

**Volume Production of Negative Ions
in the Reflex-Type Ion Source**

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(Ph.D. Thesis)

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ABSTRACT

The production of negative hydrogen ions is investigated in the reflex-type negative ion source. The extracted negative hydrogen currents of 9.7 mA (100 mA/cm^2) for H^- and of 4.1 mA (42 mA/cm^2) for D^- are obtained continuously. The impurity is less than 1%. An isotope effect of negative ion production is observed.

When anomalous diffusion in the positive column was found by Lehnert and Hoh (1960), it was pointed out that the large particle loss produced by anomalous diffusion is compensated by the large particle production inside the plasma; i.e. the plasma tries to maintain itself. "The self-sustaining property of the plasma" is applied to the reflex-type negative ion source. Anomalous diffusion was artificially encouraged by changing the radial electric field inside the reflex discharge. The apparent encouragement of negative ion diffusion by the increase of density fluctuation amplitude is observed. Twice as much negative ion current was obtained with the artificial encouragement as without.

It is found from the quasilinear theory that the inwardly directed radial electric field destabilizes the plasma in the

reflex-type ion source. The nonlinear theory based on Yoshikawa method (1962) is extended, and the anomalous diffusion coefficient in a weakly ionized plasma is obtained. "The electrostatic sheath trap", which increases the confinement of negative ions in the reflex-type ion source, is also discussed.

It has been suggested that the dissociative attachment of electrons to excited molecules might be responsible for the high negative ion production in a hydrogen plasma: Bacal and Hamilton (1979). A recent calculation by Wadehra (1979) showed a significant enhancement of the dissociative attachment rate by the vibrational excitation of the initial molecules. The largest attachment rate was given at an electron temperature of about 1.5 eV.

In our experiment, the maximum extracted negative ion current was obtained at an electron temperature of about 3 eV, which is calculated from the anomalous diffusion mechanism proposed by the author. Our experiment results are consistent with the production mechanism of negative ions through the electron volume production process proposed by Allan and Wong (1978), and Wadehra's calculated results (1979).

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Nomenclature

- c Speed of light
- e Unit charge of electron
- η Resistivity of plasma
- λ Mean free path
- \vec{k} Wave vector
- \vec{E} Electric field vector
- \vec{B} Magnetic field vector
- ω_p Plasma frequency
- ω_L Left hand cut off frequency in plasma
- ω_R Right hand cut off frequency in plasma
- λ_D Debye length
- r_L Larmor radius
- A Atomic number
- Z Production term
- f_s Density distribution of species "s"
- n_s Density of species "s"
- m_s Mass of species "s": particularly $m_e = m$ and $m_i = M$
- Ω_s Cyclotron frequency of species "s"
- T_s Temperature of species "s"
- P_s Pressure of species "s"
- D_s Diffusion coefficient of species "s"
- μ_s Mobility of species "s"
- \vec{u}_s Fluid velocity vector of species "s"
- $\vec{\Gamma}_s$ Flux vector of species "s"
- ν_e Collision frequency of electron with neutral particles.

CHAPTER 1

INTRODUCTION

In this work, we study volume production of negative ions in the reflex-type negative ion source. Production of negative ions has been important for applications in nuclear physics: Ehlers¹ (1965). Recently, the production of negative ions has been found useful to heat fusion plasmas via high energy neutral beams.

The most common method of heating toroidal plasmas, using Ohmic heating, cannot produce sufficient plasma temperature to achieve "break even": as much energy is produced as is used to heat the plasma, since it is limited by $T^{-3/2}$ dependence of plasma resistivity.

Presently, the injection of fast neutral atoms appears to be a highly attractive method for heating toroidal plasmas for bringing a fusion reactor to ignition (Fig. 1-1). The conversion of the fast-ion beam into a beam of fast neutrals to enter the magnetic field of the plasma device, and its reconversion to the ionic state by charge exchange and ionization in the plasma seem to be quite efficient. Stix² (1972) also showed that the fraction of energy which is transferred from the injected ions directly to the plasma ions by collision, is observed to be reasonably high. Ohkawa³ (1974) suggested that the use of momentum transfer due to the neutral-beam injection can drive impurity ions out of tokamak plasmas.

Neutral beam injection (NBI) appears highly competitive with other methods of heating for a full-scale thermonuclear fusion

plasma especially for reactor purposes. The required power input with neutral injection scales with the total number of particles in the plasma. The injected ions can reproduce several times their initial energy in fusion reactions while they slow down and heat the bulk plasma. A toroidal plasma heated by NBI can produce net thermonuclear power under conditions far less restrictive than Lawson's criterion: the two-component reactor suggested by Dawson et al.⁴ (1971). On the other hand, the power in radio-frequency (RF) heating methods tends to scale with the size of the device. RF-heating can also weight the energy input in favour of the more energetic particles like the case of NBI, but is less well suited to achieve optimal ion distribution functions.

The LBL multi-filament ion source has now been developed to the point of being able to supply ions in the 80- to 160 keV range. The field free type ion source, capable of 65A operation at 120 keV, has given fairly satisfactory operation for pulse lengths up to 1.5 seconds, and gives an atomic-ion (D⁺) fraction in the accelerated beam of about 65%: Berkner et al.⁵ (1979). The magnetic bucket ion source, in which the walls are lined with permanent magnets in a cusp arrangement, indicates that the atomic-ion (D⁺) fraction will be 75% to 80% at the arc power appropriate for 120 keV: Ehlers and Leung⁶ (1979).

In near-future experiments, neutral beams of higher energy and of a single atomic component might be necessary to penetrate to the center of large fusion plasma so as to drive a seed current for D.C. tokamak operation or control end plug potentials to increase the

plasma confinement time in tandem mirrors. Negative ions seem to be the only effective intermediary for efficient production of such neutral beams.

The "neutral beam" sources currently used in controlled fusion research convert the extracted positive ions into neutral atoms by electron capture in hydrogen gas. However, for high energy beams the efficiency of neutralization falls off rapidly.

For a thick gas neutralizer, the efficiency of conversion to neutrals is a function only of the ratio of the electron capture cross section of the ion to the stripping cross-section for the atom (Fig. 1-2). Since the stripping cross section falls above 50 keV, asymptotically as $\ln(E)/E$, while the capture cross section falls off much more rapidly, the efficiency of neutralization falls off faster than linearly in the beam energy.

For a negative ion beam, the neutralizer efficiency does not suffer from the same unfortunate energy dependence as for a positive ion beam (Fig. 1-3). This is essentially because the excess electron in the ion, being much less tightly bound (0.754 eV) than the first electron (13.6 eV), causes that weakly bound electron to have a much larger collisional detachment cross section in regions of higher energy. This makes the high efficiency production of neutral atom beams possible. Therefore, the higher energy one-component beams (> 200 keV) can be produced most efficiently by stripping an accelerated negative ion beam.

At LBL, the generation of negative ions through various methods has been studied: the double charge exchange production with a cesium

jet by Hooper et al.⁷ (1980), whereby low energy positive ions are converted to negative ions by two sequential electron captures in a cesium vapor target; the surface production by Leung and Ehlers⁸ (1981), where negative ions are produced by backscattering or desorption induced by energetic bombardment; electron volume production in which low-energy electrons lead to negative-ion formation by dissociative attachment or dissociative recombination by electron collisions with hydrogen molecules or hydrogen molecular ions.

This experiment is about the generation of negative ions through electron volume production, using the reflex-type discharge. Because the discharge has a small volume and high particle density, diagnostics are difficult and limited to external measurements of voltages, currents, and electrical-noise signals. Efforts have been concentrated on modifying the arc-chamber of the ion source to improve the continuous negative ion beam yield and its current density. In particular, modifications in the source geometry were sought to give satisfactory negative ion yield with limited gas flow rate through controlling the axial magnetic field and the radial electric field inside the arc-chamber.

In Chap. 2 the method of production of negative ions through electron volume processes, and related efforts will be described. Also, the basic model concept of negative ion production in the reflex-type negative ion source will be discussed. We will try to find out the mechanism through which the high intensity negative ion beam is obtained in the reflex-type ion source. We will analyze the

mechanism from two points of view; the cathode sheaths in the reflex-type ion source increase the confinement of negative ions, and the anomalous diffusion encourages the production of negative ions through the electron volume process to compensate the ions lost by the anomalous diffusion. In Chap. 3 we talk about the effect of cathode sheaths in the reflex-type ion source. The electric field in the transition region, which is the region between the cathode sheath and the plasma, is calculated using a simple model. The length of the transition region is also calculated. We will show that the electric field is strong enough to repel the negative ions. Most of our analysis and model concept are based on the anomalous diffusion theory developed by the author in Chap. 4. We will show that the radial electric field destabilizes the plasma from the quasilinear theory, and the anomalous diffusion coefficient is derived from the nonlinear theory. Our discussion of anomalous diffusion presented in Chap. 4 is related to many other works. Therefore, in Appendix 1 we will review the history of the development of diffusion theory in weakly ionized plasmas. In Appendix 2, problems of space charge effects, the Bohm sheath criterion, and sheaths are also reviewed. Those concepts are very important to understanding of weakly ionized plasmas. The experimental results and its considerations will be given in Chap. 5, and the summing-up and conclusion will be presented in Chap. 6. We will show that our experimental results and analyzed conclusion based on the anomalous diffusion theory are consistent with the electron volume production mechanism of negative ions suggested by

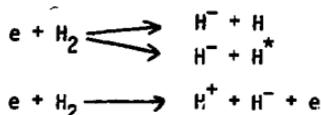
Wadehra⁹ (1979). We will also show that twice as much negative ion current is obtained experimentally with the artificial encouragement of anomalous diffusion.

CHAPTER 2
PRODUCTION OF NEGATIVE IONS

(1) Electron volume production of hydrogen negative ions

Three methods for the formation of negative ions are reviewed by Hiskes¹⁰ (1979): double charge-exchange processes, surface processes, and electron-volume processes. In this Chapter, we shall consider only the electron-volume processes.

Until recently, it has been generally considered that the main negative ion H^- production processes in hydrogen plasmas are the electron collision processes with neutral molecules in the ground state; such as dissociative electron attachment and polar dissociation.



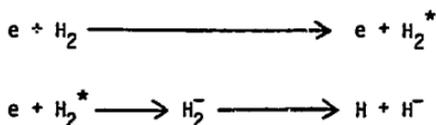
The cross sections for reactions of these kinds have been measured by Schulz¹¹ (1959); a sharp peak of the cross section with a maximum at electron energy, 14.2 eV, is attributed to the second reaction. Rapp et al.¹² (1965) also observed a similar peak of cross section. An understanding of hydrogen discharge seemed to be established.

However, the equilibrium negative ion density calculated by Nicolipoulou, Bacal and Doucet¹³ (1977) for a hydrogen plasma using the known formation and destruction processes is about 100

times lower than the density measured in their low pressure hydrogen plasma experiment. That discrepancy showed that their known H^- formation processes were not sufficient to explain the actual density of H^- ions.

However, these results were questioned because of the method of measuring the H^- density with a Langmuir probe. The anomalous high H^- density was confirmed by Bacal and Hamilton¹⁴ (1979) by photodetachment of the negative hydrogen ions and observing the rise in electron density. The recent experimental studies of dissociative attachment and vibrational excitation by Allan and Wong¹⁵ (1978) in electron collisions with H_2 and D_2 renewed the interest in negative ion production in hydrogen plasmas. It has been suggested by Demkov¹⁶ (1965), Allan and Wong¹⁵ (1978), and Wadehra¹⁷ (1978) that dissociative attachment of electrons to excited molecules might be responsible for the high negative production.

The process is interpreted in terms of the formation and dissociation of unstable states of H_2^- . The competition, between electron emission and dissociation in the molecular negative ion, could lead to a strong dependence of the dissociative attachment cross section on the mass of the hydrogen nuclei, and the initial vibrational and rotational state of the neutral molecule.



Calculations by Wadehra¹⁷ (1978) based on the above mechanism

showed a significant enhancement of the dissociative attachment cross section by vibrational or rotational excitation of the initial molecule. Particularly he looked for a mechanism capable of populating the hydrogen molecular vibration levels with $v \geq 6$.

Calculations of dissociative attachment rates are also reported by Wadehra⁹ (1979) (Fig. 2-1 and Fig. 2-2). The attachment rate coefficient at the electron temperature of 1.5 eV was taken to be on the order of $10^{-15} \text{ cm}^3 \text{ sec}^{-1}$ in a previous analysis of negative ion densities in a hydrogen plasma. Wadehra's result, however, indicate that this coefficient could be as high as $10^{-8} \text{ cm}^3 \text{ sec}^{-1}$, if there exist vibrationally excited hydrogen molecules, especially those excited in higher vibrational levels. For both H_2 and D_2 the dissociated attachment cross sections are greater than $2 \times 10^{-16} \text{ cm}^2$ for vibrational levels with thresholds below 1 eV.

The cross sections for the collisional excitation of the ground state of hydrogen molecules leading to vibrational excitation are calculated by Hiskes¹⁶ (1980). The maximum cross section to all of the vibrationally excited states is at an electron energy of about 40 eV; which is smaller than the electron energy, 60 eV, where hydrogen molecules attain the maximum ionization cross section. The vibrational levels with $v \geq 6$ require that the hydrogen molecule should have an internal energy greater than 2eV. If the molecules are "heated" to get vibrationally excited molecules, large dissociative electron attachment is expected at the relatively low electron

energy of 1 eV. Even a modest heating of the plasma can, therefore, lead to significant negative ion production.

This production mechanism may be investigated with the reflex-type negative ion source; which has been developed at LBL by Ehlers¹ (1965). The reflex-type ion source produces negative ions through volume production and it is well known that it gave the highest negative hydrogen ion current density (40 mA/cm^2) in the 1960's.

(2) Assumptions

We assume that the reflex-type negative ion source produces observable negative hydrogen atoms only through the electron-volume process. In the reflex-type ion source (Fig. 2-3), the diameter of the arc-defining hole is half of the diameter of the cylinder column. We can expect that high energy primary electrons accelerated at the cathode sheath are confined in the center part of the cylinder by the strong magnetic field; it is related to "the magnetic filter effect" proposed by Leung and Ehlers⁸ (1981). To analyze the negative ion production mechanism in the reflex-type ion source, we divide it into two regions; the central region and the surrounding region. We assume the average electron energy in the central region is higher than that in the surrounding region.

According to the production mechanism proposed by Wadehra⁹ (1979), we need both high energy electrons to produce the vibrationally excited molecules and low energy electrons to produce the negative ions by dissociative attachment to the vibrationally excited molecules. The first process has the maximum cross section at an electron energy of about 40 eV, and the later reaction has the maximum cross section at an electron energy of about 1 eV. In ordinary discharge plasmas, most electrons are produced by ionization and the electron energy is low. To get many negative ions, we also need high energy electrons; we need something to "heat" the plasma. In the reflex-type negative ion source, there are both high and low energy (temperature) electrons.

In the central region, the primary electrons ionize some neutral gas and also excite some neutral gas into vibrationally excited states. The primary electrons heat the plasma. In the surrounding region there are not so many high energy primary electrons. Nearly all of the electrons in the region are produced by ionization. Also, plasma electrons produced in the central region come there by diffusion with positive ions. The electron average energy in the region is expected to be much lower (about 1eV) than that in the central region (about 40eV): Chap. 5. In the surrounding region, the negative ion production through electron dissociative attachment to vibrationally excited molecules may be very large. The mechanism produces a high-intensity negative ion yield.

We assume that the production process of negative ions is well described by the fluid equations: the process is controlled by classical diffusion. According to a standard method to analyze the mechanism, we had to solve the continuity equation of density n_s for each species "s" in each region. The flux $\vec{\Gamma}_s = n_s \vec{u}_s$, which is used in the continuity equation, is derived from the equation of motion; \vec{u}_s is the fluid velocity vector (Appendix 1).

In the central region, we had to consider the following equations: positive ions n_p and plasma electrons n_e are produced by primary electrons (ionization), vibrationally excited molecules n_v are also produced by primary electrons and some negative ions n_n are diffused from the surrounding region. In the previous experiments by Ehlers¹ (1965), five times as much negative ion current is obtained when the arc-defining hole is

recessed from the wall of the cylinder: the surrounding region is produced by the modification besides the central region. We assume that few negative ions are produced in the central region.

$$\frac{\partial n_e}{\partial t} + \vec{\nabla} \cdot \vec{\Gamma}_e = S^+$$

$$\frac{\partial n_p}{\partial t} + \vec{\nabla} \cdot \vec{\Gamma}_p = S^+$$

where S^+ is a production term by ionization.

$$\frac{\partial n_v}{\partial t} + \vec{\nabla} \cdot \vec{\Gamma}_v = S_v^+$$

where S_v^+ is a production term of vibrationally excited molecules.

$$\frac{\partial n_n}{\partial t} + \vec{\nabla} \cdot \vec{\Gamma}_n = 0$$

In the surrounding region, we had to consider the following equations: negative ions n_n are produced by the dissociative attachment of plasma electrons n_e to vibrationally excited molecules n_v , and positive ions n_p are diffused from the central region.

$$\frac{\partial n_e}{\partial t} + \vec{\nabla} \cdot \vec{\Gamma}_e = S_e^+ - S_e^-$$

where S_e^+ is a production term of plasma electrons, and S_e^- is a loss term of plasma electrons.

$$\frac{\partial n_p}{\partial t} + \vec{\nabla} \cdot \vec{\Gamma}_p = 0$$

$$\frac{\partial n_v}{\partial t} + \vec{\nabla} \cdot \vec{\Gamma}_v = -S_v^-$$

where S_V^- is a loss term of vibrationally excited molecules.

$$\frac{\partial n_n}{\partial t} + \vec{\nabla} \cdot \vec{r}_n^+ = S_n^+ - S_n^-$$

where S_n^+ is a production term of negative ions, and S_n^- is a loss term of negative ions.

We have neglected positive and negative ion losses by recombination. In the ordinary method of analysis, space uniformity is assumed and flux gradient terms are neglected. However, the reflex-type ion source is located inside a strong magnetic field and the radial density gradient is sharp. We cannot neglect those terms. Therefore we have to solve four two-dimensional continuity equations in two regions at the same time.

(3) Model concept of analysis

The one-dimensional continuity equation for two component plasma in the reflex discharge was solved by Chen¹⁹ (1962). To decide the boundary condition in the radial direction, he assumed that particle fluxes are controlled by Bohm diffusion, and that the fluxes disappear at the wall of the cylinder; Bohm diffusion is discussed in Appendix 1. To decide the boundary condition in the axial direction, he assumed that particle fluxes are controlled by the sheath condition. He derived the primary electron current theoretically; these topics are discussed in Appendix 2.

It is very difficult to solve four two-dimensional continuity equations in two regions under proper boundary conditions. The derivation of the electric field and the treatment of production or loss terms are not easy. Above all, in the reflex-type ion source, the assumption that the production process is controlled by the classical diffusion process is suspicious. We may have to treat nonlinear equations. Therefore, we do not try to solve the equations presented in Section (2).

We will rather try to find out physically the mechanism through which a high density negative ion current is obtained in the reflex-type ion source. Then, taking advantage of the mechanism, we try to increase the negative ion current. We will show in Chap. 3 that the negative ions are well confined in the axial direction, since there are two cathode sheaths in the reflex-type ion source. We will show in Chap. 4 that the inwardly directed radial electric field

inside the reflex-type ion source destabilizes the plasma, and that the reflex-type ion source is controlled by anomalous diffusion process. Then, the anomalous diffusion coefficients are derived theoretically.

In Chap. 5 we will analyze our experimental results based on the anomalous diffusion theory presented by the author in Chap. 4. From the experimental results, we will show that the calculated negative ion density in the surrounding region increases as the calculated electron temperature decreases from 40 eV to about 1 eV (Fig. 5-10 and 5-11). However, the electron energy in the central region is still expected to be high enough to produce many vibrationally excited molecules. If the negative ion production mechanism proposed by Wadehra⁹ (1979) and his calculated results are not true, then the negative ion density should have a maximum value around an electron energy of 14.2 eV. We believe our experimental results support Wadehra's calculated results.

Then, we will also try to increase the extracted negative ion current through the encouragement of anomalous diffusion. Twice as much negative ion current was obtained with the artificial encouragement as without. The anomalous diffusion mechanism, in which we do not need to treat the production terms, can be used to increase the negative ion current in the reflex-type negative ion source.

CHAPTER 3

CATHODE SHEATHS EFFECT IN THE REFLEX-TYPE ION SOURCE

We assume the two cathode sheaths in the reflex-type ion source confine the negative ions effectively in the axial direction. In this chapter, the previously unnoticed effects of cathode sheaths in the reflex discharge are considered. Based on simple assumptions in a one-dimensional plasma, the electric field in the transition region will be calculated. Effects of the electric field in the reflex discharge will be discussed.

For a weakly ionized plasma in the positive column of the low pressure discharge, Langmuir²⁰ (1929) made the following assumptions. Most of the voltage difference from cathode to anode in typical gas discharge is concentrated in a cathode sheath; the rest of the volume is nearly field free compared to the cathode sheath, the space charge of the positive ions being neutralized by low velocity electrons. The current toward the cathode is limited by the electron and ion space charges in a double sheath near the cathode. When ions are produced uniformly throughout the space, the ions flow in nearly equal numbers to anode and to cathode; the electrons and negative ions go to the anode only.

However, during the course of his researches, Langmuir found that the electric field penetrated beyond the sheath edge, into the plasma, and accelerated ions toward the sheath. Later, the effect of this field was recognized by Bohm²¹ (1949). The region, which accelerates ions, is called the transition region.

In the case of the reflex discharge, the Bohm sheath criterion is applied only in the axial direction because of the strong magnetic field. The whole cylinder wall acts as an anode. Both ends of the reflex discharge are cathodes: one hot and the other cold. There is no anode sheath but there are cathode sheaths.

The whole of the arc voltage is concentrated in the cathode sheaths (in the order of 100 volt). In the transition region, there is still a voltage difference (in the order of 10 volts). As pointed out by Salz et al.²² (1961), the transition region in the reflex discharge penetrates far into the plasma region (much longer than a Debye length) and constitutes an electrostatic well in the middle part of the cylinder.

We try to investigate the transition region using a simple slab model (Fig. 3). The transition region is defined as a region between the plasma region and the sheath region. We assume, for simplicity, that no ion is produced there. We call the plasma side of the transition region the plasma edge ($x = 0$), and the sheath side the sheath edge ($x = L$). At the sheath edge, the plasma density is less than that in the plasma.

In the plasma region where the plasma potential is constant, both electrons and ions are in collisional equilibrium and the charge neutrality is preserved ($n_i = n_e$). In the sheath region which shields plasma, neither electrons nor ions are in collisional equilibrium. Charged particles experience no collision inside this region. A strong negative electric potential keeps both electron and ion fluxes the same to produce a steady state in the plasma.

Although ions simply fall into the cathode, electrons strike the cathode so often that they must be repelled if the distribution is to be steady. The velocity of ions entering sheaths has to satisfy the Bohm sheath criterion; ions must enter the sheath region with a velocity greater than the acoustic velocity: Bohm²¹ (1949). The space charge effect, the Bohm sheath criterion, and sheaths are discussed in Appendix 2.

In the transition region, we assume that only electrons are in collisional equilibrium and the charge neutrality is not strictly conserved. Since the shielding by the sheath is not perfect, a small portion of the strong electric field inside the sheath penetrates into this region through the sheath edge.

The small electric field accelerates ions towards the sheath so that they can satisfy the Bohm sheath criterion. However, the collisions with neutral particles prevent them from accelerating freely. We also assume that the density of primary electrons is small compared to that of plasma electrons.

For a weakly ionized plasma, the fluid equations, which are discussed in Appenix 1, are as follows; all physical quantities are defined in ordinary ways; n_e is the density of electron, P_e is the electron pressure, u_e is the fluid velocity of electron, and ν_e is the collision frequency with neutrals.

From the one dimensional equation of motion of electrons,

$$n_e m \frac{du_e}{dt} + \frac{dP_e}{dx} = -en_e E_e - n_e m \nu_e u_e \quad (3.1)$$

Since electrons are in thermal equilibrium, there is no net flux.

Since $P_e = n_e T_e$ we get the following equation:

$$-D_e \frac{dn_e}{dx} - n_e \mu_e E = 0 \quad (3.2)$$

where T_e is the electron temperature, D_e is the diffusion coefficient of electrons, and μ_e is the mobility of electrons.

Then,

$$n_e = n_0 \exp\left(-\frac{eE}{T_e} x\right) \quad (3.3)$$

where n_0 is the charge density at the plasma edge ($x = 0$).

From the results of Chen's experiment¹⁹ (1962) of the reflex discharge: $n_e/n_0 = 0.2$ at the sheath edge ($x = L$). Then, we get the following relation:

$$E = 1.6 \frac{T_e}{eL} \quad (3.4)$$

In the transition region, ions are not in thermal equilibrium anymore. Thus, we have to keep the inertia term in the equation of motion of ions. We assume that

$$\frac{1}{n_i M} \frac{dP_i}{dx} \ll 1$$

where n_i is the density of ion and P_i is the ion pressure.

Then, in the same way as for the electron case, we get the following equation,

$$\frac{du_i}{dt} + \nu_i u_i - \frac{eE}{M} = 0 \quad (3.5)$$

where ν_i is the collision frequency with neutrals.

A solution of the differential equation is

$$u_i = (u_0 - \frac{eE}{v_i M}) \exp(-v_i t) + \frac{eE}{v_i M} \quad (3.6)$$

where u_0 is the average ion velocity at the plasma edge.

Generally, when $v_i t \gg 1$, we can neglect the first term of u_i . At the sheath edge, the ion velocity is determined by the Bohm sheath criterion:

$$u_i = \sqrt{\frac{T_e}{M}} = \frac{eE}{v_i M} \quad (3.7)$$

Then, equating Eqs. (3.6) and (3.7), we get the following result

$$E = \frac{v_i M}{e} \sqrt{\frac{T_e}{M}} \quad (3.8)$$

In Chap. 6 we will show that $T_e \approx 40$ eV in the central region of the reflex-type negative ion source, and the collision frequency of ions with neutrals is in the order of $v_i = 10^7$ /sec.

Then, the electric field in the transition region is calculated from Eq. (3.8),

$$E = 65 \text{ volt/cm}$$

And the length of the transition region is calculated from Eq. (3.4),

$$L = 1 \text{ cm}$$

The reflex-type ion source is operated with an arc voltage of

300 volt. Its length is about 6 cm. The strength of the electric field seems reasonable. The length of the transition region is one-third (2 cathode sheaths) of the total length of the ion source.

When negative ions exist in the plasma, which is weakly ionized, the average temperature of negative ions is usually in the order of 1 eV. Therefore, the electrostatic well is strong enough to completely trap the negative ions, but not strong enough to trap all of the electrons which have much higher temperature. In the reflex-type negative ion source, negative ions are expected to be concentrated in the middle part of the cylinder where the ion exit slit locates. We call this phenomenon "the (electrostatic) sheath trap of negative ions". We believe this phenomenon is one of the reasons why we can get a high intensity negative ion beam in the reflex-type negative ion source, which will be discussed more precisely in Chap. 5.

CHAPTER 4

ANOMALOUS DIFFUSION IN WEAKLY IONIZED PLASMA

1) Introduction

We assume that the radial diffusion in the reflex-type ion source is controlled by anomalous diffusion mechanism. In this chapter, we are going to derive the super-critical condition and the anomalous diffusion coefficients in a weakly ionized plasma. In earlier experiments with the positive column, an instability has been observed in the presence of a strong longitudinal magnetic field. A theoretical explanation of this phenomenon has been given by Kadomtsev and Nedospasov²³ (1961) who discuss the growth of screw-shaped disturbances. The axial electric field tends to lift up the electron screw relative to the ion screw. This is equivalent to a rotation of the electron screw in the positive azimuthal directions. Due to subsequent charge separation, azimuthal electric fields will arise. This tends to drive the particles outwards with a speed of the E cross B drift and tends to destabilize the plasma. This is so-called "the screw instability". All experiments support the theory.

The instability mechanism of the reflex discharges was investigated by Hoh²⁴ (1963). He pointed out that the occurrence of a critical magnetic field in the reflex discharges may be due to an instability which is caused by the plasma particle currents flowing in the Hall direction. In a partially ionized plasma, in

which the electric and magnetic fields cross, the electrons drift more rapidly through the neutral gas than the ions. Thus, an electric space-charge field is produced in the Hall direction. This causes a subsequent drift of the charged particles in the radial direction. If this drift is in the direction of decreasing density and increasing potential, an instability may result. Hoh's theory which is called "the neutral drag instability" explained the instability mechanism well when the energy of primary electrons are relatively low: Pavlichenko et al.²⁵ (1966), and Thomassen²⁶ (1966). When the velocity of primary electrons is much faster than the thermal velocity, the effect of the beam instability has to be considered, as well as the fact that it is difficult to make the analysis. Hoh's theory may well be applied to hot cathode reflex discharges in which the primary electron energy is relatively low.

Even though the instability mechanisms of both the positive column and reflex discharge were proposed, no direct mechanism for the anomalous diffusion induced by the instability was given. On the other hand, Yoshikawa and Rose²⁷ (1962) derived an expression for a diffusion coefficient, which, in suitable limits, gives both collisional and Bohm diffusions.

Their expressions are derived from the time independent fluid equations under dominant Coulomb collisions; the motion of the ions is neglected. They considered the following physical mechanism in which a density gradient normal to the magnetic field gives rise to a Hall current. If there are small density fluctuations, a space charge field, which is electrostatic, must be set up parallel to the

Hall current to preserve its continuity. This field, crossed with the magnetic field, leads to motion in the direction of the pressure gradient. To obtain a net flux in this direction, after averaging over the fluctuations, the second order terms must be retained. The enhanced diffusion is obtained within the limits of low pressure or the limits of high magnetic field. Reasonably good agreement between Yoshikawa's theory and experiment was found by Janes and Lower²⁸ (1966). A straightforward extension of his theory, in which the ion motion is considered, was also proposed by Yoshikawa²⁹ (1970).

In Section (2), we derive the super-critical condition from the quasilinear equations in the Cartesian coordinate. We will also show that the super-critical condition is applicable for both the positive column and the reflex discharge, and that the plasma in the reflex-type ion source is unstable because of the inward directed radial electric field.

In Section (3), we derive the anomalous diffusion coefficient from the nonlinear theory developed by Yoshikawa and Rose²⁷ (1962). The electron-ion collision frequency in their method is replaced by the electron collision frequency with neutral atoms. In a weakly ionized plasma, the collisions with neutral atoms are dominant and Coulomb collisions will be neglected. We assume that the instabilities are already excited and saturated. The anomalous diffusion coefficient is obtained as the second order quantity in Fourier expansions.

2) Low frequency instabilities in a weakly ionized plasma

a) Assumptions

The well known instabilities derived from fluid equations are "the two stream instability", "the gravitational instability", and "the resistive drift instability". Here, we investigate instabilities related to anomalous diffusion, which is caused by low frequency instabilities. Both "the two stream instability" and "the gravitational (flute) instability", which need particular mechanisms to excite them, are not considered here. In a weakly ionized plasma, where Coulomb collisions are neglected, there is no pure "the resistive drift instability". However, if there is a constant 0th order electric field which exists only in a weakly ionized plasma, the electric field unstabilizes the dispersion relation even though the inertia term of ion equation is neglected. The instability is caused by charge separations in the Hall direction. The charge separations are caused by the drag force between charged particles and neutral atoms through the 0th order electric field, and the charge separations produce fluctuating higher order electric fields. The time averaged second order electric field in the Hall direction pushes particles in the perpendicular direction of the 0th order magnetic field by the $E \times B$ drift.

We attribute the instability to drift-like waves. In a weakly ionized plasma, the dispersion relation is messy since 0th order electric fields exist. We may call them "drag wave" since they are produced by the drag force.

The average first order electric fields are sinusoidal, and the

time averaged first order electric fields are therefore zero. They do not contribute to any particle transport. The steady 0th order electric fields affect plasma through the drag force. Therefore, we mention it as "the resistive drag instability" instead of "the resistive drift instability". It includes both "the screw instability" and "the neutral drag instability".

For weakly ionized plasmas, the fluid equations, which are discussed in Appendix 1, are shown below; n_s is the density of species "s", u_s is the fluid velocity of species "s", P_s is the pressure of species "s", ν_s is the collision frequency of species "s" with neutrals, and Z is the production term.

From the continuity equation,

$$\frac{\partial n_s}{\partial t} + \nabla n_s \cdot \vec{u}_s = Zn_s \quad (4.1)$$

From the equation of motion neglecting the effect of gravity,

$$n_s m_s \frac{d\vec{u}_s}{dt} + \nabla P_s = n_s q_s \left(\vec{E} + \frac{\vec{u}_s \times \vec{B}}{c} \right) - n_s m_s \nu_s \vec{u}_s \quad (4.2)$$

From the equation of state,

$$\frac{\nabla P_s}{P_s} = \gamma \frac{\nabla n_s}{n_s} \quad (4.3)$$

where γ is the ratio of specific heats C_p/C_v .

