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# A REVIEW OF MESON SPECTROSCOPY: QUARK STATES AND GLUEBALLS*

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## I. INTRODUCTION

When I was invited to give these lectures on hadron spectroscopy I was pleased and appalled. I was pleased by the invitation but appalled that the proposed topic seemed so dull and dated. After preparing these lectures the main lesson I myself have learned is that this subject is neither dull nor dated. I find myself genuinely excited by what has been accomplished and by the significance of what remains to be done. I hope these lectures will make clear the reasons for the sense of excitement I acquired in preparing them.

It is not really so surprising. Spectroscopy has always been at the heart of physics in this century — consider for instance quantum mechanics in the first quarter century and quarks in the 50's and 60's. Hadron spectroscopy would perhaps be dull and dated today if we were really in control of the theory of strong interactions. But that is far from the case. We probably do know the theory, QCD, but our understanding of its long distance dynamics is still exceedingly primitive. In these circumstances we still have a great deal to learn from the spectrum, both about dynamics and about new forms of hadronic matter, such as glueballs and multiquark states.

In the realm of dynamics an outstanding puzzle is the simplicity of the light hadron spectrum. Why do relativistic strongly-coupled bound states appear in just the configurations expected in a non-relativistic model with an instantaneous potential? This simplicity made possible the discovery of quarks as early as 1964 [1,2] but is itself still unexplained. In the bag model, which is a <u>relativistic</u> phenomenology, the low-lying states are the usual ones but excited states are predicted with exotic quantum numbers — for instance, $\bar{q}q$ states with $J^{PC} = 0^{+-}$ which never appear in the nonrelativistic model. These extra states correspond to the spurious translation modes of the

self-consistent field approximation of nuclear physics, where they are spurious because nuclei are well described by instantaneous potentials. But in QCD we don't know whether they exist because we are not in control of the dynamics. The issue is how quickly the collective field, which is the bag, responds to the motion of the quarks. This is a fundamental dynamical question, which is open both in theory and experiment.

There are other related puzzles in the light hadron spectrum, all characterized by the unexpected success of simple ideas, sometimes outrageously simple. Why does single gluon exchange correctly give the spin dependence of the light s-wave mesons and baryons?[3,4] Why do ideal mixing and the OIZ rule[5] work for the light mesons? Most recently, why do sum rules based on a short distance expansion provide a good description of the light meson spectrum.[6] All these puzzles suggest an unexpected weakness in the strength of the strong interaction. Indeed the authors of Ref. (6) have maintained the need for a smaller value of the QCD parameter $\Lambda$, which is now gaining support in other quarters.[7,8]

Turning to heavy quarkonium there are no unexpected successes. For these systems, $\bar{c}c$, $\bar{b}b$, ..., we had good reason to hope simple nonrelativistic ideas would work and, quite marvelously, they do. Using the $\psi$ and $\Upsilon$ spectra we have in effect measured the binding potential in the theoretically intractable transistion region between long ($\gtrsim$ 1 fm.) and short ($\lesssim$ .1 fm.) distances. Because the measured potential ties on smoothly to our expectations in the two limiting regions, the result is an important indirect confirmation of our understanding of QCD both at long and short distances.

The spin dependence of the potential is a difficult, still uncracked problem. Naive extrapolation of the single gluon exchange

ansatz from the light hadrons is not successful, which undercuts the significance of the apparent success for the light hadrons. Generalizations of the Breit Potential of QED assume the confining potential can be approximated by exchange of quanta of definite spin or spins, an assumption which has no close connection with current ideas about the origin of the confining force. Eichten and Feinberg[9] have carried out a more general analysis which is a useful first step toward extracting the spin structure actually implied by QCD.

What about multiquark states? — their apparent absence is another aspect of the puzzling success of the simple nonrelativistic $\bar{q}q$ spectrum. Should we have seen them or not? Indeed, have we seen them? Jaffe and Johnson have used the bag model to find a very amusing set of answers: yes and yes. That is, we should have, and we have, but we didn't know it. Using single gluon exchange to compute hyperfine splittings they find that the lowest mass $\bar{q}\bar{q}qq$ states are a collection of nine scalar mesons with the nonexotic quantum numbers of a $\bar{q}q$ nonet. The most recent data supports their reading of the $\delta$ and $S^*$ as members of this nonet (though there is a problem with the interpretation of the $\delta$). The success of the simple $\bar{q}q$ spectrum is explained by the result that most $\bar{q}\bar{q}qq$ states can fall apart into two $\bar{q}q$ pairs and are therefore too broad to produce discernible bumps in mass histograms.

Nothing illustrates more clearly the value of detailed knowledge of the hadron spectrum than the present effort to determine whether the $\bar{K}K\pi$ enhancement at 1440 MeV. is a glueball. There may be three $\bar{q}q$ states near this mass with large $\bar{K}K\pi$ decay modes: the even and odd charge conjugation axial mesons E and H' and the radially excited pseudoscalar which I call $\zeta'$. Of these only E(1420) has found its way into the little orange book of the Particle Data Group, so it is natural to consider that it might be the enhancement found last year

at SPEAR in radiative $\psi$ decay. There is however strong evidence that the SPEAR 1440 is not the $J^P = 1^+$ E(1420) of the little orange book but is instead a pseudoscalar first discovered in $\bar{p}p$ annihilation at CERN. I believe it is very likely that this state is a glueball.

With just the earlier CERN data and without the observations from radiative $\psi$ decay, it would have been difficult if not impossible to realize that this state may be a glueball. Radiative $\psi$ decay is the process of choice for glueball searches, and we could make good use of much greater statistics than we have now. To prove the glueball assignment we need to be sure that it is <u>not</u> a $\bar{q}q$ meson. For instance, if the 1440 is indeed a pseudoscalar we need to determine whether it is the still undiscovered ninth member of the radially excited $\pi'$ nonet, the $\zeta'$. I will give arguments that the 1440 does not have the properties expected of the $\zeta'$, but nothing could be more convincing than the discovery of a <u>tenth</u> pseudoscalar which does have the expected properties. The most difficult case is if the glueball and the $\zeta'$ are very near in mass and especially if they are strongly mixed. In this case it would require high quality data from many different production and decay channels to find our way through the labyrinth.

The one to two GeV region is certain to be very complicated. Just considering $\bar{q}q$ mesons there are an enormous number of states, many with confusingly similar masses and decay modes. To understand the interesting physics we will need high statistics data. Recent history teaches that each advance in statistics brings into view structure which cannot be seen in any other way. The advances of the past have brought the impressive knowledge of the mesom spectrum that is reviewed here. We are not yet at the end of this progression. It is clear that we still have much more to learn.

## II. LIGHT L = 0 MESONS

As every child learns in school, a $\bar{q}q$ pair with orbital angular momentum L and spin $S = s_1 \oplus s_2$ has total angular momentum $J = L \oplus S$, parity $P = (-1)^{L+1}$, and charge conjugation $C = (-1)^{L+S}$. For quarks $s_1 = s_2 = 1/2$ so $S = \frac{1}{2} \oplus \frac{1}{2} = 0, 1$. The spin singlet, S = 0, is antisymmetric, $\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$, while the triplet, S = 1, is symmetric, $\uparrow\uparrow, \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), \downarrow\downarrow$. Therefore the spin part of the wave function contributes $(-1)^{S+1}$ to C. The spatial wave function contributes $(-1)^L$ and the remaining factor -1 is due to Fermi statistics: after applying C to $\bar{q}q$ we get $(-1)^{L+S+1} q\bar{q} = (-1)^{L+S} \bar{q}q$. The parity is built from the spatial factor $(-1)^L$ and an extra -1 due to the opposite intrinsic parity of fermion and antifermion (the parity operator for Dirac spinors is the Dirac matrix $\gamma_0$ which has opposite signs for lower and upper components).

Therefore the quantum numbers of the possible $\bar{q}q$ mesons as a function of L are as follows:

$$
\begin{aligned}
L &= 0 \qquad J^{PC} = 1^{--},\ 0^{-+} \\
L &= 1 \qquad J^{PC} = (0,\ 1,\ 2)^{++},\ 1^{+-} \\
L &= 2 \qquad J^{PC} = (2,\ 3,\ 4)^{--},\ 2^{-+} \\
L &= 3 \qquad J^{PC} = (2,\ 3,\ 4)^{++},\ 3^{+-} \\
&\ \ \vdots
\end{aligned}
\tag{2.1}
$$

For each $J^{PC}$ there is a flavor nonet of light mesons: the isotriplet, two strange isodoublets, and two isoscalars which are orthogonal combinations of $\bar{s}s$ and $\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$. A tricky point: notice that the spin singlet is $\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$ while the isospin singlet is $\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$. The unexpected sign for the isosinglet is because[10] the isodoublet charge conjugate to (u, d) is $(-\bar{d}, \bar{u})$.

This arithmetic should not cause us to lose track of some important physics. The catalogue of $\bar{q}q$ states in Eq. (2.1) are those

expected if the force between q and $\bar{q}$ is an instantaneous potential, as in the nonrelativistic quark model. But the light mesons are manifestly relativistic systems, as I will discuss below. It is remarkable that Eq. (2.1) beautifully describes the known light meson spectrum. I will have more to say about this when I discuss the L = 1 states in the Bag model in Section III, where the possibility of $\bar{q}q$ states not found in Eq. (2.1) will be considered.

A. Ideal Mixing and the U(1) Problem

We begin with a model which I like to call the Idiot's Quark Model, IQM for short. In this remarkable model, hadrons are made of stationary, non-interacting quarks. The Hamiltonian is just mass terms,

$$H_{IQM} = m(\bar{u}u + \bar{d}d) + m_s\bar{s}s \tag{2.2}$$

so the isotriplet and isodoublet members of the nonet have masses

$$M_{I=1} = 2m \tag{2.3}$$

$$M_{I=\frac{1}{2}} = m + m_s \tag{2.4}$$

In general the I = 0 states could be

$$\begin{aligned} X &= \cos\phi\,\frac{\bar{u}u + \bar{d}d}{\sqrt{2}} - \sin\phi\,\bar{s}s \\ X' &= \sin\phi\,\frac{\bar{u}u + \bar{d}d}{2} + \cos\phi\,\bar{s}s \end{aligned} \tag{2.5}$$

but in the IQM it is easy to see that the eigenstates correspond to

$$\begin{aligned} \phi_{IQM} &= 0 \\ X_{IQM} &= \frac{\bar{u}u + \bar{d}d}{\sqrt{2}} \\ X'_{IQM} &= \bar{s}s \end{aligned} \tag{2.6}$$

since this choice diagonalizes the IQM Hamiltonian

$$\langle \bar{u}u + \bar{d}d | H_{IQM} | \bar{s}s \rangle = 0 \tag{2.7}$$

because the poor IQM has no interactions.

The remarkable thing about all this is that the mixing given by Eq. (2.6), which is called <u>ideal mixing</u>, is what actually seems to occur in most of the light $\bar{q}q$ nonets. And the reason for this is related to the "real world" version of Eq. (2.7), known as the OIZ rule,[5] after Okubo, Iizuka, and Zweig. That is, in the real world

$$\langle \bar{u}u + \bar{d}d | H_{\text{For Real}} | \bar{s}s \rangle \cong 0 \tag{2.8}$$

is small, so that typically we expect ideal mixing

$$X = \frac{\bar{u}u + \bar{d}d}{\sqrt{2}} \tag{2.9}$$

$$X' = \bar{s}s \tag{2.10}$$

Two consequences should follow from ideal mixing. First from the OIZ rule we expect

$$\Gamma(X' \to \text{strange}) \gg \Gamma(X' \to \text{nonstrange}) \tag{2.11}$$

since to get a nonstrange decay of X' there would have to be an OIZ forbidden annihilation of the $\bar{s}s$ quarks. We also expect, for a different reason,

$$\Gamma(X \to \text{nonstrange}) \gg \Gamma(X \to \text{strange}). \tag{2.12}$$

The right side of (2.12) is not OIZ forbidden (since Eq. (2.8) does not forbid $\bar{u}u \to \bar{u}s\bar{s}u$) but it is suppressed by the low probability to make $\bar{s}s$ pairs from the vacuum, reflected for instance in the small K:π ratio in the central region of rapidity in high energy hadron-hadron scattering. The distinction is that $\bar{s}s \to \pi^+\pi^-$ is OIZ forbidden but $(\bar{u}u + \bar{d}d) \to K^+K^-$ is not.

The second consequence of ideal mixing is best understood by returning momentarily to the IQM, where from (2.6) we find

$$M_{X_{IQM}} = M_{I=1} = 2m$$

$$M_{X'_{IQM}} = 2M_{I=\frac{1}{2}} - M_{I=1} = m + m_s \tag{2.13}$$

In the real world we could expect the left hand equalities to apply

$$M_X \cong M_{I=1}$$

$$M_{X'} \cong 2M_{I=\frac{1}{2}} - M_{I=1} \tag{2.14}$$

if the binding forces are flavor-blind (so to speak), that is, not just SU(3) flavor symmetric but such that the binding energy is equal for flavor octet and flavor singlet.

All this seems too simple to be of any possible relevance for relativistic, strongly interacting systems. Indeed it must have required considerable daring on the part of Messrs. O, I, and Z to realize that Eqs. (2.8)-(2.14) explain the structure of the L = 0, S = 1 vector meson nonet, where

$$\Gamma(\phi \to \bar{K}K) \gg \Gamma(\phi \to \rho\pi) \tag{2.15}$$

$$m_\rho \cong m_\omega$$

$$m_\phi \cong 2m_{K^*} - m_\rho \tag{2.16}$$

are remarkably successful. The selection rule (2.15) works at the level of two orders of magnitude in the rate (when phase space effects are removed) and the mass relations (2.16) are good to 1%! It was these facts about the vector nonet that led to the invention of the OIZ rule. It is very amusing to look for instance at Okubo's paper,[5] to see how he puts the pieces of the puzzle together.

The success of the OIZ rule and ideal mixing is still not really understood, though there are hints of possible explanations. They follow to leading order in the large $N_{Color}$ expansion[11] and in the large $N_{Flavor}$ topological expansion,[12] but neither of these N's are big enough in the real world to make us comfortable with the dominance of the leading term. They may also follow just from asymptotic freedom,[4] provided $\alpha_s$ and $\Lambda$ are small enough, smaller than we might until recently[7] have dared hope.

When we turn to the other L = 0 nonet, the pseudoscalar mesons, ideal mixing fails badly. The mass relations $m_\pi = m_\eta$ and $m_{\eta'} = 2m_K - (m_\pi \text{ or } m_\eta)$ are a disaster. Instead the success of the Gell Mann-Okubo[13] mass relation

$$m_\eta^2 \cong \frac{4}{3} m_K^2 - \frac{1}{3} m_\pi^2 \tag{2.17}$$

(which has a kosher current algebra derivation[14] that explains why the masses are squared) and low energy theorems[15] for $\eta \to \gamma\gamma$ and $\eta \to \pi\pi\gamma$ all suggest the mixing is more nearly flavor octet-flavor singlet. That is $\eta$ and $\eta'$ are more nearly mixed like $\eta_8$ and $\eta_1$

$$\eta_8 = \frac{1}{\sqrt{6}} (\bar{u}u + \bar{d}d - 2\bar{s}s)$$

$$\eta_1 = \frac{1}{\sqrt{3}} (\bar{u}u + \bar{d}d + \bar{s}s) \tag{2.18}$$

than like X and X' of Eqs. (2.9-2.10).

In high-brow circles this disaster is called the U(1) problem. The fancy name reflects tha fact that we have a failure not just of the OIZ rule and ideal mixing (which we don't understand anyway) but of more solidly based ideas about chiral symmetry and its breaking. This is a beautiful subject which would require its own set of lectures to discuss properly.[16] Here I will just say that naively in QCD with three light flavors there are nine axial currents to which PCAC should apply, and that this implies that the pion should have a light partner,[17] $m_\eta \leq \sqrt{3}\, m_\pi$, which would be degenerate with it if the OIZ rule were working. The ninth current, which is a flavor singlet, is the origin of the U(1) problem. Our understanding of the solution is that because of the chiral anomaly and gluonic configurations such as instantons, this ninth current is not really partially conserved so that the failure of $m_\eta \leq \sqrt{3}\, m_\pi$ does not contradict QCD.

It is not necessary to understand these recondite matters in order to appreciate the essential physical explanation for the mixing of $\eta$

and $\eta'$. Recall that to justify the mass relations (2.14) I had to invoke a property of the binding force which I tastelessly called "flavor-blindness", that is, that the flavor-singlet and flavor-octet binding energies are equal. If instead there is a much larger contribution to the energy in the flavor singlet channel, which is just where gluonic field configurations related to the U(1) axial current must contribute, then it is easy to see that mixing along the lines of $\eta_8 - \eta_1$ will result.

Suppose for any two light flavors, q and q', which may be the same, there is a large flavor-singlet, OIZ violating interaction energy

$$\lambda \equiv \frac{1}{3} < \bar{q}q' H' \bar{q}'q' > \tag{2.19}$$

Then including this interaction in the Idiot Quark Model, adopting quadratic masses,[14] and negelecting $m_\pi$ relative to $m_K$, $m_\pi^2 = 2m \ll m_s \cong m_K^2$, the mass matrix for the two isoscalars is

$$M \cong \begin{pmatrix} \frac{4}{3} m_s & -\frac{2\sqrt{2}}{3} m_s \\ -\frac{2\sqrt{2}}{3} m_s & \frac{2}{3} m_s + \lambda \end{pmatrix} \tag{2.20}$$

in the 8-1 basis of Eq. (2.18). The eigenstates of this matrix are defined by an angle $\theta$

$$\begin{aligned} \eta &= \cos\theta\, \eta_8 - \sin\theta\, \eta_1 \\ \eta' &= \sin\theta\, \eta_8 + \cos\theta\, \eta_1 \end{aligned} \tag{2.21}$$

which is found after diagonalizing (2.20) to be

$$\tan 2\theta = \frac{-4\sqrt{2}m_s}{3\lambda - 2m_s} . \tag{2.22}$$

The eigenvalues are

$$m_\eta^2,\ m_{\eta'}^2 = \frac{1}{2}\left(\lambda + 2m_s \pm \sqrt{\lambda^2 - \frac{4}{3}\lambda m_s + 4m_s^2}\right) \tag{2.23}$$

It is instructive to consider the limit in which $m_s = m_K^2$ is also negligible compared to $\lambda$. In this limit the mixing is exactly octet-singlet since $\theta = 0$ from (2.22) while from 2.23 we get $m_\eta^2 = 0$, $m_{\eta'}^2 = \lambda$. So qualitatively we understand the heaviness of the $\eta'$ and the 1-8 mixing pattern as a consequence of large, OIZ violations in the flavor-singlet pseudoscalar channel.

