu\rangle = s_\mu\, |\Phi_\mu\rangle \quad ,$$
$$(K + U_B)\, |\chi_\nu\rangle = t_\nu\, |\chi_\nu\rangle \quad , \tag{4}$$

one finds that up to second order in ε the single particle component of H is given by

$$H_{sp} = \int d^3r\, \psi^+(\underset{\sim}{r})\, (K + U_A + U_B)\, \psi(\underset{\sim}{r})$$
$$= \sum_{\mu\lambda} \bar{s}_{\mu\lambda}\, a^+_\mu a_\lambda + \sum_{\nu\varkappa} \bar{t}_{\nu\varkappa}\, b^+_\nu\, b_\varkappa + \sum_{\mu\nu} (T_{\mu\nu}\, a^+_\mu b_\nu + T^*_{\mu\nu}\, b^+_\nu a_\mu) \quad , \tag{5}$$

where

$$\bar{s}_{\mu\lambda} = s_\mu \delta_{\mu\lambda} - 1/8 \sum_\nu (s_\mu + s_\lambda - 2t_\nu)\, \varepsilon_{\nu\mu}\, \varepsilon^*_{\nu\lambda}$$
$$\bar{t}_{\nu\varkappa} = t_\nu \delta_{\nu\varkappa} - 1/8 \sum_\mu (t_\nu + t_\varkappa - 2s_\mu)\, \varepsilon^*_{\nu\mu}\, \varepsilon_{\varkappa\mu} \tag{5a}$$

are the renormalized single-particle energies and where

$$T_{\mu\nu} = 1/2\, \langle \Phi_\mu |\, U_A + U_B\, | \chi_\nu \rangle \tag{6}$$

causes single particle tunneling. It is the symmetric expansion (1) which keeps the renormalization small because the overlap $\varepsilon_{\nu\mu}\ \varepsilon^{*}_{\nu\lambda}$ has its maximum approximately at $s_\mu+s_\lambda-2t_\nu \approx 0$.

Assuming W to be a pairing force we evaluate the two-particle component of H. Up to second order in ε we find

$$H_{int} = H - H_{sp} = -1/4 \sum_{\varkappa\lambda\mu\nu} \delta_{\varkappa\bar{\lambda}}\ \delta_{\mu\bar{\nu}}\ (G_A a^{+}_{\varkappa} a^{+}_{\lambda} a_\nu a_\mu + G_B b^{+}_{\varkappa} b^{+}_{\lambda} b_\nu b_\mu)$$
$$+ 1/16 \sum_{\varkappa\lambda\mu\nu\zeta} [(G_A \varepsilon_{\zeta\varkappa} \varepsilon_{\bar{\zeta}\lambda} \delta_{\mu\bar{\nu}} + G_B \varepsilon_{\mu\zeta} \varepsilon_{\nu\bar{\zeta}} \delta_{\varkappa\lambda})\ a^{+}_{\varkappa} a^{+}_{\lambda} b_\nu b_\mu + h.c] \tag{7}$$

As usual $\bar{\nu}$ is related to ν by time reversal. In deriving (7) we have dropped terms leading to single-particle tunneling. We further neglected the renormalization of the pairing strengths G_A and G_B. Note the repulsive character of the two-neutron tunneling part in H_{int}. One has to keep in mind that the relative phase of the operators a and b is arbitrary. The "repulsion" is changed into an "attraction" if one makes the transformation $a \to a$ and $b \to ib$. The cross section for the transfer is, of course, invariant under this transformation.

3. Transition Probabilities

The Hamiltonian H can be split into a part H_o belonging to the unperturbed system

$$H_o = \sum_\mu s_\mu a^+_\mu a_\mu + \sum_\nu t_\nu b^+_\nu b_\nu - 1/4 \sum_{\lambda\mu\nu} \delta_{\bar{\varkappa}\bar{\lambda}} \delta_{\mu\bar{\nu}} (G_A a^+_{\varkappa} a^+_{\varkappa} a_\nu a_\mu + G_B b^+_{\varkappa} b^+_\lambda b_\nu b_\mu) \tag{8}$$

and into a part V(t) containing the overlap ε

$$H = H_o + V(t) \tag{9}$$

H_o is not explicitly time dependent whereas V(t) depends on time via the overlap ε. In the interaction representation the S-matrix is given by

$$S(t) = \hat{T} \exp \left\{ - \frac{i}{\hbar} \int_{-\infty}^{t} dt' \, V_{int}(t') \right\} \quad , \tag{10}$$

where

$$V_{int}(t) = \exp\left(\frac{i}{\hbar} H_o t\right) V(t) \exp\left(- \frac{i}{\hbar} H_o t\right) \tag{11}$$

and where $\hat{T}$ denotes the time ordering operator. The undisturbed ground state is eigenstate of H_o. This state is a product state which we assume to be given by

$$|A; B\rangle = |BCS(A)\rangle \otimes |BCS(B)\rangle \quad , \tag{12}$$

where BCS means the BCS-state of nucleus A and nucleus B. In the following we confine ourselves to spherical nuclei, each individual one having an even number of neutrons.