These equations can be solved by the procedure of linearization.

The dependent variables are separated into two parts as follows:

$$\begin{aligned} u_s &= u_{s0} + u_{s1} & n_s &= n_{s0} + n_{s1} \\ \vec{E} &= \vec{E}_0 + \vec{E}_1 & \vec{B} &= \vec{B}_0 + \vec{B}_1 \end{aligned}$$

where u_{s1} , n_{s1} , E_1 , and B_1 , are the first order terms of species "s" and they are very small. By this we mean that the amplitude of oscillation is small, and terms containing higher powers of amplitude factor can be neglected.

When only the low frequency instability (much less than ion cyclotron frequency) is considered, we need only keep the time derivative only in the continuity equation. The time derivative in the equation of motion will be negligible since the collision frequency with neutrals is large enough. We assume that the first order magnetic field B_1 is very small and that the 0th order fluid velocity u_0 is zero. We also assume that the 0th order density n_0 is uniform in both y and z direction; n_0 has a gradient only in the x direction. Since we consider only low frequency instabilities, the change caused by the instability is isothermal; that means $\gamma = 1$.

We shall omit subscripts "1" for the first order terms, and subscripts "s" for the 0th order terms. Then, we get the following two linearized first order equations with $\nabla P_s = T_s \nabla n_s$:

$$\frac{\partial n_s}{\partial t} + n_0 (\nabla \vec{u}_s) + u_s \frac{\partial n_0}{\partial x} = Zn_s \quad (4.4)$$

$$T_s \nabla n_s = n_0 q_s \left(\vec{E}' + \frac{\vec{u}_s \times \vec{B}_0}{c} \right) + n_s q_s \vec{E}_0 - n_0 m_s v_s \vec{u}_s \quad (4.5)$$

where E' is the first order electric field.

We assume that there is a constant magnetic field B_0 only in the z direction. We also assume that there are constant electric fields E_0 both in the x and z directions. We consider only the

electrostatic perturbation produced by an electric potential.

We are interested in anomalous diffusion in weakly ionized plasmas, particularly those in "the positive column" and "the reflex discharge". In this section we are concerned with mainly the source of the resistive drag instabilities in a weakly ionized plasma: "the positive column" and "the reflex discharge". We treat them under the periodic boundary condition in the Cartesian coordinates.

b) The supercritical condition

We shall omit subscript "0" for the 0th order quantities. We assume that the first order electric field is electrostatic and that it is produced by an electrostatic potential. The oscillating quantities are assumed to behave sinusoidally. From the linearized equation of motion for electrons, we get the following equations; n_e is the first order electron density, ϕ is the first order electrical potential, and u_e is the first order fluid velocity vector of electron.

$$T_e \nabla n_e = -en_e \left(\vec{E} - \phi + \frac{\vec{u}_e \times \vec{B}}{c} \right) - n_e m v_e \vec{u}_e \quad (4.6)$$

Then, we get

$$u_{ex} = b_e \left[i \frac{A}{nB} (k_y - a_e k_x) + \frac{n_e}{nB} (E_y - a_e E_x) \right]$$

$$u_{ey} = -b_e \left[i \frac{A}{nB} (k_y - a_e k_x) + \frac{n_e}{nB} (E_x + a_e E_y) \right]$$

$$u_{ez} = -v_e \frac{n_e}{n} \left(iA \frac{k_z}{n_e} - E_z \right)$$

where $\Omega_e = \frac{eB}{mc}$ and $v_e = \frac{e}{mv_e}$

$$a_e = \frac{v_e}{\Omega_e}, \quad b_e = \frac{1}{1+a_e^2}, \quad \text{and} \quad A = \frac{n_e T_e}{e} - n\phi$$

From the linearized equation of continuity for electrons,

$$\frac{\partial n_e}{\partial t} + n \nabla \cdot \vec{u}_e + u_{ex} \frac{\partial n}{\partial x} = Zn_e \quad (4.7)$$

Then,

$$-i\omega n_e + in(k_x u_{ex} + k_y u_{ey} + k_z u_{ez}) + u_{ex} \frac{\partial n}{\partial x} = Zn_e$$

We define that the perturbed electro-static waves are traveling along the z direction, which is the direction of the magnetic field, and the y direction, which is the Hall direction. For prevailing conditions, we can assume $k_x = 0$ and $E_y = 0$.

Generally for electrons with $a_e^2 \ll 1$ and $b_e = 1$, we get

$$\frac{n_e}{n} = \frac{-k_y \frac{c}{B} \frac{n'}{n} + i k_z^2 u_e}{(\omega + k_y v_{D_e}) + k_y c \frac{E_x}{B} + k_z u_e E_z + i(k_z^2 D_e - Z)} \quad (4.8)$$

where $D_e = \frac{T_e}{m v_e}$, $v_{D_e} = -\frac{T_e}{eB} \frac{n'}{n}$, and $n' = \frac{\partial n}{\partial x}$

In the same manner, we get an equation for ions assuming $k_z^2 \ll k_y^2$

$$\frac{n_i}{n} = \frac{X}{(\omega - k_y b v_{D_i}) + k_y b \frac{c E_x}{B} - k_z u_i E_z + iY} \quad (4.9)$$

where $X = -k_y b \frac{c}{B} \frac{n'}{n} - i k_y^2 a^2 b u_i$

$$Y = k_y^2 a^2 b D_i + a^2 b u_i E_x \frac{n'}{n} - Z$$

$$a = \frac{v_i}{\Omega_i}, \quad b = \frac{1}{1+a^2}, \quad \text{and } v_{D_i} = -\frac{c T_i}{eB} \frac{n'}{n}$$

$$\Omega_i = \frac{eB}{Mc}, \quad u_i = \frac{e}{M v_i}, \quad \text{and } D_i = \frac{T_i}{M v_i}$$

If we assume charge neutrality, we get the following instability criterion from the dispersion relation.

$$\begin{aligned}
 & [a^5 b^2 (k_y^2 - (\frac{n'}{n})^2) \mu_i^2 - ab \frac{k_z^4}{k_y^2} \mu_e^2] \frac{ck_y^2}{B} (-\frac{n'}{n}) (-E_x) \\
 & + [b(k_z^2 + a^2 k_y^2) \mu_e \mu_i + a^2 b k_y^2 \mu_i^2 + b k_z^2 \mu_e^2] c \frac{k_z k_y}{B} (-\frac{n'}{n}) (-E_z) \\
 & + [(a^2 b k_y \frac{c}{B} \frac{n'}{n})^2 + (k_z^2 \mu_e + k_y^2 a^2 b \mu_i)^2] Z \\
 & > [a^2 b (k_y k_z^2 \mu_e)^2 + (a^2 b k_y^2 k_z^2)^2 \mu_i \mu_e + (a^2 b k_y^2 \frac{c}{B} \frac{n'}{n})^2] D_i \\
 & + [(a^2 b k_y^2 k_z^2 \mu_i)^2 + a^2 b k_y^2 k_z^4 \mu_i \mu_e - (a^2 b k_z k_y \frac{c}{B} \frac{n'}{n})^2] D_e \\
 & + (b^2 k_y^2 k_z^2 \mu_e + a^2 b^2 k_y^4 \mu_i) \frac{e}{T_i} v_{D_i}^2 \\
 & + (a^2 b k_y^4 \mu_i + b k_y^2 k_z^2 \mu_e) \frac{e}{T_e} v_{D_e}^2
 \end{aligned}$$

Notice $(n'/n) < 0$

c) Discussion

In prevailing conditions for the positive column, it is satisfied that $v_i \gg \Omega_i (a_i \gg 1, b_i \approx 0, a_i^2 b_i \approx 1)$ and $E_z < 0, E = E_x > 0$. We can also assume that $k_y^2 \gg k_z^2, \nu_e \gg \nu_i, D_e \gg D_i$ and that $(-n'/n)^2$ is small. Then, we get the following instability criterion for the positive column.

$$\begin{aligned}
 & k_y^3 k_z \nu_e \nu_i \left(-\frac{n'}{n}\right) \left(-\frac{cE_z}{B}\right) + \left[\left(k_y^2 \frac{c}{B} \frac{n'}{n}\right)^2 + \left(k_z^2 \nu_e + k_y^2 \nu_i\right)^2 \right] Z \\
 & > a k_y^4 \nu_i^2 \left(-\frac{n'}{n}\right) \left(-\frac{cE_x}{B}\right) + \left[\left(k_y^2 k_z \nu_i\right)^2 + \left(k_y k_z^2\right)^2 \nu_i \nu_e \right] D_e \\
 & + \left[\left(k_y k_z^2 \nu_e\right)^2 + \left(k_y^2 k_z\right)^2 \nu_i \nu_e + \left(k_y^2 \frac{cn'}{Bn}\right)^2 \right] D_i \\
 & + \left(k_y^4 \nu_i \frac{e}{T_e}\right) \nu_{De}^2 \tag{4.10}
 \end{aligned}$$

In the case of the positive column, E_z and Z terms contribute to destabilize the plasma; $E_x = E_r, \nu_{De}^2, D_i$ and D_e terms contribute to stabilize the plasma. $(n'/n) < 0$ must be also satisfied. A similar relation is derived by Kadomtsev and Nedospasov²² (1961), however, the physical meaning of each term is less apparent.

When an instability sets in, many particles are lost by anomalous diffusion. Then the particle production has to be increased to compensate the particle loss to keep plasma in the same state. More charged particles (ions and electrons) are produced by the primary electrons, which are supplied from the cathode by the steady electric field E_z . Therefore, the production rate Z is

increased. The increase of electric field E_z was observed in Lehnert's experiment³⁰ (1958) when instabilities set in. The relation between the electric field E_z and the magnetic field was theoretically explained by Kadomtsev and Nedospasov²³ (1961).

Alternatively the anomalous diffusion is induced by the large particle production to reduce the particle density. On the other hand, the particle diffusions and electron diamagnetic drift try to eliminate the charge separation which causes the instability.

In prevailing conditions for the reflex discharge, it is satisfied that $v_j = \Omega_j (a_j = 1, b_j = 1/2)$; which will be explained precisely later. It is also satisfied that $E_x = E_r < 0$ and $E_z = 0$. We can assume that $k_y^2 \gg k_z^2$, $\nu_e \gg \nu_i$, $D_e \gg D_i$ and that $(-n'/n)^2$ is large in the reflex discharge: Chen¹⁹ (1962). Then we got the following instability criterion for the reflex discharge.

$$\begin{aligned}
 & \frac{1}{4} k_y^2 \nu_i^2 \left(\frac{n'}{n}\right)^2 \left(-\frac{n'}{n}\right) \frac{cE_x}{B} + \left[\frac{1}{4} (k_y \frac{cn'}{Bn})^2 + (k_z^2 \nu_e + \frac{1}{2} k_y^2 \nu_i)^2 \right] \Omega_i \\
 & > \left[\frac{1}{2} (k_y k_z^2 \nu_e)^2 + \frac{1}{2} k_y^4 k_z^2 \nu_i \nu_e + \frac{1}{4} (k_y \frac{cn'}{Bn})^2 \right] D_i \\
 & + \left[\frac{1}{4} (k_y^2 k_z^2 \nu_i)^2 + \frac{1}{2} k_y^2 k_z^4 \nu_i \nu_e - \frac{1}{4} (k_y k_z \frac{cn'}{Bn})^2 \right] D_e \\
 & + \left(\frac{1}{4} k_y^2 k_z^2 \nu_e + \frac{1}{4} k_y^4 \nu_i \right) \frac{e}{T_i} v_{D_i}^2 \\
 & + \left(\frac{1}{2} k_y^4 \nu_i + \frac{1}{2} k_y^2 k_z^2 \nu_e \right) \frac{e}{T_e} v_{D_e}^2
 \end{aligned} \tag{4.11}$$

In the case of the reflex discharge, $E_x \approx E_r$ and Z terms contribute to destabilize plasma. D_i , v_{Di}^2 and v_{De}^2 terms contribute to stabilize the plasma. We do not know about the contribution of the D_e term.

Since $E_x \approx E_r$ is negative in the reflex discharge, the plasma will be unstable when $(-n'/n)E_x \gg 0$, which is shown by Simon³¹ (1963). A similar relation is also derived by Hoh²⁴ (1963). However, neglecting the effect of a sharp density gradient, Hoh concluded that the plasma is destabilized when E_r is decreased (dv/dr is increased). Our conclusion is opposite: the plasma is destabilized when E_r is increased ($-E_r$ is decreased). In Chap. 5, we will show that this conclusion is consistent with the experimental results.

According to Hoh²⁴ (1963), the experimentally obtained critical magnetic field of most of the reflex discharges is lower than 500 gauss. In the reflex-type ion source, the magnetic field is variable from 1500 gauss to 5000 gauss. Thus, the reflex type ion source is completely in anomalous region.

When an instability sets in, many particles are lost by anomalous diffusion. Then, the number of electrons inside the column, which keeps $E_x \approx E_r$ negative is reduced. The potential in the center of the cylinder is decreased (less negative). Then the voltage drop in the cathode sheath is increased. That effect accompanies an increasing supply of the primary electrons and an increasing particle production rate Z . Alternatively the anomalous diffusion is induced by large particle production to reduce the

negative potential in the center of the cylinder.

On the other hand, the ion diffusion and the diamagnetic drifts try to eliminate the charge separation which causes the instability as in the case of the positive columns. The electron diffusion might encourage the instability. In prevailing condition for the reflex discharge, $E_z \approx 0$, so it contributes nothing.

We have investigated the cases in which $v_i \gg \Omega_i$ and $v_i \approx \Omega_i$. What happens in the case $v_i \ll \Omega_i$? This relation is usually satisfied when the pressure is very low. Then, Coulomb collisions can not be neglected, and the plasma is not weakly ionized anymore.

3) Anomalous diffusion coefficienta) Assumptions and derivation of the diffusion equations

We shall assume a macroscopically homogeneous plasma, which is subject to a homogeneous turbulence, and shall introduce a pressure gradient and electric field, both of which are small and uniform. We can make use of modified fluid equations to explain non linear effects. This method is largely based on Yoshikawa and Rose²⁹ (1962). In this case we consider collisions with only neutrals; Coulomb collisions are neglected.

We assume that instabilities are already excited and saturated. Each physical quantities in fluid equations are Fourier-expanded to include the anomalous effects. The second order electric field in the Hall direction, which is produced by instabilities, is evaluated from 0th order and 1st order equations assuming uniform electrostatic turbulences. The charged particles couple to each other through the electric field; there is no direct coupling. Also, the contribution by the ordinary diffusion based on the classical collision theory is neglected since it is expected to be small compared to the anomalous diffusion in a strong magnetic field.

In a weakly ionized plasma, the equation of motion in steady state is given as follows (Appendix 1). Then we get for each species

$$n\vec{T} - qn\vec{E} - q \frac{\vec{\Gamma} \times \vec{B}}{c} = -m\nu\vec{\Gamma}$$

The uniform magnetic field B is imposed in the z direction, and ν is the collision frequency between charged particles and neutrals.

The flux $\vec{\Gamma}$, density n , and pressure nT are defined in the ordinary

manner as those defined in Appendix 1. We have neglected the inertia term, since we considered only low frequency instabilities, this is permissible if the collision frequency with neutrals is large enough.

We have introduced the density gradient effect by replacing n by $n(1 + sx)$. Similarly, for the electric field, we introduce \vec{E}_0 , which is defined as the average value of electric field \vec{E} . \vec{E}_0 has components in both x and z directions.

Thus,

$$E_0 = \lim_{L \rightarrow \infty} \frac{1}{L} \int_0^L E dr$$

where L is the dimensional length.

we also assume that the fluctuation of the density n' is smaller than that of the average density n_0 .

Other quantities, such as electric field E and flux $\vec{\Gamma}$, which change with the external applied field are defined as follows

$$\vec{E} = \vec{E}_0 + \vec{E}'$$

$$\vec{B} = \vec{B}_0$$

$$\vec{\Gamma} = \vec{\Gamma}_0 + \vec{\Gamma}'$$

$$n = n_0 + n'$$

where \vec{E}' , $\vec{\Gamma}'$, and n' are the responses to the external effects assuming that the magnetic field B is given from outside. The effective electric field \vec{e} is defined

$$q\vec{e} = q\vec{E}_0 + T\vec{s}$$

where \vec{s} is a unit vector in the x direction.

Then the equations for the perturbed quantities are follows:

$$qn\vec{E}' + qn\vec{c} + q \frac{\vec{v}' \times \vec{B}}{c} = m\nu\vec{v}' \quad (4.12)$$

q is positive unit charge " e " for ions and negative unit charge " $-e$ " for electrons.

If there are any small density fluctuations in the Hall direction, space charge fields in that direction must be set up because of the continuity equation. These additional fluctuating fields, crossed with magnetic field, give the motions in the direction of original uniform electric field when the magnetic field passes a critical value. The uniform density gradient also causes the motion in the direction of the density gradient by the same mechanism. Therefore, the fluctuating E cross B drifts in the radial direction are produced by a density gradient or by a uniform electric field. The fluctuating drifts give a net second order flow; not a first order flow which will be averaged to zero. The additional electric fields produced by the density fluctuations have to satisfy the Maxwell equation. We consider only electro-static field since we are interested in the drift-like instabilities.

$$\vec{\nabla} \times \vec{E}' = 0 \quad (4.13)$$

If there is any small density fluctuation in the Hall direction, fluctuating electro-static fields in the Hall direction must be set up because of the continuity equation:

$$\vec{\nabla} \cdot \vec{v}' = 0 \quad (4.14)$$

The effective collision frequency ν is affected by the fluctuating electric field. We assume the effective frequency is proportional to the effective density:

$$v = v_0 \frac{n}{n_0} \quad (4.15)$$

where v_0 and n_0 are the averaged value of v and n .

We assume that the characteristic length of the fluctuations exceeds the Debye length, but small compared to the dimensional length. We proceed with the Fourier analysis, and impose periodic boundary conditions. Thus, the effect of the real boundary is neglected.

Then, the first order quantities are Fourier-expanded as follows:

$$\vec{E}' = \Sigma' \vec{E}'_k \exp(i\vec{k} \cdot \vec{r})$$

$$\vec{I}' = \Sigma' \vec{I}'_k \exp(i\vec{k} \cdot \vec{r})$$

$$n' = \Sigma' n'_k \exp(i\vec{k} \cdot \vec{r})$$

where Σ' indicates a summation over all k except $k=0$ since no electric potential fields remains in the uniform state besides \vec{E} (for a system of charged particles, this assumption corresponds to the quasineutrality in background). By definition, \vec{E}' averaged over a macroscopic plasma must be zero. And $n'_k = n'^*_{-k}$, $\vec{I}'_k = \vec{I}'^*_{-k}$, and $\vec{E}'_k = \vec{E}'^*_{-k}$. The 0th order equation is obtained by putting the above quantities into Eq. (4.12). Then, multiply Eq. (4.12) by n'_{-k} , $\exp(-i\vec{k}' \cdot \vec{r})$ integrate with \vec{r} , and sum up with k' . Then, we get

$$q \Sigma' n'_k \vec{E}'_{-k} + q n_0 \vec{E} + q \frac{\vec{I}'_0 \times \vec{B}}{c} = m v_0 \vec{I}'_0 + m v_0 \Sigma' \frac{n'_k}{n_0} \vec{I}'_{-k} \quad (4.16)$$

Next we get the k -th order equation of Eq. (4.12). We neglect that the cross terms. Then,

$$q n_0 \vec{E}_k + q n_k \vec{e} + q \frac{\vec{\Gamma}_k \times \vec{B}}{c} = m v_0 \vec{\Gamma}_k + m v_0 \frac{n_k}{n_0} \vec{\Gamma}_0 \quad (4.17)$$

The directional cosine γ , and the nondimensional quantity a , which is positive for ions and negative for electrons, are defined as follows:

$$\gamma = \frac{\vec{k}}{k}$$

$$a = \frac{v_0}{\Omega}, \quad b = \frac{1}{1+a^2}, \quad \text{and} \quad \Omega = \frac{qB}{mc}$$

And

$$E_k = E_k \gamma$$

$$\gamma_x = \gamma \sin \theta \cos \phi$$

$$\gamma_y = \gamma \sin \theta \sin \phi$$

$$\gamma_z = \gamma \cos \theta$$

$$\vec{\Gamma}_0 = (\Gamma_x, \Gamma_y, \Gamma_z)$$

E' will be evaluated from Eq. (4.13).

Eq. (4.14) can be rewritten as follows:

$$\gamma_x \Gamma_{kx} + \gamma_y \Gamma_{ky} + \gamma_z \Gamma_{kz} = 0 \quad (4.18)$$

Eq. (7), together with Eq. (8), can be solved for E_{-k} and Γ_{-k} in terms of n_k , \vec{e} , $\vec{\Gamma}_0$ to give

$$\Gamma_{-kx} = b[n_0 \frac{cE_{-k}}{B} (\gamma_y + a\gamma_x) + an_k \frac{cE_x}{B} - a \frac{n_k}{n_0} (\Gamma_y + a\Gamma_x)] \quad (4.19)$$

$$\Gamma_{-ky} = -b[n_0 \frac{cE_{-k}}{B} (\gamma_x - a\gamma_y) + n_k \frac{cE_x}{B} - a \frac{n_k}{n_0} (\Gamma_x - a\Gamma_y)] \quad (4.20)$$

$$\Gamma_{-kz} = \frac{cE_{-k}}{aB} \gamma_z n_0 + \frac{cE_z}{aB} n_k - \frac{n_k}{n_0} \Gamma_z \quad (4.21)$$

Together with Eqs. (4.19), (4.20), (4.21), we get from Eq. (4.18) the following equations:

$$n_k \frac{cE_{-k}}{B} = - \frac{a\gamma_x}{a^2 + \gamma_z^2} F'(a) - \frac{a\gamma_y}{a^2 + \gamma_z^2} G'(a) - \frac{a\gamma_z}{a^2 + \gamma_z^2} \left(\frac{n_k}{n_0}\right)^2 \Gamma_z (1+a)^2 \quad (4.22)$$

$$\text{where } F'(a) = a n_0 \left(\frac{cE_x}{B} - a \frac{\Gamma_x}{n_0} - \frac{\Gamma_y}{n_0} \right) \left(\frac{n_k}{n_0}\right)^2$$

$$G'(a) = - n_0 \left(\frac{cE_x}{B} - a \frac{\Gamma_x}{n_0} + a^2 \frac{\Gamma_y}{n_0} \right) \left(\frac{n_k}{n_0}\right)^2$$

We will take summation of Eq. (4.22) with k and will obtain the average value of the electric field in the x direction.

A dimensionless function $I(a)$ is defined as follows:

$$I(a) = \frac{a}{4} \int_0^\pi \frac{\sin^3 \theta}{a^2 + \cos^2 \theta} d\theta$$

By putting $x = \cos \theta$,

$$\begin{aligned} I(a) &= \frac{a}{4} \int_{-1}^1 \frac{1-x^2}{a^2+x^2} dx \\ &= \frac{1}{2} (1+a^2) \tan^{-1} \left(\frac{1}{a} \right) - \frac{a}{2} \end{aligned}$$

is obtained. We also define the following summation:

$$S = \sum \left(\frac{n_k}{n_0}\right)^2$$

We have assumed n_k is a function of k only; thus, the turbulence is isotropic.

Also two functions $F(a)$ and $G(a)$ are defined as follows:

$$F(a) = \Sigma' F'(a)$$

$$G(a) = \Sigma' G'(a)$$

Therefore, we get the following relation:

$$\Sigma' n_k \frac{cE_{-kx}}{B} = - F(a) I(a) \quad (4.23)$$

In the same manner:

$$\Sigma' n_k \frac{cE_{-ky}}{B} = - G(a) I(a) \quad (4.24)$$

We will now take summation with k and get average value of the electric field in the parallel direction. A dimensionless function $J(a)$ and a function $H(a)$ are defined as follows:

$$J(a) = 1 - a \tan^{-1}\left(\frac{1}{a}\right)$$

$$H(a) = (1 + a^2) \left(n_0 \frac{cE_x}{B} - a \Gamma_z \right) S$$

Therefore we get

$$\Sigma' n_k c \frac{E_{-kz}}{B} = H(a) J(a) \quad (4.25)$$

By taking the summations of Eqs. (4.19), (4.20), and (4.21) the followings are obtained:

$$\Sigma' \frac{n_k}{n_0} \Gamma_{-kx} = b F(a) + b \left(\Sigma' \frac{cn_k E_{-ky}}{B} + a \Sigma' \frac{cn_k E_{-kx}}{B} \right) \quad (4.26)$$

$$\Sigma' \frac{n_k}{n_0} \Gamma_{-ky} = b G(a) - b(\Sigma' \frac{cn_k E_{-ky}}{B} - a\Sigma' \frac{cn_k E_{-kx}}{B}) \quad (4.27)$$

$$\Sigma' \frac{n_k}{n_0} \Gamma_{-kz} = \frac{b}{a} H(a) + \frac{1}{a} \Sigma' \frac{cn_k E_{-kz}}{B} \quad (4.28)$$

From Eq. (4.16) we get the following equations

$$\Sigma' \frac{cn_k E_{-kx}}{B} + n_0 \frac{ce_x}{B} + \Gamma_y = a\Gamma_x + a\Sigma' \frac{n_k}{n_0} \Gamma_{-kx} \quad (4.29)$$

$$\Sigma' \frac{cn_k E_{-ky}}{B} - \Gamma_x = a\Gamma_y + a\Sigma' \frac{n_k}{n_0} \Gamma_{-ky} \quad (4.30)$$

$$\Sigma' \frac{cn_k E_{-kz}}{B} + n_0 \frac{ce_z}{B} = a\Gamma_z + a\Sigma' \frac{n_k}{n_0} \Gamma_{-kz} \quad (4.31)$$

Generally these summations are quite complicated but the situation is greatly simplified since we have assumed isotropic turbulence.

The final equations to determine Γ_x and Γ_y are given from Eqs. (4.23), (4.24), (4.26), (4.27), (2.29) and (4.30).

$$\begin{aligned} \Gamma_x &= \frac{1}{1+a^2} I(a) \left(n_0 \frac{ce_x}{B} - a\Gamma_x + a^2\Gamma_y \right) S - a\Gamma_y \\ &+ \frac{a}{1+a^2} \left(n_0 \frac{ce_x}{B} - a\Gamma_x + a^2\Gamma_y \right) S + \frac{a^2}{1+a^2} I(a) \left(n_0 \frac{ce_x}{B} - a\Gamma_x - \Gamma_y \right) S \end{aligned} \quad (4.32)$$

and

$$\begin{aligned} \Gamma_y = & -n_0 \frac{c\epsilon_x}{B} + \frac{a}{1+a^2} I(a) \left(n_0 \frac{c\epsilon_x}{B} - a\Gamma_x - \Gamma_y \right) S + a\Gamma_x \\ & + \frac{a^2}{1+a^2} \left(n_0 \frac{c\epsilon_x}{B} - \Gamma_y - a\Gamma_x \right) S + \frac{a}{1+a^2} I(a) \left(n_0 \frac{c\epsilon_x}{B} - a\Gamma_x + a^2\Gamma_y \right) S \quad (4.33) \end{aligned}$$

and

Also, from Eqs. (4.28) and (4.31)

$$\left(1 - \Sigma' \left(\frac{n_k}{n_0} \right)^2 \right) \left(-a\Gamma_z + n_0 c \frac{\epsilon_z}{B} \right) = 0$$

Since

$$\Sigma' \left(\frac{n_k}{n_0} \right)^2 \neq 1$$

Then, we obtain the following relation:

$$\Gamma_z = \mu n_0 \epsilon_z \quad (4.34)$$

where

$$\mu = \frac{e}{mv}$$

$$\epsilon_x = E_{0x} + \frac{I}{q} S$$

$$\epsilon_z = E_{0z}$$

$$S = \Sigma' \left(\frac{n_k}{n_0} \right)^2 = \frac{\langle n^2 \rangle_{av} - n_0^2}{n_0^2}$$

In the case of electrons in prevailing conditions in weakly ionized plasma, $a_e \cdot 10^{-3}$, so it is quite reasonable to put $a \gg 0$ in Eq. (4.32) and (4.33) as in Yoshikawa and Rose²⁷ (1962). We take the same limit: $a \gg 0$.

Then for electrons we get the following relations:

$$\begin{aligned} \Gamma_{ex} &= - \frac{\pi}{\tau_e} S \frac{cn_e^e ex}{B} + a_e \frac{cn_e^e ex}{B} \\ \Gamma_{ey} &= - \frac{cn_e^e ex}{B} \\ \Gamma_{ez} &= - n_e^e e^e ez \end{aligned} \tag{4.35}$$

In the case of ions, Yoshikawa and Rose assumed that ions are not affected by the magnetic field. This is not true in a strong magnetic field. Thus, we need further considerations.

b) Considerations

Eqs. (4.32), (4.33), and (4.34) may be applied for the positive column. Anomalous diffusion in the positive column is first observed by Hoh and Lehnert³² (1960). The theory of Kadomtsev and Nedospasov²³ (1960) adequately explains the phenomenon in terms of an unstable E cross B helical rotation in discharges with directed current. This was shown by the experiment done by Allen, Paulikas and Pyle³³ (1960).

They assumed that the collision frequency of ions with neutrals is much bigger than the cyclotron frequency, which that means $\omega \ll \nu$ in Eqs. (4.32), (4.33), and (4.34). Then, we obtain the following equations for ions.

$$\Gamma_x = n\mu_i E_x$$

$$\Gamma_y = 0 \tag{4.36}$$

$$\Gamma_z = n\mu_i E_z$$

Even though a constant electric field is introduced in the z direction in our consideration, it does not affect the flux in the x direction.

The ion flux in the radial (x) direction has a saturated value in the anomalous region. This was what was observed by Hoh and Lehnert³² (1960). Our considerations for anomalous diffusion successfully to explain the anomalous diffusion in the positive column.

The reflex discharge has relatively low pressure (10^{-2} torr) compared to the positive column (1 torr). Eqs. (4.32), (4.33) and (4.34) might also be applied to the reflex discharge. We may assume that the collision frequency of ions with neutrals is almost equal to the cyclotron frequency. This means $a_i \approx 1$ in Eqs. (4.32), (4.33), and (4.34).

The followings are obtained by putting $a_i \approx 1$ into (4.32) and (4.33):

$$\Gamma_{iy} \approx \frac{1}{2} \frac{cn_0 \epsilon_x}{B}$$

$$\Gamma_{ix} \approx \frac{1}{2} \frac{cn_0 \epsilon_x}{B}$$

When for $a_i \approx 1$, the following are obtained approximately:

$$\Gamma_{iy} \approx \frac{1}{1+a^2} \frac{cn_0 \epsilon_x}{B}$$

$$\Gamma_{ix} \approx \frac{1}{1+a^2} \frac{cn_0 \epsilon_x}{B}$$

where $\epsilon_x = E_{0x} + (T_i/e)$.

The diffusion is directed to the outside of cylindrical geometry. Therefore, the positive sign of flux is defined towards the outward direction, which is the direction of negative density gradient. The direction of fluxes, $\vec{\Gamma}_e$ and $\vec{\Gamma}_i$, have to be taken accordingly.

The following can be obtained by summing up the results:

$$\begin{aligned} \Gamma_{e\perp} &= -\alpha_e \frac{cn_e E}{B} - \alpha_e \frac{cT_e}{eB} \nabla_{\perp} n_e \\ \Gamma_{e\perp\perp} &= -\frac{cn_e E}{B} - \frac{cT_e}{eB} \nabla_{\perp} n_e \\ \Gamma_{e\parallel} &= -n_e \mu_e E \end{aligned} \quad (4.37)$$

And

$$\begin{aligned} \Gamma_{i\perp} &= \frac{(\Omega_i / \nu_i)}{1 + (\Omega_i / \nu_i)^2} (n_o \mu_i E_o - D_i \nabla_{\perp} n) \\ \Gamma_{i\perp\perp} &= \frac{(\Omega_i / \nu_i)}{1 + (\Omega_i / \nu_i)^2} (n_o \mu_i E_o - D_i \nabla_{\perp} n) \end{aligned} \quad (4.38)$$

$$\Gamma_{i\parallel} = 0$$

where " \parallel " indicates the direction of the magnetic field,

" \perp " indicates the direction of static electric field (which is perpendicular to the direction of magnetic field),

" $\perp\perp$ " indicates the Hall direction.

Then,

$$\alpha_e = \frac{\pi}{4} S_e = \frac{\pi}{4} \frac{\langle (n_e - n_{eo})^2 \rangle_{AV}}{n_{eo}^2} \quad (4.39)$$

S_e is the mean square deviation of the density fluctuation.