Returning to (2.22 - 2.23) with $m_s = m_K^2$ we find by adding the two roots the relation $m_\eta^2 + m_{\eta'}^2 = \lambda + 2m_s = \lambda + 2m_K^2$, which we can use to fix $\lambda - .67\ \mathrm{GeV}^2$. We then compute $m_\eta = 490$ and $\theta = -21^\circ$, which is a considerable improvement on $m_\eta = m_\pi$ and is in qualitative agreement with evidence that $\theta$ is small and negative. To make a better estimate of $\theta$, which corresponds to the usually quoted quark model value, we add the terms in $m_\pi^2 = 2m$ to the matrix (2.20) and multiply the off-diagonal element by a constant f to allow for the difference between the singlet and octet wave functions. When the matrix is diagonalized, we use the known values of $m_\eta$ and $m_{\eta'}$ to determine $\lambda$ and f and we then predict $\theta$. The results are reasonable: $\lambda$ is of the order quoted above, f is not too far from f = 1, and for $\theta$ we find

$$\theta_{\eta-\eta'} = -11^\circ. \tag{2.24}$$

Though the particular value of the angle may easily be uncertain by say $\pm 5^\circ$, the conclusion that $\theta$ is small and negative should be reliable. This conclusion is confirmed by the low energy theorems for $\eta \to \gamma\gamma$ and $\eta \to \pi^+\pi^-\gamma$, which follow from the chiral anomaly and current algebra. The results[15] are

$$\Gamma(\eta \to \gamma\gamma) = 164\ \mathrm{eV}.\ \left(\cos\theta - 2\sqrt{2}\,\frac{F_8}{F_1}\sin\theta\right)^2 \tag{2.25}$$

$$\Gamma(\eta \to \pi\pi\gamma) = 29\ \mathrm{eV}.\ \left(\cos\theta - \sqrt{2}\,\frac{F_8}{F_1}\sin\theta\right)^2 \tag{2.26}$$

where $F_8 \cong F_\pi$ and $F_1$ are PCAC constants for the octet and singlet axial currents. $F_1$ is expected (for instance, in the large $N_c$ expansion[11]) to have the same sign and order of magnitude as $F_8$. Experimental values are $\Gamma(\eta \to \gamma\gamma) = 324 \pm 46$ ev. and $\Gamma(\eta \to \pi\pi\gamma) = 43 \pm 6$ eV. If $\theta$ were zero Eq. (2.25) would fail by a factor 2 and if $\theta$ were positive it would fail by even more. But for instance with $F_8 = F_1$ we get 324 eV for $\theta = -8^\circ$! Because of the large factor $2\sqrt{2}\sin\theta$ which appears in the expression to be squared in (2.25), $\Gamma(\eta \to \gamma\gamma)$ is exquisitely sensitive to the sign and magnitude of $\theta$. A more general discussion which does not assume $F_8 = F_1$ and makes use of Eq. (2.26) and of $\eta' \to \gamma\gamma$ leaves unchanged the qualitative conclusion that $\theta$ must indeed be small and negativ .[15]

A second place to probe the flavor content of $\eta$ and $\eta'$ is in hadronic scattering. If we use the OIZ rule to assume in $\pi p \to \eta n$ and $\pi p \to \eta' n$ that $\eta$ and $\eta'$ are excited in proportion to their $\bar{u}u + \bar{d}d$ quark content, we find

$$\frac{\sigma(\pi^- p \to \eta' n)}{\sigma(\pi^- p \to \eta n)} = \cot^2(\theta_{\eta-\eta'} + 55^\circ) \tag{2.27}$$

so for $\theta_{\eta-\eta'} = -11^\circ$ we expect equal cross sections. The experimental ratio f the dominant spin flip cross sections is $.67 \pm .03 \pm .04$[18] or $.673 \pm .02 \pm .10$[19]. The central value gives $\theta_{\eta-\eta'} = -15^\circ \pm 1^\circ$, again confirming the conclusion that $\theta$ is small and negative. The discrepancy in magnitude is probably within the theoretical uncertainties, one of which may be considerable. To derive Eq. (2.27) we assumed the OIZ rule but the main lesson of the $\eta - \eta'$ system is that it is subject to large OIZ violating forces. Therefore the $\gamma\gamma$ and $\pi\pi\gamma$ decays may be a more reliable probe of $\theta_{\eta-\eta'}$ than the hadronic cross sections.

Qualitatively then we understand the $\eta - \eta'$ system as being determined by a large gluonic field energy which raises the singlet combination to a much higher mass than the octet. This is consistent with the fact that in QCD we expect PCAC to fail for the singlet U(1) current. However I do not want to leave the impression that the U(1) problem is fully understood. We do not understand QCD dynamics well enough to compute the magnitude or even the sign of the gluonic energy contribution to the $\eta'$ mass.

B. *Dynamical Models*

In this section I will discuss in a sketchy manner three approaches to the dynamics of the light $\bar{q}q$ mesons. These are the non-relativistic quark model, the MIT bag model, and the QCD sum rules of the ITEP group.

1. Dynamics for Optimists: Nonrelativistic Quark Model

Consider first the grounds for pessimism. Imagine that a meson with a typical hadronic radius of r = .8 fm. is a non-relativistic system of two 300 MeV. quarks bound by a harmonic oscillator potential. Then the relative momentum is

$$\langle p^2 \rangle = \frac{9}{4} \frac{1}{\langle r^2 \rangle} = (370 \text{ MeV.})^2 \tag{2.28}$$

and with nonrelativistic kinematics we find $v > c$. More generally it is clear just from the uncertainly principle that a $\bar{q}q$ bound state with radius .8 fm and reduced mas $\mu = m/2 \cong 150$ MeV. must be relativistic. There is no reason to suppose that the nonrelativistic model could be a useful approximation for the light mesons.

This has not discouraged many people from attempting to apply the nonrelativistic model to the light mesons. Encouraged by the manifestly nonrelativistic spectrum of the charmonium system, de Rujula, Georgi and Glashow[4] applied the nonrelativistic model to the light hadrons.

They obtained some amusing successes in correlating the spin dependence of the L = 0 mesons and baryons, assuming dominance of short distance single gluon exchange. In this case the effective potential is

$$< V_{Breit} >_{L=0} = - \left( - \frac{4\alpha_s}{3} \right) \frac{8\pi}{3} \frac{\vec{s}_1 \cdot \vec{s}_2}{m_1 m_2} \, |\psi(0)|^2 \qquad (2.29)$$

where $\psi(0)$ is the wave function at the origin and $s_i$, $m_i$ are the spins and masses of the quarks. This is just like positronium in QED except that $-\alpha$ is replaced $-\frac{4}{3}\alpha_s$. In positronium the minus sign is due to the opposite charge of $e^+$ and $e^-$. In QCD the analogous sign occurs because the meson is a color singlet. To see this, imagine for simplicity a theory of color SU(2). Then the QCD factor $\sum_a \lambda_1^a \lambda_2^a$ is replaced by $\vec{t}_1 \cdot \vec{t}_2$ where $t_i$ are Pauli matrices. The net color of the singlet bound state is zero, so

$$0 = < \vec{T}^2 >_{Singlet} = < \vec{t}_1^{\,2} + \vec{t}_2^{\,2} + 2\, \vec{t}_1 \cdot \vec{t}_2 >_{Singlet} \qquad (2.30)$$

and

$$2 < \vec{t}_1 \cdot \vec{t}_2 >_{Singlet} = - 2\, \vec{t}^{\,2}_{quark} = - 2 \cdot \frac{1}{2} \left( \frac{1}{2} + 1 \right) \qquad (2.31)$$

with the promised sign.

Equation (2.29) determines the $\rho$-$\pi$ and $K$-$K^*$ splittings. Just as in Eq. (2.31)

$$\vec{s}_1 \cdot \vec{s}_2 = \frac{1}{2}[S(S+1) - \frac{3}{2}] = \begin{cases} -\frac{3}{4} & S = 0 \\ +\frac{1}{4} & S = 1 \end{cases} \qquad (2.32)$$

Then Eq. (2.29) gives the right sign, $m_\rho > m_\pi$, and for the baryons it also gives $m_\Delta > m_N$. Furthermore the same parameters fit the magnitudes of the splittings for both L = 0 mesons and baryons.[4]

However, if Eq. (2.29) really describes the physics of the light mesons, it should certainly also apply to charmomium. Then we would predict

$$m_\psi - m_{\eta_c} = (m_\rho - m_\pi)\left(\frac{m_u}{m_c}\right)^2 \left|\frac{\psi_\psi(0)}{\psi_\rho(0)}\right|^2$$

$$\cong (540\ \text{MeV})\left(\frac{m_\rho}{m_\psi}\right)^2 \frac{\langle e_Q^2\rangle_\rho}{\langle e_Q^2\rangle_\psi} \left|\frac{\psi_\psi(0)}{\psi_\rho(0)}\right|^2 \tag{2.33}$$

where

$$\frac{\langle e_Q^2\rangle}{\langle e_Q^2\rangle} = \left[\frac{\frac{1}{\sqrt{2}}\left(\frac{2}{3} - \frac{-1}{3}\right)}{\frac{2}{3}}\right]^2 = \frac{9}{8} \tag{2.34}$$

In Ref. (3) it is assumed that $\psi_\psi(0) = \psi_\rho(0)$ and the result is $\sim 4$ MeV, too small by a factor 3. If instead we use the nonrelativistic relationship[20]

$$\Gamma(V \to e^+e^-) = 16\pi \frac{\alpha^2}{M_V^2} \langle e_Q^2\rangle |\psi_V(0)|^2 \tag{2.35}$$

and the measured $e^+e^-$ widths we find $\sim 440$ MeV, too large by a factor 4. The nonrelativistic model for light <u>and</u> heavy mesons with hyperfine splitting due to single gluon exchange is not a tenable hypothesis.

The failure of Eq. (2.33) may also be a problem for a wider class of models. Baryon spin structure has been economically understood with a nonrelativistic model which assumes that $\vec{L}\cdot\vec{S}$ forces are small.[21] Schnitzer has observed that this might be explained by a cancellation between single gluon exchange and a Lorentz-scalar linear confining potential.[22] A Lorentz-scalar potential does not give rise to an $\vec{s}_1 \cdot \vec{s}_2$ term in the effective nonrelativistic potential,[23] so Eq. (2.29) would also apply to this scenario.

## 2. MIT Bag Model

The bag model[3] has the advantage that it is a relativistic phenomenology of confinement. Furthermore it incorporates confinement in a way which is naturally related to ideas about the dynamical origin of confinement in QCD.[24] Quarks in color singlet combinations are

confined (by the assumed boundary conditions) to limited spatial regions in which gluon fluctuations give rise to a positive energy density B. In the spherical cavity approximation the spatial region is taken to be spherical and fixed. The quarks therefore have an energy given by the eigenvalues of the Dirac equation for an infinite spherical potential well. For massless quarks we expect this energy to be $\sim R^{-1}$ per quark because of the uncertainty principle, where R is the bag radius. In fact the lowest eigenvalue is $2.02\ R^{-1}$.

For n massless quarks or antiquarks in a bag of radius R, the total energy of the bag is then the sum of the quark energies and the volume energy due to the gluon fluctuations,

$$E = \frac{2}{R}\, n + \frac{4}{3}\pi\, R^3 B. \tag{2.36}$$

R is the value which minimizes E,

$$\frac{dE}{dR} = 0 \tag{2.37}$$

$$R = \left(\frac{2\pi B}{3n}\right)^{1/4} \tag{2.38}$$

$$E = \frac{4}{3}\,\frac{2}{R}\, n \tag{2.39}$$

Refinements of this simple picture include a mass for the strange quark, a zero-point energy due to quantum fluctuations proportional to $R^{-1}$, and hyperfine splitting from single gluon exchange.[3] The result is a description of L = 0 mesons and baryons which is as successful as the nonrelativistic quark model, though a distressingly large value of the strong coupling constant, $\alpha_s = 2.2$, is required.

### 3. QCD Sum Rules

A great deal of work has been done by the Moscow ITEP group in which the short distance structure of QCD is used to determine the light meson spectrum.[6] This work has been surprisingly

successful. It is surprising, at least to me, because I would never have imagined that a short distance expansion could be relevant to the light meson spectrum, since I thought I had learned from the deep inelastic scattering data that $\Lambda$ is of order 500 MeV. The short distance expansion is only sure to be relevant for distances r such that $r\Lambda \ll 1$, and for $\Lambda \sim 500$ MeV this means r much smaller than the typical $\sim 1$ fm. size of a light hadron.

The ITEP group has however insisted for some years that $\Lambda_{\overline{MS}}$ ($\overline{MS}$ referes to a renormalization prescription, known as "improved minimal subtraction") must be small, between 80 and 160 MeV. At the time this appeared to contradict the measurements of scaling violations, but the most recent result[7] from CDHS is $\Lambda_{\overline{MS}} \cong 200$ MeV. Also, Lepage[8] will present an analysis of $\psi$ and $\Upsilon$ data in the topical conference next week, which requires $\Lambda_{\overline{MS}}$ in the range advocated at ITEP. If $\Lambda$ is indeed this small it might explain not only the success of the ITEP approach but more generally the simplicity of the light hadron spectrum which allows it to be reasonably well described by the nonrelativistic and bag model approaches.

I will just sketch very crudely how the sum rules are derived, to give a rough feeling for what physics goes into them. Consider for example a Lorentz scalar current,

$$j(x) = \bar{q}(x)q(x), \tag{2.40}$$

and the spectal function

$$\Pi(q^2) = i \int d^4x \; e^{iqx} \; \langle Tj(x)j(0) \rangle_0 \tag{2.41}$$

The imaginary part of $\Pi(q^2)$ has a simple physical meaning: in a world in which photons were scalars $\frac{1}{q^4}$ Im $\Pi(q^2)$ would be, within kinematic factors, the total cross section for $e^+e^-$ annihilation into hadrons. For large $q^2$, Im $\Pi(q^2)$ is controlled by the short distance

region, $x^\mu \to 0$, for which the operator product expansion applies:

$$j(x)j(0) \underset{x\to 0}{\longrightarrow} \sum_n C_n(x)O_n(0) \tag{2.42}$$

$C_n$ are c-number functions of x and the $O_n$ are operators evaluated at $x = 0$. For $x \to 0$ the sum is dominated by the operators of lowest dimension, which have the most singular coefficient functions.

The operator of lowest dimension is no operator at all, $O_1 = 1$, which has zero dimension. Since the dimension of $j(x)$ is $M^3$, the singular coeffecient is $C_1 \propto 1/x^6$. In higher orders of perturbation theory $\int e^{iqx}C_1(x)d^4x$ is multiplied by a power series in $\alpha_s(q^2)$, which is useful if $q^2$ is big enough compared to $\Lambda$. The next operators have dimension 4, $\alpha_s G^a_{\mu\nu}G^a_{\mu\nu}$ and $m\bar{q}q$, and are accompanied by coefficient functions proportional to $1/x^2$ ($G^a_{\mu\nu}$ are the gluon field strength tensors, analogous to $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ in QED). To summerize, we have

$$\begin{aligned} O_1 &= 1 & C_1 &\propto 1/x^6 \\ O_2 &= \alpha_s G^a_{\mu\nu}G^a_{\mu\nu} & C_2 &\propto 1/x^2 \\ &\vdots & &\vdots \end{aligned} \tag{2.43}$$

The sum rules are obtained by expressing $\Pi$ in terms of Im $\Pi$ using a dispersion relation with Im $\Pi$ determined by the operator product expansion. The Laplace transform of $\Pi$ is then computed. The result is a sum rule of the form

$$\begin{aligned} \frac{\text{Const.}}{M^2}\int ds\, e^{-s/M^2}\Pi(s) = 1 &+ (\text{Const.})\ \alpha_s(M) \\ &+ \frac{(\text{Const.})}{M^4}\ \langle\alpha_s G^a_{\mu\nu}G^a_{\mu\nu}\rangle_0 \\ &+ \ldots \end{aligned} \tag{2.44}$$

To extract information about the spectrum from Eq. (2.44) the trick is to walk a tight-rope: choose M small enough so that the leading

resonance dominates the left hand side but big enough so the series on the right hand side stays under control.

The ITEP group reports that they are able to walk this tightrope.[6] They obtain good results for the light spectrum and most dramatically they insisted on their prediction for $m_{\eta_c} = 3.00 \pm .02$ GeV when many other theorists were contorting their models to accomodate the late deceased X(2.83). Their fits determine $\alpha_s$ (or $\Lambda$) and vaccum expectation values such as $< \alpha_s G^2 >$ and $< m\bar{q}q >$. The latter terms, which must have a nonperturbative origin since they vanish in any finite order of perturbation theory, turn out to be at least as important as the perturbative corrections for the relevant values of M.

## III. LIGHT L = 1 MESONS

Respect for the nonrelativistic quark model as a tool for taxonomy is enhanced by the successful prediction of four L = 1 nonets. The spin triplet gives rise to $J^{PC} = (0, 1, 2)^{++}$ nonets and the spin singlet to a $J^{PC} = 1^{+-}$ nonet. The best established is the tensor, $A_2$, $K^{**}$, f, f'. After years of uncertainty the $1^{++}$ axial nonet also appears to be complete: $A_1$, $Q_A$, D, E. The $1^{+-}$ nonet is almost filled, with B, $Q_B$, H identified and H' yet to be found. The most confusing nonet is the scalar, of which I think only two members have been found. The confusion is probably related to the existence of light scalar $\bar{q}\bar{q}qq$ states, which in the worst of all possible worlds might mix with the $\bar{q}q$ states.