3.1. SINGLE-NEUTRON TRANSFER

The probability for the transfer of a single neutron is given by (compare ref. 12)

$$P_1 = \sum_{\mu\nu} |M_{\mu\nu}|^2 \quad , \tag{13}$$

where

$$|M_{\mu\nu}|^2 = |-\frac{i}{\hbar}\int_{-\infty}^{+\infty} dt' \, {}_{\mu\nu}\langle A+1;\, B-1|V_{int}(t')|A;B\rangle|^2 \tag{14}$$

reads up to second order in ε

$$|M_{\mu\nu}|^2 = \frac{1}{\hbar^2}\int dt_1 dt_2 u_\mu^2 v_\nu^2 T_{\mu\nu}(t_1)\, T^*_{\mu\nu}(t_2) \times \exp\,[i(\sigma_A-\sigma_B+E_\mu+E_\nu)\,(t_1-t_2)] \quad . \tag{15}$$

Here we have introduced BCS states of the odd system defined by their matrix elements

$${}_{\mu\nu}\langle A+1;\, B-1|a_\lambda^+ b_\pi|A;\, B\rangle = u_\mu v_\nu \delta_{\lambda\mu}\delta_{\pi\nu} \quad . \tag{16}$$

The numbers u_μ^2 (v_ν^2) indicate as usual to what extent level μ (ν) of nucleus A (B) is empty (filled):

$$u_\mu^2+v_\mu^2 = 1$$

E_μ and E_ν are the single-particle energies of nucleus A and B when pairing is taken into account, i.e.

$$E_\mu = [(s_\mu-\sigma_A)^2 + \Delta_A^2]^{1/2} \quad , \tag{17}$$

where σ and Δ denote chemical potential and gap, respectively. The tunneling amplitudes $T_{\mu\nu}(t)$ defined in eq. (6) depend on time because they depend on the vector of relative distance $\underset{\sim}{r}$ between the two centers of nucleus A and nucleus B. Using the asymptotic expansion formulas of Buttle and Goldfarb[13]) we find

$$\langle \Phi_\mu | U_B | \chi_\nu \rangle = \sqrt{4\pi}\; N_\mu^* \tilde{A}_\nu \hat{j}_\mu \hat{j}_\nu \, (-)^{j_\mu + j_\nu}$$

$$\times \sum_{\ell} (-)^{1/2(\ell_\mu+\ell_\nu-\ell)} \hat{\ell} \begin{pmatrix} \ell & j_\mu & j_\nu \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \ell & j_\mu & j_\nu \\ M_\nu - M_\mu & M_\mu & -M_\nu \end{pmatrix} h_\ell^{(1)*}(i\varkappa_\mu r)(-)^{M_\mu + \frac{1}{2}} Y_{\ell\, M_\nu - M_\mu}(\hat{\underset{\sim}{r}}) \tag{18}$$

$(\ell+\ell_\mu+\ell_\nu = \text{even})$

We have used the notation of ref. 13

$$\hat{x} = (2x+1)^{1/2}$$

$$\tilde{A}_\nu = (-)^{\ell_\nu} A_\nu = \frac{\hbar^2}{2m} N_\nu \varkappa_\mu^{\ell_\mu} (\varkappa_\nu)^{-\ell_\nu - 1} \quad . \tag{19}$$

N_μ is the normalization factor of the asymptotic solution of $|\Phi_\mu\rangle$ which is a spherical hankel function.

$$\Phi_\mu(\underset{\sim}{x}) \longrightarrow N_\mu h_{\ell_\mu}^{(1)}(i\varkappa_\mu x)\; Y_{\ell_\mu M_\mu}(\hat{\underset{\sim}{x}}) \tag{20}$$

and $\varkappa_\mu = (2mB_\mu \hbar^{-2})^{1/2}$ is defined in terms of the neutron binding energy B_μ.