We also define

$$D_i = \frac{T_i}{M_i \nu_i}, \quad \Omega_i = \frac{eB}{M_i c} \quad \text{and} \quad \mu_i = \frac{e}{M_i \nu_i}$$

The mechanism which we have discussed is able to explain the anomalous diffusion in the reflex discharge. For the reflex discharge, the radial flux increases with increasing magnetic field in the anomalous region. However, at a strong magnetic field, the radial flux decreases again monotonically as observed by Bonnal et al.³⁴ (1961). This is not observed in the positive column. Since the reflex-type negative ion source is completely in the anomalous region, the mechanism which we have mentioned will be applied.

The reflex discharges are famous for having negative potentials with respect to the surrounding wall in discharge column. As shown in Eqs. (4.37) and (4.38), the inward directed electric field established inside hinders negative ions from diffusing toward the wall. It also stabilizes plasma as mentioned in Section (2). Therefore under a strong radial electric field, the anomalous diffusion might be completely suppressed even though low frequency oscillations are observed. This kind of phenomenon was reported by Chen and Cooper³⁵ (1962); they attributed the phenomenon to the short circuiting by the conductive wall. This problem is open to question.

In the case of the reflex-type negative ion source, the discharge column is recessed from the wall. Even though a strong negative potential and electric field are expected in the central region, relatively uniform potential is expected in the outside region surrounding the central region anyway. No strong electric field exists in the surrounding region. Even though the instabilities are produced in the central region, the surrounding

region is affected by the instabilities. Actually the surrounding region is diffusion dominant. The charged particles coming out from the cylinder are affected only by diffusion as follows:

$$\Gamma_e = -\alpha_e \frac{cT_e}{eB} \frac{\partial n_e}{\partial r} \quad (4.40)$$

$$\Gamma_i = - \frac{(\Omega_i/v_i)}{1+(\Omega_i/v_i)^2} D_i \frac{\partial n}{\partial r} \quad (4.41)$$

As postulated out by Yoshikawa and Rose²⁷ that the spatial variation of electric field is accompanied by a time fluctuation, S is the ratio of ac and dc power value of the electron current. We assume that the electron turbulence is periodic, $S=0.5$ and $\alpha_e = 0.39$.

For the first approximation in cylindrical geometry, we assume the following:

$$\frac{\partial}{\partial r} = -\frac{\beta}{R}$$

where β is the first zero of 0th order Bessel function, $\beta=2.42$, and R is the radius of the cylinder.

For the ion current, we define a function $f(x)$

$$f(x) = \frac{x}{1+x^2} \quad (4.42)$$

$$x = (\Omega_i/v_i)$$

$f(x)$ has a maximum value $f(x)_{\max} = 1/2$ at $x = 1$.

Therefore, Ω_i has a maximum value for $\Omega_i = v_i$.

Then

$$\Gamma_e = \alpha_e \frac{cT_e}{eB} \frac{\beta}{R} n_e \quad (4.44)$$

$$\Gamma_i = f(\Omega_i/v_i) \frac{\beta}{R} D_i n_i \quad (4.45)$$

With the maximum ion flux being achieved when $\Omega_i = v_i$, or

$$\Gamma_{imax} = \frac{1}{2} \frac{\beta}{R} n_i D_i \quad (4.46)$$

Conversely at the maximum flux, Γ_{imax}

$$\Omega_i = v_i \quad (4.47)$$

The cross field (radial) ion flux, having a maximum value with increasing magnetic field, is observed only in the reflex discharge.

Eq. (4.41) represents cross field ion flux in the anomalous region. It has a maximum point at $\Omega_i = v_i$. Considering its relatively low pressure compared to the positive column, an assumption $\Omega_i \approx v_i$ ($a_i \approx 1$) for the reflex discharge seems very reasonable. Eqs. (4.32), (4.33), and (4.34) are also showed to be applied for the reflex discharge.

This theoretical consideration presented by the author applies not only to the positive columns but also the reflex discharges. Both anomalous diffusions caused by the screw instability and by the neutral drag instability are explained at the same time.

In our calculations, the anomalous terms would not come out if we have neglected the fluctuating field in z direction. Therefore both instabilities are somehow related to the drift-like wave mentioned by the author in the previous chapter. In weakly ionized plasma, we may call them "the resistive drag instability" which is excited by "drag wave".

c) Anomalous electron transport

In the previous chapter, it was shown that the cross field (radial) electron flux is inversely proportional to the magnetic field. In our experiment, it was observed in relatively high pressure, but not in relatively low pressure. At relatively low pressure, the cross field electron flux has a maximum point with increasing magnetic field. However, we can not assume that $\Omega_e = \nu_e$, as in the case of ion flux, since the electron temperature is in the order of keV. It is not realistic.

It is found that the Debye length in relatively low pressure is close to the electron Larmor radius. In this case, an electron is expected to spend relatively long time around one particular ion. Then, the electron will oscillate with plasma frequency. When it oscillates in the plane perpendicular to the magnetic field, the high frequency oscillation has same effect as the collisions with neutrals. The increased effective collision frequency encourages diffusion perpendicular to the magnetic field.

When the plasma is in a strong magnetic field, pure plasma oscillations with electric amplitude in the cross field direction cannot exist. The plasma frequency ω_p is shifted to either righthand cutoffs frequency ω_R or lefthand cutoffs frequency ω_L . The righthand cutoffs frequency ω_R is bigger than the plasma frequency ω_p ; and the left hand cutoffs frequency ω_L is smaller than the plasma frequency ω_p .

In the previous chapter we got the cross field electron flux as follows, assuming $\Omega_e \gg \nu_e$.

$$\Gamma_{er} = - \alpha_e \frac{cT_e}{eB} \frac{\partial n_e}{\partial r} \quad (4.48)$$

If we replace v_e with ω_L or ω_R , the condition $v_e \gg \omega$ is not satisfied any more. Actually, both ω_L and ω_R are very close to the electron cyclotron frequency Ω_e . Then, $\Omega_e \approx \omega_L$ or $\Omega_e \approx \omega_R$.

We can apply the discussion of ion fluxes to electron fluxes.

We get the cross field electron flux as follows:

$$\Gamma_{er} = - \frac{(\Omega_e/\omega)}{1+(\Omega_e/\omega)^2} D_e \frac{\partial n}{\partial r} \quad (4.49)$$

where ω represents either ω_L or ω_R .

In the 1st approximation:

$$\Gamma_{er} = \frac{(\Omega_e/\omega)}{1+(\Omega_e/\omega)^2} D_e \frac{\beta}{R} n \quad (4.50)$$

Γ_{er} has a maximum value for $\Omega_e = \omega_L$ or $\Omega_e = \omega_R$. For the magnetic field which gives the maximum electron flux, we get the following relations:

$$\Gamma_{er} = \frac{1}{2} \frac{\beta}{R} n_e D_e \quad (4.51)$$

$$\Omega_e = \omega_L \text{ or } \omega_R \quad (4.52)$$

Since

$$\omega_R = \frac{\Omega_e + \sqrt{\Omega_e^2 + 4\omega_p^2}}{2}$$

from $\Omega_e = \omega_R$, we get the following answer:

$$\omega_p^2 = 0$$

This is a trivial answer.

On the other hand

$$\omega_L = \frac{\Omega_e - \sqrt{\Omega_e^2 + 4\omega_p^2}}{2}$$

from $\Omega_e = \omega_L$, we get the following answer:

$$\omega_p^2 = 2\Omega_e^2 \quad (4.53)$$

When the Larmor radius is close to the Debye length, it has been shown that the plasma oscillations perpendicular to the magnetic field encourage the diffusion in that direction.

In our experiment, Eq. (4.49) is applied in low pressure, and Eq. (4.48) is applied in high pressure. In intermediate pressure, we observed both characters.

5. Reflex type negative ion source

(1) Experimental arrangement

This investigation employed the reflex-type negative ion source in D.C. operation and the ion extracted radially or normally to the magnetic field under proper vacuum. The arc chamber of the ion source consists of a heated filament at the top of the cylinder shape anode, which has an ion exit slit and is connected with gas feed line, and an electrically insulated cold reflector cathode at the bottom of the cylindrically-shaped anode. The reflector cathode is kept in the same potential as the heated filament. The ion source was operated in a mass spectrometer arrangement with the extracting electrode at ground potential and the source structure biased negatively in order to extract negative ions, or biased positively in order to extract positive ions (Fig. 5-1).

Two Faraday cups were installed to monitor the extracted ion currents. One cup could be moved as a mass spectrometer, the other cup which is located closer to the extracting electrode is big enough to gather all particles coming through the extracting electrode (Fig. 5-2A). The electron drain current coming out through the exit slit is measured as the earth current.

The arc was generally operated with an arc current of 3A, an arc voltage of 300 V, and an extracting potential of 6 kV. The diffusion pump of the test system has a pumping speed for hydrogen of about 5000 l/sec, and the ions are extracted from the ion source into the vacuum of 10^{-5} to 10^{-4} torr. The ion exit slit size, for the ion source, is 0.025×0.150 ($7.26 \times 10^{-2} \text{ cm}^2$).

The radius of the arc defining a hole is 0.24cm, the radius of the cylindrical column is 0.48cm, and the length of the cylindrical column is 6.03cm. The whole cylindrical column acts as an anode. Both the hot and cold reflector cathodes are made of tantalum.

The hydrogen gas feed is variable to 50 cc/min, magnetic field from 1500 to 5000 gauss, and the heat current to 500A. Special attention was given to maintain an abundance of molecular gas in the immediate region of the ion exit slit. The gas is fed into the arc chamber directly in the region of the ion exit slit. In addition, the arc plasma column has been defined so as to be recessed from the ion exit slit. These arrangements allow the incoming molecular gas to surround the plasma column completely in the region where ions can be immediately extracted.

For the reflex-type negative ion source, the temperature of plasma electrons is expected to be on the order of 1eV, and the threshold energy for the ionization is 13.6 eV (15.4eV for molecules). As we discussed in Chap. 2, the injected hot primary electrons mostly ionize the hydrogen gas. The ionization is mostly done in the central region where the primary electrons are confined. Considering the relatively high gas pressure inside the ion source (10^{-1} to 10^{-2} torr, 10^{15} /cm³), the plasma is weakly ionized.

Surrounding the central region, there is a relatively cold plasma region where there are not so many primary electrons since the arc

defining holes of the top and bottom side of the cylindrically-shaped anode are recessed from the wall of the cylindrically-shape anode.

To control the radial electric field inside the plasma, the reflex-type ion source was modified. The cylindrically-shaped anode is separated into two parts so that we can change the potential between the wall anode and top part of the anode (Fig. 5-2B). Both parts are electrically insulated by two boron nitride insulators.

The ion exit slit size of the modified ion source is 35% more than old one ($9.7 \times 10^{-2} \text{ cm}^2$). Therefore the pressure inside ion source is less than that of the old one for the same gas feed. Except that point, all the other parameters of the modified ion source are as the same as those of the old one. Scaled diagrams of both old and new ion sources are shown on Fig. 5-3.

The potential between the two anodes is variable from - 75V to 75V without plasma (maximum 5A). For zero potential we can resume the old geometry. When the ion source is operated, the plasma acts as a constant current power supply. We need relatively large power to keep arbitrary potential between the two parts.

In Section (2), we analyze the experimental results with the original source using Eqs. (4.46), (4.47), (4.48), (4.51), and (4.53); these results are obtained by changing the magnetic field. In Section (3), we try to increase the negative ion current with the modified source using Eq. (4.11); we try to encourage the instabilities and anomalous diffusion by changing the radial electric field.

(2) Experimental result and Considerations

When the hydrogen gas feed increases, the extracted positive ion current and electron drain current decreases. On the other hand the extracted negative ion current first increases, and then, after reaching a maximum value, begins to decrease. The relations between the extracted charged particle current and the gas flow rate are shown on Fig. 5-4, 5-5, and 5-6.

Hydrogen molecules attain the maximum ionization cross section at the electron energy of about 60 eV, and the maximum cross section to vibrationally excited states stays at an electron energy of about 40 eV; this is calculated by Hiskes¹⁸ (1980). Maximum negative ion production rate is obtained at that electron energy if the number of cold electrons contributing dissociative attachment are not changed.

As the gas flow rate increases, the average energy of the primary electron decreases. After reaching electron energy $E_e = 60$ eV the ionization begins to decrease; the production of both positive ions and electrons begin to decrease.

On the other hand, the production of negative ions keeps increasing, and then, after reaching the maximum production rate at $E_e = 40$ eV, begins to decrease. The extracted negative ion current keeps increasing for a while as the gas flow rate increases even though the extracted positive ion current and the drain electron current decrease. At the gas flow rate (pressure) the maximum extracted negative ion current is obtained, the average energy of primary electron is about 40 eV.

Both positive and negative ions are extracted from the ion source and mass-analyzed by the traveling Faraday cup. The dominant species of the positive ion current is H^+ (90%), and the dominant species of the negative ion current is H^- (100%). We have not observed any impurities (less than 1%).

It is clear that when the magnetic field increases, the extracted positive and negative ion current first increase, and then, after reaching the maximum value, begin to decrease. The maximum extracted negative ion current of 3.6 mA (49.6 mA/cm^2) and the maximum positive current of 12.0 mA (165.3 mA/cm^2) are obtained continuously. On each curve, the maximum value of the extracted current is reached at $B = B_M$. As the gas flow rate increases (the pressure increases) the critical magnetic fields B_M are shifted toward higher field values. On the other hand, the electron drain currents behave strangely when the magnetic field increases. No consistent description can be given to them. Relations between the extracted charged particle current and the magnetic field are shown on Fig. 5-7, 5-8, and 5-9.

The investigations of ion diffusion in the reflex discharges have showed similar relations between the ion escape flux and the magnetic fields; this is observed Bonnal et al.³⁴ (1961), Pavlichenko et al.³⁶ (1964), and Thomassen¹⁶ (1966). This kind of anomalous relation between the escape flux and the magnetic fields was observed only in the reflex discharge. The order of the critical magnetic fields B_M are same in all experiments (about 500 gauss). The reflex-type ion source is operated in the magnetic

field stronger than 1500 gauss. Therefore, the extracted charged particle current across the magnetic field are controlled by diffusion mechanism, which is called the anomalous or enhanced diffusion mechanism. Then, the anomalous diffusion mechanism presented by the author in Chap. 4 may be applied.

In the negative ion production experiment the size of the ion source is very small. The source is in a strong magnetic field and it is floated in a high electric potential for the extraction of the charged particles. No direct measurement of any kind of quantities inside the ion source is possible. Diagnostics are limited to external measurements of voltage, currents and electrical-noise signals. All important quantities inside the ion source had to be estimated. Using the consideration presented by the author in Chap. 4, we are going to estimate those important quantities from the relation between the extracted charged particle currents and the magnetic fields.

The relations between the extracted ion current and the magnetic fields are expressed by Eq. (4.45). The maximum extracted currents of positive and negative ions, and the critical magnetic fields B_M of each species are given from Fig. 5-8 and Fig. 5-9.

Because, at the critical magnetic field B_M , the cyclotron frequency, Ω_i , is equal to the collision frequency ν_i , the ion temperature may be obtained by equating the following relations for the two frequencies:

$$\Omega_i \text{ [1/sec]} = 9.58 \times 10^6 \frac{B_M \text{ [1000gauss]}}{A} \quad (5.1)$$

where A is an atomic number.

$$v_i \text{ [1/sec]} = 1.38 \times 10^6 P \text{ [torr]} P_c \sqrt{\frac{T_i \text{ [eV]}}{A}} \quad (5.2)$$

where P_c is the probability of collision .

From Eq. (4.47) at the maximum point,

$$\Omega_i = v_i$$

The ion temperature is given as follows:

$$T_i = 4.82 \times 10 \frac{1}{A} \left[\frac{B_M \text{ [1000gauss]}^2}{P \text{ [torr]} P_c} \right]^2 \quad (5.3)$$

The elastic collision frequencies between the charged particles and the hydrogen gas are given by Brown³⁷ (1966). In his book, the probability of collision P_c is given as a function of temperature T_i . For each gas pressure P , different critical magnetic field B_M are given. Therefore, for each gas pressure, we get another relation between the probability of collision P_c and temperature T_i . The ion temperature T_i is given from the cross section of the two relations. Actually there is not precise data in the low ion-temperature region. We referred to the elastic cross section data by Simons et al.³⁸ (1943) and Muschlitz et al.³⁹ (1956). Their data is extrapolated into low temperature region.

Sharp increases of the probability of collision P_c are observed for both positive and negative ions below 4 eV. Both the positive ion temperature $T_i=1.5$ eV, and the negative ion temperature $T_i=1.5$ eV are given for all pressures in our experiment. Then ion densities are obtained in the following way.

From Eq. (4.46), we get the following relation:

$$\Gamma_{imax} = \frac{1}{Z} \frac{\beta}{R} n_i D_i \quad (5.4)$$

where $\beta = 2.42$ and $R = 0.24$ cm.

Since $\Omega_i = v_i$,

$$D_i \left[\frac{m^2}{sec} \right] = \frac{T_i}{AM\Omega_i} = 10 \frac{T_i [eV]}{B [1000gauss]} \quad (5.5)$$

where M is the ion mass.

Then, in the CGS unit system, the ion density n_i is obtained from Eq. (5.4) and (5.5):

$$n_i \left[/cm^3 \right] = 1.24 \times 10^9 \frac{J_{imax} [A/m^2] B [1000gauss]}{T_i [eV]} \quad (5.6)$$

where $J_{imax} = e \Gamma_{imax}$

On the other hand, the relations between the electron drain current and the magnetic fields are expressed by Eq. (4.44) or Eq. (4.50).

Under large gas flow rate (high pressure), the relation between the electron drain current and the magnetic fields obeys Eq. (4.44). Under small gas flow rate (low pressure), the relation between the electron drain current and the magnetic fields obeys Eq. (4.50). The maximum electron drain current and the critical magnetic field B_M are given from Fig. 5-7.

Because, at the critical magnetic field B_M , the cyclotron frequency, Ω_i , is related to the plasma frequency ω_p , the

electron density may be obtained by relating the following relations for the two frequencies:

$$\Omega_e [1/\text{sec}] = 1.76 \times 10^{10} B [1000\text{gauss}] \quad (5.7)$$

$$\omega_p^2 [1/\text{sec}] = 5.63 \times 10^4 n_e [/\text{cm}^3] \quad (5.8)$$

From Eq. (4.53) at the maximum point,

$$\omega_p^2 = 2 \Omega_e^2 \quad (5.9)$$

The electron density is given as follows:

$$n_e [/\text{cm}^3] = 1.95 \times 10^{11} [B_{\text{max}} [1000\text{gauss}]]^2$$

The electron temperature is obtained in the following way. From Eq. (4.52),

$$\Gamma_{\text{emax}} = \frac{1}{2} \frac{\beta}{R} n_e D_e \quad (5.10)$$

where $\beta = 2.42$ and $R = 0.24$ cm.

Since $\Omega_e = \omega_L$ or $\Omega_e^2 = (1/2)\omega_p^2$

$$D_e \left[\frac{\text{m}^2}{\text{sec}} \right] = \frac{T_e}{m \Omega_e} = 9.99 \frac{T_e [\text{eV}]}{B [1000\text{gauss}]} \quad (5.11)$$

where m is the electron mass.

Then, the electron temperature T_e is obtained from Eq. (5.10) and (5.11):

$$T_e [\text{eV}] = 1.24 \times 10^9 \frac{J_{\text{emax}} [\text{A/m}^2] B [1000\text{gauss}]}{n_e [/\text{cm}^3]} \quad (5.12)$$

where $J_{\text{emax}} = e \Gamma_{\text{emax}}$

For large gas flow rate, the electron drain current is evaluated at the weak magnetic field of $B=1500$ gauss from Fig. 5-7. For

stronger magnetic field, the electron current might to be still encouraged by the plasma oscillations. The electron density is calculated from the positive and negative ion density assuming charge neutrality. The electron temperature T_e is calculated from Eq. (4.44).

$$r_e = a_e \frac{c T_e}{e B} \frac{\beta}{R} n_e \quad (5.13)$$

where $a_e = 0.39$, $\beta = 2.42$, and $R = 0.24 \text{ cm}$

The electron temperature T_e is obtained by solving Eqs. (5.13):

$$T_e [\text{eV}] = 1.59 \times 10^9 \frac{j_{\text{emax}} [\text{A/m}^2] B [1000 \text{ gauss}]}{n_e [\text{cm}^{-3}]} \quad (5.14)$$

The calculated temperature and densities for each specie according to gas flow rate are shown in Fig. 5-10 and Fig. 5-11. Important physical values used in calculation are listed in Table 1, 2, and 3.

(3) Discussions and experiments with modified geometry

A comparison of important physical quantities with other reflex discharge experiments are shown on Table 4. Considering small size and relatively large arc current in the reflex-type negative ion source, our calculated values look reasonable. Actually there is an ambiguity in the evaluation of the collision frequency with neutral hydrogen gas. There is no precise data in the low temperature region. However, the calculated results enable us to infer something about the conditions inside the ion source.

In negative ion production experiment by Bacal and Hamilton, the electron temperature is low (0.1 to 1 eV). There is no special plasma heating mechanism to produce vibrationally excited hydrogen molecules. The validity of the calculated result by Wadehra⁹ (1979) can not be tested. Wadehra expected the largest negative ion production rate at an electron temperature of about 1 eV. If his calculated production rates are correct, the negative ion production has to increase as the plasma is cooled from high electron temperature to low electron temperature (about 1 eV). However, the production of vibrationally excited molecules should not decrease in order to keep a high dissociative attachment rate.

In the reflex-type negative ion source, high energy primary electrons are confined in the central region by the strong magnetic field. The average electron energy is about $E_e = 40$ eV. In the central region, the primary electrons ionize neutral gas and produce vibrationally excited molecules. Their production is constant since the arc current and the voltage are kept constant. In the

surrounding region, the electron temperature is lower compared to the central region. The estimated electron temperature, $T_e = 3\text{eV}$, and ion temperature, $T_i = 1.5\text{ eV}$, are obtained. The estimated plasma density is $n = 5.0 \times 10^{12} / \text{cm}^3$. Most electrons in the surrounding region are produced by ionization. Thus, they greatly contribute to the dissociative attachment of vibrationally excited molecules. Therefore, most negative ions are produced in the surrounding region. Our results are in agreement with Wadehra's result. The highest negative ion density is observed around the electron temperature of 1 eV. The maximum negative ion density is estimated to be $n = 8 \times 10^{11} / \text{cm}^3$.

The negative ions produced inside the ion source move out by anomalous diffusion mechanism. We assumed in Chap. 4 that the anomalous diffusion mechanism presented by the author is valid only in the diffusion dominant region, which is the surrounding region in the reflex-type negative ion source. Therefore all calculated physical quantities are valid only in the surrounding region.

In the reflex-type negative ion source, we assumed in Chap. 2 that there are two electron temperature components. Electrons that have higher temperature or higher average energy produce electrons that have lower temperature and vibrationally excited molecules. Electrons that have lower temperature make dissociative attachment to the vibrationally excited molecules and produce negative ions. By this mechanism, the reflex-type negative ion source can produce a high density of negative ions.

If the diffusion across the magnetic field is controlled by anomalous diffusion, we may obtain more cross field flux with encouraging instabilities. From Eq. (4.11), it is found out that the radial electric field contributes to destabilize the plasma. Then, anomalous diffusion may be encouraged. In hot cathode reflex discharges, the variation of potential is more affected by the variation along the axial direction than by the variation along the radial direction. The top part of the anode which faces the central region of the plasma determines the potential of that region, and the wall which faces the surrounding region of plasma determines the potential there (Fig. 5-3).

In order to change the radial electric field between the two parts, the top part of the anode and the wall anode are electrically separated in the modified reflex-type negative ion source (Fig. 5-2B). Using another power supply, the potential of the wall anode was biased either positive or negative in respect to the top part of the anode, which acts as a main anode.

Relations between the extracted charged particle current and the magnetic field with changing biased potential are shown on Fig. 5-12, 5-13, 5-14A, and 5-14B. From the relation of the negative ion current and the magnetic field (Fig. 5-13), the encouragement of anomalous diffusion with increasing negative bias potential is clear. The maximum negative ion current of 9.7 mA (100 mA/cm^2) was obtained, which is as large as the maximum positive ion current under same operation condition.

The relations between the biased potential and both the extracted negative ion current and its density fluctuation amplitude are shown on Fig. 5-15. The measured fluctuation was on the order of 100 kHz.

For a given magnetic field of 2500 gauss, we get twice as much negative ion current with negative biased potential of 6 volt compared to unbiased potential. On the other hand, for positively biased potential of 6 volt, we get one-fourth as much negative ion current compared to unbiased potential.

From the relation of the drain electron current (Fig. 5-12), the encouragement of anomalous diffusion with increase of negatively biased potential is clear. We have observed considerable increase of the drain electron current as the negative ion current increases or as the instabilities are encouraged.

From the relation of the positive ion current with positively biased potential (6 to 0 V), we have observed similar encouragement of anomalous diffusion as in the case of the negative ion current (Fig. 5-14A). However, similar encouragement cannot be observed from the relation of the positive ion current with negatively biased potential (0 to -6 V). Rather, discouragement is observed (Fig. 5-14B).

In the latter case with negatively biased potential (Fig. 5-14B), the positive ions produced in the central region of the plasma have to pass through the surrounding region of high negative ion density. Much of the positive ions are lost by recombination of positive and negative ions. We did not observe a large positive ion current which is expected from the anomalous diffusion of positive

ions. In the former case with positively biased potential (Fig. 5-14A), the density of negative ions in the surrounding region is not so high, thus, only few positive ions are lost through recombination process.

In Fig. 5-15 we observe increases in both the density fluctuation amplitude and negative ion current when the wall anode is more negatively biased; the instability encourages the anomalous diffusion. We believe these experimental results support our assumption that the cross field diffusion is controlled by the anomalous diffusion mechanism.

We have observed similar relations between the negative ion current and the magnetic for deuterium gas D_2 (Fig. 5-17). Maximum deuterium negative ion current of 4.1 mA (42mA/cm^2) was obtained. The relation of other extracted charged particle currents and the magnetic field with changing biased potential are shown in Fig. 5-16, 5-16A, and 5-18A for deuterium case.

The maximum deuterium negative ion current is less than half of the maximum hydrogen negative ion current. The maximum positive ion current and the drain electron current are a little more than that of deuterium. The primary electrons in the central region lose less energy by collisions in the case of deuterium gas than in the case of hydrogen gas; the average electron temperature in the central region may be a little hotter than the case of hydrogen.

The dissociative attachment rate to vibrationally excited molecules calculated by Wadehra⁹ (1979) showed distinct isotope dependence (Fig. 2.1 and 2.2). The attachment rate to the

vibrationally excited deuterium molecules is one order smaller than that of hydrogen molecules. We believe that our experimental results are consistent with Wadehra's calculation.

6. Conclusion

A hot cathode reflex-type negative ion source was investigated. Initially the main experimental results were as follows: 1) The extracted negative ion current is approximately as large as the positive ion current. 2) When the amount of hydrogen gas feed is increased, the extracted negative ion current increases initially. Then, after reaching the maximum value, the current begins to decrease. 3) When the magnetic field is increased, the extracted positive and negative ion currents increase initially. Then, after reaching the maximum value, the currents begin to decrease.

The last observation led me to look for an anomalous diffusion mechanism since that kind of phenomenon is considered as anomalous in classical diffusion theory. The extracted charged particle currents in the reflex-type negative ion source are assumed to be controlled by the diffusion mechanism or enhanced diffusion mechanism. Hoh's²⁴ (1963), and Yoshikawa and Rose's²⁷ (1962) works were reviewed and extended. In Chap. 4, the author derived the super-critical condition and the diffusion coefficients in the anomalous regime of the reflex discharge. The anomalous diffusion is attributed to the resistive drag instability which is excited by drift-like or drag waves.

The comparison with the result of the anomalous diffusion in the positive column is made; Hoh and Lehnert³² (1960). It is found that the anomalous diffusion mechanism in a weakly ionized plasma, proposed by the author in Chap. 4, is valid both for the positive

column (Eq. (4.10) and (4.36)) and for the reflex discharge (Eq. (4.11) and 4.38)).

Production of negative ions through electron-volume processes is discussed in Chap. 2. Wadehra⁹ (1979) recently showed that a significant enhancement of the dissociative attachment rate is obtained by the vibrational excitation of the initial molecules. The rate of dissociative attachment of electrons to vibrationally excited molecules has a maximum value at about $T_e = 1.5$ eV, where the highest negative ion density in hydrogen gas is expected.

In the reflex-type negative ion source, the plasma is divided into two regions; the central (hot) region, and the surrounding (cold) region. The plasma is heated in the central region, where many vibrationally excited molecules are produced. Many negative ions are produced by the dissociative attachment to the vibrationally excited molecules in the relative cold surrounding region.

Negative ions can not survive very long inside of a dense plasma. Instabilities which increase the radial diffusion, as described earlier, can increase the external H^- current. However, the increased radial diffusion does not have to be followed by decreased density. In the late 1950's anomalous diffusion in the positive column was found by Lehnert³⁰ (1958). He believed that the increased particle loss by anomalous diffusion is reflected in the increased rate of ion electron pair production. The large particle loss produced by anomalous diffusion must be compensated by the large particle production inside the plasma. The plasma has an

inherent property to maintain itself: "the self-sustaining property of plasma".

To investigate the correlation between the instabilities and the negative ion current, the reflex-type negative ion source was modified in order to change the radial electric field. As shown in Eq. (4.11), the drag instability should grow faster if the inwardly-directed radial electric field is reduced. In order to change the radial electric field, the wall anode was biased more negative in respect to the top part of the anode. Twice as much negative ion current was observed with negatively biased potential as with the unbiased potential. Simultaneously, an increase of negative ion density fluctuation amplitude, which is in the order of 100 kHz, was observed. Consequently, we can say that our model is selfconsistent.

Using the experimentally obtained critical magnetic fields for maximum H^- currents, the anomalous diffusion model was used to calculate plasma properties. The values for the electron temperature of each gas feed, and ion temperatures of $T_i = 1.5$ eV in the surrounding region are obtained. The relation between the negative ion density and electron temperature is also obtained. The observed maximum negative ion current is obtained at a relatively low calculated electron temperature of $T_e = 3$ eV. The calculated results of the reflex-type negative ion source are consistent with the production mechanism of negative ions through the electron volume production process proposed by Wadehra. However, it was not

possible to make experimental measurements of densities and temperatures for comparison.

It is interesting to note that in the reflex discharge, an electrostatic well is constituted in the middle part of the cylinder along the magnetic field. In Chap. 3, it was shown with a simple model that the electrostatic well is strong enough to trap negative ions, which have a temperature of the order of 1eV. Therefore, negative ions are concentrated in the middle part of the cylinder. In the reflex-type discharge, we can take advantage of what is called "the (electrostatic) sheath trap of negative ions" in order to get high negative ion density around the slit region.

Maximum D. C. negative ion current of 9.7 mA for H^- and of 4.1 mA for D^- were obtained, equivalent to a negative ion current density of 100 mA/cm^2 for H^- and of 42 mA/cm^2 for D^- . This is the largest negative hydrogen and deuterium current density ever obtained continuously from a volume-production source, within the author's knowledge.

APPENDIX 1
DIFFUSION OF PLASMA

The cross field diffusion of charged particles is very important for the confinement of plasmas. The diffusions in weakly ionized plasmas will be discussed in Section (1). The diffusions in fully ionized plasmas will be discussed briefly in Section (2). The summary of the diffusion coefficients will be presented at the end of Section (2). The anomalous diffusion in plasma will be discussed in Section (3). Most of arguments are concentrated to anomalous diffusion in the reflex discharges and the positive columns. The anomalous diffusion in fully ionized plasmas is discussed only the case which is related to Bohm diffusion.

(1) Classical theory of diffusion in weakly ionized plasma

In weakly ionized plasmas, as a result of electron neutral collision, the electrons do not accelerate indefinitely; but they rather reach a definite speed determined by the frequency at which collisions with neutrals occur. In the definition of classical diffusion theory in weakly ionized plasmas, only binary encounters produce particle transport; that means the collisional diffusion. We can use the kinetic equation with the Boltzmann binary collision term to evaluate the collision frequency.

In the absence of magnetic field the behavior of the plasma column was explained by the theory of Tonks and Langmuir⁴⁰ (1929). In this case, electrons would tend to diffuse out of a plasma much more readily than ions. This unequal diffusion rate immediately produces large space charges and hence an electric field in the plasma so as to reduce the electron current, but to increase the ion current to maintain space charge neutralization. The ions and electrons can no longer be expected to diffuse in a manner independent of each other. This is what is called ambipolar diffusion and the ambipolar diffusion coefficient was first obtained by Schottky⁴¹ (1924).

The motion of the charged particle in a magnetic field may be described as the sum of a gyration around the field line, a drift motion across the same lines and a motion parallel to the field. In an ionized gas or plasma the picture has to be modified by particle

interactions, such as collisions and phenomena caused by space charge. Only encounters between nonidentical particles produce a diffusion in the first order across the magnetic field. The motions are relatively slow since the plasma nearly is frozen to the magnetic field lines.

