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IN A STATISTICAL MODEL FOR NUCLEAR DISASSEMBLY

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Calculation of Complete Multi-Fragment Events in a Statistical Model for Nuclear Disassembly*

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Abstract

A quantitatively useful approximation method is developed for generating complete multi-fragment events in a statistical model for nuclear disassembly. The method consists in factorizing the exclusive microcanonical probability distribution into successively dependent one-fragment inclusive distributions, each of which is approximated by its respective truncated grand canonical equivalent.

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In this Letter we report on the development of a practical method for generating samples of multi-fragment distributions within the framework of a statistic model for nuclear disassembly. While the method is quite general and may have wide applicability, the realm of physics we have in mind is that of medium-energy nuclear collisions.

In such collisions a highly excited transient system may be formed and subsequently disassemble into many nuclear fragments. This problem has been addressed in refs. [1-2], which study the one-fragment inclusive distributions within a statistical model. Recent developments in detection technique demand that the exclusive distributions be considered, and it is of interest to calculate complete events on the basis of a statistical model. This amounts to assuming that all multi-fragment states compatible with the overall conservation laws are equally probable. [In the nuclear problem it may be necessary to divide the collision system (e.g. into participants and spectators) before invoking statistics (see ref. [3]) but this need not concern us here.] The practical task is then to generate a sample of multi-fragment events distributed statistically in accordance with the appropriate exclusive microcanonical distribution function. This undertaking is rather cumbersome, particularly when many different fragment species are included. It is therefore desirable to develop a suitable approximate method. Below we formulate the mathematical task, describe our approximation, and discuss its validity.

In an ideal exclusive measurement complete information is obtained on all the fragments in the final state. A given event, say f, is then characterized by the quantities $\{P_\alpha^{i_\alpha},\ i_\alpha \in (1,n_\alpha)\}_\alpha$. Here n_α is the multiplicity of fragments of the particular species $\alpha = n,p,d,t,\ldots$ These n_α fragments are labeled (arbitrarily) by $i_\alpha \in (1,n_\alpha)$ and their four-momenta are denoted

$P_\alpha^{i_\alpha} = (\vec{P}_\alpha^{i_\alpha}, E_\alpha^{i_\alpha})$. Because of the identity of the fragments within a given species the actual labeling is without physical significance, i.e. f is invariant under permutations of the labels i_α. The total fragment multiplicity of the event f is given by $n_f = \sum_\alpha n_\alpha$.

The set of all such events, the event space $\mathfrak{F} = \{f\}$, has certain algebraic properties, which are of great notational convenience. For example, any two events may be added, $\tilde{f} + \bar{f} = f$. By this we mean that $\tilde{n}_\alpha + \bar{n}_\alpha = n_\alpha \; \forall\alpha$ and, furthermore, that $\{\tilde{P}_\alpha^{i}, \; i_\alpha \in (1,\tilde{n}_\alpha)\} \cup \{\bar{P}_\alpha^{i_\alpha}, \; i_\alpha \in (1,\bar{n}_\alpha)\} = \{P_\alpha^{i_\alpha}, \; i_\alpha \in (1,n_\alpha)\} \; \forall\alpha$. We note that the null event, $f = 0$, which has $n_\alpha = 0 \; \forall\alpha$, is the neutral element under addition. The events with unit total multiplicity are elementary objects in the event space: any event f can be decomposed in terms of one-fragment events: $f = \Sigma f_k$ where $k = 1,\ldots,n_f)$ and $n_{f_k} = 1 \; \forall k$. It is also convenient that some events can be compared: we shall write $\tilde{f} \leqslant f$ (or $f \geqslant \tilde{f}$) iff $\exists \bar{f}$: $\tilde{f} + \bar{f} = f$, i.e. the event f encompasses the event $\tilde{f}$.

In the statistical model, the disassembly depends only on the conserved quantities of the system, which in the present case are the total baryon number A_0, the total charge number Z_0, and the total four-momentum $P_0 = (\vec{P}_0, E_0)$. The disassembling system (which shall often be referred to as the <u>source</u>) is thus characterized by the quantities $A_0 Z_0 P_0$, denoted collectively by i_0 for notational convenience, and we seek the probability distribution $p(i_0|f)$ for arriving at various final states f. The statistical assumption is that all final states compatible with the conservation laws are equally probable. Hence

$$p(i_0|f) = \mathfrak{I}(i_0)^{-1} \; \delta(i_0 - i[f]) \tag{1}$$

where $i[f] = A_f Z_f P_f$ are the values of the conserved quantities for the specified final state f. The normalization constant is determined from the condition that p be normalized,

$$\mathcal{J}(i_0) = \sum_f \delta(i_0 - i[f]) \tag{2}$$

We note that the event space $\mathcal{F}$ consists of disjoint parts, $\mathcal{F} = \cup\, \mathcal{F}_{i_0}$, where $\mathcal{F}_{i_0} = \{f \mid i[f] = i_0\}$ is the part accessible for the specified value i_0.