The matrix element $\langle \Phi_\mu | U_A | \chi_\nu \rangle$ is obtained if one interchanges μ and ν in eq. (18), replaces $\underset{\sim}{r}$ by $-\underset{\sim}{r}$ and takes the complex conjugate. We insert eq. (6) together with eq. (18) in eq. (15) and summing over the magnetic quantum numbers we obtain

$$P_1 = \frac{1}{\hbar^2} \sum_{\substack{\ell_1 m_1 n \\ (\ell+\ell_m+\ell_n=\text{even})}} u_m^2 \, v_n^2 \left[\hat{j}_m \hat{j}_n \hat{\ell} \begin{pmatrix} \ell & j_m & j_n \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \right]^2 \tag{21}$$

$$\int dt_1 dt_2 \exp\left[\frac{i}{\hbar}(\sigma_A - \sigma_B + E_m + E_n)(t_1 - t_2)\right] S_{mn}(t_1,t_2) \, P_\ell(\underset{\sim}{r}_1 \underset{\sim}{r}_2 / r_1 r_2)$$

with

$$S_{mn}(t_1,t_2) = \frac{1}{4}\left(N_m^* \tilde{A}_n \tilde{h}_\ell(\varkappa_m r_1) + N_n \tilde{A}_m^* \tilde{h}_\ell(\varkappa_n r_1)\right) \times \left(N_m \tilde{A}_n^* \tilde{h}_\ell(\varkappa_m r_2) + N_m^* \tilde{A}_m \tilde{h}_\ell(\varkappa_n r_2)\right) \tag{22}$$

and

$$\tilde{h}_\ell(x) = i^\ell h_\ell^{(1)}(ix) \tag{23}$$

being real. The argument y in the Legendre polynomial $P_\ell(y)$ is the cosine of the angle between the vector $\underset{\sim}{r}_1 = \underset{\sim}{r}(t_1)$ and the vector $\underset{\sim}{r}_2 = \underset{\sim}{r}(t_2)$. The transition probability P_1 is of course independent of an arbitrary phase factor in the normalization constants N_m. In the case of spherical nuclei considered here, normalization, single-particle energies, and occupation probabilities do not depend on magnetic quantum numbers and the indices m and n stand for all single-particle quantum numbers except the magnetic quantum numbers. The integration in eq. (21) has to be taken along the classical Coulomb trajectory, thus depending on the scattering angle θ. The differential cross-section for a single-neutron transfer then reads

$$\frac{d\sigma_1}{d\Omega} = \frac{d\sigma_R}{d\Omega} P_1(\theta) \quad , \tag{24}$$

where $d\sigma_R/d\Omega$ is the elastic Rutherford cross section. Equation (24) only holds if the probability for elastic scattering (see below) is not very different from unity.

3.2. TWO-NEUTRON TRANSFER FROM GROUND STATE TO GROUND STATE

Up to fourth order in the overlap ε the probability for two neutrons being transferred into the ground state of the system (A+2; B-2) is given by

$$P_2 = |M_2^{(1)} + M_2^{(2)}|^2 \quad , \tag{25}$$

where $M_2^{(1)}$ is the contribution from the double single-particle transfer and $M_2^{(2)}$ denotes the direct pair transfer contained in H_{int}. Using the relation

$$\langle\ A+2;\ B-2|a_\varkappa^+ a_\lambda^+ b_\nu b_\mu|A;B\ \rangle = u_\varkappa v_\varkappa u_\mu v_\mu \delta_{\bar{\varkappa}\lambda}\delta_{\nu\bar{\mu}} \tag{26}$$

we find

$$M_2^{(1)} = \frac{-1}{\hbar^2} \int_{-\infty}^{+\infty} dt_1 dt_2 \sum_{\varkappa\lambda\mu\nu} T_{\varkappa\lambda}(t_1)\ T_{\mu\nu}(t_2)\ \langle A+2;\ B-2|\hat{T}\left(a_\varkappa^+(t_1)b_\lambda(t_1)a_\mu^+(t_2)b_\nu(t_2)\right)|A;B\ \rangle$$

$$= \sum_{\substack{\ell mn \\ (\ell+\ell_m+\ell_n=\text{even})}} u_m v_m u_n v_n \left[\hat{j}_m \hat{j}_n \hat{\ell} \begin{pmatrix} \ell & j_m & j_n \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}\right]^2 \tag{27}$$

$$\times \int_{-\infty}^{+\infty} dt_1 dt_2 \exp\left[\frac{i}{\hbar}(\sigma_A-\sigma_B)(t_1+t_2) - (E_m+E_n)|t_1-t_2|\right] S_{mn}(t_1,t_2)\ P_\ell\ (\underset{\sim}{r}_1\underset{\sim}{r}_2/r_1 r_2)\ .$$

$\hat{T}$ means the time ordering operator and $a_\varkappa^+(t)$ depends on time through

$$a_\varkappa^+(t) = \exp\left(\frac{i}{\hbar} H_o t\right) a_\varkappa^+ \exp\left(-\frac{i}{\hbar} H_o t\right) \quad . \tag{28}$$