The diffusion coefficients of ions and electrons in a magnetic field are given first by Townsend⁴² (1912). Schottky's theory⁴³ (1924) of positive column of glow discharge has been extended by Tonks⁴⁴ (1939) to include the influence of an axial magnetic field. He assumed that the motion of the electrons is affected by the magnetic field, but not that of the ions. He concluded that a magnetic field may flatten the radial potential distribution or even reverse its sign and may reduce the losses of particles to the walls. Consequently, when the magnetic field is present a lower electron temperature and a smaller potential drop along the plasma column should be required to sustain a certain ion density. In the relatively weak magnetic field he used, the diffusion in the positive column was found to behave in good agreement with the collisional diffusion theory.

Diffusion of ions in a plasma across a magnetic field is shown to be not ambipolar in character in the most low pressure discharge experiment (arc discharge) by Simon⁴⁵ (1955). The ions diffuse across the field at their own intrinsic rate. Space charge neutralization is maintained by slight adjustments of the currents in the direction of the magnetic field lines. Since the ions

diffuse more rapidly across the magnetic field, an electric field arises which would reduce the ion current and which would result in a new ambipolar diffusion coefficient. A positive excess to the side wall will be balanced by a negative excess to the bottom and top wall.

In the case of the mean free paths being large compared to the dimensional length, particles stream rather than diffuse to the end electrodes. As the longitudinal electric field may be large, we may expect anisotropies in the velocity distribution of ions and electrons.

a) Diffusion in weakly ionized plasma.

In this Section, we consider fluid equations for species "s" derived from the Boltzmann binary collisions theory.

The continuity equation is given as follows:

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \vec{u}_s) = Z n_s$$

The equation of motion is given as follows:

$$m_s n_s \frac{d\vec{u}_s}{dt} + \nabla P_s = n_s q_s (\vec{E} + \frac{\vec{u}_s \times \vec{B}}{c}) - n_s m_s \nu_s \vec{u}_s$$

The state equation is given as follows:

$$P_s = n_s T_s$$

where \vec{u}_s : fluid velocity vector of species s

m_s : mass of species s

n_s : density of species s

T_s : temperature of species s

q_s : charge of species s

ν_s : collision frequency of species s with neutrals

c: light constant

We call the direction parallel to the uniform magnetic field, the parallel direction represented by a symbol "||" and the direction perpendicular to the uniform magnetic field, the perpendicular direction represented by a symbol "⊥".

We will consider only a steady state assuming that T_s is independent of position.

Putting $\vec{B} = B \hat{z}$, $\Omega_s = (q_s B / m_s c)$, $a_s = (v_s / \Omega_s)$, and $b_s = 1 / (1 + a_s^2)$, we define the diffusion coefficient D_s and mobility ν_s in both directions as follows:

$$D_s = \frac{T_s}{m_s \nu_s} \quad \nu_s = \frac{q_s}{m_s v_s}$$

$$D_{s\perp} = a_s^2 b_s D_s \quad \nu_{s\perp} = a_s^2 b_s \nu_s$$

Also we define the E cross B drift velocity u_E and the diamagnetic drift velocity u_D as follows:

$$\vec{u}_E = \frac{c \vec{E} \times \vec{B}}{B^2}$$

$$\vec{u}_{D_s} = - \frac{T_s}{n_s q_s} \frac{c}{B} (\nabla n_s \times \hat{z})$$

Then from the equation of motion, the particle flux $\vec{\Gamma}_s$ is given as follows:

$$\Gamma_{s\parallel} = n_s u_s = - D_s \nabla_{\parallel} n_s + n_s \nu_s E$$

$$\vec{\Gamma}_{s\perp} = n_s \vec{u}_{s\perp} = - D_s \nabla_{\perp} n_s + n_s \nu_{s\perp} \vec{E} + b_s n_s (\vec{u}_E + \vec{u}_{D_s})$$

Consider the spatial part of the fluid equation in cylindrical geometry (r, θ, z) (Fig. A1-1, or A2-2). The electric field is directed either outward or inward, and the density gradient is directed inward. Then assuming the azimuthal symmetry,

$$\Gamma_{s_r} = - D_{s\perp} \frac{\partial n_s}{\partial r} + n_s \nu_{s\perp} E_r + b_s n_s \frac{c E_{\theta}}{B}$$

$$\Gamma_{s_\theta} = b_s n_s \left(-\frac{cE_r}{B} + \frac{T_s}{n_s q_s} \frac{c}{B} \frac{\partial n_s}{\partial r} \right) + n_s \mu_{s\perp} E_\theta$$

$$\Gamma_{s_z} = -D_s \frac{\partial n_s}{\partial z} + n_s \mu_{s_z} E_z$$

From the equation of continuity, we get the following results:

$$\nabla \Gamma_s = Zn_s$$

or

$$\frac{\partial \Gamma_{sr}}{\partial r} + \frac{\partial \Gamma_{sz}}{\partial z} = Zn_s$$

b) Ambipolar diffusion without a magnetic field

When the plasma is much larger than a Debye length it must be quasineutral, and one would expect that the rate of diffusion of ions and electrons would somehow adjust themselves so that the two species would leave at the same rate. From the assumption that the quasineutrality $n_i = n_e$ is satisfied, and that both ion and electron fluxes are equal $\Gamma_e = \Gamma_i$, we get the ambipolar electric field as follows:

$$E = - \frac{D_e + D_i}{\mu_e + \mu_i} \frac{\nabla n}{n}$$

where $n = n_i = n_e$.

We define the ambipolar diffusion coefficient D_a , and the flux Γ is given as follows:

$$D_a = \frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e}$$

$$\Gamma = - D_a \nabla n$$

From the equation of continuity, we get the following equation:

$$\frac{\partial n}{\partial t} - D_a \nabla^2 n = Zn$$

The effect of the ambipolar field, which is produced by the fast diffusion of electrons against ion, enhances the slow diffusion of ions by a factor of two, however, the diffusion rate of the two species together is primarily controlled by the slower ions.

c) Ambipolar diffusion with a magnetic field

When there is a magnetic field in the axial direction in cylindrical geometry, the ions diffuse out primarily radially, while the electrons diffuse out primarily along the magnetic field. Therefore the condition for ambipolar diffusion in a magnetic field should be as follows:

$$\frac{\partial \Gamma_{ir}}{\partial r} + \frac{\partial \Gamma_{iz}}{\partial z} = \frac{\partial \Gamma_{er}}{\partial r} + \frac{\partial \Gamma_{ez}}{\partial z}$$

In case of an infinitely long cylindrical column, we can neglect the axial variation. Then $\Gamma_{iz} = \Gamma_{ez}$, and in the same way as the case without a magnetic field, we can get the cross field flux Γ_r .

$$\Gamma_r = \frac{a_e a_i}{1 + a_e a_i} \left(- \frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e} \frac{\partial n}{\partial r} + n \frac{c E_\theta}{B} \right)$$

We define the ambipolar diffusion coefficient, D_a' , in an infinitel long plasma, as follows:

$$D_a' = \frac{a_e a_i}{1 + a_e a_i} D_a$$

Usually $E_\theta = 0$, then,

$$\Gamma_r = - D_a' \frac{\partial n}{\partial r}$$

In the case of a finitely long cylindrical column, the diffusion across the magnetic field is not ambipolar any more. An electric field which is built up to counteract the charge separation has a greater effect on the currents in the direction of the magnetic field because of the greater conductivity in this direction. As a result ions diffuse faster than electrons across the magnetic field

and electrons diffuse faster than ions along the magnetic field. A positive excess to the side wall will be balanced by a negative excess to the bottom and top walls.

From the equation of motion and the continuity equation for each species, assuming azimuthal symmetry and quasineutrality,

$$\frac{\partial n}{\partial t} = D_{e\perp} \frac{\partial^2 n}{\partial r^2} + D_e \frac{\partial^2 n}{\partial z^2} + \nu_{e\perp} \frac{\partial}{\partial r} n E_r + \nu_e \frac{\partial}{\partial z} n E_z + n Z$$

$$\frac{\partial n}{\partial t} = D_{i\perp} \frac{\partial^2 n}{\partial r^2} + D_i \frac{\partial^2 n}{\partial z^2} - \nu_{i\perp} \frac{\partial}{\partial r} n E_r - \nu_i \frac{\partial}{\partial z} n E_z + n Z$$

We define the ambipolar diffusion coefficient in a finitely long plasma $D_{a\perp}$ as follows:

$$D_{a\perp} = \frac{\nu_e D_{i\perp} + \nu_i D_{e\perp}}{\nu_e + \nu_i}$$

Then,

$$\frac{\partial n}{\partial t} = D_{a\perp} \frac{\partial^2 n}{\partial r^2} + D_a \frac{\partial^2 n}{\partial z^2} + n Z$$

It cannot be easily separated into one-dimensional equations. Furthermore, the answer depends sensitively on the boundary conditions at the ends of the field lines.

(2) Classical theory of diffusion in fully ionized plasma

In fully ionized plasma all collisions are Coulomb collisions between charged particles. Collisions between like particles give rise to very little diffusion. However, unlike particles give rise to much diffusion in magnetic fields. Although we had to use the kinetic equation with the Fokker-Planck collision term based on stochastic process, which was discussed first by Longmire and Rosenbluth⁴⁶ (1956), we can use the MHD equation or the two wave approximation of the momentum transfer equation, which includes the effect of charged particle collisions under the modified Boltzmann binary collision theory.

The diffusion coefficient is a function of the plasma density in a fully ionized plasma, and this considerably complicates the solution of the diffusion equation. This is because the density of the scattering center is not fixed by the neutral atom density but by the plasma density itself. The conductivity is given by Spitzer⁴⁷ (1956) based on the Fokker-Planck theory, and the diffusion coefficient is given from the MHD theory with the conductivity related to the binary collision frequency; Rosenbluth and Kaufman⁴⁸ (1958).

In a fully ionized plasma, the diffusion coefficient decreases with increasing temperature. The opposite is true in partially ionized plasmas. The reason for the difference is the velocity dependence of the Coulomb cross section. The diffusion is automatically ambipolar; the generalized momentum has to be conserved between the two species. No ambipolar electric field

arises because both species diffuse at the same rate, and there is no transverse mobility. From the MHD equations,

$$\rho \frac{d\vec{u}}{dt} + \nabla p = \frac{1}{c} (\vec{J} \times \vec{B})$$

$$\vec{E}^* = \vec{E} + \frac{\vec{u} \times \vec{B}}{c} = \eta \vec{J} + \frac{1}{en} (\vec{1} \cdot \vec{u} - \vec{B} - \nabla p_e)$$

where

$$\rho = n_i M + n_e m$$

$$\vec{u} = \frac{1}{\rho} (n_i M \vec{u}_i + n_e m \vec{u}_e)$$

$$P = P_i + P_e$$

$$\eta = \frac{m \nu_{ei}}{n e^2}$$

where ν_{ei} is the ion-electron collision frequency. In a steady state, we get the following relations:

$$\nabla p = \frac{\vec{J} \times \vec{B}}{c}$$

$$\vec{E}^* = \eta \vec{J} + \frac{1}{en} \nabla p_i$$

where $P = nT$ and $T = T_i + T_e$. We assume T is independent of space.

In the first equation, the electric field does not appear explicitly because the fluid is neutral; the ions and electrons diffuse together. The second equation is called the generalized Ohm's law. We get the flux \vec{F} in a fully ionized plasma in cylindrical geometry as follows:

$$\Gamma_r = nu_r = -n \frac{c^2}{B^2} nT \frac{\partial n}{\partial r}$$

$$\Gamma_e = nu_e = -\frac{cE_r}{B} + \frac{c}{B} \frac{T_i}{e} \frac{\partial n}{\partial r}$$

Then, the diffusion constant D_{\perp} in fully ionized plasma is defined as follows:

$$D_{\perp} = n \frac{c^2 nT}{B^2}$$

and

$$\Gamma_r = -D_{\perp} \frac{\partial n}{\partial r}$$

where D_{\perp} is proportional to the plasma density.

The diffusion is automatically ambipolar in a fully ionized plasma. A plasma confined by a magnetic field carries a current in the Hall direction, which is generally the relative motion of electrons and ions. This relative motion is produced by the diamagnetic force or E cross B drifts. This relative motion must be accompanied by a friction force between the electrons and the ions to conserve momentum in that direction.

It turns out that the drift due to the effect of the friction force produces diffusion fluxes of electrons and ions which are equal in the radial direction. That means that the diffusion is inherently ambipolar.

In a uniform magnetic field, collisions between like particles give rise to no diffusion. We consider only ion-electron

collisions, which give rise to diffusion in a fully ionized plasma;
no self diffusion occurs under classical diffusion theory.

Summary of the diffusion coefficients:

Diffusion in weakly ionized plasmas

$$D_s = \frac{T_s}{m} v_s = \frac{\lambda_s^2}{\tau_s}$$

where λ_s is the mean free path.

$$D_{s\perp} = a_s^2 b_s D_s = \frac{r_s^2}{\tau_s}$$

where r_s is the Larmor radius, $a_s = (v_s/\Omega_s)$, and $b_s = 1/(1 + a_s^2)$.

$$D_a = \frac{\nu_i D_e + \nu_e D_i}{\nu_e + \nu_e} = (1 + \frac{T_e}{T_i}) D_i$$

$$D_a' = \frac{a_e a_i D_a}{1 + a_e a_i} = (1 + \frac{T_e}{T_i}) D_{e\perp}$$

$$D_{a\perp} = \frac{\nu_i D_{e\perp} + \nu_e D_{i\perp}}{\nu_i + \nu_e} = D_{i\perp}$$

Diffusion in fully ionized plasma

$$D_{\perp} = \frac{n c^2 n T}{B^2} = \frac{r_e^2}{\tau_{ei}} (1 + \frac{T_i}{T_e})$$

Bohm diffusion

$$D_B = \frac{T_e}{16eB} \approx \frac{r_e^2}{\tau}$$

where $\tau = \frac{1}{\Omega_e}$

(3) Anomalous diffusion

(a) Bohm Diffusion

A measurement of the electron diffusion coefficient across the magnetic field in the plasma of low pressure arc discharge was obtained by Bohm²¹ (1949) et al. at Berkeley. The plasma density profile was measured with a negatively biased Langmuir probe. The diffusion coefficient was given through the relaxation length (e-fold length). The arc chamber was operated around 10^{-3} torr with a magnetic field around 3000 gauss and arc voltage 150V.

Bohm's experimental result seemed incompatible with the theory of classical collisional diffusion theory. This theory required that the diffusion coefficient is inversely proportional to the square of the magnetic field, and that the relaxation length to varies as the inverse of the magnetic field.

An explanation based on oscillation was sought at that time, and the theory which was given (which has never been published) seemingly explained the magnitude of the diffusion length but required that the relaxation length vary as $B^{-1/2}$ and the diffusion coefficient vary as B^{-1} .

Bohm and his collaborators came to the conclusion that the diffusion is not consistent with collision phenomena but can be explained by "drain". Thus, later they called it "drain" or "anomalous diffusion". They have emphasized that the random fluctuation of charged density, and that the plasma oscillations may produce electric fields, which in their turn give rise to the drift motion across the magnetic field lines.

This interpretation was criticized by Simon⁴⁹ (1958), who pointed out that the transeverse diffusion of the plasma is not necessary ambipolar, and that actually, what Bohm measured is the ion diffusion coefficient. Thus, in the arc experiment, space charge neutralization can be maintained by the conducting end walls which produce an electron short circuit across the magnetic field. In Bohm²¹ (1949), it was assumed that the diffusion across the magnetic field is ambipolar, and the experimental value was compared with the theoretical value assuming ambipolar diffusion.

With this new interpretation, Simon was able to show that the arc experiment did not conflict with the classical collisional-diffusion mechanism. No additional mechanism such as plasma oscillations need to be postulated. The resultant diffusion rate is not ambipolar, however. Ions and electrons diffuse at their own intrinsic rate.

Simon's theory was promoted much more precisely; Boeschoten⁵⁰ (1964). The relation of the relaxation length $\lambda \propto 1/B$ i.e., and $1/B^2$ dependence of the ion-diffusion coefficient, seems to be established. However, the experimental value of the diffusion coefficient still shows a discrepancy of an order of magnitude with the theory. The discrepancy might be ascribed to the fact that the mean free path values and the temperature are uncertain. Another possibility is that oscillations and instabilities give rise to an additional "anomalous" diffusion which is compatible with the observation $\lambda \propto 1/B^{1/2}$. Therefore the possibility of $1/B$ diffusion was not excluded. Also the diffusion of electrons was not

determined from the above considerations, and the electrons might well move across the magnetic field by some anomalous diffusion mechanism as proposed by Hoh⁵¹ (1962). Anomalous transverse electron diffusion was reported by Zharinov⁵² (1960).

Spitzer⁵³ (1960) derived a formula of Bohm's diffusion coefficient assuming that ion acoustic waves, generated by currents along the magnetic field give rise to large amplitude fluctuating electric fields normal to the lines of force. Taylor⁵⁴ (1961) obtained, from general stochastic considerations, a formula for the perpendicular diffusion coefficient, which can recover both the classical and Bohm diffusion coefficient. Ichimaru and Rosenbluth⁵⁵ (1971) recovered both diffusion coefficients considering low-frequency long-wave length fluctuations.

The rate of particle diffusion across a strong magnetic fields is, in contrary to diffusion along the magnetic field, proportional to the frequency of collision with non identical particles. Microscopic-anomalous diffusion mentioned above can thus be visualized as a consequence of additional particle collisions with fluctuating electric fields. It can also be regarded as made of the E cross B drift motions of the particles, where E is associated with microscopic instabilities. This type of diffusion is important when particle collisions are rare, i.e., when the plasma is quite "collision less".

When the plasma is unstable microscopically, transport of large portions of plasma often takes place. In such a case, the state of the plasma is often convective rather than diffusive. For a bounded

plasma, however, such convective transport of plasma can be described as an equivalent anomalous diffusion. Generally the convective motion of the plasma soon becomes associated with irregular, large-amplitude, and multiharmonic fluctuations produced by drift wave instabilities. The corresponding particle transport can be termed as turbulent diffusion (convection), which is observed in fully ionized plasmas.

Yoshikawa and Rose²⁷ (1962) have derived an expression which in suitable limits gives both collisional and Bohm diffusion; The motion of the ion is neglected. They consider the following physical mechanism; a density gradient normal to the magnetic field gives rise to a Hall current. If there is a small density fluctuation, a space-charge field must be set up parallel to the Hall current to preserve its continuity. This field crossed with the magnetic field leads to motion in the direction of the pressure gradient. To obtain a net flux in this direction, after averaging over the fluctuations, second order terms must be retained. We expect to obtain anomalous Bohm diffusion in the limit of low pressure or high magnetic field.

Yoshikawa and Rose carried out experiments in a hollow cathode low pressure arc discharge (10^{-3} torr). The plasma thus generated is highly ionized (30%) so that Coulomb collisions are dominant. They tried to avoid the short circuiting effect by letting the magnetic field line terminate at a glass wall where no plasma is visible. The experiment seems to favor anomalous diffusion up to 1000 gauss.

In experiments of Q-machine where the plasma is produced without currents, the limit set by the fluctuation effect is, however, low enough to make the presence of Bohm's anomalous diffusion incompatible with the measured relaxation length as shown by D'Angelo and Rynn⁵⁶ (1961).

No conclusive measurements have been made on the diffusion rate of plasma particles across the magnetic field in the plasma under low pressure arc discharge. Anomalous diffusion is well reviewed by Boeschoten⁵⁰ (1964), and by Hoh⁵⁷ (1962).

(b) Anomalous diffusion in the positive column

A measurement of the ion diffusion coefficient across the magnetic field in the positive column of glow discharge was done by Lehnert and Hoh³² (1960). The positive column works around a pressure of 1 torr. The mean free paths of ions and electrons are usually very small as compared to the dimension of the discharge tube. Typical discharge parameters are the plasma density around 10^9 to 10^{13} /cm³, the electron temperature of some electron volts and the ionization degree 10^{-7} to 10^{-3} . Usually, Coulomb collisions are very infrequent.

The purpose of Lehnert's³⁰ (1958) experiment is to study diffusion across a magnetic field in a configuration which is free from the short circuit effect proposed by Simon⁴⁵ (1955). For the purpose, a long cylindrical plasma column (4m) with a homogeneous magnetic field along the axis has been chosen. At reasonably high pressures or at sufficiently long tube lengths, the short circuiting effect of the conducting end walls, such as that which occurs in an arc plasma, vanishes. The transverse diffusion is ambipolar. Then Lehnert made an important discovery. The experiments were shown to agree with the classical collision diffusion theory of ambipolar diffusion, but only in a range of magnetic fields up to a certain critical value. Above this value an instability was found to set in and the results indicated a diffusion rate much greater than that given by the classical diffusion theory.

In contradiction to the arc discharge experiment, the radial density distribution in the positive column, immersed in a strong

axial magnetic field, is independent of the magnetic field. (The relaxation length is the same). Nevertheless, the particle flux to the wall is expected to be greatly decreased as a consequence of the classical theory. The reduced particle loss reflects itself in the diminished rate of ion-electron pair production. Lehnert thought that this requires a lower electron temperature and a decreased axial electric field. Thus, the transverse diffusion coefficient can be estimated from the measured axial electric field. Schottky's theory⁴¹ (1924) of the positive column in glow discharge has been extended by Tonks⁴⁴ (1939) to include the influence of an axial magnetic field properly. Later Lehnert³⁰ (1958) rigorously re-examined various aspects of the theory, and he derived that the axial electric field is connected with electron temperature through the energy balance relation. The transverse diffusion coefficient is expressed as a function of the axial electric field.

The effect of the axial magnetic field is to decrease the transverse diffusion coefficient, the electron temperature and also the axial electric field. In spite of criticisms, like that of Ecker⁵⁸ (1961), experiments have shown good agreement with this theory. Therefore the axial electric field may be regarded as a good indication for the large change in the particle loss. Then he made the important discovery that the positive column suddenly became unstable and that the transverse diffusion increased greatly when the axial magnetic field exceeded a certain critical point.

With stereo-streak photographs, Allen, Paulikas and Pyle³³ (1960) found that the current in the positive column concentrated in a rotating screw shaped channel when the magnetic field just passed its critical value. A description of the positive column, based on the super critical theory for magnetic field, has been given by Kadomtsev and Nedospasov²³ (1960). The super critical theory showed that energy and plasma could be balanced at an oscillating steady state, with a plasma screw rotating. Therefore it is called "screw instability".

In their theory, the collision frequency of ion is assumed to be much larger than their gyro-frequency since the pressure is relatively high (1 torr). Since ions are assumed to be unaffected by the the magnetic field, they lag behind the rotation of the electron distribution. A physical interpretation of the instability mechanism of Kadomstev and Nedospasov was given by Hon and Lehnert⁵⁹ (1961). The theory was further extended by Hoh⁵¹ (1962) and by Johnson and Jerde⁶⁰ (1962).

The axial electric field E_z tends to "lift up" the electron screw relative to the ion screw. This is equivalent to a rotation of the electron screw in the positive θ direction. Due to a subsequent charge separation, an azimuthal electric field will arise, which tends to drive the particles outwards with the E cross B drift and to destabilize the plasma.

On the other hand, the ambipolar electric field tends to rotate the electron screw in the negative θ direction relative to the ion screw. Under the prevailing experimental conditions, no field

reversal takes place near the critical magnetic field. This rotation counteracts the charge separation produced by the longitudinal electric field. Additional stabilizing effects are provided by diffusion and conduction, which tend to smooth out the perturbed density and potential distributions and to suppress their asymmetry.

As the second order effect, the regular and random azimuthal electric fields can drive the charged particles across the magnetic field in addition to the normal diffusion process. In the present case, as the instability set in, the external axial electric field is converted to a regular azimuthal electric field through the agency of a plasma screw. And, as pointed by Ecker⁵⁸ (1961), the plasma temperature is expected to increase above the critical magnetic field since it is the function of axial electric field.

The various experimental results with the positive column in glow discharge, contrary to the experiments with the plasma in low pressure arc discharge, the findings are very much the same, and are satisfactorily explained theoretically. In particular, the ratio of the ion current, collected by a probe at the wall, to that at the center decreases with increasing magnetic field in a characteristic manner until the critical field is reached, and after which it increases. At this critical field, there is an increase in low and high frequency noise. Also, the power needed by the plasma to maintain a constant density decreases until the critical field is reached and increases thereafter.

Simon³¹ (1963) pointed out that the mechanism of Kadomtsev and Nedospasov could also be present in plasma of low pressure arc

discharge (10^{-3} torr) in an uniform magnetic field. A somewhat similar phenomena had been observed in Neidigh's experiment⁶¹ (1958) of a low pressure arc discharge . The diffusion obeyed the classical laws as long as the background pressure remained above a critical point. Below this critical pressure the discharge changed abruptly to a new mode, in which an asymmetric potential distribution rotated uniformly around the direction of the magnetic field. A similar anomalous effect was also observed by Zharinov⁶² (1962).

Interestingly, in Zharinov's arc discharge experiment, the electron probe current suddenly increases by an order of magnitude at a certain magnetic field whereas the ion current remains unaltered. In the plasma of low pressure arc discharge, we no longer have an applied electric field parallel to the magnetic field, which produces the drift of electrons through the ions. Instead, it is the direct streaming of ions and electrons out of the plasma to the end walls which provides our mechanism. The electrons tend to stream out of the plasma much more rapidly than the ions do, owing to their greater thermal velocity. An ambipolar electric field develops at once, which greatly reduces the difference in the two flow rates. However, exact cancelation does not occur. Thus, there is a net streaming of the ions through the electrons in the direction of the field lines, and a helical perturbation may be unstable as in the case of the positive column.

Besides the difference in the origin of the streaming in the positive column, there is continuous ionization of the neutral going

on, in the arc discharge, and plasma is created by ionization only along the axis of the cylindrical arc chamber. Away from the axis we have only diffusion and streaming. Another difference is relative low pressure regions (10^{-3} torr) for the arc discharge. The radial density distribution in the positive column is a function only of the tube radius, and is independent of the applied magnetic field, while in the arc discharge, the radial density distribution depends vitally upon the external field strength.

(c) Anomalous diffusion in the reflex discharge

The anomalous diffusion, observed by Lehnert in the positive column in an uniform magnetic field, was adequately explained and demonstrated to be well in accordance with the experimental results.

On the other hand, in a low pressure (10^{-2} torr) cold cathode reflex discharge contained in a glass chamber, Bonnal et al.³⁴ (1961) found that the escape flux increases with the magnetic field, even though it decreases with the increasing magnetic field according to the classical diffusion until a critical magnetic field. They also found that the radial anomalous diffusion decreases again with the increasing magnetic field at magnetic field much larger than the critical field. The intense emission of noise by the discharge above the critical field is detected for both high frequency and for low frequency components.

It was emphasized that the axial electric field, which is necessary to induce the screw instability, is absent in the plasma of the reflex discharge since there is no direct current in the axial direction. Consequently, a different instability mechanism must be sought. This led to the problem of studying the diffusion process in the reflex discharge.

Bingham and Chen⁶³ (1962) detected a helical instability of the plasma column of a hot cathode reflex discharge in a metal chamber. They showed that the plasma column becomes azimuthally asymmetric in a strong magnetic field and that it rotates at a speed cE/B . Chen and Cooper³⁵ (1962) investigated particle diffusion in the hot cathode reflex discharge (10^{-2} torr) contained in a metallic

chamber, in the course of a study of low frequency density oscillations. They reported that the diffusion does not exhibit any unusual behavior up to a magnetic field of 7900 Gauss, even though they recognized a low frequency instability at a magnetic field about 750 Gauss. Fushova et al.⁶⁴ (1964) disagreed with the result of Chan and Cooper, and established that in low pressure (10^{-4} torr) the flux of charged particles near the wall, in a hot cathode reflex discharge contained in a metallic chamber, does in fact start to increase at some critical value of the magnetic field.

The potential distribution of the reflex discharge have been calculated by Chen¹⁹ (1962). In his calculation the variations of all quantities in the axial direction have been neglected, and the discharge has been divided into plasma and sheath regions; the double sheath, at the hot emitting cathode, and the sheath, at the cold reflecting cathode at the far end of the cylindrical tube, have been considered. The source term included allows for ionization of the background gas by both primary electrons emitted by the cathode and by the plasma electrons. Similar calculation including the axial variation is done by Brzhechko et al.⁶⁵ (1968). The measurement of minimum wave length for this type of instability based on Chen's calculation is done by Bingham⁶⁶ (1964).

Pavlichenko et al.³⁶ (1964) investigated diffusion in a cold cathode reflex discharge contained in a glass chamber, and established that the anomalous behavior of the particle flux near the wall (anomalous diffusion) is caused by a microscopic helical

instability originating in the drift motion of the plasma in the crossed electric and magnetic fields. They mentioned that it is striking that there is no increase of diffusion in a strong magnetic field; such no increase was noted by Chen and Copper³⁵ (1962), in their experiments with the reflex discharge contained in a metallic chamber, despite the existence of low frequency oscillation. These oscillations do not lead to increased diffusion, and this may be a result of the stabilizing influence of the metallic shielding on development of instabilities.

Thomassen²⁶ (1966) calculated the diffusion coefficient from the fluctuation spectrum in a hot cathode reflex discharge above the critical magnetic field. It was shown that the diffusion from the classical collision process is negligible in comparison with the random walk type transport due to these fluctuations. A small magnetic well is created with Ioff bars. It suppresses the low frequency instability and reduces the loss rate. A similar experiment was done by Dubovoi et al.⁶⁷ (1966) with a cold cathode reflex discharge.

Hoh²⁴ (1963) carried out a theoretical analysis of the stability of a weakly ionized plasma in the crossed electric and magnetic field in low pressure region (10^{-2} torr). In the axis and cylindrical symmetric plasma, with no external axial electric field, with an inwardly directed electric field and pressure gradient, and with an axial magnetic field, electrons and ions drift in the azimuthal direction because of the E cross B drift and diamagnetic drift. Because of the diamagnetic drift, electrons

drift much faster than ions in the azimuthal direction when the direction of electric field and that of density gradient is opposite, which is pointed out by Simon³¹ (1963).

In a weakly ionized plasma, the drifting ions and electrons collide with the neutral molecules of the background gas and experience a fictional retarding force. The ions, because of their greater mass, lose more momentum in these collisions and hence, tend to lag behind the electrons; this will be encouraged by both the E cross and B diamagnetic drifts.

This difference in drift velocity gives rise to a current in the direction of the electron drift velocity, and the azimuthal electric field is also induced; thus giving rise to radial transport. In the rotating frame in the plasma, the electric field is not constant but is fluctuating. Neither the steady nor the first order fluctuating components give rise to the net radial drain, but the second order fluctuating component gives rise to a mean square radial displacement which will not be small. When the amplification of the perturbation produced by the charge separation is larger than the effect of diffusional dissipation of the perturbation, a low frequency instability occurs, which is called "three fluid" or "neutral drag instability."

The screw instability is driven by an axial electric field, whereas, the neutral drag instability is driven by an inward directed radial electric field which is not always achieved in the positive column. Hoh²⁴ (1963) mentioned that above the critical magnetic field, the anomalous diffusion induced by the instability

seems to be small. This can be understood as that the anomalous diffusion effectively represents an enhanced transverse mobility, and, hence, a decreased radial electric field. The value of the critical magnetic field was estimated for the weakly ionized cases, and found to be in satisfactory agreement with the values observed experimentally.

The striking feature of this anomalous diffusion is a report by Pavlichenko et al.³⁶ (1964) that these increases in diffusion are accompanied by a decrease of the electron temperature of the plasma. This contradicts with the result obtained by the positive column. On the other way, results for a plasma with higher degree of ionization is contradictory to Hoh's theory. In a cold cathode reflex discharge contained in a metallic chamber, the critical magnetic fields observed by Pavlichenko et al.²⁵ (1966) are several times smaller than those predicted by Hoh. The absence of a radial electric field in this dense plasma column is reported by Brzhechko and Pavlichenko⁶⁵ (1968). In their experiment, the anomalous diffusion was observed under conditions in which a radial electric field in the plasma was essentially absent.

Dushin et al.⁶⁸ (1967), Brzhechko and Pavlichenko⁶⁵ (1968) and Brzhechko et al.⁶⁹ (1969) associated the anomalous diffusion of the plasma discharge with the observed drift instability of an unhomogeneous plasma in the reflex discharge. However, the turbulent kinetic micro-instabilities, like drift instabilities in a strong magnetic field, cannot lead directly to an increase in the

flux of charged particles across the magnetic field. They concluded that the observed low frequency oscillations are the result of the drift instability in an unhomogeneous plasma. An important role may be played by beams of oscillating electrons, which can excite electron plasma oscillations of large amplitude as suggested by Briffod et al.⁷⁰ (1964) in their cold cathode reflex discharge experiment.

