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THE RELATION BETWEEN FORM FACTORS AND THE SCALING FUNCTIONS IN ELECTROPRODUCTION AND NEUTRINO REACTIONS*

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September 26, 1972

ABSTRACT

We show that the relation between the asymptotic behavior of the transition and isovector elastic form factors and the threshold behavior of the scaling functions suggested by Drell and Yan, West, and Bloom and Gilman follows from precocious scaling and the equal-time algebra of currents. The relation for the isoscalar form factor depends on the nature of the J = 0 fixed pole.

* Supported in part by the U. S. Atomic Energy Commission.

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†† Supported also by the Air Force Office of Scientific Research, Office of Aerospace Research, United States Air Force, under Contract No. F44620-70-C-0028.

Drell and Yan [1] and also West [2] have suggested on the basis of parton models that the rate of decrease of the elastic electromagnetic form factors of the nucleon as [3] $Q^2 \to \infty$ may be correlated with the rate at which the scaling function $F_2(\omega)$ vanishes as $\omega = 2\nu/Q^2 \to 1$. They suggested that if the form factor (defined in Eq.(16 deceases as

$$G^2(Q^2) \xrightarrow[Q^2 \to \infty]{} (1/Q^2)^n \tag{1}$$

and if

$$F_2(\omega) \xrightarrow[\omega \to 1]{} (\omega - 1)^p , \tag{2}$$

then

$$n = p + 1. \tag{3}$$

Later, Bloom and Gilman [4] introduced a new scaling variable $\omega' = \omega + m^2/Q^2$ (m = nucleon mass) and showed that experimentally $\nu W_2(\nu,Q^2)$ scales faster in ω' than in ω, and that together with background contributions, the resonance peaks follow the scaling curve $F_2(\omega')$ in the average (i.e., the resonances build up the scaling curve). They suggested Eq.(3) from these observations; they further argued that Eq. (3) is true also for the form factors for the transition between the nucleon and resonances, defined in the narrow width approximation by

$$W_2(\nu,Q^2) = \delta\left(\nu - (Q^2 + M^2 - m^2)/2\right) G_R^{\,2}(Q^2) . \tag{4}$$

In this paper we will first see how experimentally observed fast scaling plus causality and the mass spectral conditions, as embodied in the Deser-Gilbert-Sudarshan [5] (DGS) representation,

implies Eq. (3) for the transition form factors. Then utilizing the Adler-Dashen-Gell-Mann-Fubini (or simply Fubini) sum rule, we will derive Eq. (3) with a trivial modification for the elastic isovector form factor. It will be shown that the validity of Eq. (3) for the isoscalar form factor depends on the nature of the right signature fixed pole at $J = 0$.

The DGS representation for $W_2(\nu,Q^2)$ is written as

$$W_2(\nu,Q^2) = \int_0^\infty d\mu \int_{-1}^{1} db\ \sigma(\mu,b)\ \delta(-Q^2 + 2b\nu - \mu)\ \epsilon(\nu + bm^2)\ , \tag{5}$$

or

$$\nu W_2(\nu,Q^2)/m^2 = \frac{1}{2m^2}\int_0^\infty d\mu\ \sigma\left(\mu, \frac{Q^2+\mu}{2\nu}\right) \tag{6}$$

for Q^2 and ν positive. To be precise, this form is appropriate for the W_2 amplitude of the axial vector currents. It is customary to write the DGS representation for the vector current amplitudes with an explicit factor of Q^2 in front due to the conservation of current [6]. However, the final results are unchanged [7], so we will go on with the representation given by Eq. (5). If $\sigma(\mu,b)$ is unimportant for $\mu \gtrsim \mu_{max}(b)$, then

$$\nu W_2(\nu,Q^2)/m^2 \xrightarrow[\substack{Q^2 \gg \mu_{max}(\omega^{-1}) \\ \text{with } \omega \text{ fixed}}]{} \frac{1}{2m^2}\int_0^\infty d\mu\ \sigma(\mu,\tfrac{1}{\omega})\ ,$$