For the direct pair transfer we obtain

$$M_2^{(2)} = -\frac{i}{8\hbar}\sum_{\substack{\ell mn \\ (\ell+\ell_m+\ell_n=\text{even})}}(\Delta_A u_n v_n+\Delta_B u_m v_m)\left[\hat{j}_m\hat{j}_n\hat{\ell}\begin{pmatrix}\ell & j_m & j_n\\ 0 & \frac{1}{2} & -\frac{1}{2}\end{pmatrix}\right]^2 \tag{29}$$

$$\times\int_{-\infty}^{+\infty}dt\left|\frac{2m}{\hbar^2}\left[N_n^*\tilde{A}_m\tilde{h}_\ell(\varkappa_n r) - N_m\tilde{A}_n^*\tilde{h}_\ell(\varkappa_m r)\right]/(\varkappa_m^2-\varkappa_n^2)\right|^2 .$$

In deriving eq. (29) we have used the relation[13])

$$(\varkappa_\mu^2-\varkappa_\nu^2)\varepsilon_{\nu\mu} = \frac{2m}{\hbar^2}\left[\langle\ \Phi_\mu|V_B|\chi_\nu\ \rangle - \langle\ \Phi_\mu|V_A|\chi_\nu\ \rangle\right] \tag{30}$$

together with eq. (18). We also used the gap equation

$$\Delta_A = G_A\sum_{\mu>0}u_\mu v_\mu \tag{31}$$

for nucleus A and for nucleus B.

3.3. ELASTIC CHANNEL

The elastic amplitude M_o differs from unity because of inelastic processes. There are two second order contributions. The first arises from the remormalization of the single particle energies (eq. (5a)) and the second is due to the diagonal part of the tunneling operator $T_{mn}T_{pq}$. Writing

$$M_o = 1 - M_o^{(1)} - M_o^{(2)}$$

we find

$$M_o^{(1)} = \frac{i}{4} \sum_{\substack{\ell mn \\ (\ell+\ell_m+\ell_n=\text{even})}} (v_m^2 - v_n^2)\ (s_m - t_n) \left[\hat{j}_m \hat{j}_n \hat{\ell} \begin{pmatrix} \ell & j_m & j_n \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \right]^2 \tag{32}$$

$$\times \int_{-\infty}^{+\infty} dt \left| \frac{2m}{\hbar^2} \left[N_n^* \tilde{A}_m \tilde{h}_\ell(\varkappa_n r) - N_m \tilde{A}_n^* \tilde{h}_\ell(\varkappa_m r) \right] / (\varkappa_m^2 - \varkappa_n^2) \right|^2$$

and

$$M_o^{(2)} = -\frac{1}{2} \sum_{\substack{\ell mn \\ (\ell+\ell_m+\ell_n=\text{even})}} \left[\hat{j}_m \hat{j}_n \hat{\ell} \begin{pmatrix} \ell & j_m & j_n \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \right]^2 \tag{33}$$

$$\times \int_{-\infty}^{+\infty} dt_1 dt_2\ S_{mn}(t_1,t_2)\ Z_{mn}(t_1,t_2)\ P_\ell\ (\underset{\sim}{r}_1 \underset{\sim}{r}_2 / r_1 r_2) \quad ,$$

where $S_{mn}(t_1,t_2)$ is defined in eq. (22) and

$$Z_{mn}(t_1,t_2) = \exp\left[i\ (\sigma_A - \sigma_B)\ (t_1 - t_2) - i\ (E_m + E_n) |t_1 - t_2| \right] \tag{34}$$

$$\times \left(v_m^2 u_n^2 \theta(t_1 - t_2) + u_m^2 v_n^2 \theta(t_2 - t_1) \right) + \{ t_1 \longleftrightarrow t_2 \} \quad .$$

The function $\theta(t)$ is the step function. We recall that summation index m refers to nucleus A and index n to nucleus B. The differential cross section for the elastic scattering is given by

$$\frac{d\sigma_{e\ell}}{d\Omega} = \left| 1 - M_o^{(1)}(\theta) - M_o^{(2)}(\theta) \right|^2 \frac{d\sigma_R}{d\Omega} \equiv P_o \frac{d\sigma_R}{d\Omega} \tag{35}$$

4. Scattering of ^{196}Hg on ^{200}Hg Below the Coulomb Barrier

For the numerical evaluation we use the following approximations:

a) The normalization constants N_m of the asymptotic wave functions are calculated with Morinigo wave functions[4]). The effective nuclear radius R which separates the nuclear interior from the asymtotic region is chosen as $R = (1.2M^{1/3} + 1.2)$ fm, where M is the nuclear mass number.