Thomassen's experiment⁷¹ (1968), in the cold cathode reflex discharge in low pressure (10^{-3} torr) contained in a glass chamber; was confined to low magnetic fields from 100 to 500 Gauss. In this range, a large amplitude oscillation appears while the density begins to drop, indicating the importance of these drift waves for understanding the loss process. High frequency beam plasma instabilities occurring in bursts and low frequency resistive drift waves are both found.

The neutral drag instability, which is driven by the radial electric field, was examined in detail in a hot cathode reflex discharge, which is free from the axial electric field. It is also free from velocity anisotropic instabilities, which exist in the cold cathode reflex discharge. There are two distinct modes of operation of the reflex discharge. The plasma is created by the primary beam of particles, which stream down to the column, and are reflected at each end of the potential barriers. The discharge characteristics are very different when the beam density is a significant fraction of the plasma density, or when the beam velocity greatly exceeds the thermal velocity of the electrons. The

former occurs at low pressures and the latter with cold cathodes where discharge voltage exceeds 500 V. In these cases, there are strong two-stream instabilities, which are claimed to dominate the discharge characteristics and the transports by Briffod et al.⁷⁰ (1964). In a hot cathode reflex discharge in the order of 10^{-2} torr pressure, these interactions are weak or absent even though the discharge voltages are in the order of 100 V; Thomassen²⁶ (1966). The cold cathode reflex discharge is a very complicated plasma, exhibiting several types of oscillations and relaxations simultaneously; this diffusion process is not yet completely understood.

APPENDIX 2

INTERACTION OF PLASMA WITH BOUNDARIES

In actual plasma experiments, the plasma is contained in a vacuum chamber of finite size. When ions and electrons hit the wall, they recombine and are lost. This problem can be treated from two distinct ways. When the plasma density is very small, the maximum electron and ion currents that can flow from a cathode to an anode are limited by the space charges. This problem will be discussed in Section (1). When the plasma density is not small any more, the electron and ion currents cannot flow directly from the cathode to the anode anymore. The electron and ion currents to the wall are decided by the sheath boundary condition between the plasma and the wall. This problem will be discussed in Section (2). In Section (3), we discuss about sheaths, which are strong electric fields produced between the plasma and the wall. When the cathode is hot, it emits electrons. The primary electron current from the cathode to the plasma is limited by the space charges in the sheath. This problem will be also discussed in Section (3).

(1) Space charge effects(a) The space charge limited flow

Let us consider two infinite parallel planes A (negatively charged) and B (positively charged). When no electrons are emitted from A, the potential between the two plates varies linearly between the two plates. (Fig. A2-1). As the temperature of A is raised electrons are emitted. We assume that the electrons move with constant velocity across the space. Then there will be, in the unit volume, a space charge ρ equal to J/v , where v is the velocity of the electrons. If the velocities are uniform, the space charge will be uniform. From the Poisson's equation

$$\frac{d^2V}{dx^2} = -4\pi\rho$$

If we consider ρ constant and negative, the potential distribution between the two plates takes the form of a parabola. If the temperature of plate A is increased still further, the electron current increases so that the potential curve finally becomes a parabola with a horizontal tangent at the surface of A. If the electron has no initial velocity, the current cannot increase beyond this point. Any further increase of current would make the potential curve at the surface of A slope downwards and the electrons would be unable to move against this unfavorable potential gradient. The effect of the space charge limits the current.

Let V be the potential at a distance x from the plate A.

$$\frac{1}{2} m v^2 = eV$$

$$J = \rho v$$

Then

$$\left(\frac{dV}{dx}\right)^2 - \left(\frac{dV}{dx}\right)^2 = 8\pi J \left(\frac{2mV}{e}\right)^{1/2}$$

If there is an opposing potential gradient at the surface of plate A, then no current flow. In the case of the saturation current

$$\left(\frac{dV}{dx}\right)_0 = 0$$

Then, we get the space charge limited flow.

$$J = \frac{\sqrt{2}}{9\pi} \left(\frac{e}{m}\right)^{1/2} \frac{V^{3/2}}{x^2}$$

(b) Effect of initial velocity on the space charge limited flow

Let us take into account the effect of initial velocities when the current is limited by the space charge, assuming that the electrons have a Maxwell distribution.

According to Maxwell's law, the number of N of electrons emitted per unit time per unit area with velocity u is $n(u)$ and the total emission N is given by

$$\frac{\partial n}{\partial u} = n'(u) = N \frac{mu}{T} \exp\left(-\frac{mu^2}{2T}\right)$$

$$N = \int_0^{\infty} n'(u) du \quad [1/\text{sec}][1/\text{cm}^2]$$

There is a potential minimum V in front of cathode at x' (Fig. A2-2).

Then the space charge is

$$\rho = e \int \frac{n'(u)}{v} du$$

In region β ($x > x'$)

$$\rho_{\beta} = 2e \int_{u'}^{\infty} \frac{n'(u)}{v} du$$

In region α ($x < x'$)

$$\rho_{\alpha} = 2e \int_{v'}^{u'} \frac{n'(u)}{v} du + e \int_{u'}^{\infty} \frac{n'(u)}{v} du$$

The current to the cathode is

$$J = e \int_{u'}^{\infty} n'(u) du$$

And

$$v^2 = u^2 + \frac{2eV}{m}$$

$$v'^2 = -\frac{2eV}{m}$$

$$u'^2 = -\frac{2eV'}{m}$$

where V' is the minimum potential with negative value. Lets us consider space charge in region B ($x > x'$). Then,

$$\rho = e \int_{u'}^{\infty} \frac{n'(u)}{v} du$$

Then let us change variables as follows:

$$u'^2 \rightarrow v^2 - \frac{2eV}{m}$$

$$n'(u) \rightarrow N \frac{mV}{T} \exp\left(-\frac{m}{2T} \left(v^2 - \frac{2eV}{m}\right)\right)$$

$$u'^2 \rightarrow \frac{2e(V - V')}{m}$$

Putting variables as follows, and defining an error function $S(x)$

$$x^2 = \frac{m}{2T} v^2$$

$$v_M^2 = \frac{2T}{\pi m}$$

and

$$S(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-y^2) dy$$

Then, we get the following relation:

$$\rho = \rho_M \exp(n) S(\sqrt{n})$$

where

$$\rho_M = N \frac{e}{v_M} \exp\left(\frac{eV'}{T}\right)$$

$$n = e\left(\frac{V - V'}{T}\right)$$

We define also a new error function $P(x)$ as follows:

$$P(x) = 1 - S(x)$$

Therefore depending on whether $x - x'$ is negative or positive, we get the following relation:

$$\rho = \rho_M \exp(\eta)(1 \pm P(\sqrt{\eta}))$$

Putting this into the Poisson equation, we get the following differential equation:

$$\frac{d^2 \eta}{dx^2} = \left(\frac{T}{e}\right)^2 4\pi\rho_M \exp(\eta)(1 \pm P(\sqrt{\eta}))$$

Finally, we get the following integral equation:

$$\xi = \int_0^{\eta} \frac{1}{[\exp(\eta) - 1 \pm \exp(\eta) P(\sqrt{\eta}) \mp \frac{2}{\pi} \sqrt{\eta}]^{1/2}} d\eta$$

where

$$\xi = \frac{e}{T} \sqrt{8\pi\rho_M} (x - x')$$

The upper or the lower sign are to be taken according as $x - x'$ is negative or positive, respectively. This integration is calculated by Langmuir⁷² (1923). According to Langmuir, taking into account the second term, the space charge limited flow is given as follows.

$$J = \frac{\sqrt{2}}{9\pi} \left(\frac{e}{m}\right)^{1/2} \frac{(V - V')^{3/2}}{(x - x')^2} \left(1 + \frac{2.66}{\sqrt{\eta}}\right)$$

where V has the minimum V' at $x = x'$.

(c) The space charge limited flow with positive charges

The maximum electron current that can flow from a given hot cathode to an anode in high vacuum is limited by the space charge of the electrons. If even a small amount of gas is present, the positive ions formed tend to neutralize the electron space charge and this allows the current to increase until the current becomes limited only by the electron emission from the cathode as determined by its temperature. The rate at which the ions need to be produced in order to neutralize the space charge is usually less than one percent of the rate at which the electrons flow from the cathode. If we consider hot cathode in a gas, we see that electrons as well as ions are present in the positive ion sheath.

Let the cathode C emit a surplus of electrons without appreciable initial velocities to anode A of potential V_A

$$J_0 = \frac{\sqrt{2}}{9\pi} \left(\frac{e}{m}\right)^{1/2} \frac{V_A^{3/2}}{a^2}$$

We define m as the mass of electron and M as the mass of positive ion. Let us introduce positive ions uniformly distributed over a plane B of potential V_B . Positive ions flow to the cathode C of potential zero. Because of the partial neutralization of electron space charge the electron current will increase to a new value. In the same way as in the case of space charge limited flow, we define the electron current as J_e and the positive current as J_p . We define the electron space charge as ρ_e and the positive ion space charge as ρ_p .

$$\rho_e v_e = J_e \quad , \quad \rho_p v_p = J_p$$

$$\frac{1}{2} m v_e^2 = eV \quad , \quad \frac{1}{2} M v_p^2 = e(V_B - V)$$

We define variables as follows:

$$\alpha = \left(\frac{J_p}{J_e}\right) \left(\frac{M}{m}\right)^{1/2}, \quad \lambda = \frac{x}{a}, \quad \phi = \frac{V}{V_A} \quad \text{and} \quad \phi_B = \frac{V_B}{V_A}$$

Putting above variables into the Poisson equation, we get the following equation:

$$\frac{d^2 \phi}{d\lambda^2} = \frac{4}{9} \frac{J_e}{J_0} \left[\frac{1}{\sqrt{\phi}} - \frac{\alpha}{\sqrt{\phi_B - \phi}} \right]$$

Since the electron current is limited by space charge,

$$\left(\frac{d\phi}{d\lambda}\right)_{\lambda=0} = 0$$

Then, we get the following differential equation:

$$\frac{d\phi}{d\lambda} = \frac{4}{3} \left(\frac{J_e}{J_0}\right)^{1/2} \left[\phi^{1/2} + \alpha \left((\phi_B - \phi)^{1/2} - \phi_B^{1/2} \right) \right]^{1/2}$$

Let us consider the case when ions are emitted from anode, $\phi_B = 1$. When $\alpha = 1$, the potential gradient at the anode is zero and the positive current as well as the electron current is thus limited by space charge. Even an unlimited supply of positive ions available at the anode is not capable of neutralizing the electron space charge, because the positive ion current cannot become more than a definite fraction of the electron current, this fraction (when $\alpha = 1$) being equal to the square root of the ratio of the mass of the electron to that of the ion. When $\alpha = 1$, the potential

distribution is symmetric about its central point, $\lambda = 1/2$ and $\phi = 1/2$. Between the cathode and this central point there is an excess of negative space charge, while from the central point to the anode there is an excess of positive charge. Then, we get the following integral equation:

$$\lambda = \frac{3}{4} \left(\frac{J_0}{J_e} \right)^{1/2} \int_0^\phi [\phi^{1/2} + \alpha ((1-\phi)^{1/2} - 1)]^{-1/2} d\phi$$

The ratio J_e/J_0 is found at the anode where $\lambda = 1$ and $\phi = 1$.

$$\left(\frac{J_e}{J_0} \right)^{1/2} = \frac{3}{4} \int_0^1 [\phi^{1/2} + \alpha ((1-\phi)^{1/2} - 1)]^{-1/2} d\phi$$

The numerical integration is done by Langmuir²⁰ (1929). The electron current increases as more positive ions are emitted from the anode, until the positive ion current also becomes limited by space charge. When this occurs the electron current and the positive ion current are each 1.86 times as great as the currents of electrons or ions that could flow if carriers of the opposite sign were absent.

$$J_{e\max} = 1.86 J_0$$

$$J_{i\max} = \left(\frac{m}{M} \right)^{1/2} J_{e\max}$$

The numerical calculation done by Langmuir²⁰ (1929) also shows that the ions have a maximum effect in increasing the electron current when they are introduced at a point which is 4/9 of the distance from the cathode to the anode. When a large intensity of ionization is involved throughout all or a large part of the space between the electrodes, like the case of low pressure discharge

plasmas, we cannot analyze it in this way any more. We had to consider it as an interaction of plasma with an electrode.

(2) The plasma sheath equation and the Bohm sheath criterion

Plasmas have tendency to maintain themselves in a neutral and field free state in spite of any forces to change it. A sheath, which is able to shield the plasma region from the strong electric fields that would be caused by any electrode, is one of the examples of that property. Although the shielding is so good that the sheath edge is very near the plasma potential, it is not quite perfect, and a small portion of the potential drop which occurs between the electrode and the plasma may penetrate beyond the sheath edge.

In order to produce a steady state in plasma, ions and electrons must reach the wall at a rate equal to that at which they are formed. This happens automatically with the ions, which simply fall into the wall quite steadily. As for electrons, however, they strike the walls so often that the majority must be repelled if the distribution is to be steady. The potential of the wall finally adjusts itself to such a value that the number of electrons energetic enough to go over the barrier is equal to the number produced.

Most of this potential drop occurs not in the plasma but across the wall sheath. The plasma field, compared with the sheath field, is so small that it produces negligible changes of potential over the distance of many sheath thickness. Therefore it may be assumed that the plasma potential is constant, at least in so far as the processes involved in sheath formation are concerned. However, the plasma fields cannot be completely neglected, because over the long distances that they cover, they are able to accelerate positive ions

up to appreciable energies of the order of the plasma electron temperature.

In the plasma approximation, the distribution of ion energy can be replaced by a stream of ions, all of which possess the mean energy of the distribution. Since the electrons are approximately in thermal equilibrium, they have Maxwell distributions. Therefore the distribution of electron energy can also be replaced by a stream of electrons, all of which possess the mean thermal velocity, but at the edge of the sheath, the direction of electrons can be distributed only over a hemisphere. Then the electron current is given as follows:

$$J_e = \frac{1}{2} \rho_e v_e = \frac{1}{2} \rho_e \left(\frac{2T_e}{\pi m} \right)^{1/2}$$

For ions, the velocity distribution is not Maxwellian. Then the ion current is given as follows:

$$J_i = \rho_i v_i = e n_p \left(\frac{2E_i}{M} \right)^{1/2}$$

where n_p is the plasma density at the sheath.

According to the experimental result by Langmuir and also later by Bohm, the normal components of velocity of the ions that reach the edge of a cathode are roughly that of a Maxwellian distribution corresponding half of electron temperature, which is called the Bohm sheath criterion.

$$E_i = \frac{1}{2} T_e$$

Then, the ion current is given as follows:

$$J_i = en_p \left(\frac{T_e}{M} \right)^{1/2}$$

Therefore at the edge of the sheath, the space charges are given as follows:

$$\rho_e = J_e \left(\frac{2\pi m}{T_e} \right)^{1/2}$$

$$\rho_i = J_i \left(\frac{M}{T_e} \right)^{1/2}$$

Consider a one dimensional semi-infinite plasma in which the electron temperature T_e is much bigger than the ion temperature T_i and the mean free path is much bigger than the Debye length λ_D . The sheath criterion is valid for a wide range of conditions. The fact that the ion velocity near a boundary should be given by the electron, rather than the ion temperature, is physically reasonable when $T_e \gg T_i$, since the force driving the ions into the wall is provided by the plasma pressure $n_0 T_e$ (Notice here the ion temperature T_i is different from the ion energy E_i .)

According to Chen's way⁷³ (1961), we seek time-independent solutions of the Poisson's equation in the region within a distance (λ collisional mean free path) of the boundary where the particles are assumed to suffer no collisions. Let us choose a point approximately $(1/2)\lambda$ away from the wall to be the origin, $x = 0$ $V = 0$. The equations of motion for free-fall to negative V are obeyed for many lengths λ_D on either side of this point of negative potential V .

$$n_e = n_{e0} \exp\left(\frac{eV}{T_e}\right)$$

The subscript "o" will denote quantities at $x = 0$. Ions arrive at $x = 0$ with a uniform energy eV_0 toward the wall.

From the continuity equation,

$$n_i = n_{i0} \frac{1}{\sqrt{1 + \left(\frac{V}{V_0}\right)^2}}$$

From Poisson's Equation,

$$\frac{d^2V}{dx^2} = -4\pi e(n_i - n_e)$$

We define variables as follows:

$$n = -\frac{eV}{T_e}, \quad \xi = \frac{x}{\lambda_D}, \quad a = \frac{n_{i0}}{n_{e0}} \quad \text{and} \quad n_i = -\frac{eV_0}{T_e}$$

Then, we get the so-called "the plasma sheath equation".

$$\frac{d^2n}{d\xi^2} = \left[\frac{a}{\sqrt{1 + (n/n_i)}} - \exp(-n) \right]$$

Numerical solutions for this equation have been given by Self⁷⁴ (1963). In the plasma approximation ($n_i = n_e$), this formulation leads to an integral equation for the potential which has also been calculated by Self⁷⁵ (1965). However the plasma approximation can not give a continuous transition from the plasma to sheath region.

At $x = 0$ or $\xi = 0$

$$n = 0$$

Then we get the following integral equation:

$$n = \pm \int_0^\xi \frac{d\xi'}{[4an_i[(1 + (n/n_i))^{1/2} - 1] + 2(\exp(-n) - 1) + (dn/d\xi)^2]^{1/2}}_{\xi=0}$$

For the case $\frac{dn}{d\xi} = 0$ at the arbitrary point, we get the following relations at $\xi = 0$ and $n = 0$ for an arbitrary a and n_i :

$$\left(\frac{d^2n}{d\xi^2}\right)_{\xi=0} = a - 1$$

$$\left(\frac{dn}{d\xi}\right)_{\xi=0} = 0$$

Actually it is very difficult to solve the plasma-sheath equation.

We assume that for small ξ we can expand n as follows:

$$n = A + B\xi + C\xi^2 + D\xi^3$$

We assume that the ξ^3 term is very small, so we assume $D = 0$.

When $a = 1$ at $\xi = 0$ ($n_{i0} = n_{e0}$) we get $A = B = C = 0$. Then

$$n = 0$$

This result represents the plasma region.

When $a < 1$ at $\xi = 0$ ($n_{i0} < n_{e0}$)

$$n = -\frac{1}{2}(1-a)\xi^2$$

η decreases as ξ increases. This does not represent the sheath region.

When $a > 1$ at $\xi = 0$ ($n_{i0} > n_{e0}$)

$$\eta = \frac{1}{2} (a - 1) \xi^2$$

η increases as ξ increases. This represents the sheath region.

Both results are shown in (Fig. 3-3).

When n_i is too small, $\eta = f(\xi)$ becomes oscillatory. At the critical n_i , $\eta = f(\xi)$ has an inflection point at $\xi \neq 0$. At the inflection point $d\eta/d\xi = 0$ and $d^2\eta/d\xi^2 < 0$.

When $\xi \neq 0$

$$\frac{d^2\eta}{d\xi^2} = a \frac{1}{(1 + \eta/n_i)^{1/2}} - e^{-\eta} < 0$$

Then we get the following relation:

$$n_i \leq \frac{1}{2a} \frac{1 - \exp(-\eta)}{a \exp(\eta) - 1}$$

This result is for the oscillatory case: where $a > 1$ and $1 - ae^{-\eta} < 0$.

Let us consider two functions $f(\eta)$ and $g(\eta)$ defined as follows (Fig. A2-4).

$$f(\eta) = 1 - \exp(-\eta)$$

$$g(\eta) = 2an_i(a \exp(\eta) - 1)$$

For large n_i , there is no solution for $f(\eta) = g(\eta)$. $d\eta/d\xi$ is never zero except at the origin. For small n_i , there are two solutions, corresponding to a negative value for $(d\eta/d\xi)^2$ and an

oscillatory behavior for n .

Then the critical n_i is given by the following equations.

$$f(n) = g(n)$$

$$f'(n) = g'(n)$$

If we define $x = \sqrt{2n_i}$, we get the following equation:

$$ax^2 - 2ax + 1 = 0$$

And

$$x = a \pm \sqrt{a^2 - a}$$

We get the Bohm sheath criterion.

$$n_i \geq \frac{1}{2} a^2 \left(1 \pm \sqrt{\frac{a-1}{a}}\right)^2$$

where $a > 1$ but $a - 1 \ll 1$.

It is the characteristic of the plasma-sheath equation that when n_i becomes about the order of 1, the curve develops catastrophically in the distance of the order of λ_D . Hence if $\lambda \gg \lambda_D$, the value of a must be very close to unity to enable n to stay close to zero for such a long distance. Beyond λ from the wall, the collision changes the form of the plasma-sheath equation.

Then, generally the Bohm sheath criterion is true for $a - 1 \ll 1$.

$$n_i > \frac{1}{2} a^2 \approx \frac{1}{2}$$

Thus by the time ions reach the region of free fall, they must have somehow or other acquired an energy greater than $(1/2)T_e$. Then, how did they acquire this energy?

The positive ions are supposed to have negligible energy when they are formed, and in weakly ionized plasma, it is quite unlikely

that they get energy through collisions with electrons. They acquire energy as corresponding to the electric fields through which they pass. Then, where is the electric field?

Actually n is an integration form of ξ , and it is not easy to expand in ξ polynomials. In real plasma at the sheath edge $\xi = 0$, $dn/d\xi \neq 0$. It was discovered by Tonks and Laugmur⁴⁰ (1929) that the electric field penetrated beyond the sheath edge, into the plasma, and accelerated ions towards the sheath. The region is called "the transition region". At the sheath edge, the plasma density is less than those in the plasma. Bohm²¹ (1949) showed that the ion current depends on the electron temperature, and not the ion temperature, because the electron temperature determines the strength of the electric field which draws the ions towards the sheath.

If there are some negative ions in the plasma which have the same mass and almost the same temperature as positive ions, they cannot come into the sheath since they will be repelled by the electric field in the transition region. Instead electrons which have larger energy and speed come into the sheath. Therefore the boundary condition in weakly ionized plasma for negative ions must be decided very carefully.

(3) Sheaths and the space charge limited flow

(a) Sheaths

The theory of sheath in a gas discharge was first given by Tonks and Langmuir⁴⁰ (1929). In this theory the electrons are assumed to be in thermal equilibrium. The ions are assumed to be created at rest throughout the plasma and to fall freely, without collision, to the walls where they recombine with electrons. From the equation of motion for the ions and the Poisson's equation, a differential integral equation is written for the potential, which is called "the plasma sheath equation."

Actually there exist two distinct theories of sheaths, the Schotky's ambipolar diffusion theory (Appendix 1) and the Tonks-Langmuir free fall theory. Both theories are based on the idea that ion and electron fluxes to the wall should be the same to preserve the charge neutrality of the wall. The former has usually been commonly used when the mean free paths for both electron-neutral and ion-neutral collisions are much less than the dimensional length, while the latter is usually applied to low pressures when the mean free path for ion-neutral collision is in the order of the dimensional length. The two theories predict quite disparate density and potential profiles, each of which is in reasonable agreement with experimental measurements.

Both theories assume that the plasma is quasi-neutral, and thus, do not account for the sheath formation of the plasma boundary. However, the free-fall theory leads mathematically to a plasma-sheath boundary at a finite radius, where the potential and

plasma density are finite but their gradients are infinite. At this boundary the ion velocity has to satisfy the Bohm sheath criterion.

On the other hand, the ambipolar diffusion theory does not give rise to a plasma-sheath boundary; the plasma density goes to zero and the potential and velocities become infinite at a finite radius. The isothermal ambipolar diffusion theory, modified by the inclusion of the nonlinear ion inertia term, give results in close agreement with the free all theory, and the theory bridges the gap between the low and high pressure regimes in a continuous fashion: Self and Ewald⁷⁶ (1966).

In typical gas discharges involving a large intensity of ionization throughout a volume, the whole of the voltage difference from cathode to anode is concentrated in a cathode sheath (region between the cathode and the plasma) in the order of 100 volt, in which there is a positive ion space charge; the rest of the volume is nearly field free compared to the cathode sheath, the space charge of the positive ions being neutralized by low velocity electrons. In the transition region between the two regions, there is the voltage difference (in the order of 1 volt) to satisfy the Bohm sheath criterion. The voltage difference in an anode sheath (region between the anode and the plasma) is very small (in the order of 1 volt).

If the filament is hot, than the current is limited by the electron and ion space charges in a "double sheath" near the cathode. The transition region is strong enough to repel low temperature negative ions. When ions are produced uniformly

throughout space there will be a maximum potential in the space, but this usually exceeds the anode potential by not more than a volt or so, and thus, the ions flow, in nearly equal numbers, to anode and cathode, while the electrons and negative ions go to the anode only.

(b) The space charge limited flow in a double sheath

An electrode which is at a negative potential (order of 100 volt) with respect to the plasma, repels all the ultimate electrons which move towards it, except those that have a sufficient component of energy normal to the surface of the electrode, enabling them to move against the retarding field. The sheath is an electron sheath. The positive ions in the plasma that move towards the electrode are collected by it.

If the cathode is heated so that it emits electrons, these flow out through the positive ion sheath and if this electron current becomes sufficiently great it will neutralize the positive ion space charge in the immediate neighborhood of the cathode. Since the electrons start with negligible velocities from the cathode, the conditions are the same as those postulated when we calculated the effect of ions liberated at the anode while the electron currents flow from a hot cathode. In the present case it is the sheath edge, functioning as anode, that emits a surplus of ions while a limited number of electrons is emitted from the cathode.

Let the cathode, at potential $-V$ with respect to the plasma, first be at such low temperature that it emits no electrons. We let a_0 denote the sheath thickness under these conditions. For this purpose we replace J_0 by J_p , V_a by V , m_e by M_p , and a by a_0 in the formula of space charge limited flow with positive charges derived in section (1-C).

$$J_p = \frac{\sqrt{2}}{9\pi} \left(\frac{e}{M_p}\right)^{1/2} \frac{v}{a_0^2}{}^{3/2}$$

And

$$\alpha = \frac{J_e}{J_p} \left(\frac{m_e}{M_p}\right)^{1/2}$$

The effect of these electrons in neutralizing the ion space charge cannot cause an increase in J_p for this current is fixed by the plasma, but manifests itself by changing the sheath thickness so that this becomes "a" instead of "a₀". The relation between a and a₀ is calculated by Langmuir³⁹ (1929). As the cathode temperature is raised, the electron current density J_e increases and is equal to the electron emission from the cathode until α becomes equal to unity and the current becomes limited by the space charge. A further increase in electron current cannot occur. The cathode is then covered by a double layer and the ratio of the electron current to the ion current is equal to (M_p/m_e) . Even a change in the cathode voltage will not cause a change in the electron current if the positive ion current J_p remains constant. In the cathode drop, velocity of ions adjusts itself to such a value that J_p in the plasma bears the proper ratio to the electron current.

When negative ions exist, which are produced by electron attachment to neutrals in the plasma, they are repelled by the potential in the transition as mentioned in section (2). They do not come into the double sheath, and do not affect the positive ion

current J_p . Therefore we can still treat the double sheath as a two component problem.

(c) Primary electron current from the cathode

Any group of charged particles entering the sheath with a given velocity distribution will contribute to the space charge, at any point within the sheath, the amount $\int (1/v)dI$. The total space charge ρ at any point depends only on the potential at that point. In the case of the double sheath where the electron and ion current are both limited by the space charge, and where $dV/dx = 0$ at both the inner and outer edges, we have

$$\int_{V_M}^{V_S} \rho dv = 0$$

V_M and V_S are the potentials at these two edges. At the places where $dV/dx = 0$, one of these is at the minimum potential region very close to the cathode and the other is close to the outer edge of the sheath. For the present we will define it as a place where $dV/dx = 0$ and $\rho = 0$ so $d^2V/dx^2 = 0$, too. Within the sheath there are three groups of carriers to be considered; the positive ions from the plasma, the ultimate electrons from the plasma and the electrons emitted by the hot cathode. Negative ions do not come into the sheath as mentioned in section (b).

Plasma ions

If ρ_{ps} is the space charge of these ions at S with kinetic energy E_p , then their space charge ρ_p at a point of potential V is given from the continuity equation as follows.

$$\rho_p = \rho_{ps} \left(\frac{E_p}{V_s - V + E_p} \right)^{1/2}$$

$$H_p = \int_{V_M}^{V_s} \rho_p dv = 2\rho_{ps} E_p^{1/2} [(V_{sM} + E_p)^{1/2} - E_p^{1/2}]$$

where $V_{sM} = V_s - V_M$: the potential between S and M.

$$\rho_{ps} = J_p \left(\frac{M_p}{2E_p} \right)^{1/2}$$

Plasma electrons

The electron distribution will be Maxwellian with a temperature T_e .

$$\rho_e = \rho_{es} \exp\left(-\frac{V - V_s}{T_e}\right)$$

$$H_e = \int_{V_M}^{V_s} \rho_e dv = E_e \rho_{es}$$

$$\rho_{es} = J_{es} \left(\frac{2\pi m_e}{T_e} \right)^{1/2}$$

Electrons from the cathode

If the cathode emits more electrons than can flow across the sheath, then there will be a potential minimum V_M at a short distance from the cathode surface.

If T_c is the temperature of the cathode, then V_M may be calculated from

$$J_{eM} = J_c \exp\left(-\frac{V_M - V_c}{T_c}\right)$$

J_c is the current density of the saturated emission from the cathode corresponding to its temperature T_c , and J_{eM} is the current density which gets past the potential minimum and is thus the electron current density passing through the sheath. We use an error function $S(x)$ defined in section (2), and E_c instead of T_c implying energy given by the cathode. Then, we get the following relations:

$$\rho_c = \rho_{cM} \exp(\beta) S(\sqrt{\beta})$$

$$\beta = \frac{V - V_M}{E_c}$$

when $x > 2$

$$\exp(x^2) S(x) \approx \frac{1}{\sqrt{\pi}} \frac{2x}{2x^2 + 1}$$

$$\rho_c \approx \rho_{cM} \left[\frac{E_c}{\pi(V - V_M + E_c)} \right]^{1/2}$$

ρ_{cM} is the space charge density of the emitted electrons at the point of minimum potential.

$$\begin{aligned} H_c &= \int_{V_M}^{V_S} \rho_c dV = E_c \left[\rho_{cs} - \rho_{cM} + 2\rho_{cM} \left(\frac{V_{SM}}{\pi E_c} \right)^{1/2} \right] \\ &= 2\rho_{cs} V_{SM} \left[1 - \left(\frac{\pi E_c}{4V_{cM}} \right)^{1/2} + \frac{E_c}{V_{SM}} + \dots \right] \end{aligned}$$

$$\rho_{cs} = J_{eM} \left[\frac{m_e}{2(V_{SM} + E_c)} \right]^{1/2}$$

Then

$$H_p = H_e + H_c$$

And at sheath edge S

$$\rho_{ps} = \rho_{es} + \rho_{cs}$$

Finally, we get the following equation:

$$\frac{J_{eM}}{J_p} = \left(\frac{m_p}{m_e}\right)^{1/2} \frac{1 - (E_p/(V_s - V_M))^{1/2} (1 + (E_e/2E_p)) + (E_p/2(V_s - V_M))}{1 - (\pi E_c/4(V_s - V_M))^{1/2} - ((E_e - E_c)/2(V_s - V_M))}$$

When $1/2 E_c < E_p < (V_s - V_M)$, an increase in either E_p or E_c causes a decrease in J_{eM} . Thus the effect of the initial velocities of both the ions and the electrons at the sheath edge is to decrease the electron current that can flow from a hot cathode.

From the Bohm sheath criterion $E_p = (1/2)T_e$, the ion current is given as follows:

$$J_p = en_s \left(\frac{T_e}{M}\right)^{1/2}$$

where n_s is the plasma density at the sheath edge.

Defining $V = V_s - V_M$, $\phi = (V/T_e)$, we get an important function $f(\phi)$ as follows:

$$f(\phi) = \left(\frac{\pi}{2}\right)^{1/2} \left[\frac{\phi - \sqrt{2\phi} + 0.25}{\phi - 0.5 \sqrt{\pi\phi(T_c/T_e)} - 0.5(1 - (T_c/T_e))} \right]$$

This function is calculated by Chen¹⁹ (1962).