$$\equiv F_2(\omega)\ , \tag{7}$$

where $\omega = 2\nu/Q^2$ is the Bjorken scaling variable (see Fig. 1). The precocious scaling observed in SLAC-MIT experiment indicates that $\mu_{max}(b)$ $(0 < b < 1)$ is very small. If [8]

$$F_2(\omega) \xrightarrow[\omega\to 1]{} c(\omega - 1)^p\ , \tag{8}$$

we see from Eq. (7) that $\sigma(\mu,b)$ must have the behavior

$$\sigma(\mu,b) \xrightarrow[b\to 1]{} \hat{\sigma}(\mu,1)(\tfrac{1}{b} - 1)^p, \quad (\mu \neq 0) \tag{9}$$

where $\hat{\sigma}(\mu,b)$ is regular and nonzero at $b = 1$ [9]. Hence,

$$F_2(\omega) \xrightarrow[\omega\to 1]{} (\omega - 1)^p \int_{0+\epsilon}^\infty d\mu\ \hat{\sigma}(\mu,1)/2m^2\ . \tag{10}$$

Before arguing about the elastic form factor, let us first see what Eq. (9) implies for the transition form factors, which we define as

$$G_R^{\ 2}(Q^2) = \int_{\nu_R - M\Gamma/2}^{\nu_R + M\Gamma/2} d\nu\ W_2(\nu,Q^2)\ , \tag{11}$$

where $\nu_R = (Q^2 + M^2 - m^2)/2$ with M and Γ being the resonance mass and width, respectively. This definition includes the background under the resonance. Whether or not the relative magnitude of the resonances versus the background remains nonvanishing as $Q^2 \to \infty$ is a dynamical question we can not answer here, but the previous analysis [4] indicates that it does. If we substitute Eq. (5) into Eq. (11),

and assume we can interchange the orders of integration [7], we have

$$G_R^{\,2}(Q^2) = \int_0^{M^2-m^2+M\Gamma} d\mu \int_{b_1(Q^2)}^{b_2(Q^2)} \frac{db}{2b}\,\sigma(\mu,b)\ , \tag{12}$$

where

$$b_1(Q^2) = (Q^2+\mu)/(Q^2+M^2-m^2+M\Gamma)$$

$$b_2(Q^2) = \mathrm{Min}[1,\ (Q^2+\mu)/(Q^2+M^2-m^2-M\Gamma)]\ . \tag{13}$$

(The boundaries are illustrated in Fig. 2.) As Q^2 increases, the region of integration is pushed closer and closer to the boundary $b = 1$, the same region probed by $F_2(\omega)$ as $\omega \to 1$. For $Q^2 \gg M^2 - m^2 + M\Gamma$, we can use the $b \longrightarrow 1$ behavior Eq. (9), yielding

$$G_R^{\,2}(Q^2) \xrightarrow[Q^2 \gg M^2-m^2+M\Gamma]{} \left(\frac{1}{Q^2}\right)^{p+1} \frac{1}{2(p+1)}$$

$$\times \left[\int_0^{\mu_1} d\mu\ (\mu_1-\mu)^{p+1}\,\hat{\sigma}(\mu,1) - \int_0^{\mu_2} d\mu\ (\mu_2-\mu)^{p+1}\,\hat{\sigma}(\mu,1)\right]\ , \tag{14}$$

where $\mu_{1,2} = M^2 - m^2 \pm M\Gamma$. This is the Bloom-Gilman relation Eq. (3) for the transition form factor. It might look at first sight as if this were a trivial result as long as $G_R^{\,2}(Q^2)$ includes the background contribution, but it is not so. An obvious counter example is $\nu W_2(\nu,Q^2) \to a(\omega-1)^p + b(\omega-1)^q/Q^2$ $(p > q+1)$ near $\omega = 1$, for

which $F_2(\omega) \to a(\omega-1)^p$ as $\omega \to 1$, but $G_R^{\,2}(Q^2) \to (1/Q^2)^{q+2}$ as $Q^2 \to \infty$.