b) The difference quotient which appears in eqs. (29) and (32) is replaced by the symmetrized differential quotient.

$$\frac{2m}{\hbar^2}\left(N_n^* \tilde{A}_m \tilde{h}_\ell(\varkappa_n r) - N_m \tilde{A}_n^* \tilde{h}_\ell(\varkappa_m r)\right)/(\varkappa_m^2 - \varkappa_n^2)$$

$$\approx -\frac{1}{2} F_{mn}\left[\frac{d}{d\varkappa_m}\left(\varkappa_m^{\ell_m+\ell_n+1} \tilde{h}_\ell(\varkappa_m r)\right) + \frac{d}{d\varkappa_n}\left(\varkappa_n^{\ell_m+\ell_n+1} \tilde{h}_\ell(\varkappa_n r)\right)\right] \quad , \tag{36}$$

where

$$F_{mn} = N_n^* N_m / \left[\varkappa_m^{\ell_m+1} \varkappa_n^{\ell_n+1} (\varkappa_m + \varkappa_n)\right] \quad .$$

By numerical comparison with the exact expressions one finds that this assumption is reasonable. In order to simplify the integrals we consider only the exponential behavior of the function $\tilde{h}_\ell(\varkappa r)$

$$\tilde{h}_\ell(\varkappa r(t)) \approx \tilde{h}_\ell(\varkappa r_o) \exp\left\{\varkappa [r_o - r(t)]\right\} \quad , \tag{37}$$

where r_o is the distance of closest approach, i.e. the classical turning point. Within the same approximation we replace the arguments of the Legendre polynomials by 1, i.e. by their values at the turning point. In eq. (37) we expand the

exponent up to second order in the time t and perform all integrals analytically. In view of the rapid decrease of $h_\ell(\varkappa r)$ with increasing r these approximations also seem to be justified[14]).

c) The single-particle energies are taken from ref. 15. The configuration space consists of 13 levels ranging from the $1h_{9/2}$ level up to the $3d_{3/2}$ level. We used the pairing rotational model[16]) to determine Δ and G from the two-neutron separation energies in the mercury region. For ^{200}Hg we obtain $\Delta_A = 1$ MeV and $G_A = 0.11$ MeV, in the case of ^{196}Hg we find $\Delta_B = 1.17$ MeV and $G_B = G_A$. The chemical potential for BCS nuclei is defined by

$$\sigma = -\frac{1}{4}\,[S(2,N) + S(2,N+2)] \quad , \tag{38}$$

where S(2N) stands for the separation energy of two neutrons in a nucleus with N neutrons. From this relation the values $\sigma_A = -7.17$ MeV and $\sigma_B = -7.85$ MeV are deduced for ^{200}Hg and ^{196}Hg, respectively.

In fig. 2 we show the differential cross sections for the total one-neutron transfer in the reaction ^{200}Hg(^{196}Hg,^{195}Hg)^{201}Hg and for the transfer of two neutrons into the ground states of ^{194}Hg and ^{202}Hg in the reaction ^{200}Hg(^{196}Hg,^{194}Hg)^{202}Hg at $E_{cm} = 560$ MeV. Using the radii for nucleus A and B according to section 4a we find the Coulomb barrier at $E_{cb} = 562$ MeV. The amplitudes for the two-neutron transfer in the reaction B(A,A+2)B-2 are equal in BCS to those of B(A,A-2)B+2. The cross section for the reaction ^{200}Hg(^{196}Hg,^{198}Hg)^{198}Hg, however, is symmetric around $\theta_{cm} = 90°$ because of the two identical nuclei in the final configuration. Figure 1 also contains the function

$$F = P_o + P_1 + P_2 + P_{-1} + P_{-2} \quad , \tag{39}$$

where P_o is the probability for elastic scattering along the Coulomb trajectory

as defined in eq. (35). The quantities P_1 and P_2 have been introduced in section 3; they are the probabilities for the transfer of one or two neutrons to ^{200}Hg whereas P_{-1} and P_{-2} denote the transfer of one or two neutrons to ^{196}Hg. A necessary condition for perturbation theory to be valid is that F does not deviate much from unity.

In fig. 3 the energy dependence of the differential cross sections is plotted for $\theta_{cm} = 120°$. Again perturbation theory is limited to $F \approx 1$.

5. Discussion

We first point out that our results should not be interpreted quantitatively. There are considerable uncertainties coming from the single-particle wave functions: If the normalization constants N_m are changed by a factor 2 then the absolute values for the cross sections for single-neutron and two-neutron transfer are changed by a factor of 16 and 256, respectively. Also uncertainites in the BCS wave functions are not irrelevant. A truncation of the configuration space would reduce the theoretical cross sections for the two-neutron transfer considerably. Therefore, our results should be compared with future experiments in terms of relative cross sections.