We finally get the primary electron current from the cathode as follows:

$$J_{eM} = f(\phi) n_s v_e$$

where

$$v_e = \left(\frac{2T_e}{\pi m} \right)^{1/2}$$

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Table 1: Measured positive ion parameters

Gas flow rate [cc/min]	Pressure [10^{-2} torr]	Maximum current [mA]	Maximum current density [10^3 A/m ²]	Critical magnetic field [10^3 Gauss]	Positive ion density [10^{12} /cm ³]
5	1.7	10.0	1.46	2.3	2.8
10	3.2	11.6	1.60	3.0	4.0
15	5.2	10.9	1.50	4.0	5.0
20	6.8	12.0	1.65	4.0	5.5
25	8.6	10.0	1.38	4.0	4.6
30	10.6	9.3	1.28	4.3	4.6
35	13.0	—	—	—	—

Table 2: Measured negative ion parameters

Gas flow rate [cc/min]	Pressure [10^{-2} torr]	Maximum current [mA]	Maximum current density [10^3 A/m ²]	Critical magnetic field [10^3 Gauss]	Negative ion density [10^{12} /cm ³]
5	1.7	--	--	--	--
10	3.2	1.7	2.3	2.0	0.4
15	5.2	2.4	3.3	3.0	0.8
20	6.8	3.3	4.6	4.3	1.6
25	8.6	3.6	5.0	4.0	1.7
30	10.0	3.4	4.7	4.0	1.6
35	13.0	2.9	4.0	4.5	1.5

Table 3: Measured electron parameters

Gas flow rate	Pressure	Electron current and density	Critical magnetic field	Electron temperature	Electron density
[cc/min]	[10^{-2} torr]	[mA]	[10^3 Gauss]	[eV]	[10^{12} /cm ³]
5	1.7	13.8	2.0	43.8	0.8
10	3.2	10.9	2.8	25.2	1.5
15	5.2	9.4	3.5	17.1	2.4
20	6.8	4.8	—	3.0	3.8
25	8.6	4.3	—	3.5	2.9
30	10.0	3.7	—	3.0	3.0
35	13.0	3.3	—	—	—

For gas rate 5, 10 and 15cc/min. these numbers are the maximum currents. The others are currents at $B=1500$ Gauss.

For gas flow rate 20, 25, 30, and 35 cc/min. No critical magnetic fields.

For gas flow rate 20, 25, 30 and 35 cc/min. $n_e = n^+ - n^-$ where n^+ (positive) and n^- (negative) ion density.

Table 4: Comparison of important physical quantities

	Chen et al. (1962)	Brifford et al. (1963)	Pavlichenko et al. (1964) (1966)	Thomassen (1966) (1968)	Reflex type negative ion source
P [torr]	1.5×10^{-2}	1.4×10^{-2} 5.7×10^{-3}	5×10^{-3} 10^{-3}	3×10^{-3} 10^{-2}	10^{-2}
I [A]	0.4	0.2 0.1	0.3 1.0	0.3 0.3	3
L [cm]	55	14 85	70 100	66 130	6
R [cm]	0.65	0.75 1.5	2.2 1.75	2.05 1.5	0.24
T_e [eV]	3	3 2.5	20 50	— 5	3
T_i [eV]	0.4	0.4 0.4	— 0.5	— —	1.5
n [/cm ³]	3×10^{11}	4.5×10^{11} 10^{11}	5×10^{11} 10^{12}	10^{11} 10^{11}	5×10^{12}

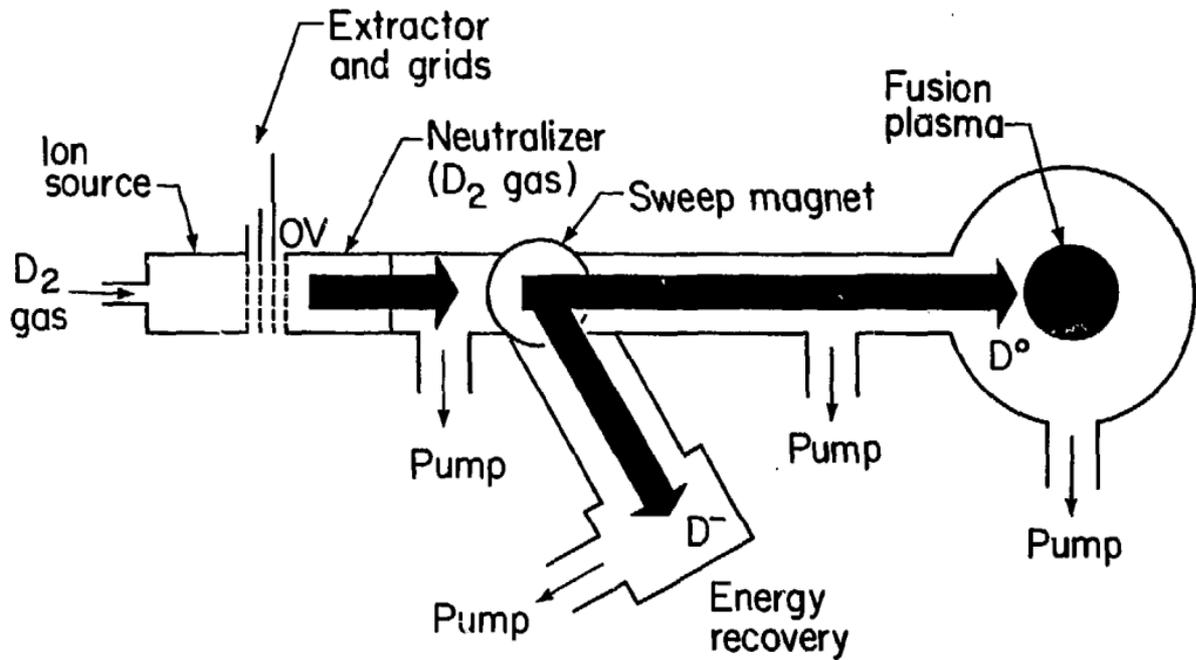
Figure Captions

- Fig. 1-1 Schematic of neutral beam injection system.
- Fig. 1-2 Electron capture cross section σ_{10} and electron loss cross section σ_{01} for hydrogen ion and atom in H_2 gas.
- Fig. 1-3 Neutralization efficiency η for hydrogen and deuterium ions in each gases.
- Fig. 2-1 Rate of dissociative electron attachment to various vibrational states of H_2 : Wadehra⁹ (1979).
- Fig. 2-2 Rate of dissociative electron attachment to various vibrational states of D_2 : Wadehra⁹ (1979).
- Fig. 2-3 Conceptual view of the central part and surrounding part of the reflex-type negative ion source.
- Fig. 3 Potential distribution around the sheath boundary with a transition region, and its effect to charged particles in the reflex-type negative ion source.
- Fig. 5-1 Experimental geometry of the reflex-type negative ion source.
- Fig. 5-2A Experimental arrangement for the reflex-type negative ion source before it is modified.
- Fig. 5-2B Experimental arrangement for the modified reflex-type negative ions source.
- Fig. 5-3 Twice scaled diagram of the original (left) and the modified (right) reflex-type negative ion source.
- Fig. 5-4 Relations between the electric drain current and the gas flow rate for various values of magnetic field for H_2 gas.

- Fig. 5-5 Relations between the extracted positive ion current and the gas flow rate for various values of magnetic field for H_2 gas.
- Fig. 5-6 Relations between the extracted negative ion current and the gas flow rate for various values of magnetic field for H_2 gas.
- Fig. 5-7 Relations between the electron drain current and the magnetic field for various values of gas flow rate for H_2 gas.
- Fig. 5-8 Relations between the extracted positive ion current and the magnetic field for various of gas flow rate for H_2 gas.
- Fig. 5-9 Relations between the extracted negative ion current and the magnetic field for various values of gas flow rate for H_2 gas.
- Fig. 5-10 Relations between the electron temperature and the gas flow rate for H_2 gas.
- Fig. 5-11 Relations between the density of each species and the gas flow rate for H_2 gas.
- Fig. 5-12 Relations between the electron drain current and the magnetic field with changing the bias potential for H_2 gas.
- Fig. 5-13 Relations between the extracted negative ion current and the magnetic field with changing the bias potential for H_2 gas.

- Fig. 5-14A Relations between the extracted positive ion current and the magnetic field with changing the positive bias potential for H_2 gas.
- Fig. 5-14B Relations between the extracted positive ion current and the magnetic field with changing the negative bias potential for H_2 gas.
- Fig. 5-15 Relation between the bias potential and the extracted negative ion current (left), and relation between the bias potential and the density fluctuation amplitude (right) for H_2 gas.
- Fig. 5-16 Relations between the electron drain current and the magnetic field with changing the bias potential for D_2 gas.
- Fig. 5-17 Relations between the extracted negative ion current and the magnetic field with changing the bias potential for D_2 gas.
- Fig. 5-18A Relations between the extracted positive ion current and the magnetic field with changing the positive bias potential for D_2 gas.
- Fig. 5-18B Relations between the extracted positive ion current and the magnetic field with changing the negative bias potential for D_2 gas.
- Fig. A1-2 The diamagnetic and the E cross B drifts in the positive column.
- Fig. A1-2 The diamagnetic and the E cross B drifts in the reflex discharge.

- Fig. A2-1 Potential distribution between two infinite parallel planes, A (negatively charged) and B (positively charged), the plane A emits no electrons and the current from the plane A is space charge limited.
- Fig. A2-2 Potential distribution with its minimum near hot cathode when the emitted electrons have an initial velocity.
- Fig. A2-3 Potential distribution near the sheath boundary obtained from the plasma-sheath equation with simple assumptions.
- Fig. A2-4 Behavior of the function f and g as n_i is varied.



XBL 824-496

Fig. 1-1

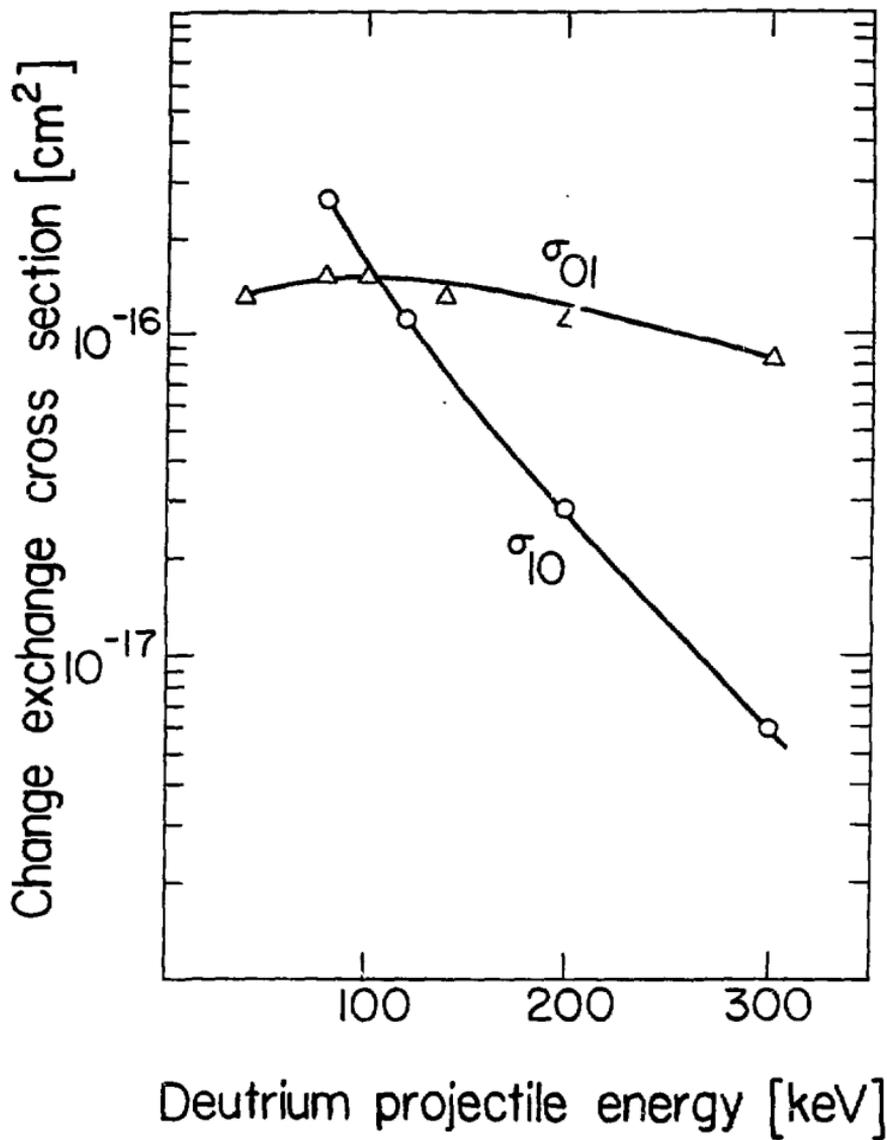


Fig. 1-2

XBL 824-490

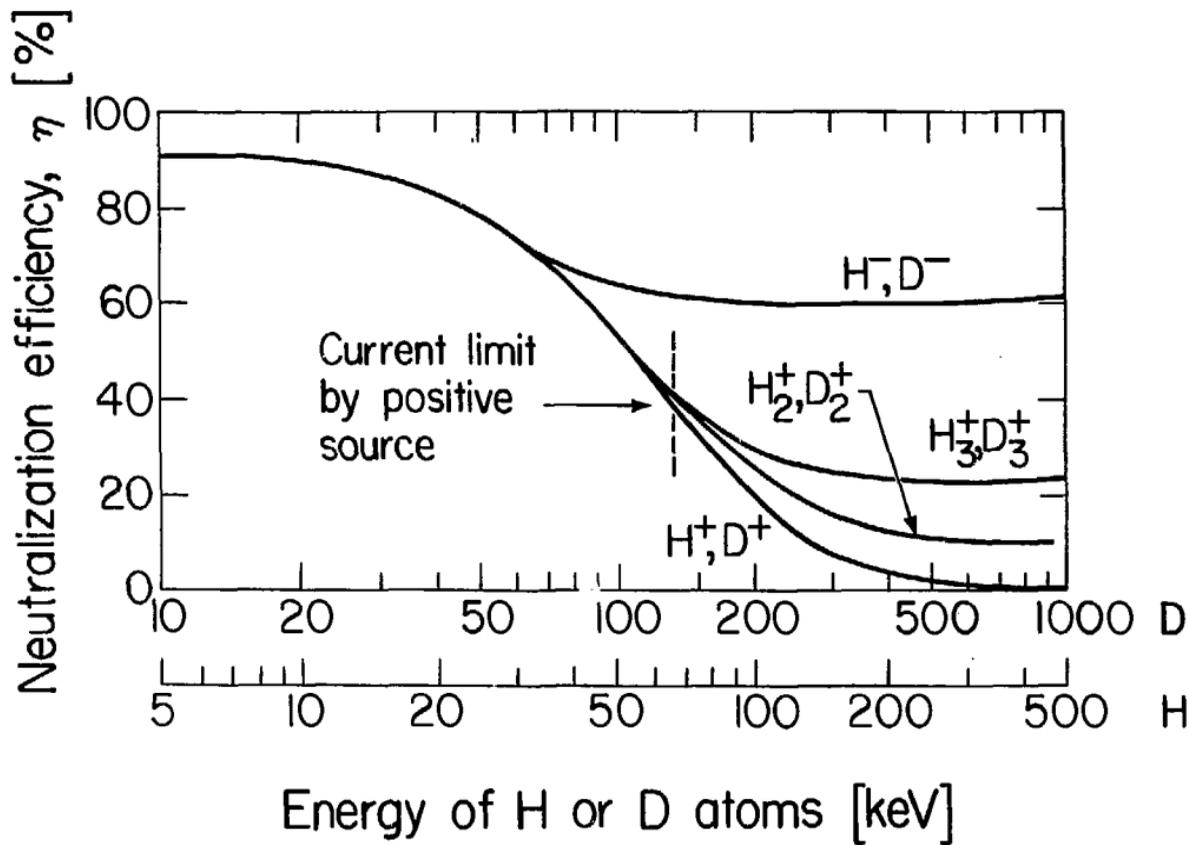


Fig. 1-3

XBL 824-489

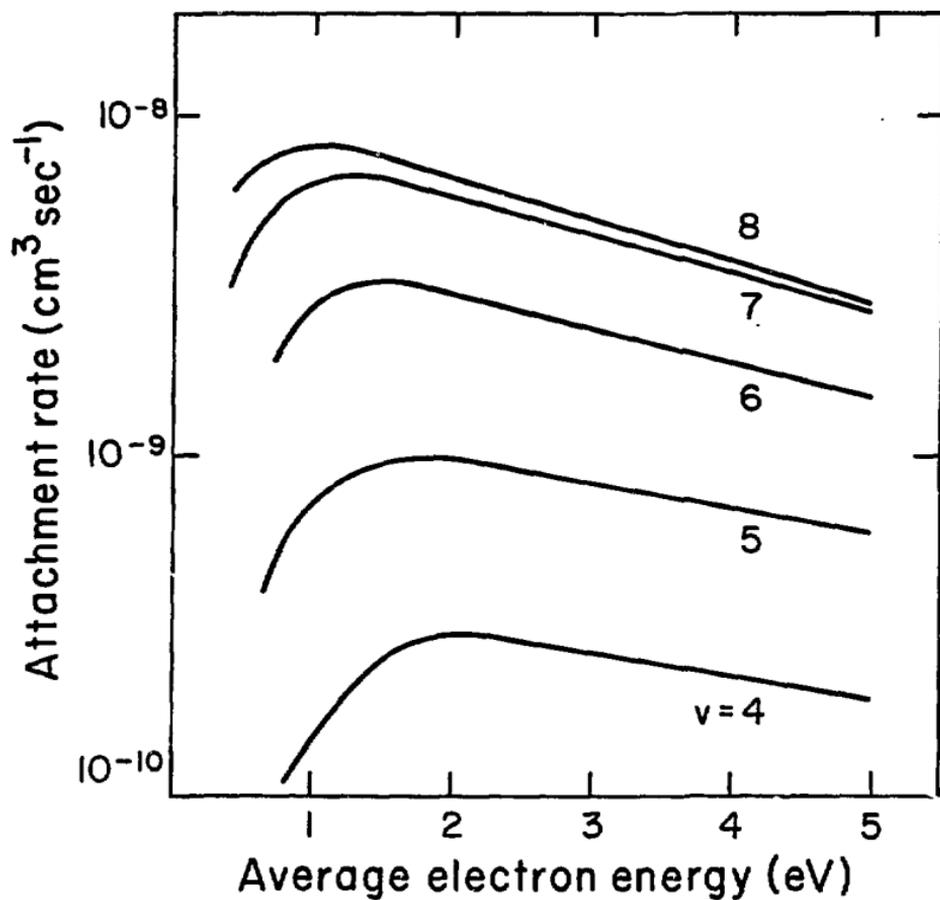


Fig. 2-1

XBL 825-594

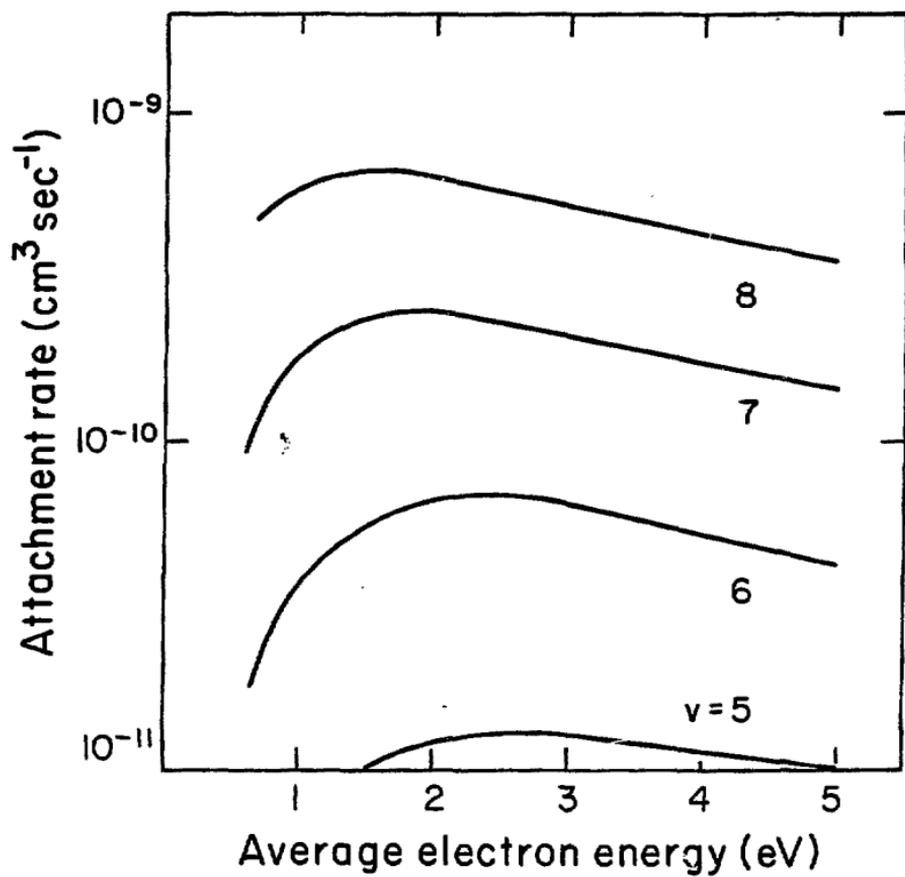
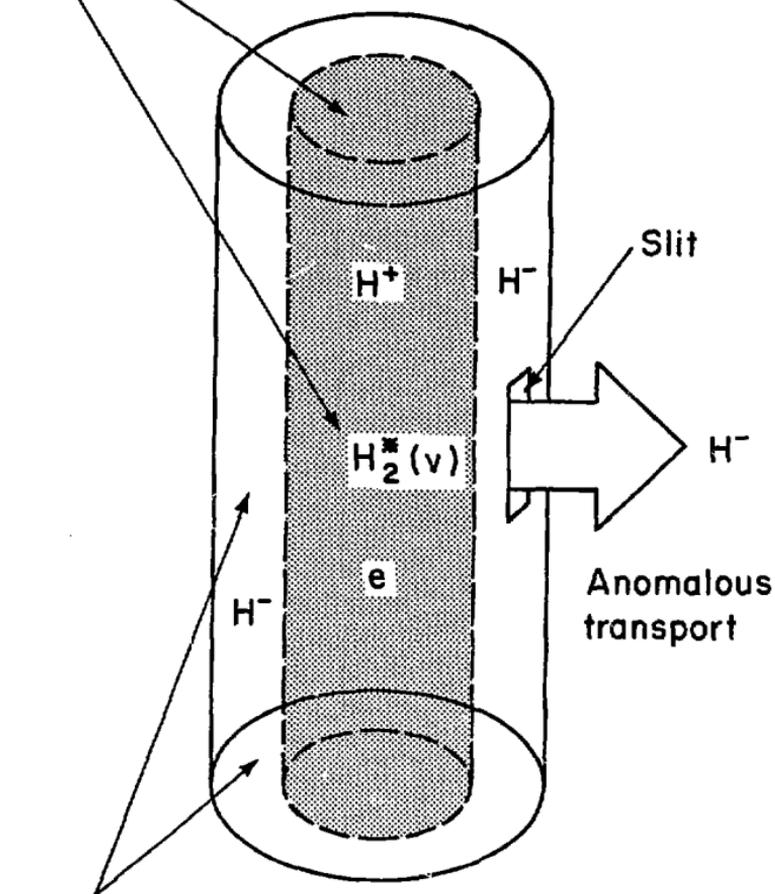
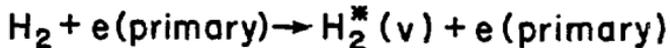
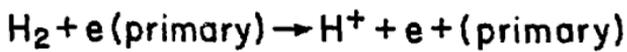


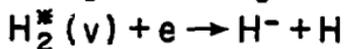
Fig. 2-2

XBL 825-593

Central (hot) region

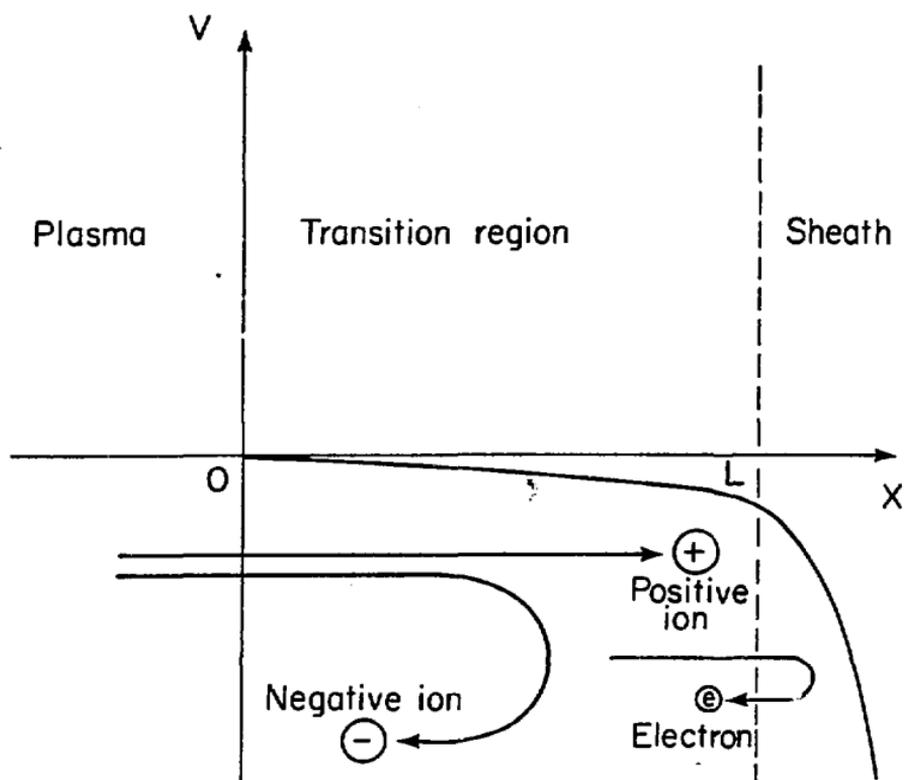


Surrounding (cold) region



XBL 825-592

Fig. 2-3



XBL 824-519

Fig. 3

Reflex type Ion source

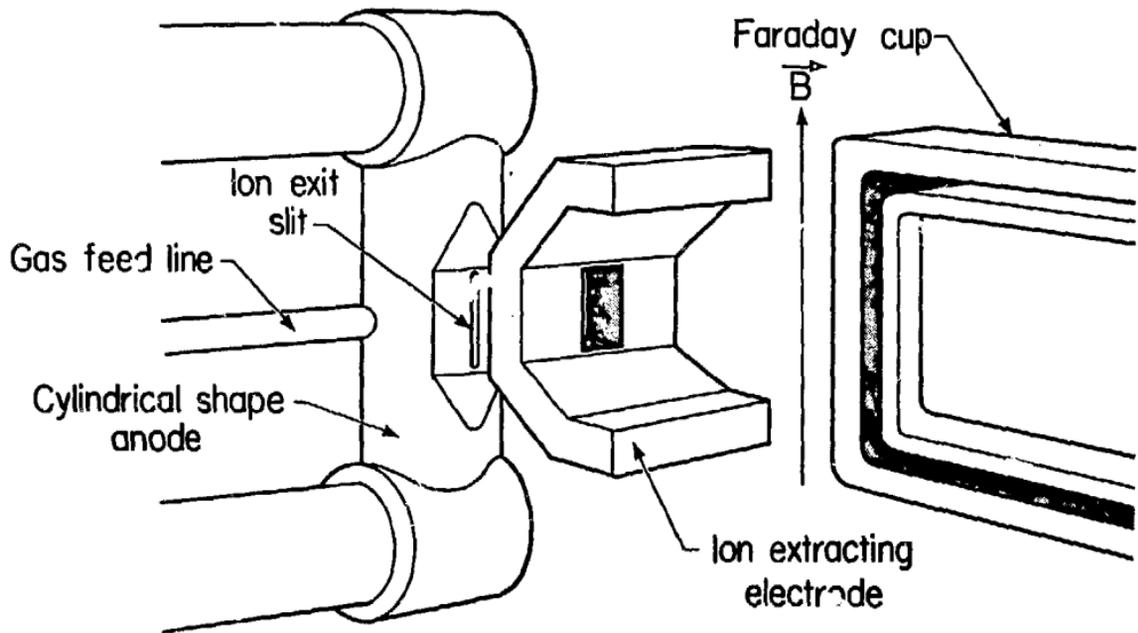
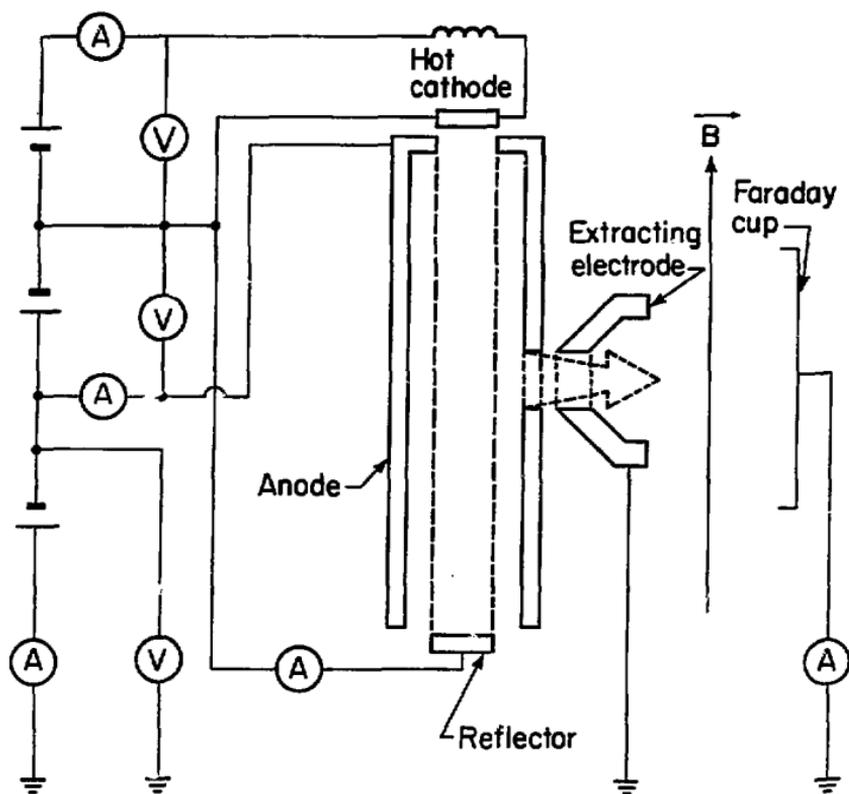