We can <u>not</u> carry out the same calculation for the elastic form factors, because $G^2(Q^2)$ is generated from the single point $(\mu,b) = (0,1)$, where $\sigma(\mu,b)$ is expected to be highly singular and may include even derivatives of δ functions [10]. Instead we must invoke an additional relation between the Born term and the continuum. One such relation is the Fubini sum rule for the charged currents

$$1 = G^2(Q^2) + \int_{\nu_0}^{\infty} d\nu\ W_2(\nu,Q^2)\ , \tag{15}$$

where ν_0 is the value of ν at the πN threshold, the charged currents $(\Delta S = 0)$ are normalized as $[J_0^{\dagger}(\vec{x},0),\ J_0(0)] = V_0^{(3)}\delta(\vec{x})$, $W_2(\nu,Q^2)$ on the right-hand side is <u>crossing-symmetric</u> in ν. The form factor $G^2(Q^2)$ reduces to

$$[G_E^{\,2}(Q^2) + (Q^2/4m^2)\,G_M^{\,2}(Q^2)]/(1+Q^2/4m^2) \tag{16}$$

in case of the charged vector current. It should be noted that G_E and G_M are the <u>isovector</u> form factors. Substituting the DGS representation (5) into (15) and carrying out the integral over ν yields the result that $1 - G^2(Q^2)$ is given by an integral of $\sigma(\mu,b)/2b$ over the shaded region in Fig. 3. Hence,

$$\frac{dG^2(Q^2)}{dQ^2} = -\frac{1}{2(Q^2+\bar{m}^2-m^2)}\int_0^{\bar{m}^2-m^2} d\mu\ \frac{\bar{m}^2-m^2-\mu}{Q^2+\mu}$$

$$\times\ \sigma\left(\mu,\ \frac{Q^2+\mu}{Q^2+\bar{m}^2-m^2}\right)\ , \tag{17}$$

where $\bar{m} = m + m_\pi$. Again, for $Q^2 \gg \bar{m}^2 - m^2$ the integral is restricted to b near 1, so we can use the asymptotic behavior of $\sigma(\mu,b)$ near $b = 1$, Eq. (9), yielding

$$G^2(Q^2) \sim \text{const.} + \left(\frac{1}{Q^2}\right)^{p+1} \frac{1}{2(p+1)} \int_0^{\bar{m}^2-m^2} d\mu$$

$$\times\ (\bar{m}^2 - m^2 - \mu)^{p+1}\ \hat{\sigma}(\mu,1)\ . \qquad (18)$$

The constant term must vanish if $G^2(\infty) = 0$. This establishes the relation (3) except that <u>p describes the threshold behavior of $F_2^-(\omega) - F_2^+(\omega)$</u>. This will be tested in the inelastic neutrino reactions, if the vector and axial vector scaling functions are equal.

In order to extend our method to the isoscalar current, we must find some relation valid for all $Q^2 > 0$ between the Born term and the continuum of the physical electromagnetic structure function. The sum rule giving the $J = 0$ right signature nonsense fixed pole may be useful for this purpose [11]:

$$\gamma(Q^2) = \nu_B\, G^2(Q^2) + \int_{\nu_0}^{\infty} d\nu\ \nu \left\{ W_2(\nu,Q^2) - \sum_{\alpha_i > 0} (\text{Regge term}) \right\}\ , \qquad (19)$$

where $\nu_B = Q^2/2$. It is known that the Regge residue $\beta_i(Q^2)$ behaves like $(1/Q^2)^{\alpha_i - 1}$ as $Q^2 \to \infty$. If we can subtract the Regge terms without changing the threshold behavior near $\omega = 1$ of the integrand of Eq.(19), we can derive by the same technique as before

$$G^2(Q^2) \sim \quad \text{Max} \left\{ \left(\frac{1}{Q^2}\right)^{p+1},\ \frac{\gamma(Q^2)}{Q^2} \right\}\ . \qquad (20)$$

This is similar to Eq.(18). Many analyses [12] have been done for the Q^2 dependence of $\gamma(Q^2)$, and they indicate that $\gamma(Q^2)/Q^2$ changes sign between $Q^2 = 0$ and 1 GeV^2, but that it is almost constant above 1 GeV^2.