The two-neutron transfer shows an interesting interference pattern because of the fact that the imaginary parts of the amplitudes for double single-transfer and for pair transfer are opposite in sign. The shallow minimum in the two-neutron transfer cross section belongs to the node of the imaginary part. The break-up of a pair becomes more important for shorter coupling times because then the oscillating behavior of the exponent in eq. (27) is less relevant. Shorter coupling time means higher energies or larger scattering angles. Unfortunately, perturbation theory fails to describe the details of the second rise. The cross section for a single neutron will also deviate from the exponential law[17]) when nuclear distortions invalididate perturbation theory. Non-perturbative theories have been discussed[18]) in the literature although they again are restricted to the assumption that Coulomb excitation does not couple too strongly to nuclear excitation. Qualitatively, Coulomb excitation leads to a damping of the elastic channel. For quantitative results one has to treat both transfer and Coulomb excitation in a quantum mechanical way[10]).

We briefly consider some kinematical effects. The overlap between the single-particle wave functions of the two nuclei generally depends on the velocity of the relative motion. For sub-Coulomb reactions this dependence can be neglected[19]). Next, it is clear that heavy-ion transfer reactions are connected with a transfer of angular momentum. There are two contributions. First, the angular momentum is changed because of the polarization of the two cores and, secondly, the transferred particle itself carries along an angular momentum in its relative motion. For bombarding energies below the Coulomb barrier it is difficult to calculate the optimum value[20]) for the transferred angular momentum.

Finally, we mention that recoil effects[20]) can be neglected in the scattering of two almost identical nuclei. This means that the most favourable Q-value for the neutron transfer is approximately zero, a rather well met property of the reactions we discussed in this paper.

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Appendix

We show that for identical nuclei the symmetric expansion (1) in section 2 is equivalent to the introduction of localized, orthonormal states. For simplicity we assume that there is only one state $|\phi\rangle$ in nucleus A and one state $|\chi\rangle$ in nucleus B. Since the two nuclei are identical the two eigenstates of the coupled system are

$$|\psi^{(+)}\rangle = (2(1+\varepsilon))^{-1/2}\,(|\phi\rangle + |\chi\rangle) \tag{A1}$$

and

$$|\psi^{(-)}\rangle = (2(1-\varepsilon))^{-1/2}\,(|\phi\rangle - |\chi\rangle) \quad , \tag{A2}$$

where $\varepsilon = \langle\phi|\chi\rangle$ is the overlap of the two wave functions (which we assume to be real). The two eigenstates $|\psi^{(+)}\rangle$ and $|\psi^{(-)}\rangle$ are neither localized in nucleus A nor in nucleus B. The combinations

$$|\bar{\phi}\rangle = 2^{-1/2}\,(|\psi^{(+)}\rangle + |\psi^{(-)}\rangle) \tag{A3}$$

and

$$|\bar{\chi}\rangle = 2^{-1/2}\,(|\psi^{(+)}\rangle - |\psi^{(-)}\rangle) \tag{A4}$$

are, however, localized. Using (A1) and (A2) we can express $|\bar{\phi}\rangle$ and $|\bar{\chi}\rangle$ by the unperturbed states $|\phi\rangle$ and $|\chi\rangle$. We make a power series expansion in ε and get the expansion (1) of section 2

$$|\bar{\phi}\rangle = (1 + \tfrac{3}{8}\varepsilon^2)\,|\phi\rangle - \tfrac{1}{2}\varepsilon|\chi\rangle + \ldots \tag{A5}$$

$$|\bar{\chi}\rangle = (1 + \tfrac{3}{8}\varepsilon^2)\,|\chi\rangle - \tfrac{1}{2}\varepsilon|\phi\rangle + \ldots \tag{A6}$$

The generalization to identical nuclei with an arbitrary number of states is straightforward.

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Figure Captions

Fig. 1. Schematic picture of two nuclear potential wells A and B separated by the distance r. Horizontal lines denote the single particle levels near the fermi levels.

Fig. 2. Differential cross sections for the total single-neutron (sn) transfer $^{200}Hg(^{196}Hg,^{195}Hg)^{201}Hg$ and for the ground state two-neutron (tn) transfer $^{200}Hg(^{196}Hg,^{194}Hg)^{202}Hg$ at E_{cm} = 560 MeV. The function F, defined in eq. (39) depends on θ_{cm} and is a measure of the failure of perturbation theory.

Fig. 3. Differential cross sections at $\theta_{cm} = 120°$ as a function of the bombarding energy E_{cm}. The symbols sn, tn are defined in fig. 2. Again F, depending now on E_{cm}, is a measure of the failure of perturbation theory.