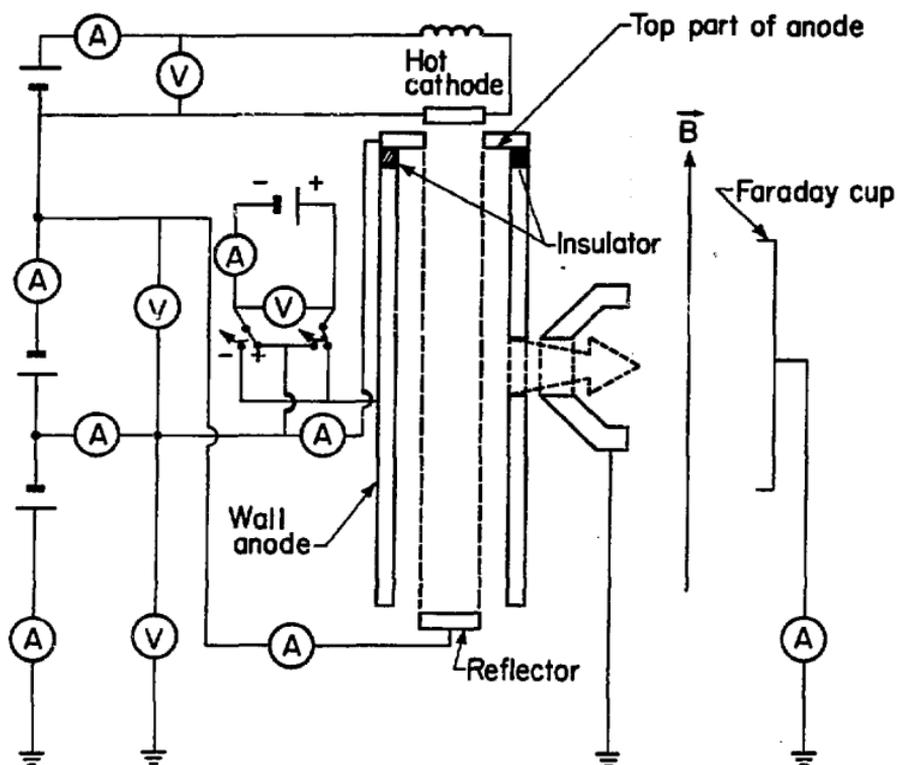
Fig. 5-1

XBL 824-491



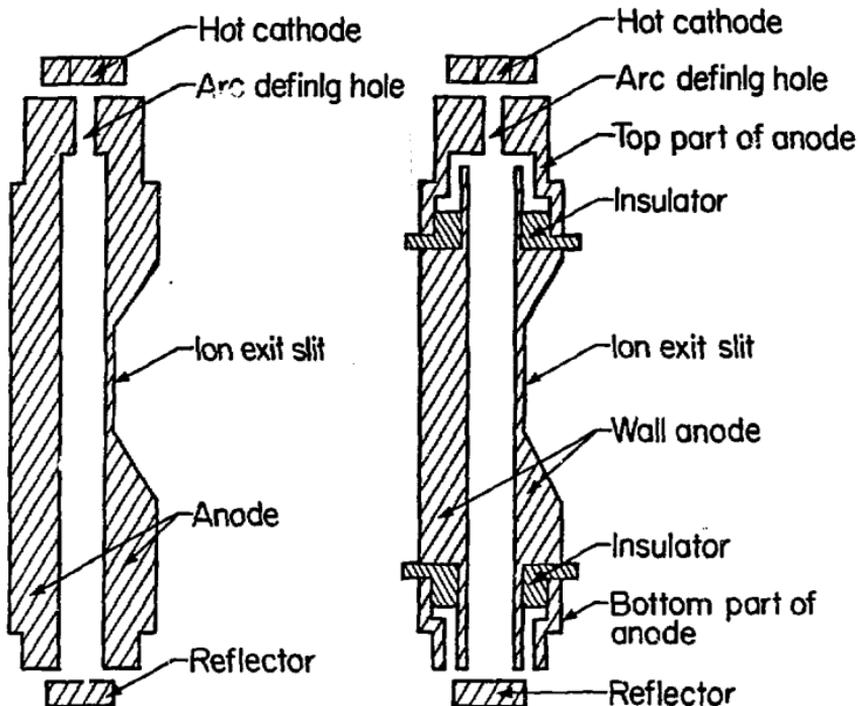
XBL 824-497

Fig. 5-2A



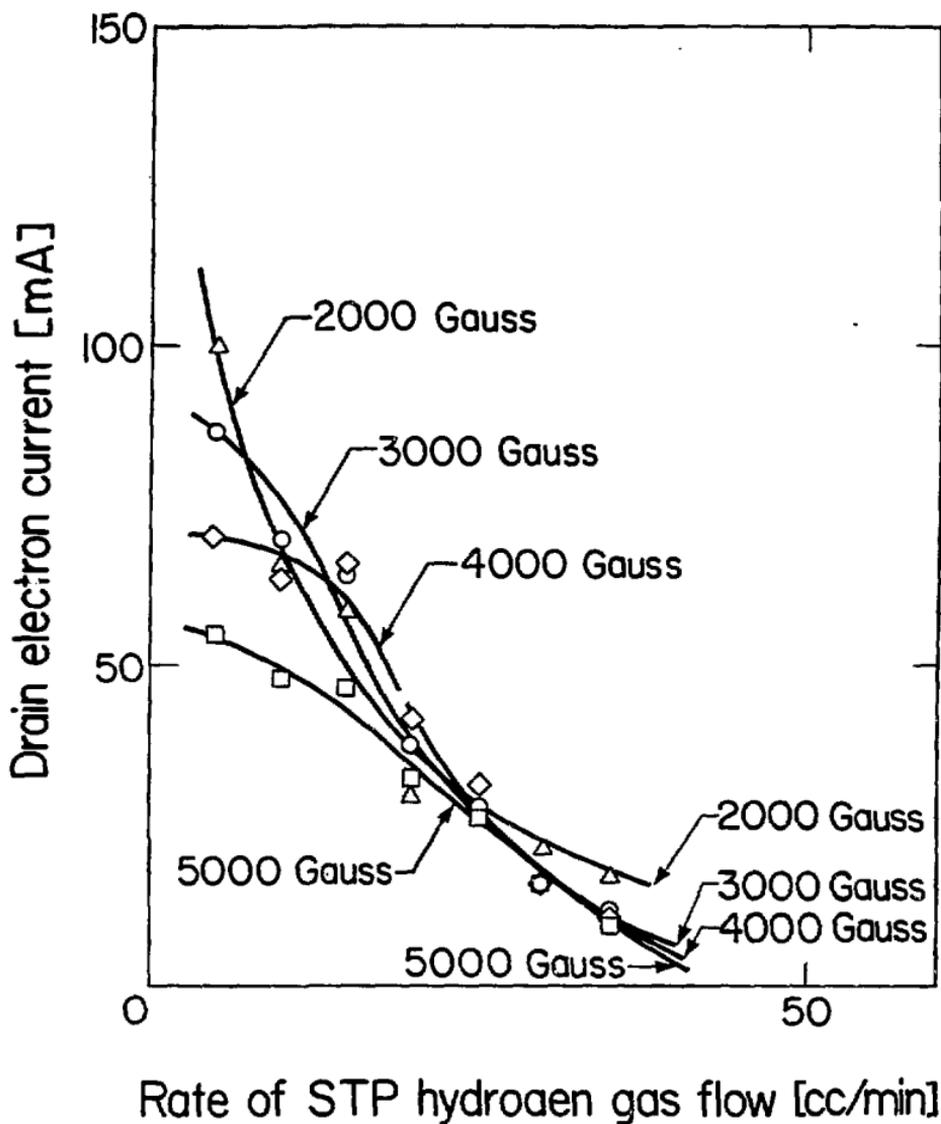
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Fig. 5-2B



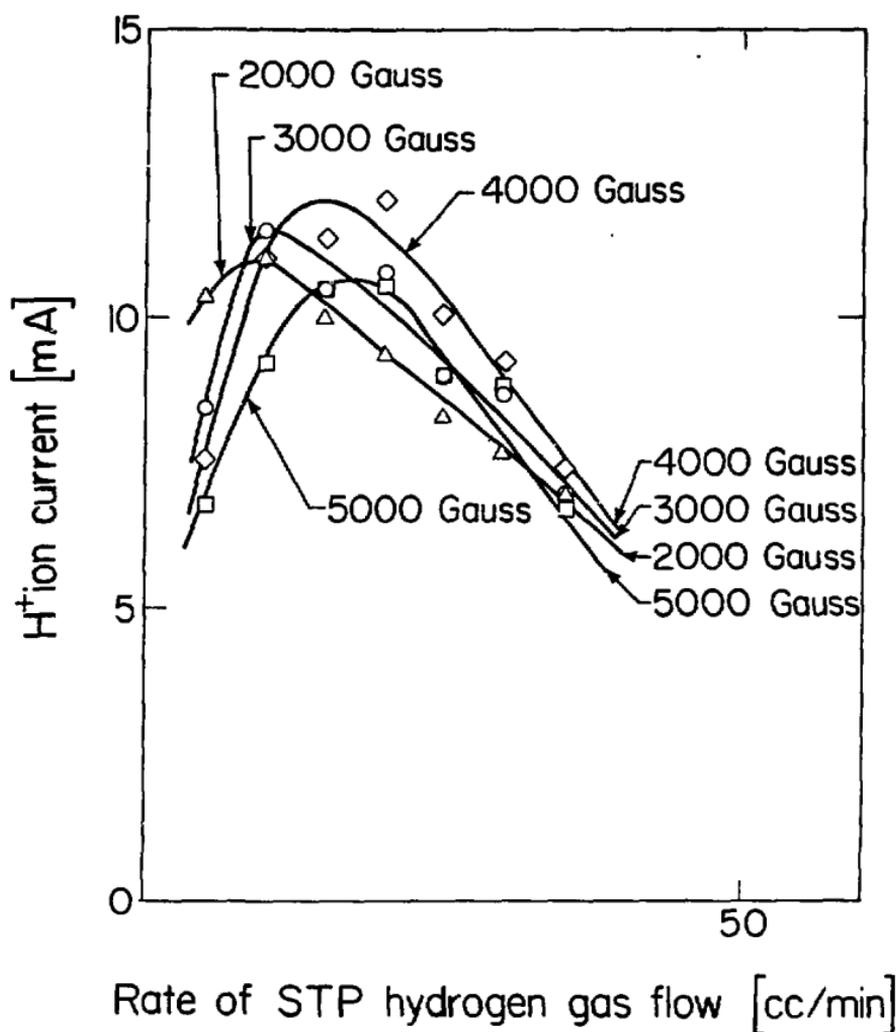
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Fig. 5-3



XBL 824-511

Fig. 5-4



XBL 824-504

Fig. 5-5

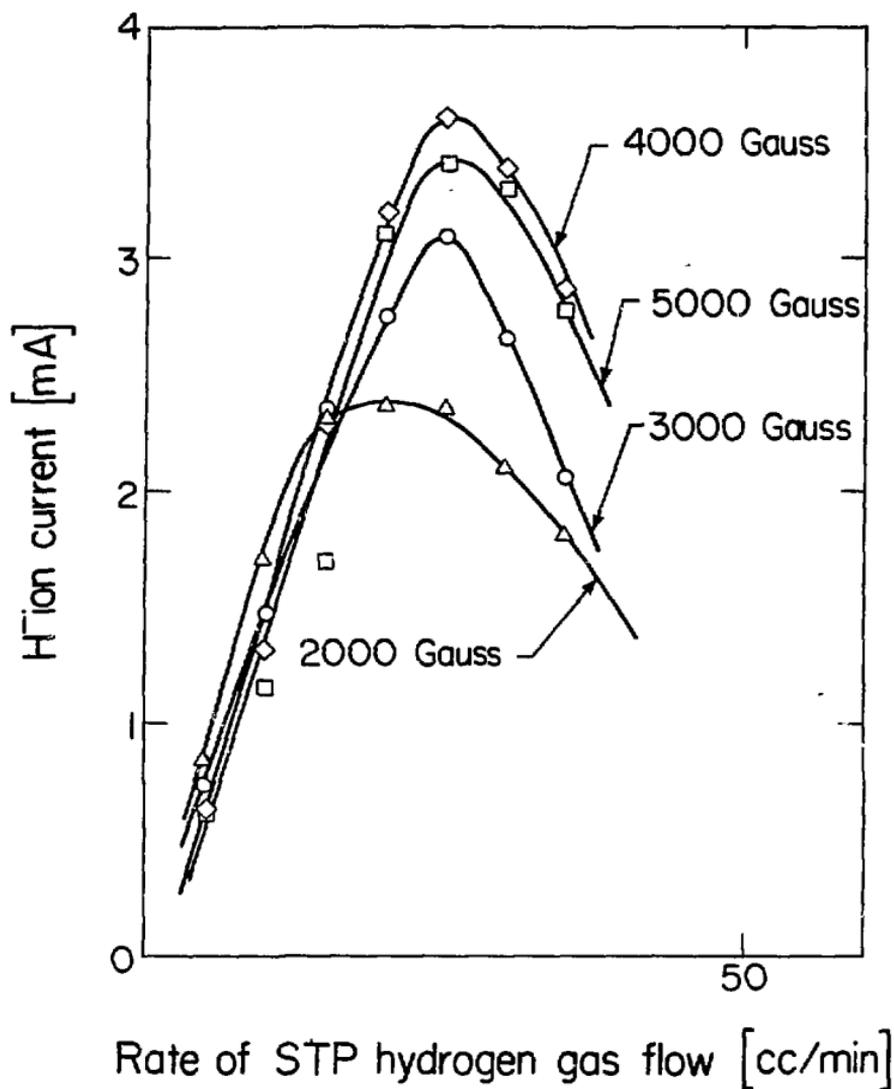
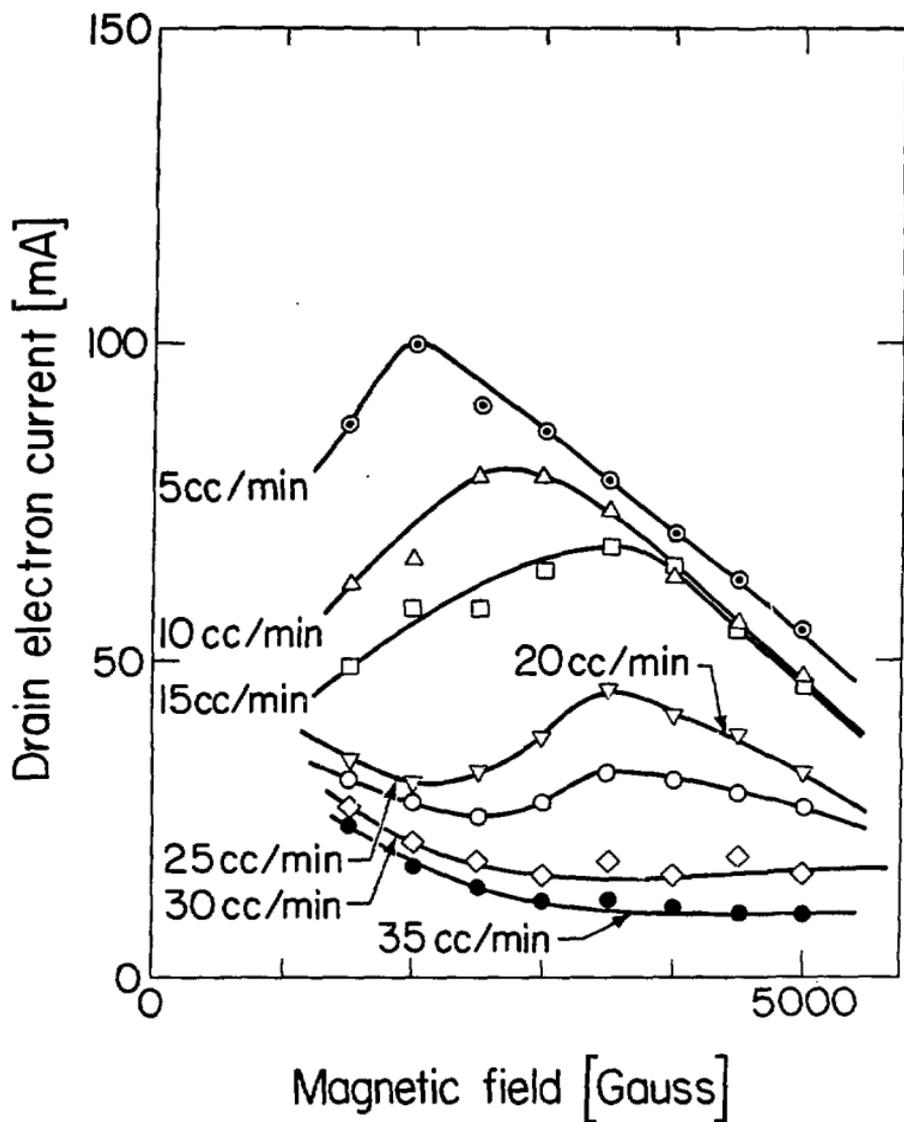


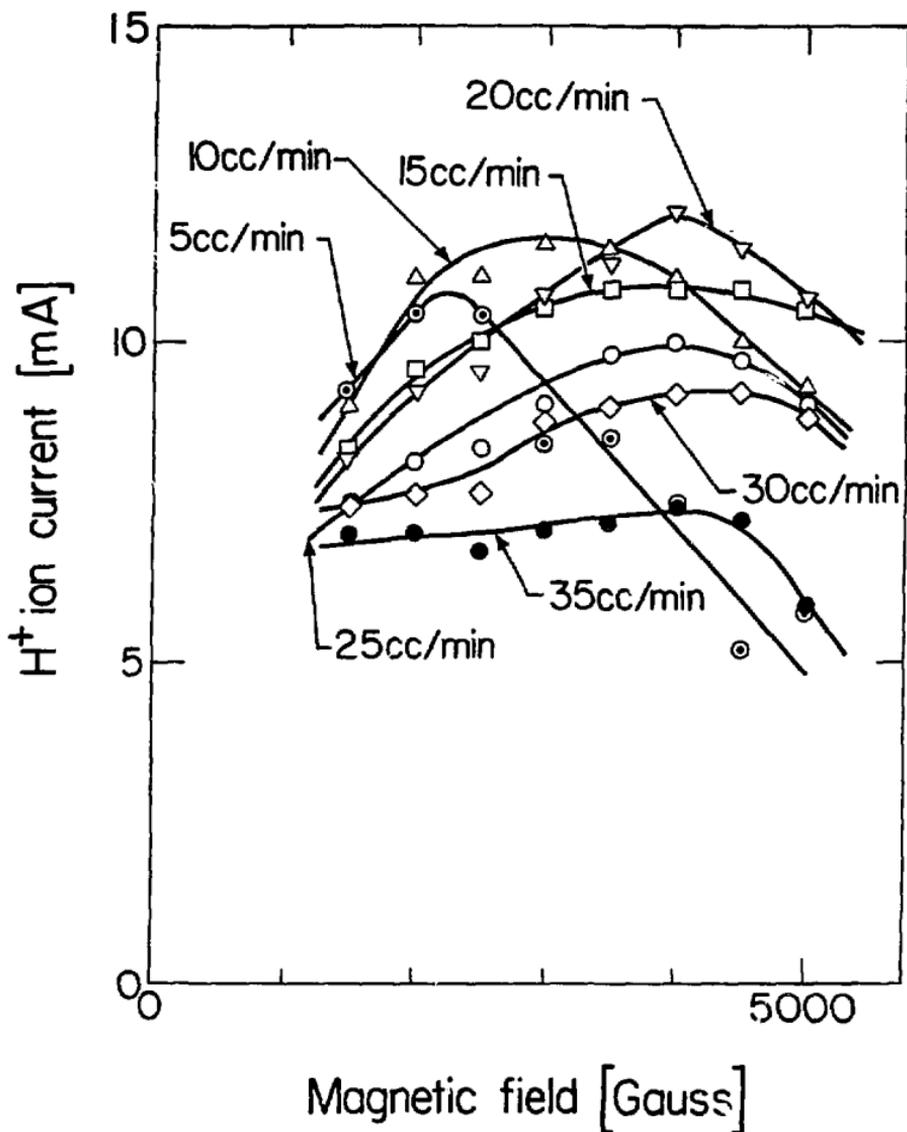
Fig. 5-6

XBL 824-510



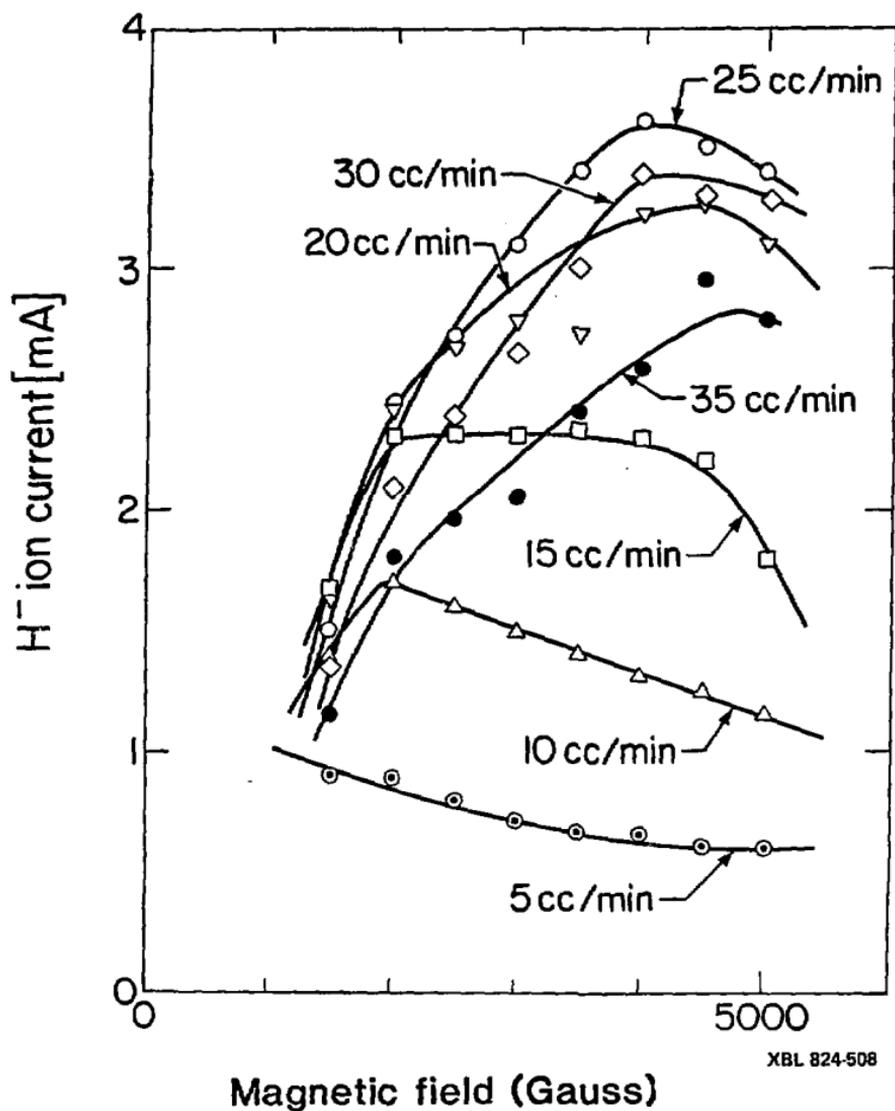
XBL 824-507

Fig. 5-7



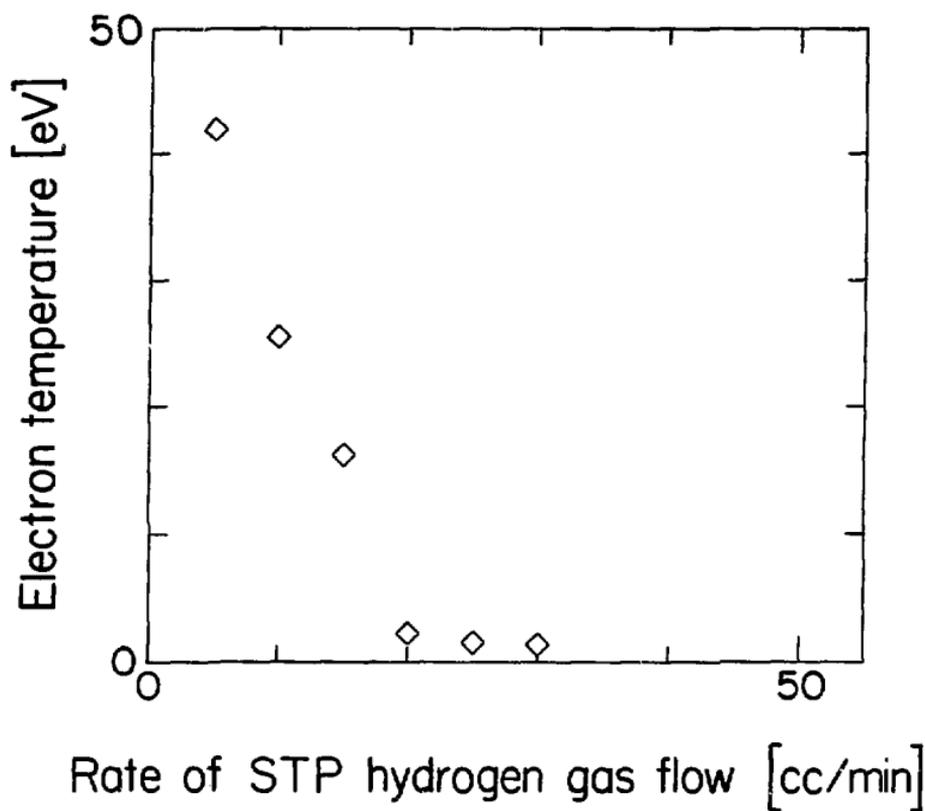
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Fig. 5-8



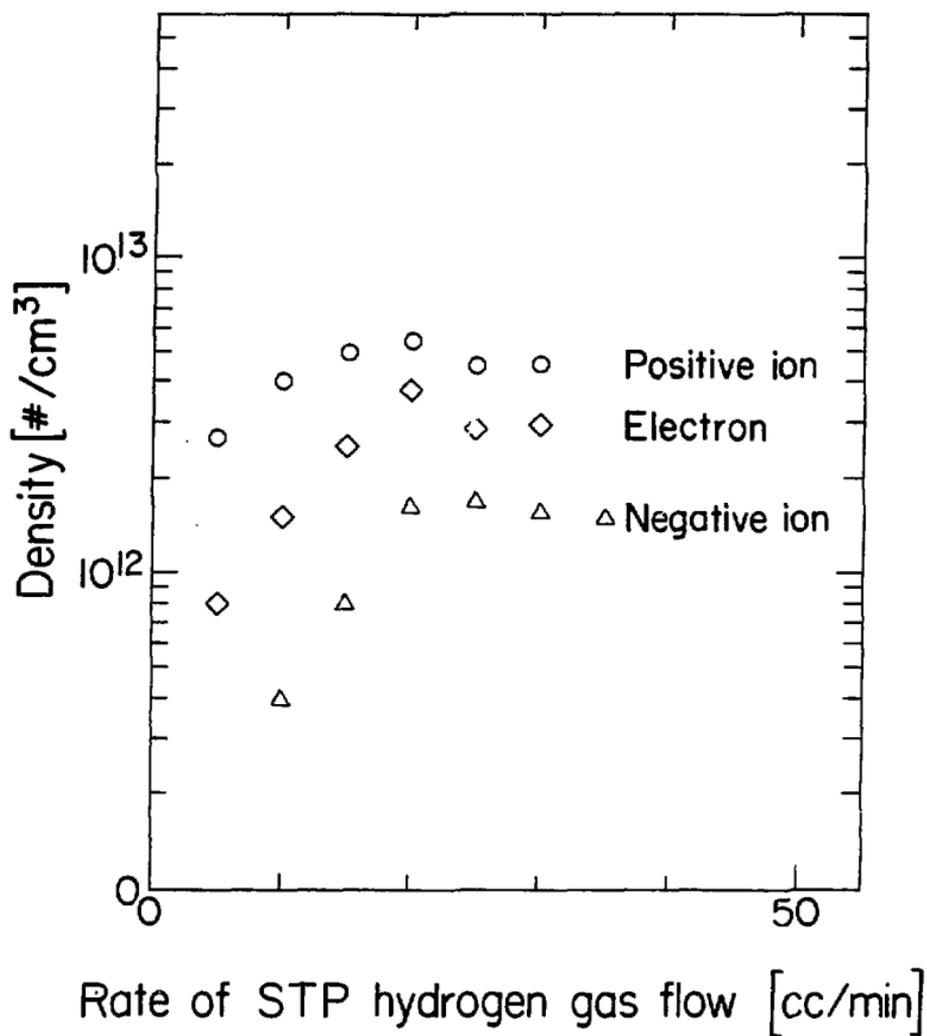
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Fig. 5-9



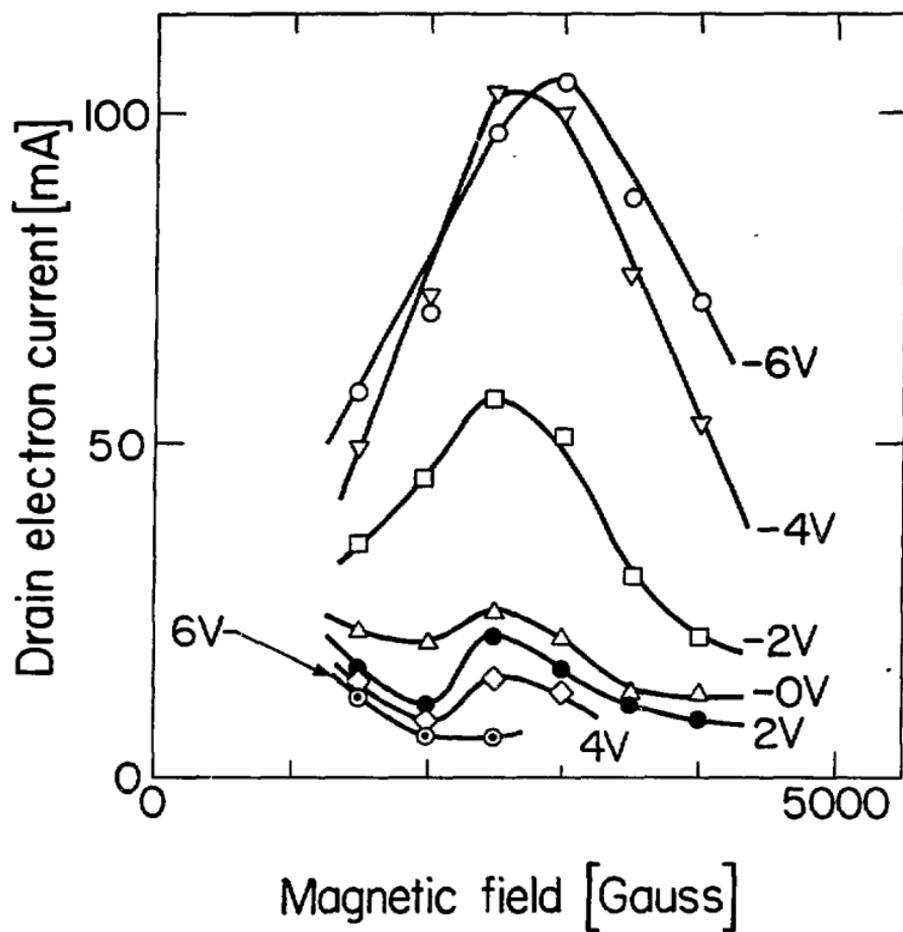
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Fig. 5-10



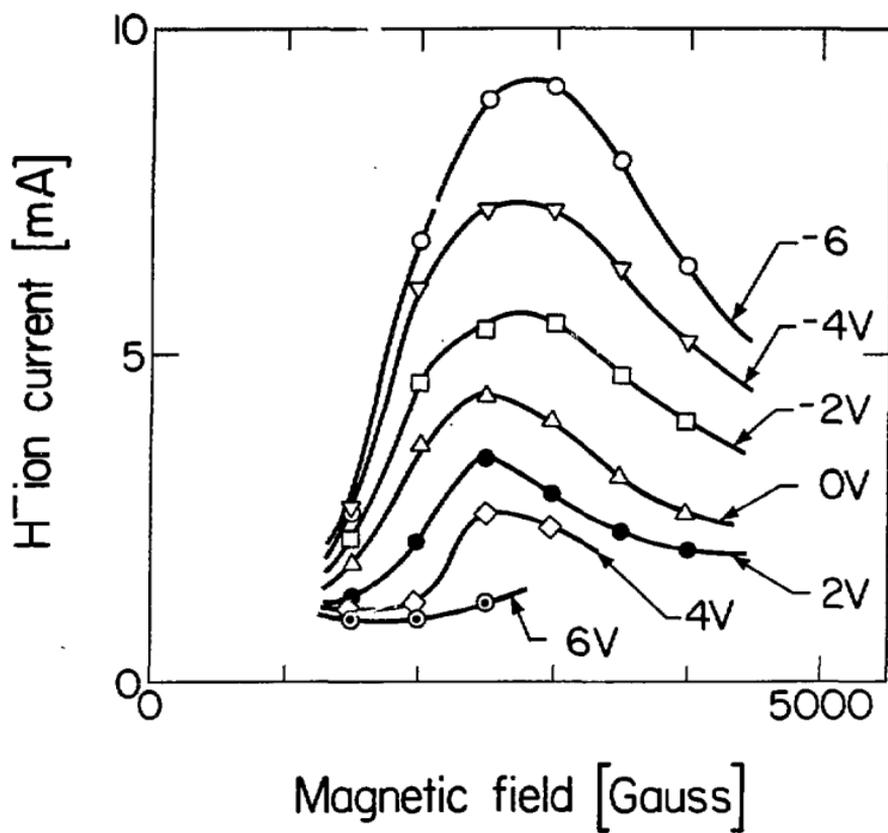
XBL 824-502

Fig. 5-11



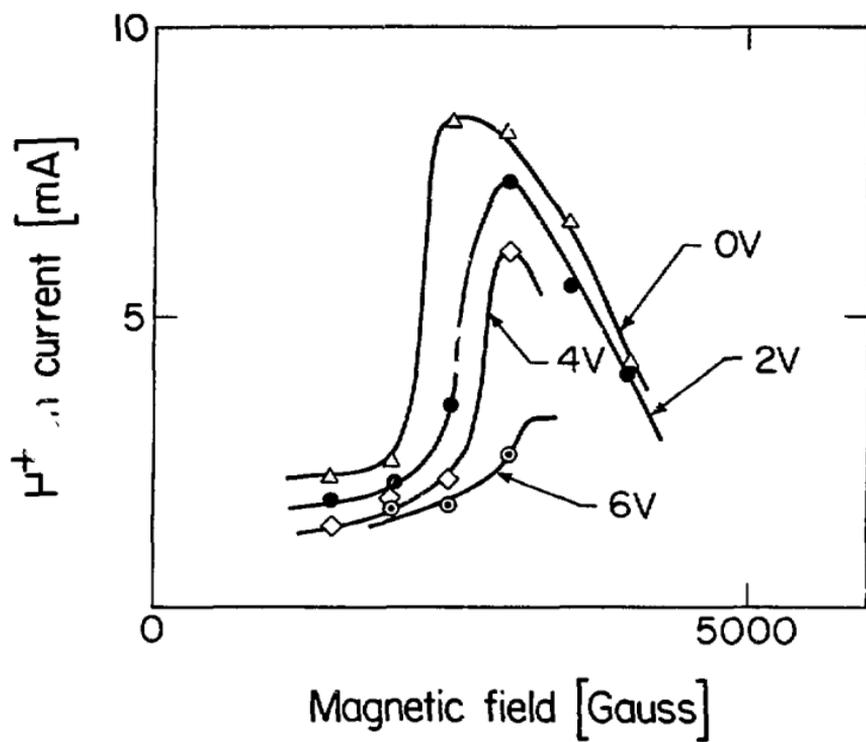
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Fig. 5-12