We will conclude with a few remarks: (i) In Eq.(10) $\hat{\sigma}(\mu,1)$ is integrated from $\mu = 0$ to ∞, whereas in Eqs.(14) and (18) the integral extends only to a finite value. Our result depends on $\hat{\sigma}(\mu,1)$ being nonzero below this finite value. Experimentally, scaling sets in very early (at least for electroproduction), as early as $Q^2 = 1$ GeV^2 outside of the resonance region. This, along with the assumption that $F_2(\omega)$ goes to zero smoothly as $\omega \to 1$ (power law behavior), implies that in fact $\hat{\sigma}(\mu,1) \simeq 0$ for $\mu \gtrsim 0.5$ GeV^2. (ii) The asymptotic behavior of $G^2(Q^2)$ for resonances does not set in until $Q^2 \gg M^2 - m^2$. This is consistent with the argument made by Elitzer [13] that a dipole behavior of each transition form factor is compatible with Bloom-Gilman type scaling only if the dipole mass increases with the resonance mass. (iii) As is seen in our derivation, the interesting prediction by Bloom and Gilman that $F_{2p}(\omega)/F_{2n}(\omega) \to (\mu_p/\mu_n)^2$ as $\omega \to 1$ does <u>not</u> follow. This is because there is no simple relation between the coefficients in, for example, Eqs. (10) and (18).

[Note added: After completion of this work, we learned that R.A.Brandt and W.-C. Ng (New York University Preprint NYU 14/72) have derived Eq.(3) for the transition form factor using essentially the same method.]

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7. See P. Langacker and M. Suzuki (to be published) for the detail of the vector current calculation.
8. We assume here the precocious scaling and the power law behavior of $F_2(\omega)$ near threshold are true for the axial vector current, too.
9. We expect $\sigma(\mu,b)$ to be highly singular at $(\mu,b) = (0,1)$, as is required to generate a Born term. Hence, Eq.(9) is not valid at $\mu = 0$. However, all our integrals of $\sigma(\mu,b)$ avoid the point $(\mu,b) = (0,1)$, so there is no difficulty.
10. The point $(\mu,b) = (0,1)$ is not included in Eq.(12). See Footnote 9 and Fig.2.
11. Existence of the right signature fixed pole at $J = 0$ is known for vector currents at this moment. This discussion applies specifically to the vector current case, instead of the axial vector case.
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FIGURE CAPTIONS

Fig. 1. The line integral in the (μ,b) plane that generates $F_2(\omega)$ as $\omega \to 1$.

Fig. 2. The integration region that generates $G_R^2(Q^2)$. The two boundary lines are given by $\mu = -Q^2 + b(Q^2 + M^2 - m^2 \pm M\Gamma)$.

Fig. 3. The line integral that generates $-dG^2(Q^2)/dQ^2$. The shaded region generates $1 - G^2(Q^2)$.