We would expect from any of the considerations of Section II A that the $A_1$ and B nonets should be ideally mixed. If one does gluon counting,[4] then the scalars and tensors might be less ideal, since they can mix by two (on-shell) gluons while the J = 1 states need three. Furthermore the scalar states may be susceptible to the special vacuum configurations that contribute to the non-ideal mixing of the pseudoscalar nonet (ie, the U(1) problem). Here the trace anomaly[25] may play a role analogous to the chiral anomaly for the pseudoscalars.

I will discuss these four nonets in turn and then conclude the section with a discussion of a very interesting problem which arises if we try to apply the Bag model to the L = 1 states.

### A. The Four L = 1 Nonets

$2^{++}$ nonet: The tensor nonet is now the best understood of the L = 1 nonets. The nonet is ideal to a reasonable approximation. $\Gamma(f' \to \bar{K}K)$ is about an order of magnitude larger than $\Gamma(f' \to \pi\pi)$, even though the phase space favors $\pi\pi$ by a factor 4, and

$\Gamma(f \to \pi\pi) \gg \Gamma(f \to \bar{K}K)$ also holds. The mass relations Eq. (2.13) hold to a good approximation, $m_{A_2} \cong m_f$ to 3% and $m_{f'} + m_f \cong 2m_{K^{**}}$ to 2%. The mixing angle computed from the masses is $25^\circ \pm 4^\circ$ where $35^\circ$ would be ideal.

<u>$1^{++}$ nonet</u>: After years of wondering whether there is an $A_1$, two high statistics experiments confirm its existence, though at a mass higher than the 1.1 GeV usually discussed in the past. The highest statistics experiment, performed by the ACCMOR collaboration, sees the $A_1$ at $m_{A_1} = 1280 \pm 30$ with $\Gamma_{A_1} = 300 \pm 30$ in $\pi^- p \to \pi^-\pi^-\pi^+ p$.[26] Unlike all previous experiments, this one has sufficient statistical power to extend to large momentum transfer ($|t| \sim .7$ GeV$^2$) where the $\rho\pi$ threshold enhancement (Deck effect) at $\sim$1.1 GeV no longer contributes. The data from this experiment is shown in Fig. (3.1). The other experiment,[27] the same one which discovered $\zeta(1275)$, sees the $A_1$ in a charge exchange channel $\pi^- p \to \pi^+\pi^-\pi^0 n$, with $m_{A_1} = 1240 \pm 80$ and $\Gamma = 380 \pm 10$. A combined fit to both sets of data (with the ACCMOR data having by for the greatest weight) gives $m_{A_1} = 1230 \pm 30$ and $350 \pm 60$.[28]

The rest of the nonet is the strange $Q_A(1340)$ (to be discussed in the next subsection on the $1^{+-}$ nonet) and the isoscalars D(1285) and E(1420). The large mass for the $A_1$ is attractive theoretically since it is consistent with the ideal mixing mass relations, $m_{A_1} \cong m_D$ and $m_{A_1} + m_D \cong 2m_E$. The dominance[29] of $E \to K^*K$ and the fact[30] that $B(D \to \bar{K}K\pi) \sim 10\%$ are also consistent with ideal mixing; in fact, the $\bar{K}K\pi$ decays of D need not reflect an $\bar{s}s$ component since they are consistent with proceeding by $D \to \delta\pi$, and $\delta$ could be formed by a find state interaction via $D \to \eta\pi\pi \to \delta\pi$. Here $\delta$ is an isovector $0^{++}$ meson which will be discussed below.

There are two problems with this neat picture of the $1^{++}$ nonet. One is that a CERN experiment[31] sees $Kp \to Dn$ but not $Kp \to En$. This

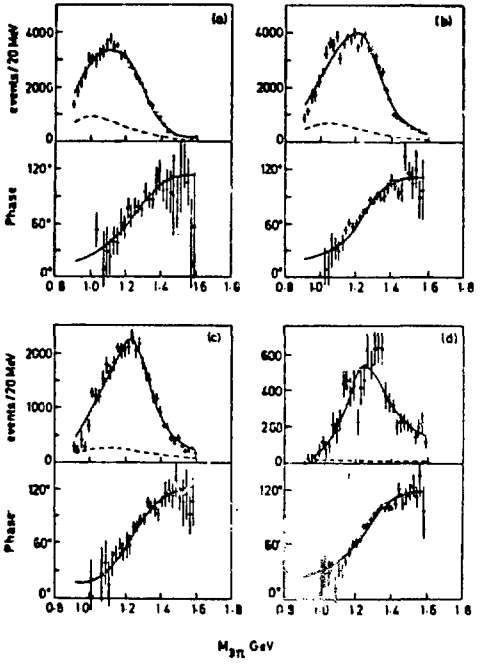


Figure 3.1. (From Ref. (26)). The intensity and phase of $1^+S\ \rho\pi$ for the following momentum transfers:

(a) $0.0 \leqslant |t'| \leqslant 0.05$, (b) $0.05 \leqslant |t'| \leqslant 0.16$,

(c) $0.16 \leqslant |t'| \leqslant 0.3$, (d) $0.3 \leqslant |t'| \leqslant 0.7$ GeV.$^2$

Solid curves are fits to resonant $A_1$ plus Deck effect; dashed curves are Deck intensity alone. The apparent peak at ~1.1 moves to ~1.3 as $|t'|$ increases and the Deck effect becomes unimportant.

cannot be if $E \cong \bar{s}s$ and $D \cong \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$. The LASS group at SLAC will soon be able to make a statement about this problem.

The second problem is the consistency of the new large mass for the $A_1$ with the data[32] from $\tau \to \pi\pi\pi\nu$ which was advertised as giving $m_{A_1} \cong 1.1$ GeV. In fact this $\tau$ decay data can be fit with a peak at 1180 MeV.[33] And because of the limited phase space, the apparent $A_1$ mass in $\tau$ decay is shifted down from the true mass.[33] I examined this effect for a range of widths for the $A_1$ <u>and</u> also to explore the sensitiv[illegible] in Table (3.1), where I have assumed $m_{A_1} = 1.23$ GeV.[28] and studied

| $\Gamma_{A_1}$ (GeV) \ $M_\nu$ (GeV) | 0 | .25 | .3 | .4 | .5 |
|---|---|---|---|---|---|
| .3 | 1.22 | 1.21 | 1.21 | 1.20 | 1.17 |
| .4 | 1.21 | 1.20 | 1.20 | 1.18 | 1.15 |
| .5 | 1.20 | 1.19 | 1.18 | 1.17 | 1.13 |

Table 3.1. The position of the $\rho\pi$ peak in $\tau \to \nu A_1 \to \nu\rho\pi$ for $m_{A_1} = 1.23$, as a function of the $A_1$ width and the $\nu_\tau$ mass.

the position of the $\rho\pi$ peak in $\tau \to \nu_\tau A_1 \to \nu_\tau \rho\pi$ as a function of the $A_1$ total width and the $\tau$ neutrino mass. The present experimental $2\sigma$ upper limit on the $\nu_\tau$ mass is 250 Mev.[34] For a given value of

$\Gamma_{A_1}$ the difference between $m_\nu = 0$ and $m_\nu = 250$ MeV. is only 10 MeV. in the position of the $\rho\pi$ peak. It takes larger values of $m_\nu$ to have a significant effect on the peak position. The position of the peak is rather sensitive to the $A_1$ width. For $m_\nu = 0$ and $\Gamma_{A_1} = 500$ MeV. the peak is shifted downward by 30 MeV to 1200 MeV. This is not far from the successful fit to the $\tau$ data in Ref. (33), which had the apparent peak at 1180 MeV. We need higher statistics $\tau$ decay data to know whether there is really a conflict with the large $A_1$ mass value of the pion scattering experiments.

The electromagnetic decays $\psi \to \pi\pi\pi\pi$ are an interesting channel in which to study the $A_1$. In the negative G-parity channel $\psi \to A_1\rho \to \rho\pi\rho \to 5\pi$ the $A_1$ signal may be overwhelmed by a large $A_2$ signal, since both $\psi \to A_1\rho$ and $\psi \to A_2\rho$ are s-wave decays. But in the positive G-parity $4^-$ channel $\psi \to A_2\pi$ is a d-wave decay and therefore greatly suppressed relative to $\psi \to A_1\pi$ which is s-wave. These positive G-parity final states are formed predominantly when $\psi$ decays via a virtual photon. The contribution to the hadronic width of the $\psi$ from this mechanism is

$$\Gamma(\psi \to \gamma^* \to \bar{q}q) = R_{hadron} \cdot \Gamma(\psi \to e^+e^-)$$

$$\cong 12 \text{ KeV.}$$

or about 20% of all hadronic decays. (An estimate of the rate for the exclusive width $\Gamma(\psi \to \gamma^* \to \pi\pi\pi\pi)$ is given in my paper cited in Ref. (95).) (I thank G. Gidal for a discussion.)

<u>$1^{+-}$ nonet</u>: Here 8/9 of the nonet is in place: B(1231), $Q_B$(1340) and H(1190). There are large uncertainties in the mass of the H,[35] which was established in the channel $\pi^-p \to \pi^+\pi^-\pi^0 n$ by the same ZGS experiment[27] which I have already mentioned in three other contexts. If I follow my prejudice for an ideal nonet, then the H' should be

found in $\bar{K}K\pi$ with a mass $m_{H'} \cong 2m_{Q_B} - m_B \cong 1450$ MeV. As I will discuss in Section VII, these may be <u>four</u> $\bar{K}K\pi$ resonances in this mass region: E(1420), H' and two pseudoscalars, one a $\bar{q}q$ radial excitation and the other a glueball. Of these only H' has negative charge conjugation.

The quantum numbers of the $1^{+-}$ and $1^{++}$ nonets differ only by C- parity, which prevents the strong mixing of the C eigenstates. But only three particles of a nonet are C eigenstates, so C cannot prevent the mixing of the other six. $A_1^{\pm}$ and $B^{\pm}$ are however G parity eigenstates, G = -1 and +1 respectively, so they also cannot mix strongly. If SU(3) flavor were as good a symmetry as isospin, then $\mathcal{G}$, the SU(3) analogue of G, would similarly forbid the strong mixing of $Q_A$ and $Q_B$. But the substantial breaking of SU(3) flavor, which in QCD is just due to the quark masses, means that substantial mixing can occur.[36] The $Q_A(1340)$ and $Q_B(1340)$ are in fact not the observed eigenstates: their masses are the result of an analysis[37] which I will now briefly describe.

Two $J^P = 1^+$ strange mesons are observed experimentally,[35] called $Q_1(1270)$ and $Q_2(1410)$. They are what we observe, so they are the eigenstates of the full Hamiltonian. Their most striking difference is that

$$\Gamma(Q_1 \to \rho K) \gg \Gamma(Q_1 \to K^*\pi) \tag{3.1}$$

whereas

$$\Gamma(Q_2 \to K^*\pi) \gg \Gamma(Q_2 \to \rho K) \tag{3.2}$$

$Q_A$ and $Q_B$ are the $\mathcal{G}$ eigenstates. The physical eigenstates $Q_1$ and $Q_2$ are expressed in terms of $Q_A$ and $Q_B$ by a mixing angle $\phi$,

$$Q_1 = \cos\phi \; Q_A + \sin\phi \; Q_B$$

$$Q_2 = -\sin\phi \; Q_A + \cos\phi \; Q_B \tag{3.3}$$

In the A - B basis the mass matrix is

$$M = \begin{pmatrix} m_A & \varepsilon \\ \varepsilon & m_B \end{pmatrix} \tag{3.4}$$

where

$$\varepsilon = \langle A|H|B \rangle . \tag{3.5}$$

The angle $\phi$ is found by diagonalizing M,

$$\tan 2\phi = \frac{2\varepsilon}{m_A - m_B} \tag{3.6}$$

$\phi$ and $m_{A,B}$ are fixed roughly just by SU(3) symmetry and the decay pattern (3.1) and (3.2).[37] Using Eq. (3.3) and flavor SU(3), the amplitudes for the suppressed decays are

$$M(Q_1 \to K^*\pi) = \frac{1}{2}\, gA \cos\theta + \frac{3}{\sqrt{20}}\, gB \sin\theta \tag{3.7}$$

$$M(Q_2 \to \rho K) = -\frac{1}{2}\, gA \sin\theta - \frac{3}{\sqrt{20}}\, gB \cos\theta \tag{3.8}$$

where $g_{A,B}$ are reduced matrix elements that could be determined from other decays (such as $B \to \omega\pi$ and $A_1 \to \rho\pi$). But independently of $g_{A,B}$ if we interpret (3.1) and (3.2) to mean that (3.7) and (3.8) should vanish we find $\tan\theta \cong 1/\tan\theta$ or

$$\theta \cong 45^o. \tag{3.9}$$

In Eq. (3.6) this implies $m_A \cong m_B$. Since the trace of M, Eq. (3.4), is invariant under diagonalization, we have finally

$$m_A \cong m_B \cong \frac{1}{2}\,(m_1 + m_2) = 1340 \text{ MeV}. \tag{3.10}$$

<u>$0^{++}$ nonet</u>: The $0^{++}$ nonet can only be described as a zoo. Difficulties are practical and conceptual. The practical difficulty is the presence of large s-wave backgrounds (everybody loves an s-wave,

especially phase space) and masking by states of higher spin. The conceptual problem is that there may be low-lying s-wave $\bar{q}\bar{q}qq$ scalars[38] as well as the expected p-wave $\bar{q}q$ scalar nonet.

There are now nine well established scalar states with the right quantum numbers to fill a nonet. But these nines states do not a nonet make! They are the isovector $\delta(980)$,[39] the isoscalars $S^*(980)$ and $\varepsilon(1425)$,[4-] and the strange quartet $K(1425)$.[41] The degeneracy of $\delta$ and $S^*$ suggests an ideal nonet with $S^* = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$ and $\varepsilon = \bar{s}s$, except that both $\delta$ and $S^*$ couple very strongly to $\bar{K}K$ (though they are below $\bar{K}K$ threshold) and $\varepsilon$ couples more strongly to $\pi\pi$ than $\bar{K}K$ by at least a factor 10.[42]

The most likely explanation is that $\delta$ and $S^*$ are not members of this $L = 1$ nonet but "something else" (see the next section). Then $\varepsilon(1425)$ and $K(1425)$ may be part of the real p-wave $\bar{q}q$ nonet, with the isovector and one more isoscalar yet to be found. In fact, a candidate for the missing isoscalar has been observed at 1770 MeV. in a BNL experiment on $\pi^- p \to K_s K_s n$.[43] The isovector could correspond to a $K_s K^-$ enhancement in the 1300 GeV region seen in a phase shift analysis of $\pi^- p \to K^- K_s p$.[44]

B. A Dynamical Puzzle: L = 1 States in the Bag Model

After the generally encouraging results for the s-wave states in the bag model it was hoped that predictions for the p-wave states would "provide a [further] crucial test...."[3] This hope has not been fulfilled because of a fundamental and so far intractable problem. I suspect that the solution, must be sought outside the Bag model, in the QCD dynamics of bag formation.

In the Bag model angular excitations are classified by j - j coupling rather than the L - S coupling of the nonrelativistic model. The p-wave mesons are found by putting one quark in the $S_{1/2}$ cavity

eigenmode and the other in the $P_{1/2}$ mode.[45] Then for example a tensor meson with $J = J_z = 2$ is formed from $\bar{q}_s^{\uparrow} q_{p(+1)}^{\uparrow}$ where "↑" denotes $s_z = +1/2$ and p(+1) denotes the $j_z = +1$ component of the $q_p$ mode. The trouble is that there are two such states, the other being $\bar{q}_{p(+1)}^{\uparrow} q_s^{\uparrow}$. There is then a C-parity doubling with two C eigenstates

$$\frac{1}{\sqrt{2}}(\bar{q}_s^{\uparrow} q_{p(+1)}^{\uparrow} \pm \bar{q}_{p(+1)}^{\uparrow} q_s^{\uparrow}). \tag{3.11}$$

In addition to the desired $J^{PC} = 2^{++}$ nonet we have found a $2^{+-}$ nonet, degenerate in our approximation with the $2^{++}$ nonet. The $2^{+-}$ configuration is exotic in that it cannot be constructed from $\bar{q}q$ in the nonrelativistic quark model. There is no experimental evidence for such states.

In addition to the four L = 1 nonets of the nonrelativistic model, $(0, 1, 2)^{++}$ and $1^{+-}$, we expect four additional degenerate nonets, $(0, 1, 2,)^{+-}$ and $1^{++}$. There is no experimental evidence for any of these extra states in the mass range of the known p-wave nonets.

It is easy to see why the static cavity approximation gives these extra states but not so easy to see what to do about it. The extra modes are most easily visualized in the L - S scheme. In addition to the familar mode in which an L = 1 $\bar{q}q$ pair has its center of mass at rest relative to the cavity there is another mode in which an L = 0 $\bar{q}q$ pair moves in a p-wave with respect to the cavity. The existence of the cavity in the Bag model gives rise to the new degrees of freedom which are not present in the two-body $\bar{q}q$ spectrum of the nonrelativistic potential model.

The same problem occurs in the self-consistent field approximation in nuclear physics, and there it <u>is</u> easy to see what to do about it.

In that case the analogue of our cavity is the effective field exerted on any given nucleon by its neighbors. The nucleus is a nonrelativistic system and the binding force is described to a good approximation by an instantaneous potential. The fixed field ansatz is then an artifact, because the real physics is that the self-consistent field changes much more quickly than the neuleons move. Therefore the extra states are also an artifact, they are spurious, and the right prescription in nuclei is to throw them away.

In hadrons the answer is not so clear. We do not want to assume a light hadron is a nonrelativistic system bound by an instantaneous potential. Rather there is a dynamical question here for which we must look to QCD for the answer. The question is what is the time scale $\tau_v$ of QCD vacuum fluctuations relative to $\tau_q \sim 10^{-23}$ sec., the time for a quark to move a characteristic hadronic distance? If $\tau_v \gg \tau_q$ the vacuum adjusts very slowly to the motion of the quarks, so the static cavity approximation is good and the extra states should exist with masses near those of the usual states. The other extreme $\tau_v \gg \tau_q$ approximates the instaneous potential and the extra states are essentially spurious (or exist at a very high mass). In between there is a continuum of possibilities.