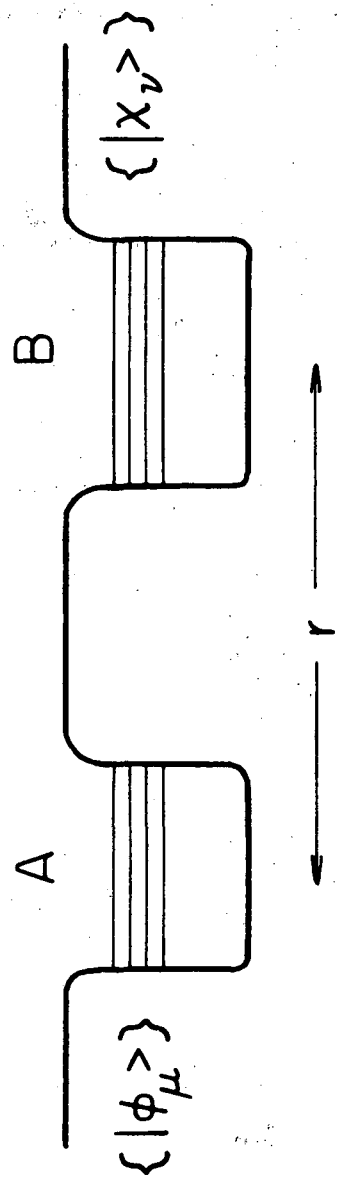

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Fig. 1

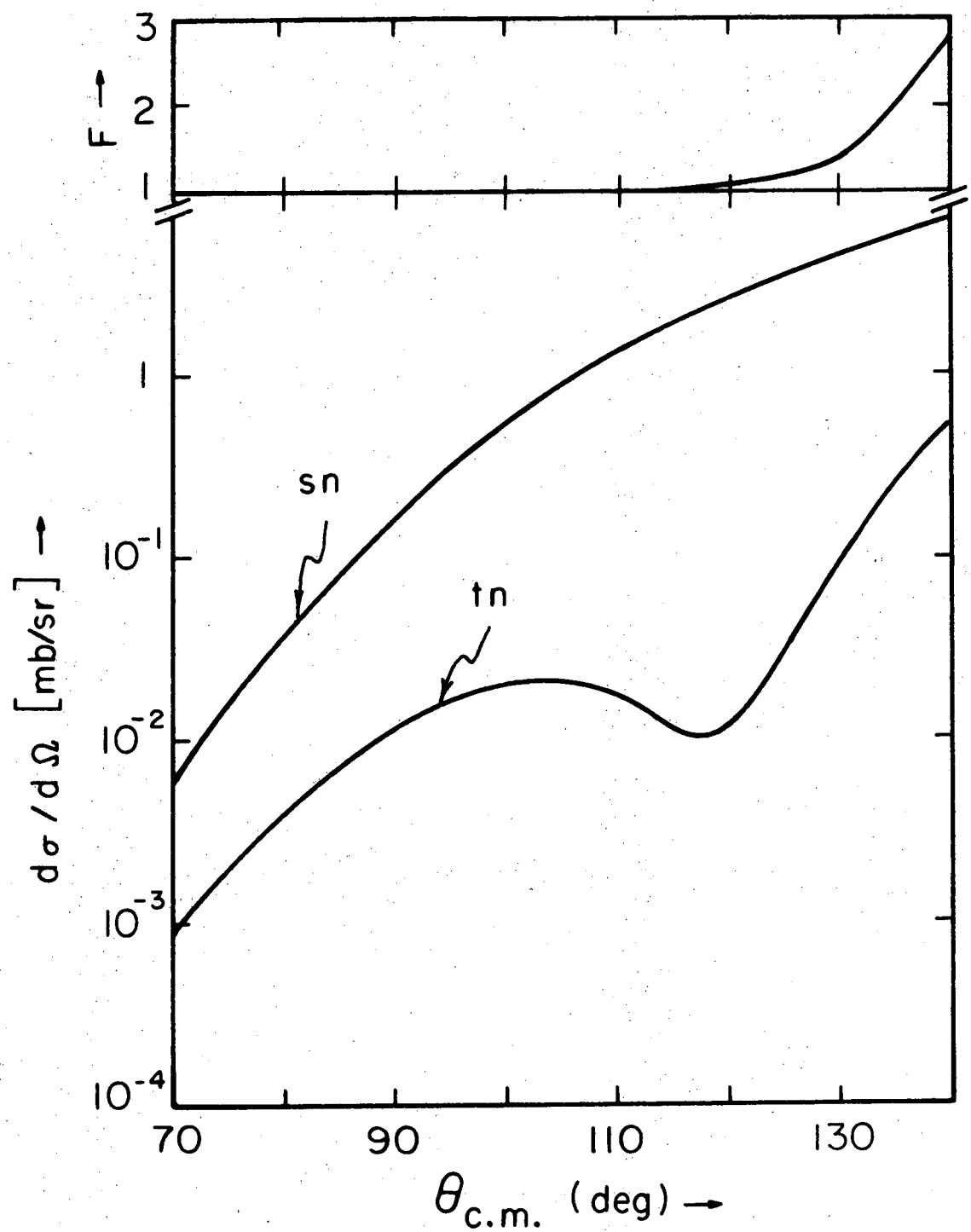

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Fig. 2