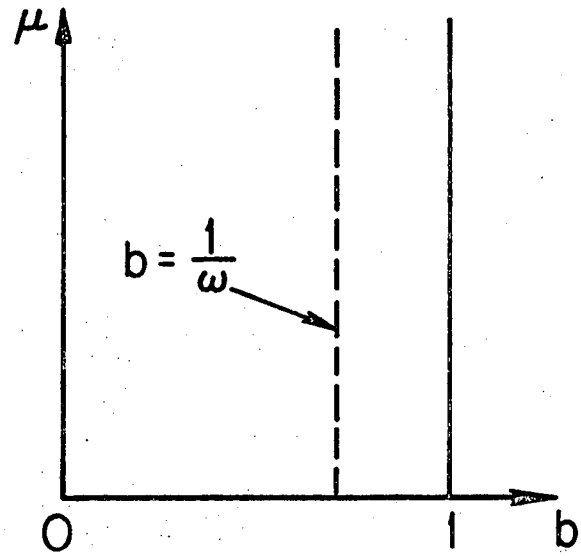

Fig.1

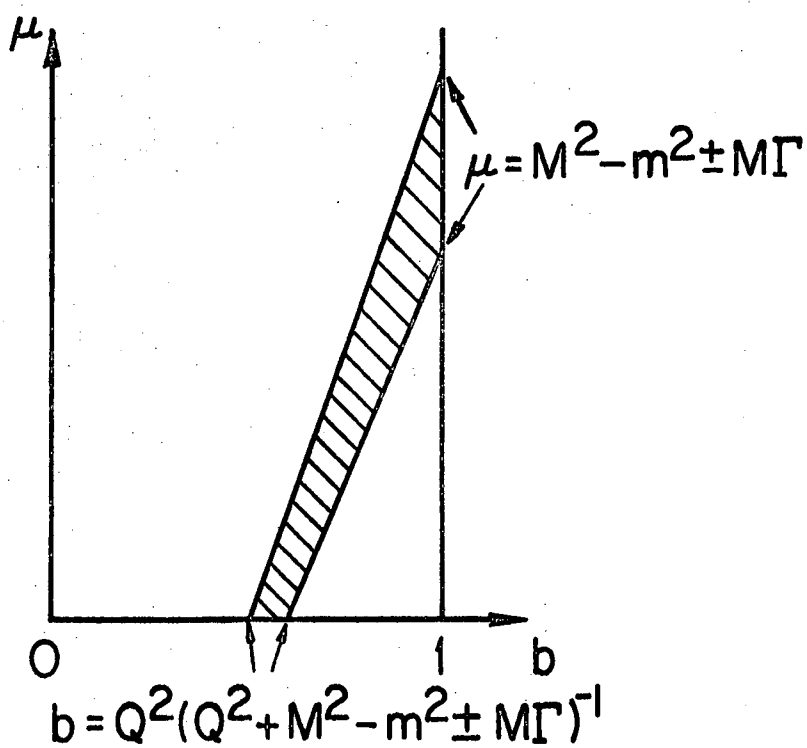

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Fig. 2

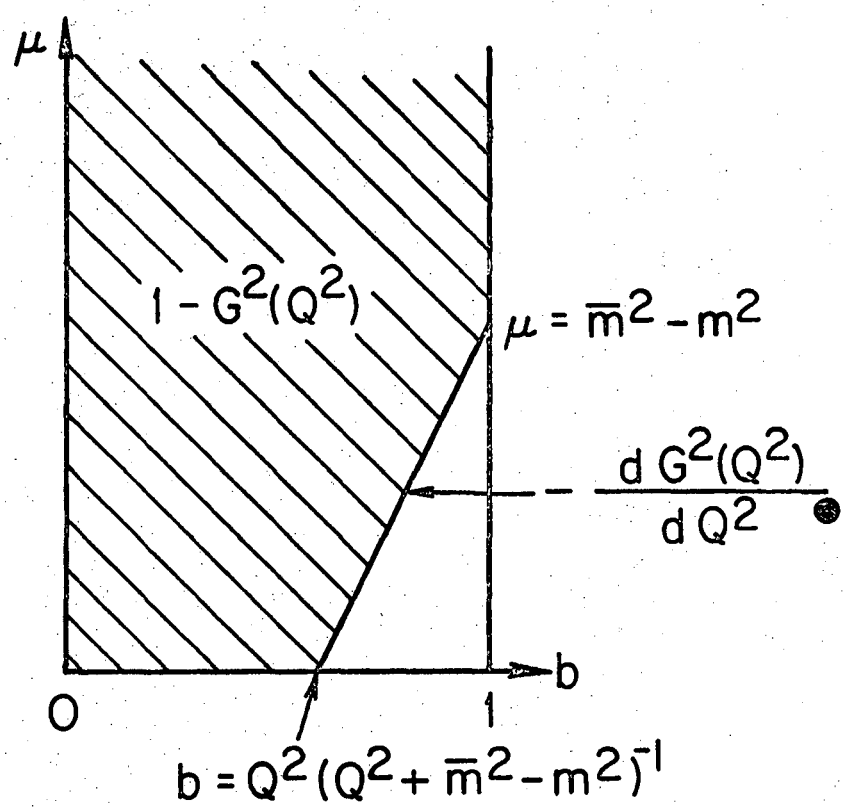

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Fig.3

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