It is clear that for heavy enough quarks we have $\tau_q \gg \tau_v$ and the extra states are spurious. For the light hadrons the two scales are probably of the same order of magnitude, though I wouldn't know how to guess if $\tau_q/\tau_v$ were, say, 3 or 1/3. This is the most complicated possibility. It is surely not excluded by present experiments that the extra states are a few hundred GeV above the known p-wave nonets, and it is worth looking for them. The best signature is the exotic quantum numbers $J^{PC} = 0^{+-}$ and $2^{+-}$ since $J^{PC} = 1^{+-}$ states could be radial excitations.

Although I described the extra states in the cavity rest frame, they can as well be viewed from the $\bar{q}q$ rest frame as cavity excitations. They are collective modes of the color flux that binds the quarks in QCD. For states of large J where a string model may apply they would correspond to excitations of the string.[46]

## IV $\bar{q}\bar{q}qq$ EXOTICS

It was possible to discover quarks from the hadron spectrum known in the early 60's because the known states could be identified with simple $\bar{q}q$ mesons and qqq baryons. Many years later and with many more states discovered, the simple classification scheme is still remarkably successful. Success is always gratifying but this success is also puzzling. What about more complicated states, such as the four quark exotics made of $\bar{q}\bar{q}qq$? Does QCD predict their existence or not? Exotic quantum numbers, such as Q = 2 or S = 2, would make them easy to detect. Is it a success or a failure that they have not yet been found?

A neat solution to this puzzle has been given in the Bag model.[38,47] The solution has two parts:

1) The lowest-lying $\bar{q}\bar{q}qq$ states do not have exotic quantum numbers, but form nonets with the same net quantum numbers as $\bar{q}q$ nonets — they are called "crypto-exotic" nonets.[47]

2) Most of the $\bar{q}\bar{q}qq$ states — both the truly exotic and the crypto-exotic — can "fall apart" into two constituent color-singlet $\bar{q}q$ mesons and are consequently too broad to detect as S-matrix poles. The existence of the low-lying crypto-exotic nonets is implied by the hyperfine splitting due to single gluon exchange, the same approximation discussed in Section II which gives a good qualitative description of the L = 0 hadrons. In this approximation, it is not hard to see why the $\bar{q}\bar{q}qq$ ground state turns out to be a $J^{PC} = 0^{++}$ scalar nonet.[48]

The quark eigen-modes are classified by the group $SU(3)_{color} \times SU(2)_{spin} \times SU(3)_{flavor}$. It is useful to consider $SU(6)_{color-spin}$ which contains $SU(3)_{color} \times SU(2)_{spin}$ and to classify states by $SU(6)_{color-spin} \times SU(3)_{flavor}$. Where $\underset{\sim}{\sigma}$ and $\vec{\sigma}$ denote the

eight color and three spin matrices, the energy-shift due to single gluon exchange is

$$\Delta E = -K\frac{\alpha_s}{R}\sum_{i,j} < \lambda_i \cdot \lambda_j \, \vec{\sigma}_i \cdot \vec{\sigma}_j > \tag{4.1}$$

K is a flavor-dependent constant and the sum is over all $\bar{q}q$, qq, and $\bar{q}\bar{q}$ pairs (i, j). In analogy to the SU(2) relation for a $\bar{q}q$ bound state

$$\vec{s}_1 \cdot \vec{s}_2 = -\frac{1}{2}\,[S^2_{tot} - s_1^2 - s_2^2] \tag{4.2}$$

the expectation value in Eq. (4.1) may be rewritten in terms of $SU(6)_{color-spin}$ Casimir operators

$$\Delta E = K\frac{\alpha_s}{R}\,[\,\frac{1}{2}\,C_6(TOT) - C_6(qq) - C_6(\bar{q}\bar{q}) + \ldots]. \tag{4.3}$$

For simplicity I have displayed only the largest terms in Eq. (4.3); contributions of $SU(2)_{spin}$ and $SU(3)_{color}$ Casimir operators are omitted. $C_6$ is the sum of the squares of the 35 SU(6) generators, the analogue of $S(S+1) = \sum_i \vec{\sigma}_i^2$ for SU(2). $C_6$ dominates Eq. (4.3) just because SU(6) has more generators than SU(3) and SU(2).

The quantum numbers of the ground state are easily obtained from Eq. (4.3) and Fermi statistics. Since $C_6(qq)$ and $C_6(\bar{q}\bar{q})$ appear with a minus sign we want to maximize them. The largest Casimir for a diquark is obtained from the <u>symmetric</u> representation, the 21 in $6 \times 6 = 21 + 15^*$, (just like in SU(2), $2 \times 2 = 3 + 1$, where the triplet is symmetric and the scalar antisymmetric). But if the diquark is symmetric under $SU(6)_{color-spin}$, Fermi statistics require that it be antisymmetric under $SU(3)_{flavor}$, <u>ie</u>, in the 3* of $3 \times 3 = 6 + 3^*$. Therefore qq is in the flavor 3*, $\bar{q}\bar{q}$ is in the flavor 3, and the ground state $\bar{q}\bar{q}qq$ is in a flavor nonet, $3 \times 3^* = 8 + 1$!

The spin of this nonet is determined by $C_6(TOT)$, the first term in Eq. (4.3). Since it contributes positively to $\Delta E$ we want to

minimize it. This is achieved when the total state is an $SU(6)_{color-spin}$ singlet, in which case it is also a singlet of $SU(2)_{spin}$, that is $J = 0$. P and C are then positive because all four constituents are in an s-wave. The conclusion is that the lowest-lying $\bar{q}\bar{q}qq$ states have precisely the same quantum numbers as the $J^{PC} = 0^{++}$ nonet formed from $\bar{q}q$ in a p-wave!

Although this crypto-exotic nonet has the same net quantum numbers as the p-wave scalar nonet, its exotic quark content give it properties very different from the $\bar{q}q$ nonet. The quark content and estimated masses are shown in figure (4.1). Notice in particular the degenerate isoscalar and isotriplet at 1100 MeV., which are just the usual ideally mixed non-strange isoscalar and isotriplet <u>plus an $\bar{s}s$ pair.</u> Unlike the non-strange isoscalar and isotriplet of a $\bar{q}q$ nonet, these $\bar{q}\bar{q}qq$ states will couple strongly to $\bar{K}K$.

This last observation should start bells ringing — in connection with the peculiarities of the scalar mesons discussed in Section III. Remember that the isotriplet $\delta(980)$ and the isoscalar $S^*(980)$ do not make good partners for the $\varepsilon(1400)$ because they couple strongly to $\bar{K}K$ while $\varepsilon(1400)$ couples most strongly to $\pi\pi$. I argued that $\varepsilon(1400)$ and $\kappa(1400)$ are good candidates for the p-wave nonet but that $S^*$ and $\delta$ are not and must be "something else." The cryptoexotic nonet appears to be just the "something else" we were looking for.[38,47]

We suppose then that the states estimated to be at 1100 MeV. in Fig. (4.1) are in fact the $\delta$ and $S^*$ at 980 MeV. $S^*$ is then composed of $\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)\bar{s}s$ and is below threshold for its fall-apart decays to $\bar{K}K$ and $\pi\pi$, though it is presumeably responsible for the observed $I = 0$ $\bar{K}K$ threshold enhancement. The principal decay $S^* \to \pi\pi$ is then OIZ forbidden, which explains the narrow $S^*$ width — <u>e.g.</u>, $14 \pm 5$ MeV. according to Gidal et al.[40] and 8 MeV. according to Irving et al.[42]

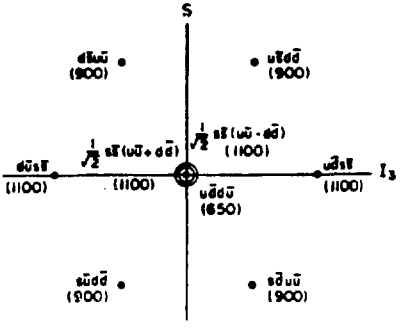


Figure 4.1, from Ref. (48). The lightest $\bar{q}\bar{q}qq$ exotics: the $J^{PC} = 0^{++}$ crypto-exotic nonet. The quark content and masses are shown.

Just because it is degenerate with the S* and also couples strongly to $\bar{K}K$, there is a very strong case for the crypto-exotic assignment of the $\delta$. What about the other two predicted crypto-exotics, the $\kappa(900)$ and $\varepsilon(650)$? There is no evidence for them in standard $K\pi$ and $\pi\pi$ phase shift analyses.[30] This is not so surprising if we realize that, unlike S*, they are predicted to be far above their fall-apart thresholds, at 630 and 280 MeV. respectively. Even if their masses are overestimated by ~100 MeV at 900 and 650, they are still far enough above their fall-apart thresholds to be unobservably broad, with widths of the order of their masses. Although such states would be unobservable as S-matrix poles, it may be possible to verify their existence by using a "P-matrix" analysis of the data.[49]

This is a neat explanation of the known peculiarities of the scalar mesons, but it leaves a question about the assignment of the $\delta$. The $\delta$ has equal fall-apart decay amplitudes to $\bar{K}K$ and to $\eta_s\pi$,[47] where $\eta_s$ denotes the $\bar{s}s$ component of $\eta$. If I assume, as I have for $\varepsilon$ and $\kappa$ that $\Gamma_\delta$ would be of order $m_\delta$ if $\delta$ could decay freely to $\bar{K}K$ and $\eta_s\pi$, then a crude estimate for $\Gamma_\delta$ is $m_\delta$ multiplied by its fall-apart probability to $\eta\pi$. This is just

$$\Gamma_\delta \cong \frac{1}{2} \sin^2(\theta + 55^\circ)\, m_\delta \tag{4.4}$$

since $\sin^2(\theta + 55^\circ)$ is the probability that $\eta$ is an $\bar{s}s$ pair, where $\theta$ is the mixing angle defined in Eq. (2.21). For the preferred $\theta = -11^\circ$ this gives $\Gamma_\delta \cong 250$ MeV., five times large than the 50 MeV. width quoted in the PDG tables.[30] To make Eq. (4.4) consistent with 50 MeV., we need $\theta \cong -35^\circ$. I find this an unpalatably large departure from the standard value, which, as discussed in Section II A, is supported by 1) the mass formula, 2) low energy theorems for $\eta \to \gamma\gamma$, $\pi\pi\gamma$, and 3) the ratio $\sigma(\pi p \to \eta n) / \sigma(\pi p \to \eta' n)$.

Although Eq. (4.4) is only a crude order of magnitude estimate, it is distressing that it disagrees with the quoted width by a factor ~5. There is however another interpretation of the data, proposed several years ago by Flatté[50] (so that collusion with today's theorists would have required tachyons). The 50 MeV. width quoted in the PDG tables is obtained from the width seen in $\delta \to \eta\pi$.[39] Flatté observes, using a simple model, that if $\delta$ actually has a much larger width its apparent width in $\delta \to \eta\pi$ could be small because of "cusps" formed by the opening and closing of the $\bar{K}K$ channel. Fitting the data with his model he finds that the true width could be as large as 300 MeV.

If correct this view would remove the only outstanding puzzle in the picture of the crypto-exotic nonet. The test of Flatté's model is to measure $\Gamma_\delta$ in $\delta \to \bar{K}K$ where the true width should appear. This means accumulating enough statistics to extract the width from the shape of the $l = 1$ s-wave $\bar{K}K$ threshold enhancement. The best available data[51] does not have enough statistics to decide the question — it is shown in figure (4.2). If $\Gamma_\delta$ were as large as, say, 150 MeV., it would be acceptable given the crude "derivation" of Eq. (4.4).

Is it possible that higher mass $\bar{q}\bar{q}qq$ states might also be observable as ordinary resonances? The answer could be yes if there are other $\bar{q}\bar{q}qq$ states which are below the threshold for fall-apart decay. Among the states considered by Jaffe[47] there is a second $J^{PC} = 0^{++}$ nonet, denoted by 9*, with precisely the same quark content as the $\delta$ - S* nonet. This 9* differs from the $\delta$ nonet by "recoupling coefficients" which cause it to prefer fall-apart decays to two vector mesons over decays to two pseudoscalars by a factor $\sim(.56/.18)^2 = 10$. Therefore if these states were below the vector-vector thresholds, their widths would be suppressed by an order of magnitude and they could be observable as

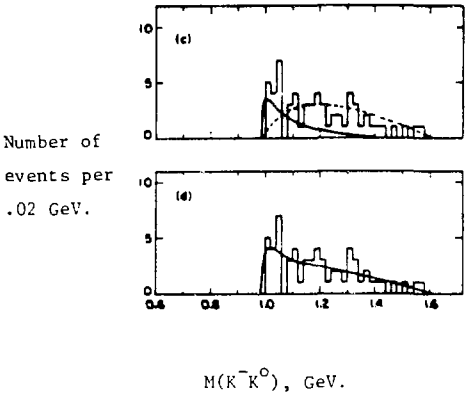


Figure 4.2. $K^-K^0$ mass distribution from Ref. (39). Top curves represent phase space (dashed) and the $\delta$ contribution (solid). Bottom curve is the sum.

ordinary "mass-bumps". Jaffe's estimate of their masses is $(\bar{u}u\bar{d}d) = 1450$, $(\bar{s}u\bar{d}d) = 1600$, $[\bar{s}s(\bar{u}u + \bar{d}d)] = 1800$, all below their vector-vector thresholds. In particular the $\bar{u}u\bar{d}d$ state could be responsible for the $\rho\rho$ threshold enhancement recetnly observed[52] in $\gamma\gamma \to \rho\rho$, much as S* and $\delta$ may be responsible for the $\bar{K}K$ threshold enhancements. If this is correct then $\omega\omega$ should also occur in the ratio $\rho\rho:\omega\omega = 3:1$, and the state should also be seen in the $\pi\pi$ and $\eta\eta$ channels in the ratio $\pi\pi:\eta\eta = 12:1$ (in computing this ratio I assume the standard $\eta - \eta'$ mixing angle $-11^{\circ}$).

Other possible observable states on Jaffe's list are a $J^{PC} = 2^{++}$ nonet with estimated masses 1650, 1800, 1950. These states only fall apart to two vectors, though by gluon exchange they could decay to two pseudoscalars. The estimated masses are above the fall-apart thresholds, but not by very much, and the estimates could easily be high — as they are for $\delta$ and S*. Thus the $\bar{u}u\bar{d}d$ state is also a candidate for the $\rho\rho$ enhancement especially if its actual mass is below the estimated 1650 MeV. The branching ratios for this state would also satisfy $\rho\rho:\omega\omega = 3$ and $\pi\pi:\eta\eta = 12$. Its three body decays might be as prominents as its $\pi\pi$ and $\eta\eta$ decays, since both occur by virtue of gluon exchange. This state together with others on Jaffe's shopping list have been proposed by B. Li and K. Liu as the cause of the observed $\rho\rho$ enhancement.[53] They use vector meson dominance to relate the vector-vector and $\gamma\gamma$ widths of these states.

So far only the s-wave $\bar{q}\bar{q}qq$ states have been investigated, with $J^{PC} = (0, 1, 2)^{++}$. It would be interesting to study the p-wave spectrum to see if any could have masses below their fall-apart thresholds. However to approach this question in the bag model it would be necessary to confront the problem discussed in Section III B.

## V. A CATALOGUE OF HIGHER $\bar{q}q$ EXCITATIONS

In sections II and III we have seen the results of impressive experimental progress that has led to the almost complete assignment of six $\bar{q}q$ nonets: two s-wave and four p-wave. This is interesting for what it teaches us about how quarks bind but also because it establishes a matrix of known states up to ~ 1.5 GeV against which we can look for the deviations that would represent new physics. An example is the discussion of $\bar{q}\bar{q}qq$ exotics in the preceding section: it is the understanding of the known "matrix" of $\bar{q}q$ states which enabled us to recognize $\delta$ and S* as "something else". Similarly, in Section VII I will argue on the basis of what we know about the $A_1$ nonet, discussed in Section III, and the $\pi'$ nonet, to be discussed in this section, that the $\bar{K}K\pi$ enhancement seen at 1440 MeV in $\bar{p}p$ annihilation at rest and in $\psi \to \gamma X$ is probably a pseudoscalar glueball.

To understand the new physics above ~1.5 GeV. it is important to have a similar grasp of that part of the $\bar{q}q$ spectrum. This is a very tall order. In this section I will briefly describe the progress that has been made. I will do little more than list the observed radial and orbital excitations and present references to the experimental literature.

### A. Radial Excitations

As the reader should convince himself, $J^{PC} = 0^{-+}$ states cannot be constructed from $\bar{q}q$ states with $L > 0$ in the nonrelativistic quark model. Therefore in the nonrelativistic model new $J^{PC} = 0^{-+}$ states can only be interpreted, if they are $\bar{q}q$ states, as radial excitations. Low spin states are usually the hardest to detect, so it is impressive that evidence for 8/9 of an excited pseudoscalar nonet is already in place. The radial ground state is denoted by the principal quantum number N = 1, so these are (N = 2, L = 0) states.

The candidates for this nonet are listed in Table (5.1). The same

| | M | Γ | Decays |
|---|---|---|---|
| $\pi'$ | ~1270 | ~580 | $\varepsilon\pi, \rho\pi$ |
| $K'$ | 1400-1450 | ~250 | $\varepsilon K, K^*\pi(\rho K?)$ |
| $\zeta$ | 1275 | 70 | $\eta\pi\pi(\delta\pi, \varepsilon\eta)$ |

Table 5.1. Candidates for 8/9 of the radially excited pseudoscalar nonet. (See text for references to experimental papers.)

high statistics diffractive $\pi p$ scattering data[26,35] (from ACCMOR) that gave evidence for the $A_1$ at 1280 also appears to show very broad resonant behavior in the $IJ^{PC} = 1, 0^{-+}$ $\varepsilon\pi$ and $\rho\pi$ channels. A fit to that data[28] yielded $m_{\pi'} \cong 1270$ MeV. The K' was seen in $\varepsilon K$ in the LASS spectrometer at SLAC[54] and more recently in $\varepsilon K$ and $K^*\pi$ by the ACCMOR collaboration at CERN.[35] The mass is reported from 1400 to 1450 MeV. Finally one isoscalar condidate has been seen under the D(1285) in the reaction $\pi^- p \to \eta\pi^+\pi^- n$, by the same ZGS experiment[55] that also observed the $A_1$ and H mesons[27] in $\pi^+\pi^-\pi^0$. I call this particle the $\zeta(1275)$, in honor of the ZGS, as explained in figure (5.1) which also illustrat the perils of trying to fool around in Physics Review Letters.