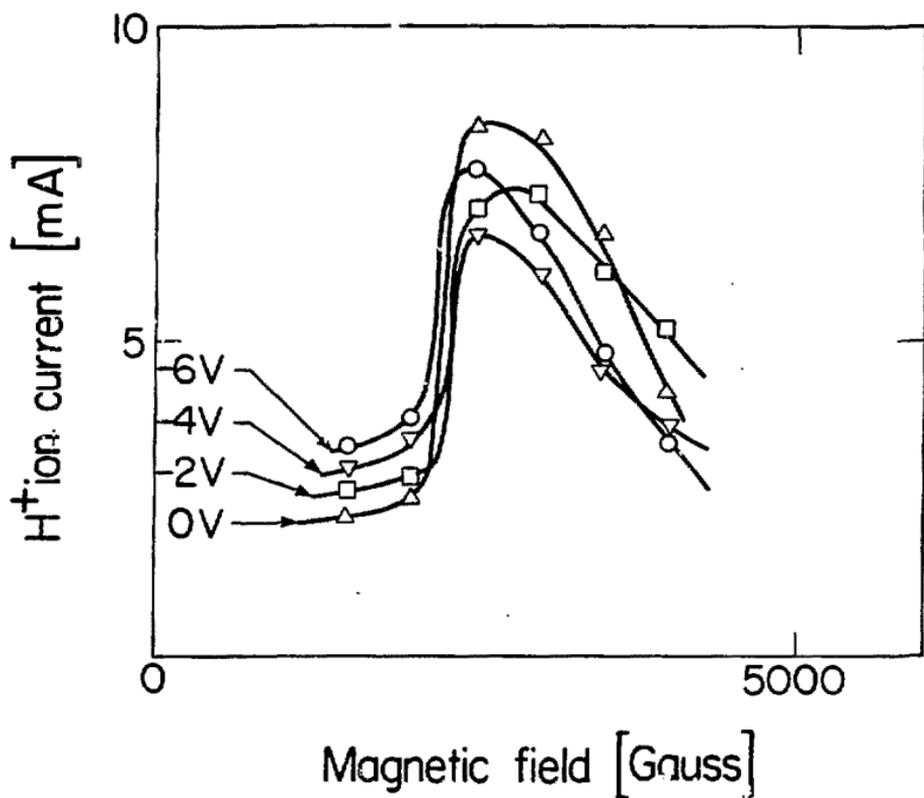
XBL 824-512

Fig. 5-13



XBL 824-513

Fig. 5-14A



XBL 824-518

Fig. 5-14B

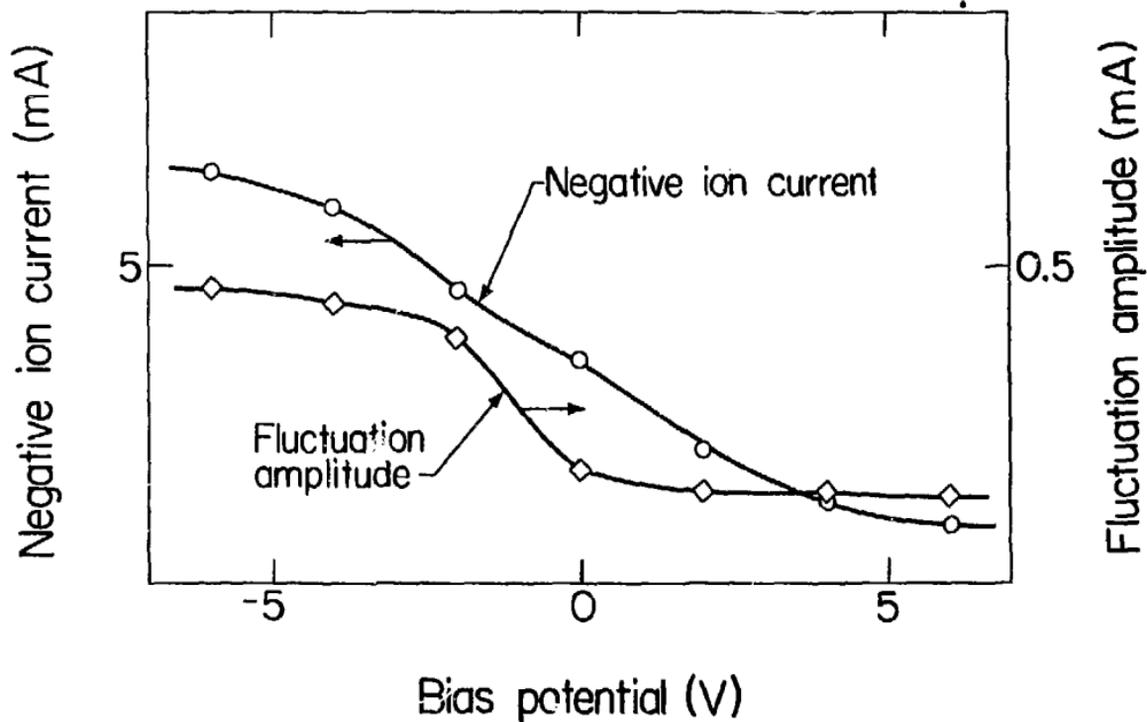
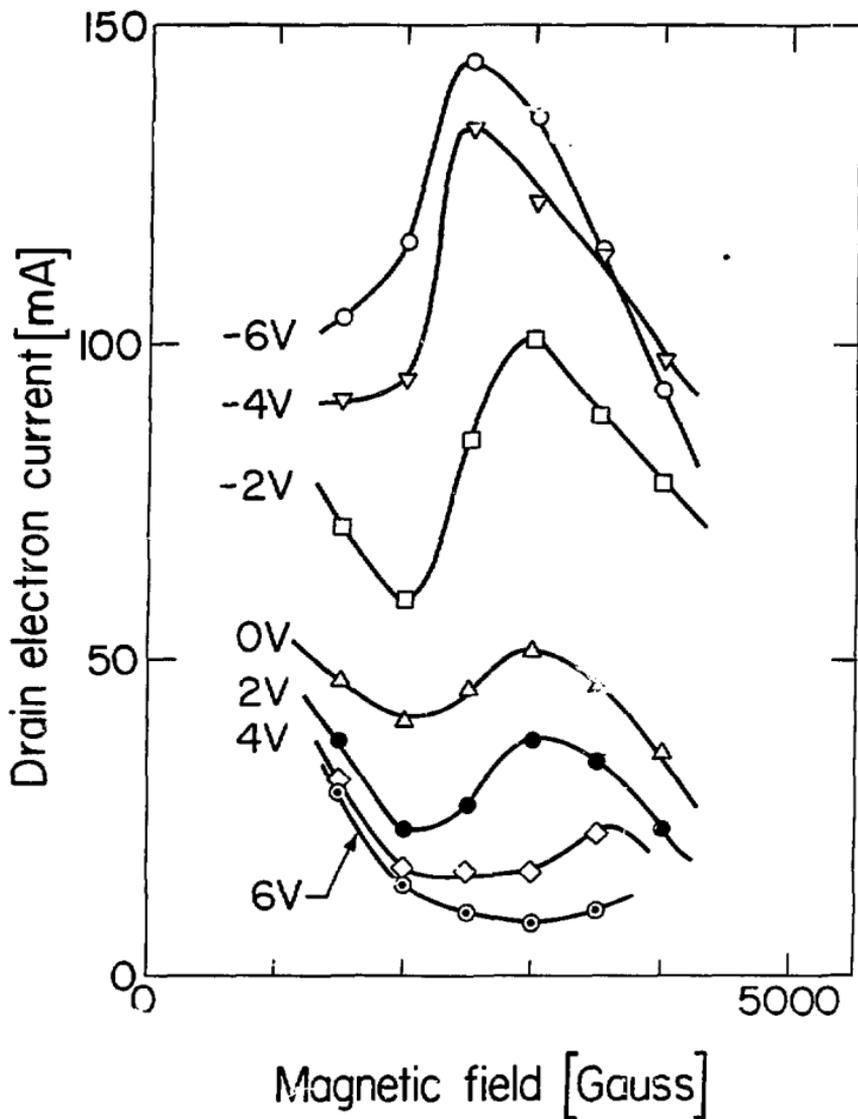


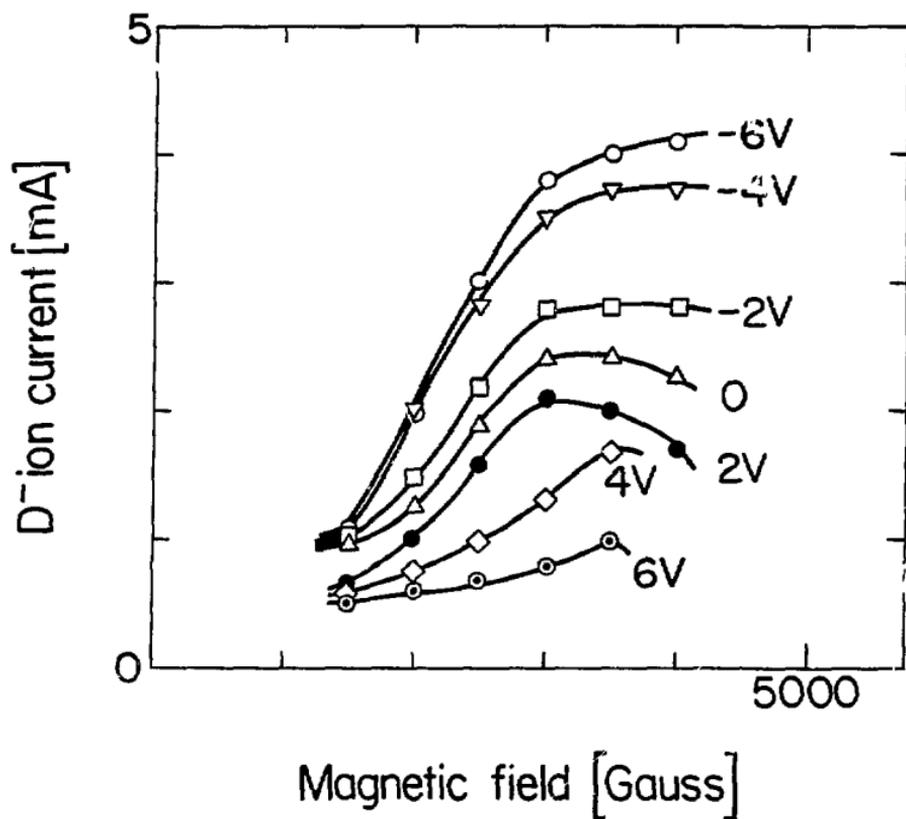
Fig. 5-15

XBL 824-506



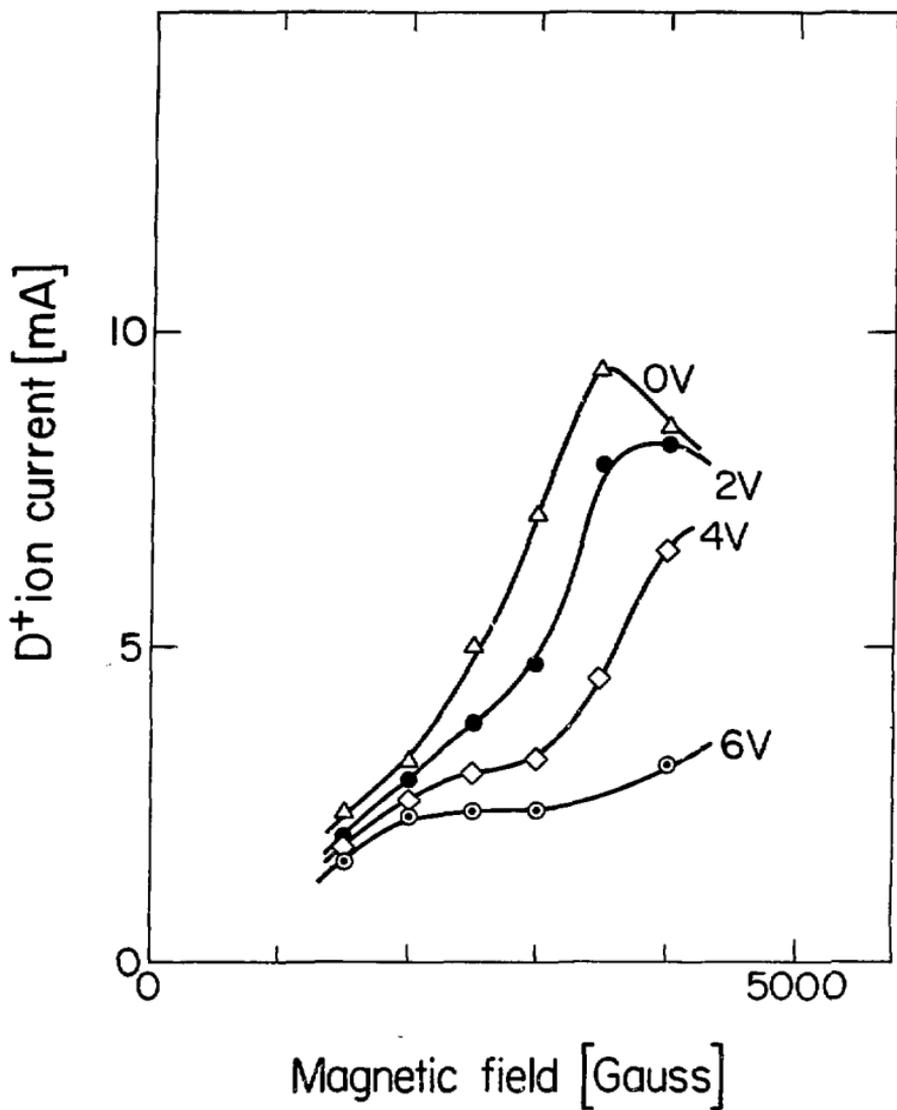
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Fig. 5-16



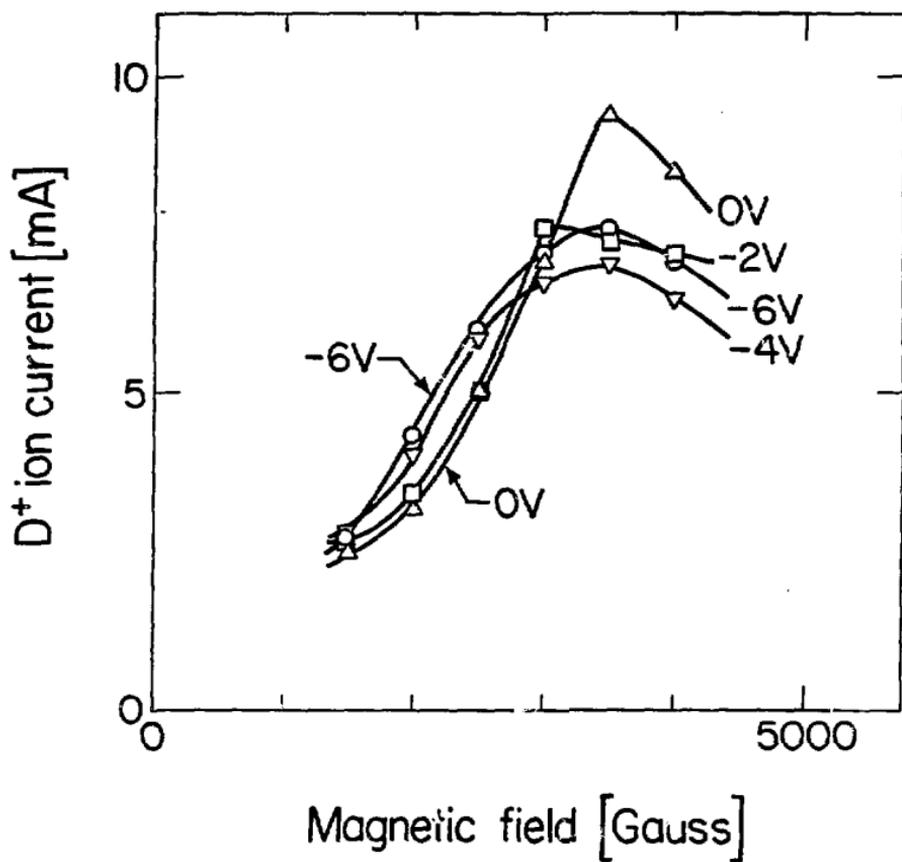
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Fig. 5-17



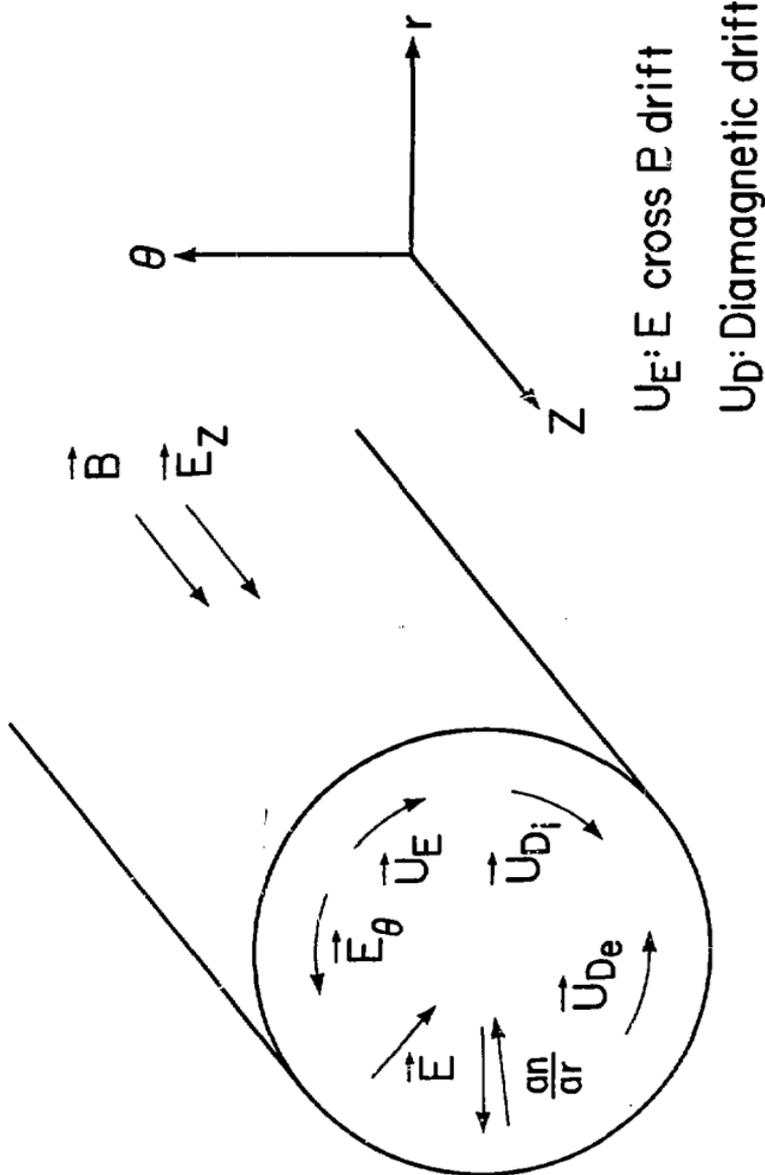
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Fig. 5-18A



XBL 824-514

Fig. 5-18B



XBL 824-492

Fig. A1-1

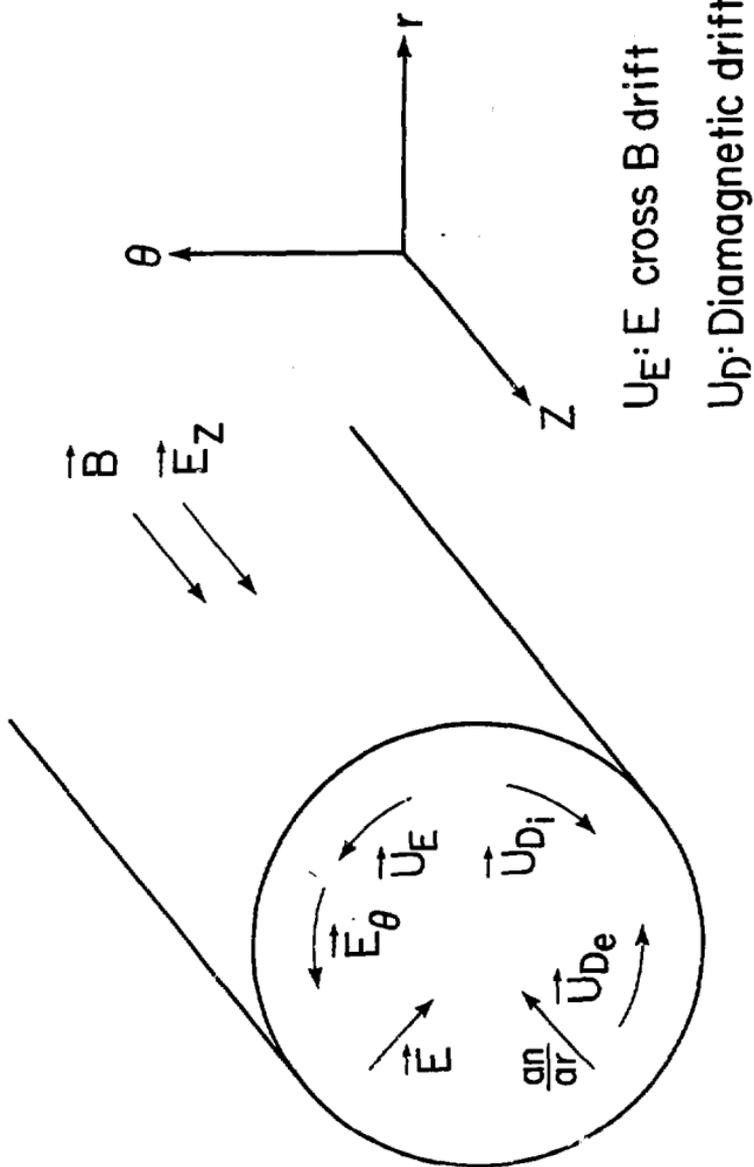
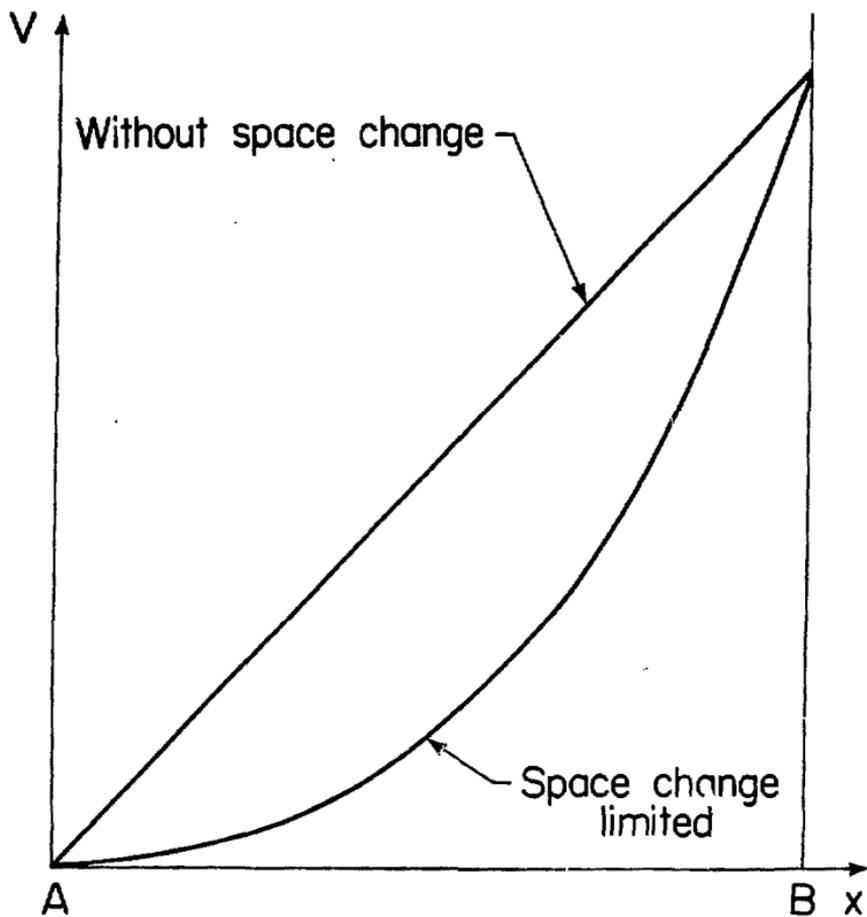
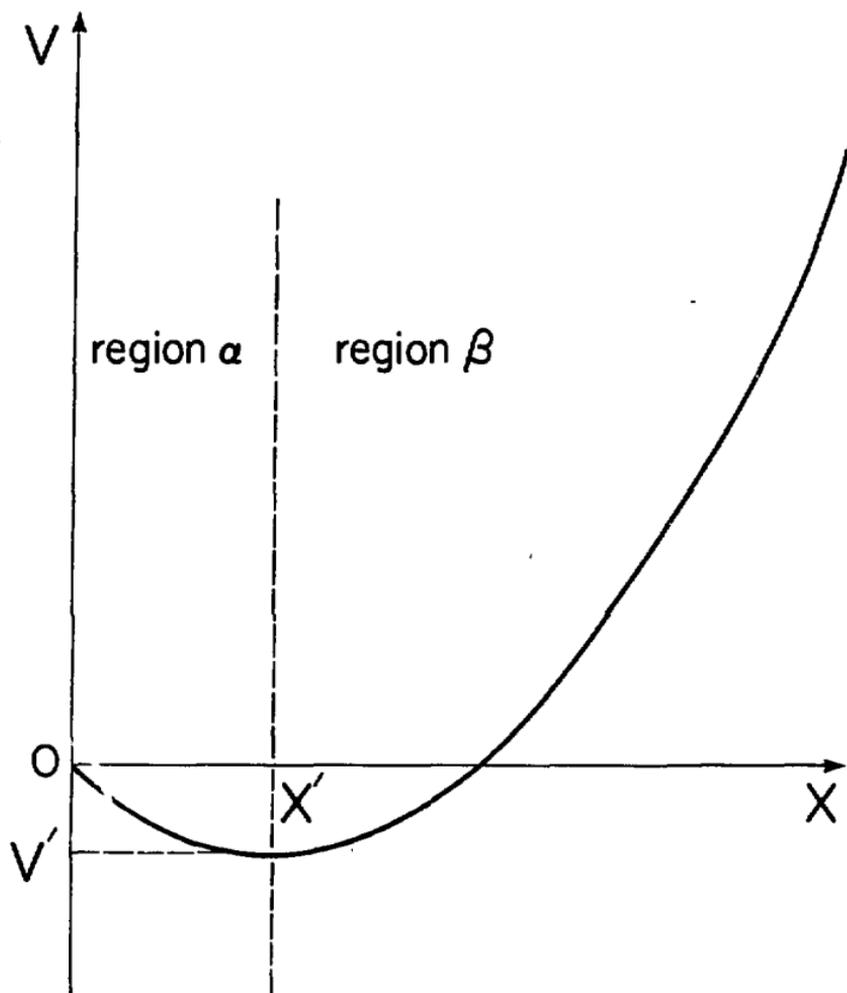


Fig. A1-2



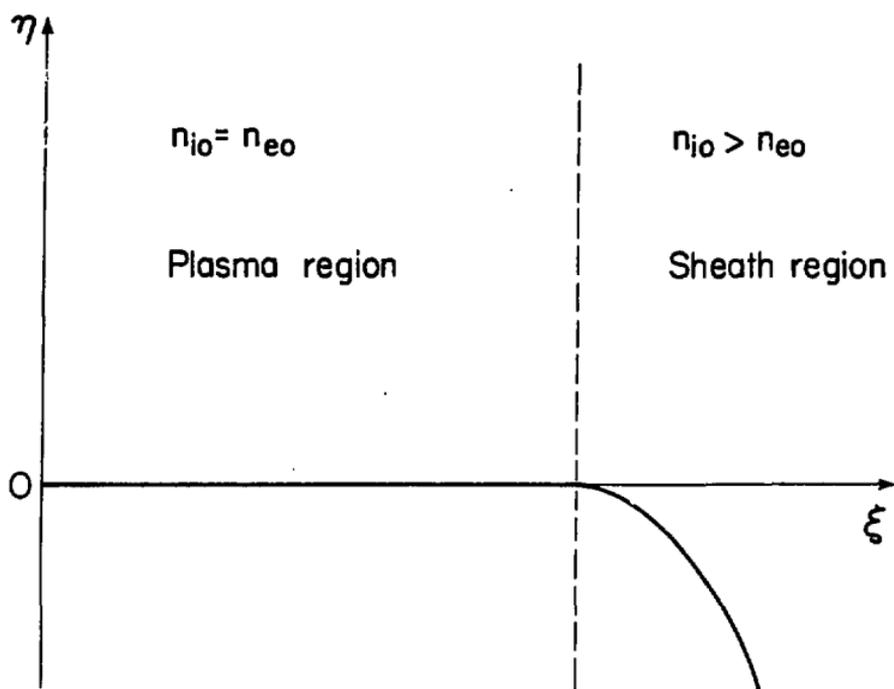
XBL 824-500

Fig. A2-1



XBL 824-498

Fig. A2-2



XBL 824-501

Fig. A2-3

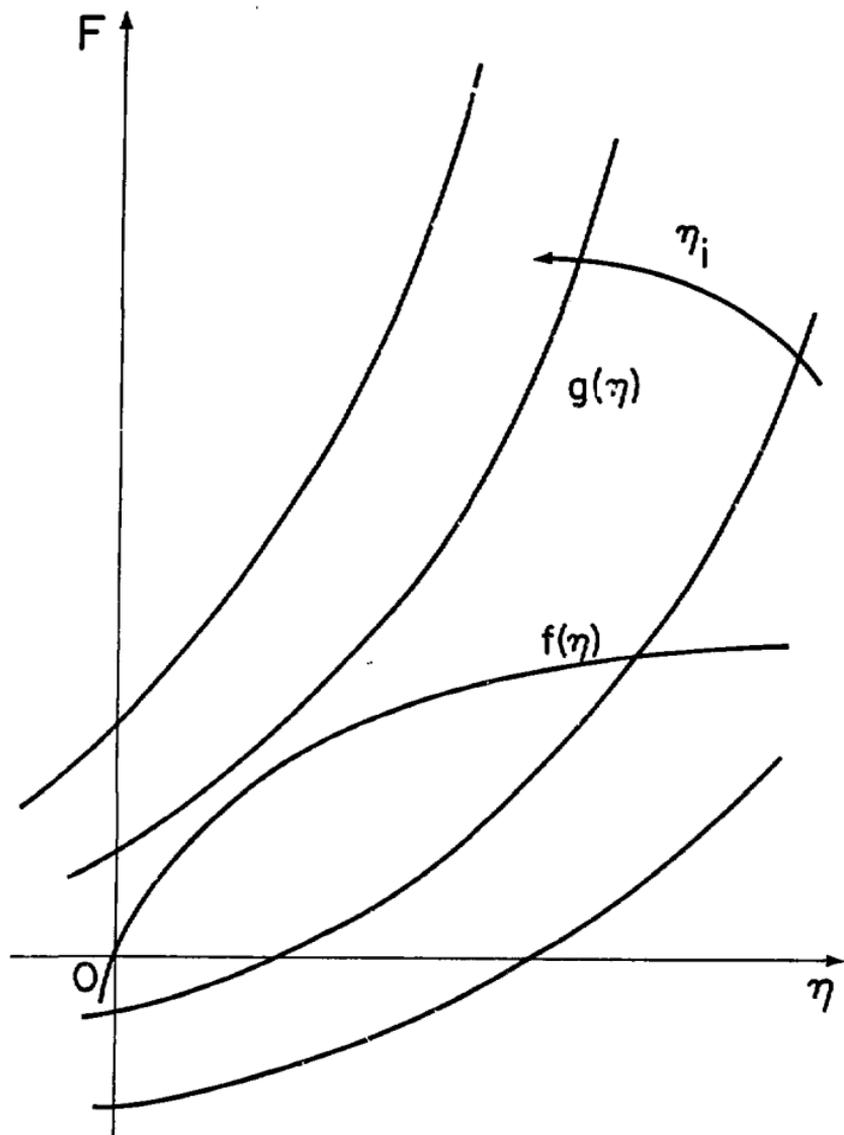


Fig. A2-4

XBL 824-503