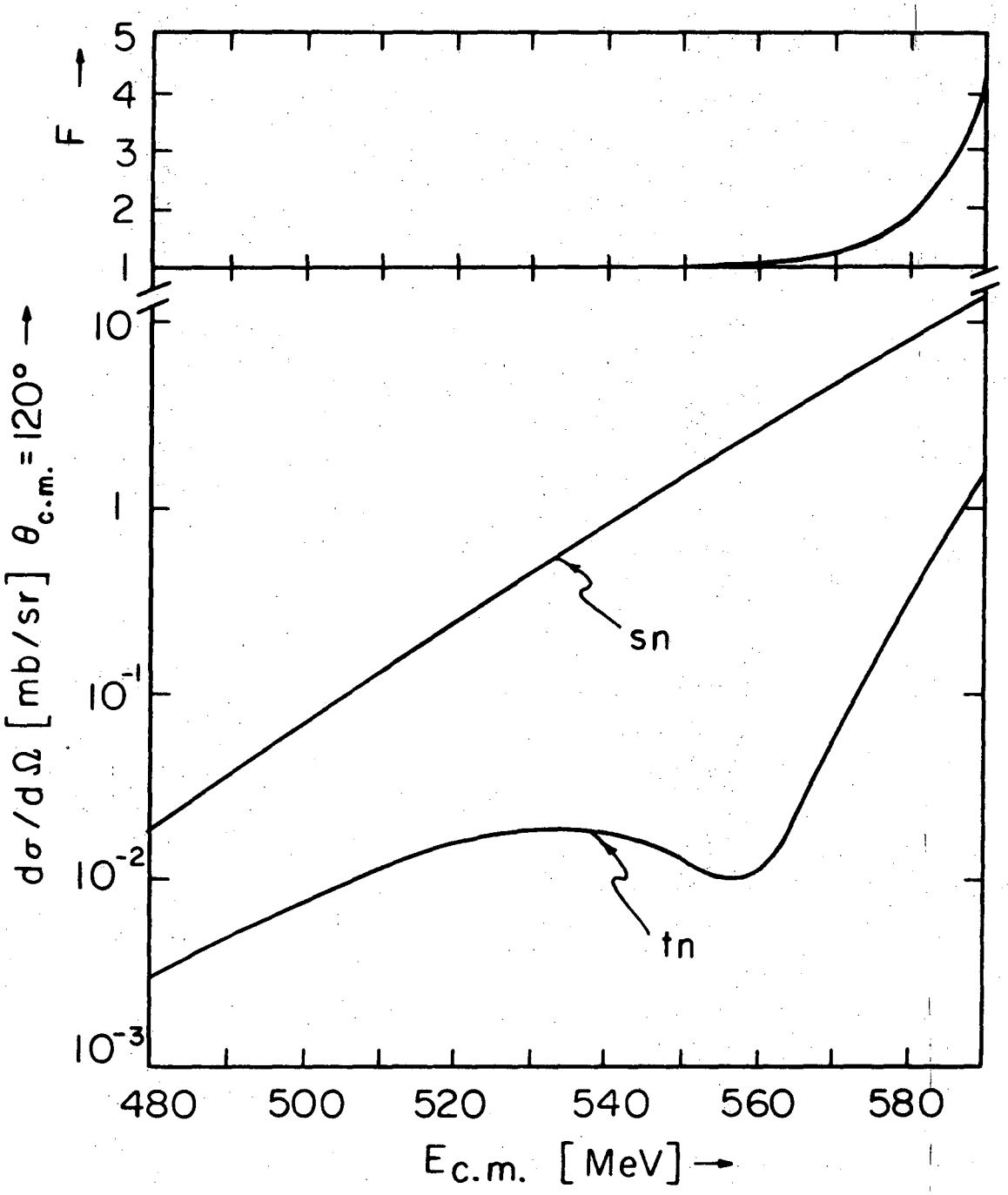

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Fig. 3

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