Because it plays an important part in the discussion of the glueball candidate in Section VII, the evidence for $\zeta(1275)$ is shown in Figs. (5.2) and (5.3). It is likely that many presumed observations

If we suppose G is distinct from E and $J^P(G) = 0^-$, how can we decide if G is a glueball? If it is not a glueball, then it is most likely to be a radially excited $\bar{q}q$ meson. There are already two excellent candidates for an excited $J^{PC} = 0^{-+}$ nonet, K'(1400) and $\zeta(1275)$ ($\zeta$ is named in honor of the ZGS, R. I. P. - it is called $\eta(1275)$ in the data card listings of Ref. (26)). The $\zeta$ was observed in a partial wave analysis of $\pi^- p \to \zeta n \to \eta\pi^+\pi^- n$ in the very sensitive $\pi p$ experiment[21] which did not see a significant $\eta\pi\pi$ signal near 1.4 GeV. We might hypothesize that G and $\zeta$ are the two isoscalars in the nonet.

(a) Before

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If we suppose $G$ is distinct from $E$ and $J^P = 0^-$ for $G$, how can we decide if $G$ is a glueball? If it is not a glueball, then it is most likely to be a radially excited $\bar{q}q$ meson. There are already two excellent candidates for an excited $J^{PC} = 0^{-+}$ nonet, $K'(1400)$ and $\zeta(1275)$ [the $\zeta$ was named in honor of the zero-gradient synchrotron at Brookhaven National Laboratory—it is called $\eta(1275)$ in the data card listings of Ref. 26]. The $\zeta$ was observed in a partial-wave analysis of $\pi^- p \to \zeta n \to \eta\pi^+\pi^- n$, in the very sensitive $\pi p$ experiment[21] which did not see a significant $\eta\pi\pi$ signal near 1.4 GeV. We might hypothesize that $G$ and $\zeta$ are the two isoscalars in the nonet.

(b) After

Figure 5.1 from Ref. (99). Explanation for the name $\zeta$ before and after editorial treatment, including second coming of the ZGS to Long Island.

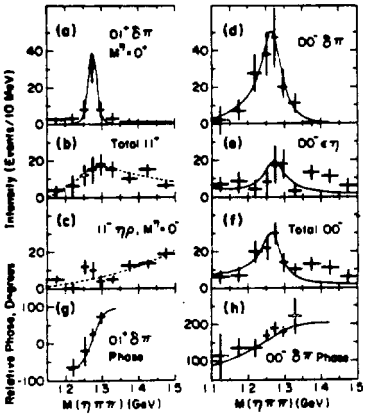


Figure 5.2, from Ref. (55). Results of phase shift analysis. (a) - (f) are intensities of labeled partial waves. (g) is the phase of $01^+$ $\delta\pi$ relative to $11^+$ $\delta\pi$. (h) is the phase of $00^-$ $\delta\pi$ relative to $00^-$ $\varepsilon\eta$. Curves are fits which include D and $\zeta$.

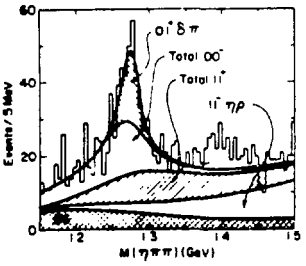


Figure 5.3, from Ref. (55). The experimental mass spectrum compared to the intensity curves of Fig. 5.2.

of D(1285) based on $\eta\pi\pi$ mass histograms alone were in fact observations of both D and $\zeta$, since only a spin analysis could separate the two signals. A large $\zeta$ component in these experiments is indicated by the tendency to report widths larger than the $\Gamma_D \cong 10$ MeV seen in Ref. (55).

Notice that $\zeta$ appears clearly in $\delta\pi \to \eta\pi\pi$ and perhaps also in "$\varepsilon$"$\eta$. (Here "$\varepsilon$" obviously cannot refer to $\varepsilon(1400)$ but rather to a fit to the I = 0 s-wave dipion phase shift.) It is however not at all certain that there is really structure in the $\varepsilon\eta$ channel. Except for the single low bin at 1280 the $\varepsilon\eta$ data could simply be rising smoothly from threshold — bear in mind that the data points in Fig. (5.2) are not corrected for acceptance, which falls by roughly a factor two from 1.28 to 1.4.[56] If the low point at 1280 were raised by $\sim 2\sigma$ there would be no evidence of any structure.

On the other hand, if there really is structure in the $\varepsilon\eta$ channel then there might be a second particle at $\sim 1.4$ GeV. I will call this possible object the "glitch" or gl(1.4). Notice it does not appear at all in $\delta\pi$. I will have more to say about gl(1.4) in Section VII. It is essential to repeat this experiment with good acceptance at 1.4 GeV to see if gl(1.4) really exists. If it does, one possibility is that it is the missing ninth member of the nonet. Notice however in Table (5.1) that $\pi'$ and $\zeta$ have the same mass. This suggests ideal mixing, in which case the ninth member of the nonet, which I will call $\zeta'$, should have a bigger mass,

$$m_{\zeta'} \cong 2m_{K'} - m_{\zeta} \tag{5.1}$$

yielding $m_{\zeta'} \cong 1.5 - 1.6$ GeV for $m_{K'}$ from 1.40 to 1.45 GeV.

In addition to these pseudoscalars four other states have been observed whose only nonrelativistic $\bar{q}q$ assignments are as radial

excitations. Three are radial excitations of p-wave states, (N = 2, L = 1): they are the scalar K'(1850),[41] the $1^{++}$ $A_1'(1650)$,[35] and the $1^{+\pm}$ Q'(1750)[35] which is a mixture of $Q_A'$ and $Q_B'$ analogous to the N = 1 Q mesons discussed in Section III A. The fourth unambiguous radial excitation is an (N = 2, L = 2) state! It is the $2^{-+}$ $A_3'$ (2100),[35] a radial excitation of the L = 2 $A_3$(1710).

In addition there are several states which in the nonrelativistic $\bar{q}q$ model could be orbital or radial excitations. These include enough particles to fill a $J^{PC} = 1^{--}$ nonet, which could however be classified as (L = 2, N = 1) or as (L = 0, N = 2). Most likely both nonets are present in the same mass region. For instance $\rho'$(1600) might actually be two states, at ~ 1530 and ~ 1690 MeV.[57] Other candidates for these nonets are $\omega'$(1640 - 1700),[57] $\phi'$(1900),[57] and perhaps K*(1700).[41] Another ambiguous object is a $2^{++}$ $\pi f$ resonance[35] at 1700 MeV., which could be an (N = 2, L = 1) radial excitation of the f(1270) or a purely orbital excitation, (N = 1, L = 3).

B. L ≥ 2 Orbital Excitations

In addition to the just mentioned ambiguous cases, there are several known L ≥ 2 excitations. The Particle Data Group tables[30] contain entries for 8/9 of the leading L = 2 $J^{PC} = 3^{--}$ nonet consisting of g(1700), $\omega$(1670) and K*(1753). Other L = 2 states are the $2^{-+}$ $A_3$(1670)[58] and the $2^{-\pm}$L(1820),[35] analogous to the $1^{+\pm}$ Q system. The leading L = 3 $4^{++}$ nonet is also 8/9 filled, containing $A_2^*$ (2.00),[59] h(2070)[30] and K*(2070).[41,59]

## VI HEAVY QUARKONIUM

Unlike the light mesons whose simplicity is still mysterious, we expect the quarkonium states of c, b, and heavier quarks to be truly accessible to nonrelativistic approximations. If we repeat the crude harmonic oscillator estimate, which yielded discouraging results in Section II B for the light quarks, we find for $m_c = 1.5$ GeV and $\sqrt{\langle r^2 \rangle} = .5$ fm. that $p = \frac{3}{2}\sqrt{1/\langle r^2 \rangle} = 450$ MeV. and $v^2 \cong 1/4$. So for $\psi$,T, and heavier systems the nonrelativistic model begins to be a serious approximation. The slowly moving heavy quarks are bound in this approximation by an instantaneous potential, calculable in principle from QCD. We may use the experimental quarkonium spectrum to "measure" this potential and to compare it with our theoretical expectations. This program has begun beautifully, as I will describe below.[60]

### A. The Smoothness Hypothesis

Consider first the spin independent potential $V_0(r)$. What do we know about it theoretically? Most reliably we know at short distances that it should approach the Coulomb potential due to single gluon exchange. For small enough values of r, say

$$r \leqslant \frac{1}{2\text{GeV}} \cong .1 \text{ fm.} \tag{6.1}$$

we have

$$V_0(r) \cong -\frac{4}{3}\frac{\alpha_s(r^{-1})}{r}$$

$$\cong \frac{8\pi}{27 r \ln(r\Lambda)} \tag{6.2}$$

where the group theory factor 4/3 is explained by Jackson in Ref. (20). As I have indicated in preceding sections, $\Lambda$ may be of order 200 MeV or perhaps even smaller.

Equation (6.2) follows from the validity of perturbation theory at short distances in QCD — i.e., asymptotic freedom. At large distances our ideas about confinement suggest, though less precisely and less reliably, that the potential is rising linearly, say for

$$r \gtrsim 1 \text{ fm}. \tag{6.3}$$

that

$$V_0 = Kr \tag{6.4}$$

The constant K is calculable in principle but not yet in practice. It may however be estimated from the slope of the light meson Regge trajectories,

$$K \cong 1/2\pi\alpha'_{\text{Regge}} \tag{6.5}$$

by a plausible argument based on the correspondence limit of quantum mechanics, presented below.

The transistion region .1 fm $\leqslant r \leqslant$ 1 fm. is probably the most complicated and the most intractable theoretically. Here we can now do no better than make a reasonable guess, which I've called the "smoothness hypothesis," that there is a smooth transistion between the asymptotic long and short distance regimes. In fact $\psi$ and $\Upsilon$ live in the transistion region; their spectra are determined primarily by distances .1 fm $< r <$ 1 fm. By "measuring" $V_0$ in this region we are not probing directly either the long or short distance physics, but we are testing both regimes indirectly, by way of the smoothness hypothesis. The transistion potential determined in this way does connect smoothly to the expected long and short distance potentials.

Their are a host of few-parameter potentials which all succeed in describing the $\psi$ and $\Upsilon$ spectra. Some of them are listed in Table (6.1). They include wildly different analytic forms which would imply

| V(r) | Reference |
|---|---|
| $-\frac{A}{r} + Kr + V_o$ | 61 |
| $-\frac{4}{3}\int\frac{d^3q}{(2\pi)^3}\, e^{i\vec{q}\cdot\vec{r}} \cdot \frac{1}{\vec{q}^2} \cdot \frac{16\pi^2}{\ln(1 + \frac{\vec{q}^2}{\Lambda^2})}$ | 62 |
| $-\frac{4}{3}\frac{\alpha_s}{r} + \frac{32\pi(\alpha_s B)^{1/4}}{3} + C(\alpha_s^3 B)^{1/4}$ | 63 |
| $A \ln(r/r_o)$ | 64 |
| $a + br^x \quad x = 0.1$ | 65 |
| Inverse scattering construction | 66 |

Table 6.1. Examples of potentials which successfully fit $\psi$ and $\Upsilon$ spectra and are indistinguishable in the region probed by those spectra.

wildly different behaviors if extrapolated into the long or short distance regimes. But the important point is that for the region which is "measured" by the $\bar{c}c$ and $\bar{b}b$ spectra they are virtually indistinguishable, from one another and from the potential constructed directly from the spectrum using the inverse scattering methods.[66] This is evident in Fig. (6.1)[67] which includes potentials (61) and (65) from Table (6.1) and a refined version of potential (62) (see also a similar figure containing more potentials in the review by Eichten[60]).

To check the smoothness hypothesis it is easiest to consider the Cornell[61] potential

$$V_{\text{Cornell}} = -\frac{A}{r} + Kr + \text{constant} \tag{6.6}$$

which has a form that manifestly interpolates smoothly. Smoothness is confirmed if the constants A and K agree with Eqs. (6.2) and (6.5). The fits give $A_\psi = .52$, $A_\Upsilon = .48$, and $K = 1/(5.5\ \text{GeV}^2)$ which imply $\alpha_s(\psi) = .39$, $\alpha_s(\Upsilon) = .36$ and, using Eq. (6.5), $\alpha'_{\text{Regge}} = .87\ \text{GeV}.^{-2}$ It is encouraging that the coupling constant "runs" in the right direction, $\alpha_s(\Upsilon) < \alpha_s(\psi)$. The rather large values for $\alpha_s(\psi)$ and $\alpha_s(\Upsilon)$ are not surprising since the effective coupling here is evaluated not at the quark mass but at the smaller scale of the bound state momentum (which would be $\alpha_s m_Q$ in a ladder approximation[68]). Given the approximations involved, the smoothness hypothesis is an impressive success.

It is also intriguing that the Richardson potential[62] gives a somewhat better fit than the Cornell model, and with only the single parameter $\Lambda$ (and the quark masses). See Gottfried's review[60] for a discussion.

The connection between the string tension K and $\alpha'_{\text{Regge}}$, Eq. (6.5), is obtained by a classical argument,[69] justified by the correspondence limit of quantum mechanics which applies for sufficiently large angular

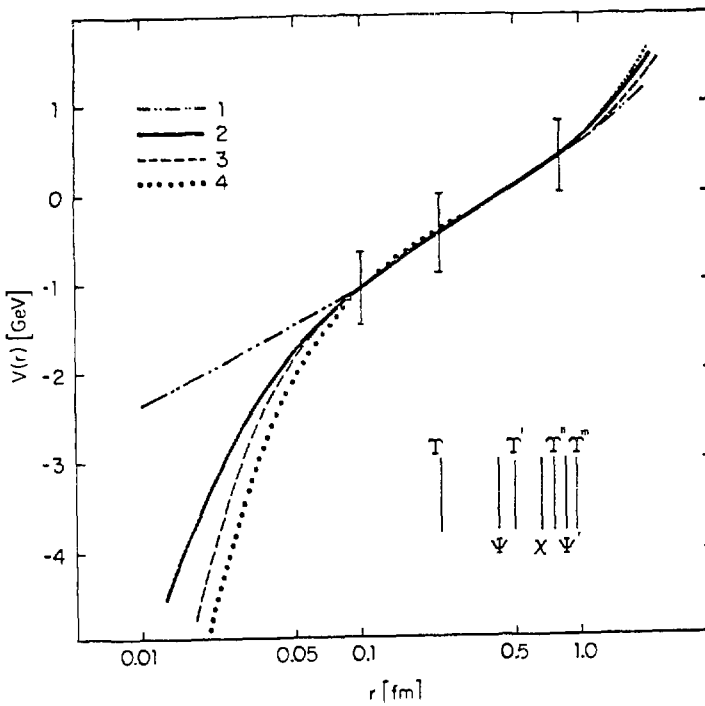


Figure 6.1, from Ref. (67). Four potentials which successfully fit the $\psi$ and $\Upsilon$ spectra: (1) $a + br^x$, (2) Richardson plus higher order corrections, (3) logarithmic interpolation from Coulomb to linear, (4) Cornell. $\psi$ and $\Upsilon$ states are displayed at their mean-squared radii.

momen a. In this limit a high spin meson is regarded as a rigid rotor, the "rod" in QCD being the flux tube whose mass per unit length is the string tension K. where L is the length of the tube and $\omega$ its angular velocity, the classical relativistic energy is

$$E = \int_{-L/2}^{L/2} \frac{K d\ell}{\sqrt{1-\omega^2\ell^2}} = \frac{\pi}{2} KL \tag{6.7}$$

and t· angular momentum is

$$J = \int_{-L/2}^{L/2} \cdot \ell \frac{\omega \ell K d\ell}{\sqrt{1-\omega^2\ell^2}} = \frac{\pi}{8} KL^2. \tag{6.8}$$

In obt ining these results it is assumed that the tip of the rotor moves the speed of light, $\omega L/2 = 1$.[70] The slope of the Regge trajec ory is then

$$\alpha'_{\text{Regge}} = \frac{J}{E^2} = \frac{1}{2\pi K} . \tag{6.9}$$

ing K determined by the fit to the Cornell potential in Eq. (6.9) we get $\alpha'_{\text{Regge}} = .87\ \text{GeV}^{-2}$, which is comfortably in the range for $\alpha'_{\text{Regge}}$ taken rom the meson masses discussed in Section V. For instance, from the ρ rajectory, consisting of ρ(776), $A_2$(1317), and g(1700) we get

$$\frac{1}{1.317^2 - .776^2} = .88\ \text{GeV}^{-2}$$

or

$$\frac{2}{1.70^2 - .776^2} = .87\ \text{GeV}^{-2}$$

[illegible] and $\gamma^4$ [illegible] $\text{GeV}^{-2}$

Like the puzzle posed by the success of the crude nonrelativistic model of the light hadrons, it is also puzzling that this crude picture of the linear Regge trajectories works so well for such modest values of J.

## B. Spin Dependence

The spin dependence of the heavy quark potential is a challenging problem which tests our mastery of QCD dynamics. It may be a tractable problem because for heavy quarks there is a manageable expansion in powers of inverse quark masses. We can focus on the leading terms which are proportional to $m_Q^{-2}$. The most general form of the spin dependence in the leading order is[9]

$$V_{Spin} = \left(\frac{\vec{s}_1 \cdot \vec{L}}{2m_1^2} + \frac{s_2 \cdot \vec{L}}{2m_2^2}\right) \frac{1}{r}\frac{dV_1}{dr} + \frac{\vec{L} \cdot (\vec{s}_1 + \vec{s}_2)}{m_1 m_2} \frac{1}{r}\frac{dV_2}{dr}$$

$$+ \frac{1}{m_1 m_2} \left(\vec{s}_1 \cdot \hat{r}\, \vec{s}_2 \cdot \hat{r} - \frac{\vec{s}_1 \cdot \vec{s}_2}{3}\right) V_3$$

$$+ \frac{2}{3m_1 m_2}\, \vec{s}_1 \cdot \vec{s}_2\, V_4 \tag{6.10}$$

where $\vec{r} = r_1 - \vec{r}_2$, $L = \vec{r} \times \vec{p}$, and $\vec{p}$ is the center of mass momentum. The four terms are known as the Thomas, spin-orbit, tensor, and spin-spin force respectively.

In QED for a Coulomb potential due to single photon exchange the $V_i$ are given by

$$V_1 = V_2 = -\frac{\alpha}{r} \tag{6.11a}$$

$$V_3 = \frac{1}{r}\frac{dV_1}{dr} - \frac{d^2V_1}{dr^2} \tag{6.11b}$$

$$V_4 = \nabla^2 V_2 \tag{6.11c}$$

Substituted into Eq. (6.10) this giv s the usual Breit potential for positronium,

$$V_{Breit} = \frac{3}{2}\frac{\vec{L} \cdot (\vec{s}_1 + \vec{s}_2)}{m^2}\frac{\alpha}{r^3} + \frac{1}{m^3}\left(3\vec{s}_1 \cdot \hat{r}\, \vec{s}_2 \cdot \hat{r} - \vec{s}_1 \cdot \vec{s}_2\right)\frac{\alpha}{r^3}$$

$$+ \frac{8\pi}{3}\frac{\vec{s}_1 \cdot \vec{s}_2}{m^2}\,\delta(\vec{r}) \tag{6.12}$$

For QCD at short distances where single gluon exchange dominates Eqs. (6.11) and (6.12) apply with the replacement[20]

$$\alpha \to \frac{4}{3}\alpha_s \tag{6.13}$$

Gromes[71] has tabulated the analogous potentials generated by exchange of elementary scalar, pseudoscalar, axial vector, and tensor quanta.

Eichten and Feinberg[9] have found the most general exact relationship which can hold among the $V_i$ in QCD. It is

$$V_4 = \nabla^2 V_2 \tag{6.14a}$$

and

$$V_1 = V_0 + \tilde{V}_1 \tag{6.14b}$$

The second relation means that there must be a contribution to the Thomas term determined by the spin independent potential, $V_0$, which has been "measured" as discussed above. $\tilde{V}_1$ is in general arbitrary. Equations (6.14) are obtained from a general analysis of the potential (constructed from the large time limit of the Wilson loop). By comparing the contributions of single gluon exchange and instantons to the $V_i$, Eichten and Feinberg show that no stronger statement than Eqs. (6.14) can be generally valid. In general in QCD there are no fewer than three potentials, say $\tilde{V}_1$, $V_2$, $V_3$, which, in addition to $V_0$, determine the spin dependence.

From Eq. (6.14b) we see that the optimistic ansatz that one gluon exchange dominates all spin dependence[4] is very unlikely, since there must be a Thomas contribution from the long-range, spin-independent confining potential.

In an attempt to construct a useful ansatz, Eichten and Feinberg observe that in the $\bar{Q}Q$ center of mass the unknown potentials $\tilde{V}_1$, $V_2$, $V_3$ are due to color magnetic fields <u>only</u>. According to a popular hypothesis, based on an analogy with superconductivity, confinement

is due to color-electric forces.[72] Then in this view confinement is accomplished by $V_0$, the only term which gets contributions from the color-electric field. Eichten and Feinberg suggest as a minimal assumption that only color-electric forces are long range and that $\tilde{V}_1$, $V_2$, $V_3$ are determined purely by short-distance magnetic fields which are dominated by single gluon exchange. With this assumption we have

$$\tilde{V}_1 = 0 \tag{6.15a}$$

since the single gluon contribution to $V_1$ is already contained in $V_0$, and

$$V_2 = -\frac{4}{3}\frac{\alpha_s}{r} \tag{6.15b}$$

$$V_3 = 4\frac{\alpha_s}{r^3} \tag{6.15c}$$

as in Eq. (6.11). Together with Eq. (6.14) the spin-dependent potential (6.10) is completely determined in terms of the "measured" $V_0$ with no free parameters. The only long-range spin-dependent force is the Thomas term determined by $V_0$ according to (6.14b).

The picture of confinement underlying this ansatz is based on a property of ordinary superconductors: a superconductor of electric charge repels magnetic flux. When magnetic flux is forced into a superconductor it does not spread in the usual way of Coulomb fields but is restricted to a narrow tube whose interior is in the normal (non-superconducting) state.

This dynamics could be the origin of confinement in QCD if we reverse the roles of electric and magnetic fields.[72] Suppose the vacuum, so-called "empty space", is a superconductor for color magnetic charges. Quarks carry color-electric charge and this vacuum wishes to repel the electric flux emanating from them. So the field lines between a widely separated Q and $\bar{Q}$ in a color singlet will <u>not</u> spread and give

a 1/r Coulomb potential. Instead the color E field is forced into a flux tube which connects the Q and $\bar{Q}$, and we get a linear potential like the Cornell model, where K is just the energy per unit length of the flux tube. For widely separated quarks, such as states of high J, we have then a picture resembling a string model where the "string" is the color-electric flux tube. For quarks which are not so far apart, such as the s-wave mesons for example, the flux "tube" is probably more like a spherical region than an elongated tube, and we have a picture which may explain the Bag model.

This is not however a proof of the Eichten-Feinberg conjecture, even assuming the validity of the superconductor analogy. There might for instance be excitations of the flux tubes which create non-perturbative magnetic effects.[73]

If we do pursue the Eichten-Feinberg conjecture, the spin dependent potential for $\bar{Q}Q$ is

$$V_{Spin} = \frac{1}{2m^2}\vec{L}\cdot\vec{S}\,\frac{1}{r}\frac{dV_0}{dr} + \frac{4\alpha_s}{3m^2}\left(\frac{\vec{L}\cdot\vec{S}}{r^3} + \frac{8\pi}{3}\vec{s}_1\cdot\vec{s}_2\,\delta(\vec{r}) + \frac{3\vec{s}_1\cdot\vec{r}\;\vec{s}_2\cdot\vec{r} - \vec{s}_1\cdot\vec{s}_2}{r^3}\right) \qquad (6.16)$$

Taking $V_0 = V_{Cornell}$ they then obtain $m_\psi - m_{\eta_c} = 130$ MeV in reasonable agreement with the experimental value 116 ± 9 MeV. The splitting between the $^3P_J$ $\chi$ states and the center of gravity (which is $\frac{m_0 + 3m_1 + 5m_2}{9}$) is computed to be (37, -29, -94) MeV. for J = (2, 1, 0) while the experimental values are (30 ± 2, -14 ± 2, -109 ± 3) MeV. These predictions may however be sensitive to higher order corrections in $\alpha_s$ to be discussed by Lepage.[8] Independent of these $O(\alpha_s)$ corrections they predict

$$\frac{M_{\psi'} - M_{\eta'_c}}{M_{\psi} - M_{\eta_c}} = \left| \frac{\psi_{2s}(0)}{\psi_{1s}(0)} \right|^2$$

$$= \frac{M^2_{\psi'} \Gamma(\psi' \to e^+e^-)}{M^2_{\psi} \Gamma(\psi \to e^+e^-)} \tag{6.17}$$

which implies $M_{\psi'} - M_{\eta'_c} = 80 \pm 15$ MeV. (in agreement with the value announced later in the conference of 92 ± 5 MeV[74]).

The analogous predictions should be appreciably more reliable for the $\Upsilon$ system. In the Cornell model for instance $\langle v^2 \rangle_{\psi} \cong .2$ and $\alpha_s(\psi) \cong .31$ whereas $\langle v^2 \rangle_{\Upsilon} \cong .1$ and $\alpha_s(\Upsilon) \cong .23$. Both relativistic and radiative corrections are more manageable for the $\Upsilon$.

Another approach to the spin dependence is to assume A) that the confining potential is a Lorentz scalar and B) that it can be abstracted from the nonrelativistic reduction of the Bethe-Salpeter equation for the exchange of an elementary scalar meson. This does allow a good fit of the $\bar{c}c$ $\chi$ masses.[75]

A proposed test of this hypothesis is that the D and B mesons have inverted p-wave multiplets, $M(^3P_0) > M(^3P_1) > M(^3P_2)$.[76] The argument is that the spin-orbit force would be

$$V_{\text{Spin-Orbit}} = \frac{\vec{L} \cdot \vec{S}}{4m^2} \left( -\frac{K}{r} + \frac{4}{3} \frac{\alpha_s}{r^3} \right) \tag{6.18}$$

The term $-K/r$ is the effect of the confining potential and the minus sign follows from assumptions A) and B). The second term $4\alpha_s/3r^3$ is due to gluon exchange. If the first term dominates the p-wave multiplets will indeed be inverted. Taking $K = \frac{1}{6}$ GeV$^2$, to fit the spin-independent features of the spectrum, and for $r = .8$ fm the right side of Eq. (6.18) is proportional to $\sim (-1 + \frac{\alpha_s}{2})$ and for $\alpha_s < 2$ the multiplets will invert. However the running coupling $\alpha_s(k)$

should be evaluated not at $k = m_D$ but at a scale characterized by the internal momentum of the bound state.[68] Since the reduced mass of a light-heavy meson $\bar{q}Q$, like D or B, is $\mu \cong m_q$ the relevant scale is small, $\alpha_s$ is not in the perturbative regime, and the sign is not clear (nor is the adequacy of single gluon exchange). For r large enough, which is to say for a sufficiently high orbital excitation, the $-K/r$ term will eventually dominate over $\alpha_s/r^3$,[77] though whether this means $L = 2$ or $L = 3$ is not clear to me.

I should confess to a prejudice against the approach characterized by assumptions A) and B). Even if A) is correct and the potential $V_0$ is a Lorentz scalar, it seems to me that the idea B) that the spin dependence is what we would get from exchange of an elementary scalar quantum has little connection to our intuition about the dynamics of confinement.

Since $\bar{Q}q$ mesons have small reduced masses, $\mu = m_q m_Q/(m_q + m_Q) \cong m_q$, they are relativistic systems with the typical 1 fm. size characteristic of the light mesons. Notice however in Eq. (6.10) that the spin-orbit, tensor, and spin-spin forces are all proportional to $1/m_Q$ and that only the Thomas force has a contribution which is not so suppressed. If this holds to higher orders in the relativistic corrections (plausible though I'm not aware of a proof) then for a $Q\bar{q}$ system with $m_Q \gg m_q \cong \mu$ we have[9]

$$V_{Spin} = \vec{L} \cdot \vec{s}_q \, V_a(r,\mu) + \frac{\mu}{m_Q} (\vec{s}_q \cdot \vec{s}_Q \, V_b(r,\mu) + \ldots) \tag{6.19}$$

Since $V_a$, $V_b$ are independent of $m_Q$ for large enough $m_Q$, Eq. (6.19) implies scaling laws which relate the spin splittings of different $\bar{Q}q$ mesons. For instance the B and D hyperfine splittings $(m(^3S_1) - m(^1S_0))$ should be related by

$$m_{B^*} - m_B = \frac{m_c}{m_b}(m_{D^*} - m_D)$$

$$\cong 50 \text{ MeV.} \tag{6.20}$$

C. The Most Serious Problem

I do not want to leave this subject without mentioning the most serious outstanding problem. This is the failure by a factor two of the predictions for the rates of the radiative transistions $\psi' \to \gamma\chi$ and $\chi \to \gamma\psi$.

In earlier data it appeared that not only the magnitudes but also the ratios of the rates for $\psi' \to \gamma\chi_{0,1,2}$ disagreed with the theoretical expectation. But in the most recent data from the Crystal Ball the ratios are in reasonable agreement with the factor $(2J + 1)K^3$ expected for the dominantly electric-dipole transistions of the nonrelativistic model. Normalizing to $(2J+1)K^3$ the experimental ratios for $\Gamma_0 : \Gamma_1 : \Gamma_2$ are reported by Porter[74] as $1{:}00 \pm .07{:}.99 \pm .08 = 1.31 \pm .10$. The worst ratio, $\chi_2 : \chi_0$, is only 30% and $2\sigma$ from the expected value of 1.

The problem of the magnitude persists, for instance in both the "naive" version of the Cornell model, which predicts $\Gamma_{0,1,2}$ = 50, 45, 29 keV., and with the coupled-channel corrections which give $\Gamma_{0,1,2}$ = 43, 34, 24 keV.[61] For comparison the Crystal Ball group now reports $\Gamma_{0,1,2}$ = 22, 18, 16 keV with statistical errors from 7 to 10% and an overall uncertainty in normalization of less than 15%.[74]

Arafune and Fukujita[78] suggest that strong interaction corrections, which are known to have large effects on other decay rates,[8] may be responsible for this problem. They use the Breit potential, Eqs. (6.12 - 3), to incorporate the effect of transverse gluon exchange. Perturbing in the Breit potential they find that the wave functions

are distorted so badly that they and the $E_1$ rates cannot be reliably estimated. For the ratios among the $E_1$ rates they find manageable corrections, though their results are in poorer agreement with the new data[74] than is the uncorrected $(2J + 1)K^3$. The qualitative conclusion that the wave functions are more sensitive to strong interaction corrections than the spectrum may explain the failure to predict the magnitude of the $E_1$ transistions.

## VII GLUEBALLS

Certainly the most exciting development in meson spectroscopy during the last year is the possibility that we have found evidence for a glueball here, at SPEAR, in data from the Mark II[79] and Crystal Ball[80] collaborations. I will discuss the evidence for this interpretation of the $\bar{K}K\pi$ enhancement at 1440 MeV. after a brief review of what theory can offer in the way of glueball phenomenology. You will see that the theoretical problem is so difficult that we are forced to rely on the most simple, general, model independent ideas to interpret the data. The knowledge of the light $\bar{q}q$ meson spectrum reviewed in Sections II and III plays a crucial role in this analysis.

### A. *Theoretical Models*

In the future, calculations with a space-time lattice may well lead to quantitative understanding of the spectrum. On the lattice the glueball spectrum is more accessible than the $\bar{q}q$ spectrum, because of the special problem of including fermions. Bhanot[81] has estimated the glueball mass scale by relating it to the string tension (see Section VI) in the strong coupling limit. In particular he studied the correlation between two closed flux loops as a function of their separation. The "glueball mass" determines the scale of the exponential fall-off

$$< U(r)U(0) > \propto e^{-mr} \tag{7.1}$$

His result is m = 1.4 ± .7 GeV., a reasonable scale. However this is not necessarily the mass of any particular glueball but rather corresponds to a weighted average of all the states which can be exchanged. In addition it is not possible to extract the spins and parities of the contributing states.

The bag model[38,82] and the nonrelativistic potential model[83,84] have also been used to study the glueball spectrum. It will be

clear from the preceding lectures that I view these approaches as sometimes useful but limited phenomenological guides to the light $\bar{q}q$ spectrum. Applied to the glueball spectrum their reliability is even more sharply limited. In the bag model the bag constant B need not be the same for glueballs as for $\bar{q}q$ mesons, though that is assumed in the calculations I will review. The larger color charge of a gluon suggests that B is actually bigger for glueballs. Similarly the strength of the potential in the nonrelativistic model is not known from $\bar{q}q$ physics and is probably larger; in addition, there is no reliable way to estimate the "constituent gluon mass" which is a necessary ingredient of this approach. A further very serious difficulty which afflicts both bag and potential models is that the spin-dependent forces are likely to be larger — and therefore even harder to estimate reliably — for glueballs because of the larger spin and color charge of the gluon.

In the bag models calculations[38,82] free, massless (and therefore transverse) gluons are confined to a static spherical cavity.[85] The single gluon modes are then transverse electic, $P = (-1)^{L+1}$, and transverse magnetic, $P = (-1)^{L}$. The energies of the three lowest modes, in terms of the radius R of the cavity, are

$$\begin{aligned} TE_1: &\quad L^P = 1^+ \quad E = 2.74/R \\ TE_2: &\quad L^P = 2^- \quad E = 3.96/R \\ TM_1: &\quad L^P = 1^- \quad E = 4.49/R \end{aligned} \tag{7.2}$$

The ground state glueball is then constructed from two $TE_1$ modes. Since it is a color singlet, Bose statistics require that it be the <u>symmetric</u> combination of the two $TE_1$ modes. Therefore the quantum numbers are $J^{PC} = 0^{++}, 2^{++}$. Minimizing E(R) as described in section II, these states are found at M $\cong$ .96 GeV.[82]

The first excited states are made from $TE_1 \times TE_2$ with $J^{PC} = (1,2,3)^{-+}$ and $TE_1 \times TM_1$ with $J^{PC} = (0, 1, 2)^{-+}$. However we again encounter the problem discussed at the end of Section III for the p-wave $\bar{q}q$ states, which I argued requires understanding the time scale of the QCD vacuum. In Ref. (82) the authors in effect adopt the approximation that the vacuum response time is very short — as in an instaneous potential approximation — so that the "extra" states are spurious and should be discarded. These are just the states obtained by putting the ground states, $J^{PC} = (0, 2)^{++}$, in a p-wave with respect to the cavity, that is, $(0, 2)^{++} \times 1^{-} = (1, 1, 2, 3)^{-+}$. My suspicion, as expressed in Secton III for the light $\bar{q}q$ states, is that these extra states are not really spurious but may exist at some higher mass, since I doubt that the vacuum response time is much smaller than the time for a gluon or light quark to move a typical hadronic distance, O(1 fm.).

In any case, subtracting these four spurious (or higher mass) states from the initial list of six, we are left with two states, $J^{PC} = (0, 2)^{-+}$. The mass of these two states is estimated at $M \cong 1.3$ GeV.[82]

Spin forces will break the degeneracy of the $(0, 2)^{++}$ ground state and the $(0, 2)^{-+}$ first excited state. These are large and very difficult to estimate. Hyperfine splitting may drive the scalar, $0^{++}$, to a mass near zero, so that it mixes with the vacuum, and to compute the mass of the first scalar glueball we would have to solve this very difficult mixing problem.[86]

The nonrelativistic potential model has also been used to study the glueball spectrum. In this approach it is necessary to assume that the bound gluons acquire a constituent mass, which is not known. In this case the gluons might also be longitudinally polarized,

which changes the predicted glueball quantum numbers from the bag model where the gluons are purely transverse.

Barnes[84] argues that only the transverse gluons are really present[87] and therefore obtains the same states in his spectrum as are found in the Bag model (provided the "extra" Bag model states are discarded). However the ordering of the states is different than in the Bag model. Including single gluon exchange, the lightest states are a degenerate scalar and pseudoscalar, $J^{PC} = 0^{\pm +}$, and the first excitation is a tensor, $J^{PC} = 2^{++}$.

Many other authors have assumed that the longitudinal modes are present [83,88,89] This approach gives the same list of states as would obtain in the Bag model if the "extra" states were retained at their naive values[90] (i.e., where they would be if the vacuum response time were very, very slow). The total spin of the two gluons is

$$S = 1 \oplus 1 = (0, 2)_{\text{Sym.}} + (1)_{\text{Antisym.}}$$

For L = 0 the color singlet gg state must be symmetric by Bose symmetry, giving $J^{PC} = (0, 2)^{++}$ ground states, as in the Bag Model. For L = 1 the color singlet and Bose requirements force us to choose the antisymmetric spin wave function, S = 1, so that the first excited states are $J^{PC} = (0, 1, 2)^{-+}$. Notice that $J^{PC} = 1^{-+}$ is an exotic combination in that is never occurs in the $\bar{q}q$ spectrum in the non-relativistic approximation. These states have been called "oddballs" by Carlson et al.[91] who have studied their properties. Oddball quantum numbers can also arise (1) in the three gluon sector and (2) in the Bag model as "bag excitations" of gg or $\bar{q}q$ — i.e., the "extra" states discussed here and in Section III.

The ITEP group has recently applied their sum rule technique

to the glueball spectrum.[92] The results are dramatically different from those I have just reviewed: while the $J^{PC} = 2^{++}$ glueball is estimated at $\sim 1\frac{1}{2}$ GeV, the scalar and pseudoscalar are estimated at much larger masses, $\sim 4$ GeV. More precisely, these are the values of the "critical masses" in the respective gg channels. The critical mass is defined as the value of M at which the "nonperturbative" $M^{-4}$ terms in Eq. (2.44) are of order 10% of the leading perturbative terms. The critical mass is large in the $0^{\pm+}$ gg channels because the leading perturbative terms are small.

Because the vacuum expectation values which appear in $O(M^{-4})$ in Eq. 2.44, such as $\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle$, must arise by nonperturbative mechanisms, the authors argue that the critical mass should be associated roughly with the bound state masses in the appropriate channel. But the connection is evidently not tight, as illustrated by the fact that in the pion channel they find a critical mass of $\sim 1\frac{1}{2}$ GeV, much larger than $m_\pi$.

B. Some Good Questions Without Good Answers

In the preceding subsection I reviewed theoretical work on the glueball spectrum. It should be clear that there is now no reliable, precise set of predictions which can be used to determine whether any particular newly discovered state is a glueball. Our understanding of the dynamical properties of glueballs is even less well developed then our primitive understanding of the $\bar{q}q$ spectrum.

A key question is how wide we expect glueballs to be. If they are too broad they might never be seen, like for instance the majority of the $\bar{q}\bar{q}qq$ states discussed in Section IV. Folklore has it that glueball widths are typically the geometric mean of OZ allowed and OIZ suppressed decay widths,[89,93]

$$\Gamma_{\text{Glueball}} \sim \sqrt{\Gamma_{\text{OIZ allowed}} \cdot \Gamma_{\text{OIZ Suppressed}}}. \tag{7.3}$$

This estimate is true in the $SU(N)_{Color}$ theories to the leading order in $1/N_{Color}$,[11] since for a glueball $G = |gg>$ and a meson $M = |\bar{q}q>$ the decays to two $\bar{q}q$ mesons obey

$$\Gamma(G \to M_1M_2) \propto 1/N^2_{Color} \tag{7.4}$$

$$\Gamma(M \to M_1M_2)_{\text{OIZ allowed}} \propto 1/N_{Color} \tag{7.5}$$

$$\Gamma(M \to M_1M_2)_{\text{OIZ suppressed}} \propto 1/N^3_{Color} \tag{7.6}$$

Equation (7.3) is also suggested by unitarity since glueballs may be intermediate states in OIZ forbidden decays, such as

$$\text{Im} <\bar{s}s|\bar{u}u> \propto <\bar{s}s|G> <G|\bar{u}u> + \ldots \tag{7.7}$$

From Eq. (7.7) it appears that the glueball decay amplitudes are suppressed by the square root of the OIZ suppression factor for $\bar{q}q \to \bar{q}'q'$ amplitudes. However there are also OIZ allowed intermediate states which contribute to the right side of Eq. (7.7), such as in

$$\text{Im} < \phi|\rho\pi> \propto <\phi|\bar{K}K> <\bar{K}K|\rho\pi> <\phi|G> <G|\rho\pi> + \ldots \tag{7.8}$$

Equation (7.8) raises a familiar puzzle: $<\phi|\bar{K}K>$ and $<\bar{K}K|\rho\pi>$ are both OIZ allowed, so either there are cancellations among several terms on the right side of Eq. (7.8) or $<\bar{K}K|\rho\pi>$ is small for a reason other than the OIZ rule. A mechanism for cancellations among quark intermediate states has been proposed in the context of dual models.[12] Or $<\bar{K}K|\rho\pi>$ might itself be small for the same reason as the inequality (2.12), also not a consequence of the OIZ rule.

In general there are many different ways in which Eq. (7.8) might be satisfied. If there are cancellations either among glueball intermediates or between them and quark intermediates, than $\Gamma_{glueball}$ might be much larger than Eq. (7.3).[94] If there are no such cancellations of glueball intermediates then Eq. (7.3) would actually be an upper bound.

Another dynamical question of crucial importance in our ability to identify glueballs is the extent to which they are mixed with $\bar{q}q$ mesons. The mixing angle between an aboriginal glueball $G_0(gg)$ and quark meson $M_0(\bar{q}q)$ is

$$\tan 2\theta = \frac{2\langle G_0|M_0\rangle}{m_{G_0} - m_{M_0}} \tag{7.9}$$

The numerator depends on the details of the wave functions and also on the preceding, unanswered question: if glueballs are very broad the mixing is very large and vice-versa. There is not much to say about the denominator except that it depends on the luck of the draw. Since the field is very crowded, with many states in the 1-2 GeV region, there is a good chance than any $G_0$ will have an $M_0$ of the same quantum numbers within striking distance. Only a soothsayer would attempt to answer this question in general. It must be confronted on a case by case basis.

Finally, what about the electromagnetic couplings and decays of glueballs, in particular, the coupling to two photons? Naively since the aboriginal glueball $G_0$ is made of electrically neutral gluons, we expect small electromagnetic couplings. But this question is clearly related to the preceding (unanswered) ones. In addition we must remember that the glueball candidates are likely to be in the 1-2 GeV region which is filled with <u>excited</u> $\bar{q}q$ states. These excited states will also tend to have suppressed $\gamma\gamma$ couplings.

## C. The Glueball Candidate at 1440

It is clear from the preceding that we don't yet have detailed theoretical guidance to the spectrum or the important dynamical properties of glueballs. It is also clear from the first two lectures that there is likely to be a numbing abundance of quark states in the 1-2 GeV. region where we might expect to find the first glueballs. How then will we recognize a glueball if we do happen to see one?

I believe at this moment the only answer is to concentrate on the generic, qualitative properties which a glueball must have, almost just by definition. I know of two such properties:

A) Glueballs will be produced prominently in hard gluon channels.

B) Glueballs do not "fit" into $\bar{q}q$ multiplets.

These properties are almost pure tautology. Indeed B) is a tautology and A) is guaranteed provided that the glueballs contain valence gluons (as in the bag and nonrelativistic models discussed above).

Because of property A) radiative $\psi$ decays are a prime glueball hunting ground. In perturbation theory[95]

$$\Gamma(\psi \to \gamma X) = \Gamma(\psi \to \gamma gg) + O(\alpha_s^3) \tag{7.10}$$

$$\frac{\Gamma(\psi \to \gamma gg)}{\Gamma(\psi \to ggg)} \cong \frac{16\alpha}{5\alpha_s}\left(1 + O(\alpha_s)\right) \tag{7.11}$$

which implies

$$B(\psi \to \gamma X) \cong (6\text{-}10)\% \tag{7.12}$$

The two gluons in Eq. (7.10) are in a color singlet and may "resonate" to form a glueball. Therefore any prominent new state in this channel should be examined to see if it has a plausible assignment in the $\bar{q}q$ spectrum.

In the Spring of 1980 the Mark II collaboration announced a large signal[79] seen subsequently in the Crystal Ball[80] with a rate[96]

$$B(\psi \to \gamma(\bar{K}K\pi)_{1.44}) = (4.0 \pm 0.7 \pm 1.0) \cdot 10^{-3}. \qquad (7.13)$$

This is a very large rate, as large in just the $\bar{K}K\pi$ mode as the $\eta'$ is in all its modes, $\eta'$ being previously the most prominent state in $\psi \to \gamma X$. So property A) is certainly satisfied. What about property B)? The early publications referred to this effect as E(1420), the $\bar{s}s$ member of the $A_1$ nonet, discussed in Section III, which decays predominantly to $\bar{K}K\pi$. But Scharre[97] has noted the dominance of $\delta\pi \to \bar{K}K\pi$ in the SPEAR data, as opposed to the dominance of $K^*K \to \bar{K}K\pi$ in the pion scattering data[29] which established the E as a $J^P = 1^+$ state. A theoretical argument also suggests immediately that the state seen at SPEAR is probably not the E: a $J^P = 1^+$ state would be suppressed in $\psi \to \gamma X$ since it would not couple[98] to the two massless gluons which dominate according to Eq. (7.10).

D. E(1420) and G(1440)

Several theoretical papers have suggested, with varying degrees of belligerence, that the state seen at SPEAR is not[82,99] or might not be[100] or is[101] the E(1420). In my case[99] the conclusion that the SPEAR 1440 is not the E(1420) was based on examining the complete experimental history of the so-called "E", which goes back to the early 1960's. Even without the SPEAR data there is strong evidence from the earlier experiments that at least two different states were being observed. One is E(1420), the $J^P = 1^+$ partner of the $A_1$ discussed in Section III. The second, which I and the authors of Ref. (82) idependently called G(1440), is the state seen at SPEAR, probably a pseudoscalar, $J^P = 0^-$. G(1440) was probably first observed in the early 60's, in a $\bar{p}p$ annihilation experiment,[102] whose members named it E, for the first resonance discovered in Europe. If my

conclusions are correct they actually did not observe the $J^P = 1^+$ particle which is today called E(1420) in the tables of the Particle Data Group.

The experimental record is like a jigsaw puzzle which won't fit together. Keeping only the most reliable experiments there still appear to be several irreconcilable contradictions.

1) Highly believable experiments report different spin-parities: $1^+$ in $\pi p$ scattering[29] and $0^-$ in $\bar{p}p$ annihilation at rest.[102] In the latter $J^P$ was measured in two independent ways. The determination based on the angular distribution between $\delta$ and $\pi\pi$ in $\bar{p}p \to$ "E"$(\pi\pi) \to \delta\pi(\pi\pi)$ is particularily convincing.

2) The low-background Dalitz plots from SPEAR and two studies of $\bar{p}p$ annihilation at rest[104,105] are extremely similar — enhanced $\delta$ regions, no strong K* bands — and very different from the K*- dominated plot seen in $\pi p$ scattering.[29] The latter data does have a large background, but the impression of K* dominance is confirmed by an analysis which includes side-band background subtraction.

3) A $\bar{p}p$ annihilation experiment in flight[106] reports a five standard deviation signal for $\eta\pi\pi$ while no $J^P = 1^+$ $\eta\pi\pi$ signal is seen by a much more sensitive $\pi p$ experiment.[55] In radiative $\psi$ decay there is a possible indication of an $\eta\pi\pi$ signal which requires further study.[107]

4) In $\pi p$ scattering[29] and in $\bar{p}p$ annihilation in flight[108] the E(1420) is accompanied by D(1285) with a substantially larger (factor 5 to 10) rate for the D. This is what we'd expect from the OIZ rule if E and D are approximately ideally mixed as discussed in Section III. But in $\bar{p}p$ annihilation at rest and in radiative $\psi$ decay there is no sign of D, despite the

prominence of the so-called "E" signals. In $\psi \to \gamma X$ the most serious constraint is posed by the $\eta\pi\pi$ data,[107] since $\eta\pi\pi$ is a 50% mode of D(1285).[30]

This evidence and more is summarized in Table (7.1). The table does have a consistent interpretation only if two different states are involved. One, seen in $\pi p$ scattering, is the E(1420) of the Particle Data Group table, a $J^P = 1^+$ state which decays predominantly to K*K and is probably predominantly an $\bar{s}s$ state. The second, seen in $\psi \to \gamma X$ and in $\bar{p}p$ annihilation at rest, is a $J^P = 0^-$ state which decays to $\bar{K}K\pi$ and $\eta\pi\pi$ — both with substantial $\delta\pi$ components — and does not decay copiously to K*K. While E dominates the $\pi p$ data and G dominates the $\psi \to \gamma X$ signal and the data from $\bar{p}p$ annihilation at rest, the presence of $\eta\pi\pi$ signals and of a large D signal suggest that both G and E are produced substantially in $\bar{p}p$ annihilation in flight ($p_{LAB} \geqslant 700$ MeV.).

But why should only G be seen in $\bar{p}p$ annihilation at rest while G, E, and D are all produced for $p_{LAB} \geqslant 700$ MeV? This conclusion may seem contrived, but on further reflection it confirms the impression that we have found how the pieces of the puzzle fit together. It is in fact just what we would expect for the proposed $J^{PC}$ assignments. Consider the reaction

$$(\bar{p}p)_{rest} \to X\pi\pi$$

where X is a positive charge conjugation eigenstate, C(X) = +. (The final state $X\pi^0$ is not allowed by J, P, and C if C(X) = + and if X has abnormal spin-parity, $J^P = 0^-, 1^+, \ldots$ .) The initial $\bar{p}p$ state may have quantum numbers $J^{PC} = 0^{-+}$ or $1^{--}$. For the dipion in an s-wave the initial $\bar{p}p$ state must be $J^{PC} = 0^{-+}$ by C invariance and then only for $J^P(X) = 0^-$ can X be in an s-wave relative to the

TABLE I: E vs. G

| | $\bar{K}K\pi$ | $\eta\pi\pi$ | $J^P$ | D(1285) | |
|---|---|---|---|---|---|
| $\pi p$ | $K^*K$ | No | $1^+$ | D >> E | |
| $(\bar{p}p)_{rest}$ | $\delta\pi$ (+ $K^*K$?) | | $0^-$ | No | |
| $(\bar{p}p)_{flight}$ | | Yes ($\delta\pi$) | $1^+/0^-$ | D >> E | |
| $\psi \to \gamma X$ | $\delta\pi$ | Indication ($\delta\pi$) | | No | |

Table 7.1. E versus G. Summary of experimental results discussed in the text.

dipion. For $J^P(X) = 1^+$ either the dipion must be in (at least) a p-wave (possible for $\pi^+\pi^-$ but not for $\pi^0\pi^0$) or X must be in a p-wave relative to the s-wave dipion. In either case (and especially in the latter) there is a formidable angular momentum barrier for $\bar{p}p$ annihilation at rest into $E\pi\pi$ and $D\pi\pi$, which is no longer effective in $X\pi^+\pi^-$ for $p_{LAB} \geqslant 700$ MeV. (We then expect the suppression of E and D to hold out to larger $p_{LAB}$ for $X\pi^0\pi^0$ than for $X\pi^+\pi^-$.) The conclusion then is that the seemingly peculiar pattern observed in $\bar{p}p$ annihilation at rest and in flight is a simple kinematical consequence of the $J^{PC}$ assignments of G, E, and D.

Another kinematical consideration which suggests $J^P(G) = 0^-$ is the fact that $G \to \delta\pi$ would then be the only allowed two body s-wave decay, just as $E \to K^*K$ is the only allowed two body s-wave decay if $J^P(E) = 1^+$. The decays $E \to \delta\pi$ and $G \to K^*K$ are p-wave processes and are therefore suppressed by the small available phase space.

Two other arguments also suggest $J^P(G) = 0^-$. One[82,99] is that the dominant partial waves for the di-gluon in $\psi \to \gamma gg$ are[109] $0^+$, $0^-$, $2^+$ of which only $0^-$ is consistent with abnormal spin-parity required by $G \to \delta\pi$. The second[99,100] is the special preference for $\bar{K}K\pi$ decays which a pseudoscalar glueball might uniquely have, because at the quark level it would prefer annihilation to $\bar{s}s$ over $\bar{u}u + \bar{d}d$ by a factor $m_s/m_u$ in the amplitude (like $\pi \to \mu\nu$ is enhanced over $\pi \to e\nu$). The $\bar{s}s$ pairs would often hadronize to s-wave $\bar{K}K\pi$, which by final state interaction would form $\delta\pi$ some but not all of the time. Therefore, in contrast to some suggestions[110] we do not expect G to decay in an SU(3) symmetric fashion nor do we require the ratio $G \to \bar{K}K\pi/G \to \eta\pi\pi$ to correspond precisely to $\delta \to \bar{K}K/\delta \to \eta\pi$ (which is not very well known in any case). Rather the first ratio should be $\geqslant$ the second and there may be more K mesons than predicted by SU(3) symmetry. These are

special properties of a pseudoscalar glueball and are consistent with what is known experimentally.

(Two weeks after these lectures were given the Crystal Ball group presented a spin-parity analysis of the 1440 $\bar{K}K\pi$ enhancement in radiative $\psi$ decay.[96] The results favor $J^P = 0^-$ over $J^P = 1^+$ at the 99% level, and they also confirm the earlier claim[97] of $\delta\pi$ dominance.)

E. G(1440) and $\zeta'$(?): Is G a Glueball?

If we accept the conclusion of the previous section that the state seen at SPEAR, G(1440), is a pseudoscalar and not the axial vertor E(1420), then we must again ask whether it satisfies property B) — i.e., does it "fit" in a $\bar{q}q$ multiplet. The only possible[99] assignment is to identify G with the still missing ninth member of the radially excited $\pi'$ nonet, which I discussed at some length in Section V. The eight observed states are the isotriplet $\pi'$(1270), the strange quartet K'(1400-1500), and the isoscalar $\zeta$(1275). In Section V I used the name $\zeta'$ to refer to the missing isoscalar. The question now is whether G is in fact $\zeta'$. I believe the answer is that G is not $\zeta'$, for reasons given below.

Since radiative $\psi$ decay to isovectors is severely suppressed,[111] G(1440) is certain to be an isoscalar. For the same reason[111] it must be predominantly an SU(3) flavor singlet. The previously known isoscalar, $\zeta$(1275), seen in $\pi^- p \to \zeta n \to \eta\pi\pi n$, is not seen in $\psi \to \gamma\eta\pi\pi$: it should appear at the mass of D(1285), where no signal has been seen.[107] Therefore if we assume for the moment that $G = \zeta'$, then the data implies

$$\Gamma(\psi \to \gamma\zeta') \gg \Gamma(\psi \to \gamma\zeta). \tag{7.14}$$

Although this inequality appears to be much stronger than the analogous

inequality for $\eta'$ and $\eta$, it is possible that it might be explained by assuming the mixing to be approximately singlet-octet like $\eta$ and $\eta'$,

$$\zeta' \cong \zeta_1$$

$$\zeta \cong \zeta_8 \tag{7.15}$$

where

$$\zeta_1 = \frac{1}{\sqrt{3}} (\bar{u}u + \bar{d}d + \bar{s}s)$$

$$\zeta_8 = \frac{1}{\sqrt{6}} (\bar{u}u + \bar{d}d - 2\bar{s}s) \tag{7.16}$$

Equation (7.15) would mean that $\zeta'$ is essentially the radial excitation of $\eta'$. But then it is peculiar that $\Gamma(\psi \to \gamma\zeta')$ is as large as $\Gamma(\psi \to \gamma\eta')$, both because of the smaller available phase space <u>and</u> because the radial excitation should couple more weakly to two gluons (a la the van Royen-Weisskopf model of $\bar{q}q$ meson formation[112]). If G has other important decays, such as $\eta\pi\pi$ or $\pi\pi\pi\pi$, in addition to the established $\bar{K}K\pi$ mode, then $\Gamma(\psi \to \gamma G)$ is substantially bigger than $\Gamma(\psi \to \gamma\eta')$ and the problem is even more severe.

This remark can be made quantitative. The rate for radiative decay of a vector quarkonium $V(\bar{Q}Q)$ to a pseudoscalar quarkonium of a different flavor $P(\bar{Q}'Q')$, $V(\bar{Q}Q) \to \gamma + P(\bar{Q}'Q')$, has been computed[113] in weak binding approximation. Applied to $\eta'$ and its excitation $\zeta'$ the result is[114]

$$\frac{\Gamma(\psi \to \gamma\zeta')}{\Gamma(\psi \to \gamma\eta')} = \left(\frac{K_{\zeta'}}{K_{\eta'}}\right)^3 \left(\frac{M_{\eta'}}{M_{\zeta'}}\right)^2 \frac{E_{\zeta'}}{E_{\eta'}} \cdot \left| \frac{\psi_{\zeta'}(0)}{\psi_{\eta'}(0)} \right|^2 \tag{7.17}$$

where $K_P$ and $E_P$ are the pseudoscalar momentum and energy in the $\psi$ rest frame and $\psi_P(0)$ is the pseudoscalar $\bar{q}q$ wave function at the origin. If $G = \zeta'$ then Eq. (7.17) and the experimental inequality $\Gamma(\psi \to \gamma G) \geqslant \Gamma(\psi \to \gamma\eta')$ would imply

$$|\psi_{\zeta'}(0)|^2 > 3\,|\psi_{\eta'}(0)|^2 \tag{7.18}$$

which makes G a most unlikely candidate to be the excitation of $\eta'$.

The argument is not complete because the binding corrections may be of the same order as the essentially kinematic factors in Eq. (7.17). It is important to know the approximate magnitude or even just the sign of the binding corrections. It is however reassuring that Eq. (7.17) gives a reasonable account of the ratio $\Gamma(\psi \to \gamma\eta')/\Gamma(\psi \to \gamma\eta)$; using[115] $|\psi_{\eta'}(0)/\psi_{\eta}(0)|^2 = \cot^2(11^\circ)$, we find 7 for the $\eta'$ to $\eta$ ratio.

The second argument against the assignment $G = \zeta'$ involves the $\pi p \to \eta\pi\pi n$ data which was discussed at some length in Section V. This is the experiment[55] which discovered $\zeta(1275)$ and observed a possible psuedoscalar signal in $\eta''\varepsilon'' \to \eta\pi\pi$ at ~1400 MeV., which in Section V I called the "glitch" or gl(1400). In the $J^P = 0^-$ $\delta\pi \to \eta\pi\pi$ channel there was no hint of a resonance near 1400. Since we would expect $G \to \eta\pi\pi$ to have a significant $\delta\pi$ component, as the observed indication in $\psi \to \gamma\, G \to \gamma\eta\pi\pi$ indeed has[116], it is unlikely that gl(1400) is G(1440) and it appears that $\pi p \to Gn$ was not seen by Stanton et al. We can obtain a very conservative upper bound on $\sigma(\pi p \to Gn \to \eta\pi\pi n)$ by assuming that the events in the gl(1400) bump are due to G(1440). In this case a <u>rough</u> estimate of the rapidly changing acceptance[56] yields

$$\frac{\sigma(\pi p \to Gn \to \eta\pi\pi n)}{\sigma(\pi p \to \zeta n \to \eta\pi\pi n)} \leq 0.4 \tag{7.19}$$

Since G(1440) is probably not gl(1400), it is likely that the ratio is actually $\ll 0.4$.

How does this inequality compare with what we would expect if G were $\zeta'$? The SPEAR data then requires 1-8 mixing, Eq. (7.15), which means that $\zeta'$ is $\sim \frac{1}{3}\,\bar{s}s$ and that $\zeta$ is $\sim \frac{2}{3}\,\bar{s}s$ . Then the OIZ rule

implies that

$$\frac{\sigma(\pi p \to \zeta' n)}{\sigma(\pi p \to \zeta n)} \cong 2 \tag{7.20}$$

I have already observed in Section II that the analogous prediction for $\eta'$ and $\eta$ is reasonably successful; it implies $\theta = -15^\circ$ or, conversely, if we fix $\theta = -11^\circ$ we find $\sim 1.4$ instead of 2. Since $\eta\pi\pi$ is only an OIZ allowed decay for the $\bar{u}u + \bar{d}d$ components, neglecting phase space we'd expect $B(\zeta' \to \eta\pi\pi)/B(\zeta \to \eta\pi\pi) \cong 2$ and for the right side of (7.19) we'd expect = 4, a factor 10 above the conservative experimental upper bound. However $\zeta \to \bar{K}K\pi$ is severely constrained by the available phase space. Assuming that $\bar{K}K\pi$ and $\eta\pi\pi$ are the dominant decays and taking account of three body phase space I find instead of the ratio two that

$$\frac{B(\zeta' \to \eta\pi\pi)}{B(\zeta \to \eta\pi\pi)} \cong 1.1 \tag{7.21}$$

Then for the cross sections observed at the ZGS we expect

$$\frac{\sigma(\pi^- p \to \zeta' n \to \eta\pi\pi n)}{\sigma\ \pi^- p \to \zeta n \to \eta\pi\pi n)} \cong 2 \tag{7.22}$$

a factor 5 (and probably even $\gg 5$) larger than the experimental upper bound. As I emphasized in Section V, it is important to repeat these measurements in an experiment with acceptance optimized for 1.4 GeV and higher.

These two arguments suggest that G(1440) does not fit into a $\bar{q}q$ multiplet. Could it be a $\bar{q}\bar{q}qq$ state? Four quark states would not be produced at larger rates than typical $\bar{q}q$ states in $\psi \to \gamma X$. And a pseudoscalar $\bar{q}\bar{q}qq$ cannot be constructed from the orbital ground state, $L = 0$, but requires at least $L = 1$. Such states are not easily studied in the Bag model because of the difficulty discussed at the end of Section III. Like the $L = 0$ $\bar{q}\bar{q}qq$ states, most of the $L = 1$ states are probably too broad to observe as ordinary resonances.

I have argued that G(1440) is probably a glueball because A) it is very prominent in a hard gluon channel and B) it does not "fit" into any $\bar{q}q$ multiplet. If G were prominent in a second hard gluon channel the case would be strengthened. Just as the leading particle in a charmed quark jet is a charmed hadron, we expect the leading particle in a gluon jet to often (though not always) be a glueball.[88,117] It would be interesting to examine gluon jets for leading G's.

The weakest part of my argument is B), since I have had to argue that G does not have the properties we would expect $\zeta'$ to have. The best proof would be to find the real $\zeta'$. The apparent degeneracy of $\pi'(1270)$ and $\zeta(1275)$ suggests ideal mixing, in which case $m_{\zeta'} \sim 1.5$ - 1.6 GeV as in Eq. (5.1), and the dominant decay would be $\zeta' \to \bar{K}K\pi$. The task will be most difficult if the mixing is not ideal and $\zeta'$ is lurking in the 1400 - 1500 MeV mass region. In this case we have four states in this region which decay to $\bar{K}K\pi$ to disentangle: E, H', G, and $\zeta'$. A hint that this may be the case is the report from the original $\bar{p}p$ annihilation experiment[102] that the $\bar{K}K\pi$ width is 80 ± 10 MeV., broader than (though not yet inconsistent with) the ~50 MeV width seen at SPEAR. Perhaps gl(1400) is $\zeta'$ and both $\zeta'$ and G are produced in $\bar{p}p$ annihilation at rest.

To unravel such a complicated situation we need to construct a G - $\zeta'$ table, Table (7.2), analogous to the E - G table, Table (7.1). Right now most of Table (7.2) is empty and of the six entries, three are speculative. I have made (premature) guesses in the right column about the dominant states in each channel. G and $\zeta'$ may both appear in $\bar{p}p$ annihilation because of the anomalously large width and the need for $\delta\pi$ and K*K in the fit to the Dalitz plot.[116] The other guesses are based on the preceding dicussion. If G and $\zeta'$ are

| | $\bar{K}K\pi$ | $\eta\pi\pi$ | $\gamma\gamma/\rho\gamma$ | $\zeta(1275)$ | |
|---|---|---|---|---|---|
| $\bar{p}p$ (esp. at rest) | $\Gamma \sim 80 \pm 10$ $\delta\pi(+K^*K?)$ | | | | $G + \zeta'?$ |
| $\pi p$ | | $\varepsilon\eta?$ | | Yes | $\zeta'?$ |
| $\psi \to \gamma X$ | $\delta\pi$ | $\delta\pi?$ | | No | G |
| Kp | | | | | |

Table 7.2. G versus $\zeta'$. See text for discussion.

ideally mixed $\zeta'$ production will be suppressed relative to $\zeta$ in $\pi p$, $\bar{p}p$ and $\gamma\gamma$ scattering.

Premature guesses aside, the important point is that by completing Table (7.2) we can disentangle G from $\zeta'$, including the difficult question of mixing. The success of the naive prediction for $\eta' \to \gamma\gamma$ and two estimates of $\eta'$ - G mixing[119] all suggest that $\eta'$ - G mixing is small. But $\zeta'$ - G mixing could be appreciable if $m_G - m_{\zeta'}$ is very small.

F. Other Glueball Candidates

At this moment there are no other glueball candidates for which as strong a case can be made as for G(1440). Some authors[89,120] have speculated that S*(980) may be a glueball since it is not a good fit to the $L = 1$ $J^{PC} = 0^{++}$ nonet. I have discussed the peculiarities of this nonet in Section III and in Section IV argued for the conclusion obtained in the bag model that S* and $\delta$ are $\bar{q}\bar{q}qq$ states. The arguments for this interpretation are quite convincing: S* does have the properties we expect of an $\bar{s}s(\bar{u}u + \bar{d}d)$ state. There is no comparable evidence favoring the glueball interpretation, and the failure to observe $\zeta \to \gamma S^*$ is evidence against it.

The newly discovered[52] enhancement near the threshold in $\gamma\gamma \to \pi\pi$ has also been proposed as a glueball candidate. Layssac and Renard[121] argue that it may be a pseudoscalar glueball. Using the data they estimate the $\gamma\gamma$ width to be $\sim 8$ keV. This would be a surprisingly large width for a glueball, being larger than other typical second order electromagnetic decays such as $\eta' \to \gamma\gamma$, $\omega \to e^+e^-$, $\rho' \to e^+e^-$, $\phi \to e^+e^-$ and $f \to \gamma\gamma$. If it is not produced in radiative $\psi$ decay as strongly as G(1440), then it is most likely either a $\bar{q}q$ meson or perhaps a $\bar{q}\bar{q}qq$ state. I argued for the latter interpretation of this state in Section IV.

Donohue, Johnson, and Li[82,94] make the interesting suggestion that there may be a tensor glueball near the f, with which it is strongly mixed. This would explain the experimental bound $\Gamma(\psi \to \gamma f) \gtrsim (5\text{-}10) \cdot \Gamma(\psi \to \gamma f')$ which contradicts the expectation based on ideal mixing that $\Gamma(\psi \to \gamma f) \cong 2\Gamma(\psi \to \gamma f')$. The f signal would be enhanced because of its glueball admixture. The $\bar{q}q$ and gg states would mix dominantly via the $\pi\pi$ channel so that when the mass matrix is diagonalized one of the eigenstates will decouple from $\pi\pi$. The way to discover the second state is to look in other decay channels, such as $\bar{K}K$ and perhaps $\eta\eta$. One would hope to see either two discernible peaks or, if not, a peak with a noticeably different shape and width than observed in $\pi\pi$.

A related suggestion, from J. Rosner,[122] is motivated by the failure of SU(3) and SU(6) predictions for the f which might be explained if it has a sizeable glueball component. He finds that the orthogonal admixture, which he calls $f_\perp$, should have a mass between 1.45 and 1.87 GeV., closer to f' than to f. Since mixing with the f' was not included, the results are not self-consistent. A complete analysis requires study of the three body mixing between f, f', and $f_\perp$.

## VIII CONCLUSION

I hope I have conveyed a sense of the impressive progress that is being made in meson spectroscopy and of the interesting issues that remain. Continued experimental study of the spectrum in the complicated 1-2 GeV. region is essential, since only with this detailed knowledge can we find and interpret the deviations which represent new physics. Observation of exotic quantum numbers could be due to $\bar{q}\bar{q}qq$ states, bag or string excitations, or glueballs. But all of these examples of new physics can also produce states with the same quantum numbers as nonrelativistic $\bar{q}q$ model states. In fact it is likely that the first discovered glueballs and $\bar{q}\bar{q}qq$ states do not have exotic quantum numbers. In this case only a very thorough understanding of the $\bar{q}q$ spectrum will enable us to interpret what we have found.

The charmonium and bottomonium spectra have provided striking confirmation of our ideas about QCD dynamics. The potential which fits the data interpolates smoothly between short distance behavior compatible with asymptotic freedom and long distance behavior consistent with naive ideas about confinement and light meson Regge trajectories. The principal remaining challenge is to understand the spin dependence of the potential. Heavy $\bar{Q}Q$ systems are the ideal laboratory in which to approach this problem since the effects occur in a power series in $1/m_Q^2$.

I argued in Section IV for the thesis of Jaffe and Johnson that $\delta$ and $S^*$ are probably $\bar{q}\bar{q}qq$ states. I find this hypothesis to be very attractive, despite the problem of the $\delta$ width which might be resolved by higher statistics studies in the $\bar{K}K$ channel. After these lectures were given, the Crystal Ball group presented data[96] for an $\eta\eta$ enhancement at 1640 MeV. in $\psi \to \gamma\eta\eta$. While this could be a $\bar{q}q$, gg,

or $\bar{q}\bar{q}qq$ state, I have argued elsewhere[123] for the likelihood of one of the two latter possibilities. My favorite guess is that it is a four quark state with the same flavor content as the S*, $\bar{s}s(\bar{u}u + \bar{d}d)$. If this is correct it should decay equally to $K^+K^-$, $\bar{K}^oK^o$, and $\eta\eta$ and have no other large two body decay modes.

Finally we are on the verge of identifying the first of the long-awaited glueballs. The first glueball may have been seen but not recognized in a study of $\bar{p}p$ annihilation at rest, performed at CERN in the early 60's. The possibility of recognizing it today as a glueball depends crucially on its recent rediscovery at SPEAR in radiative $\psi$ decay. Nothing can better illustrate the importance of the study of spectrosocopy than the extent to which we are forced to draw on detailed knowledge of the meson spectrum in order to decide whether the 1440 is indeed a glueball. In the case of the 1440 we must disentangle at least two and probably four $\bar{K}K^-$ states in this mass region. Two have been found, E and G, and two remain, H' and $\zeta'$. The problem of identifying G as a glueball is essentially reduced to the spectroscopic problem of finding $\zeta'$ and completing the $\pi'$ nonet.

Two areas of experimental work are crucial in the discussion of the glueball candidate. One is the general study of meson spectroscopy in the 1-2 GeV. region and the second is the study of radiative $\psi$ decay. If these areas continue to get the attention they deserve, they will continue to be high-yield gold mines in the future. Explication of the low-lying glueball spectrum may require that we increase the statistical power in both of these areas by another order of magnitude.

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