The exclusive distribution p pertains to the situation where the specification of the final state is complete, corresponding to an exclusive measurement. When only partial specification of the final state is made, as is most often the case, the relevant quantity is the corresponding inclusive distribution $\hat{p}$. This quantity can be obtained from the exclusive distribution by integrating over the unobserved quantities. In particular, when the partial measurement is such that complete information is obtained for some of the fragments, the inclusive distribution is given by

$$\hat{p}(i_0|\tilde{f}) = \sum_{f \succcurlyeq \tilde{f}} p(i_0|f) \tag{3}$$

Here the observed partial event is denoted by $\tilde{f}$ and the summation is over all final states f that encompass the specified observation $\tilde{f}$. It is simple to express the inclusive distribution in terms of the phase space integrals (2):

$$\begin{aligned} \hat{p}(i_0|\tilde{f}) &= \sum_{\bar{f}} p(i_0|\tilde{f} + \bar{f}) \\ &= \mathcal{J}(i_0)^{-1} \sum_{\bar{f}} \delta(i_0 - i[\tilde{f}] - i[\bar{f}]) \\ &= \mathcal{J}(i_0)^{-1}\, \mathcal{J}(i_0 - i[\tilde{f}]) \end{aligned} \tag{4}$$

i.e. the inclusive probability for the partial event $\tilde{f}$ is equal to the corresponding complementary phase space integral $\mathcal{J}(i_0 - i[\tilde{f}])$ divided by the total phase space integral $\mathcal{J}(i_0)$.

By combination of (2) and (4) it is possible to factorize $p(i_0|f)$ into simpler quantities:

$$p(i_o|f) = \mathfrak{I}(i_o)^{-1}\ \delta(i_o - i[\tilde{f}] - i[\bar{f}])$$

$$= \mathfrak{I}(i_o)^{-1}\ \mathfrak{I}(i_o - i[\tilde{f}])\ \mathfrak{I}(i_o - i[\tilde{f}])^{-1}\ \delta((i_o - i[\tilde{f}]) - i[\bar{f}]) \qquad (5)$$

$$= \hat{p}(i_o|\tilde{f})\ p(i_o - i[\tilde{f}]|\ \bar{f})$$

where $\tilde{f} + \bar{f} = f$. This relation expresses the fact that the exclusive probability for obtaining the event f is equal to the inclusive probability for obtaining a part of the event, $\tilde{f} \leqslant f$, times the exclusive probability for obtaining the complementary event $\bar{f} = f - \tilde{f}$, given that $\tilde{f}$ has already been obtained.

By repeated use of the above relation (5), it is possible to factorize the exclusive probability p into inclusive probabilities. In particular, by decomposing the specified final multi-fragment event f in terms of elemenary one-fragment events f_k, $p(i_o|f)$ can be factorized into one-fragment inclusive distributions:

$$p(i_o|f = \sum_{k=1}^{n_f} f_k) = \hat{p}(i_o|f_1)\ p(i_1|\sum_{k=2}^{n_f} f_k) = \ldots$$

$$= \prod_{k=1}^{n_f} \hat{p}(i_{k-1}\ |f_k) \cdot p(i_{n_f}\ |0) \qquad (6)$$

Here we have defined $i_k \equiv i_{k-1} - i[f_k]$ for $k \in (1,n_f)$. The exclusive factor $p(i_{n_f}|0)$ vanishes unless the quantities specified by i_{n_f} all vanish, thus guaranteeing that the event f is in fact accessible by the disassembling system characterized by i_o.

The factorization (6) of the exclusive multi-fragment distribution p into inclusive one-fragment distributions is particularly convenient when one seeks to generate a statistical representation of p, i.e. a sample {f} of

multi-fragment events which are statistically distributed in event space according to the probability density $p(i_0|f)$. To accomplish this task, one may proceed as follows.

Each event f is considered as a sum of elementary one-fragment events, $f = \Sigma f_k$. To generate an event f, one first makes a random selection of the term f_1 on the basis of the inclusive probability distribution $\hat{p}(i_0|f_1)$. Once f_1 has been selected, the remaining part of the event is known to be characterized by the quantities $i_1 = i_0 - i[f_1]$. The next term f_2 is subsequently selected on the basis of $\hat{p}(i_1|f_2)$, and the further reduced residual event can be characterized. This procedure is iterated until no residual system remains. [That this is guaranteed to happen at some point follows from the fact that only actually accessible states are constructed by this procedure: In eq. (5) the inclusive probability $\hat{p}$ is only nonvanishing if in fact the specified event $\tilde{f}$ is part of an actually accessible event f, and thus $\tilde{f}$ has an accessible counterpart such that $\tilde{f} + \bar{f} = f$. Therefore, the outlined procedure has unit efficiency (i.e. one need never abort the construction procedure and start over again).] In this way a single event f is constructed. By employing the procedure repeatedly, a sample {f} of desired size can be generated.

The procedure described above is a mathematically valid way of generating a representative sample of the exact many-fragment distribution $p(i_0|f)$. However, it requires the exact one-fragment inclusive distributions, which are cumbersome to calculate, particularly when several excitable fragment species are included.

Some degree of approximation is therefore necessary. Fortunately, one-fragment distributions, which are the only ones required in the procedure, are much easier to approximate than more exclusive quantities. It is

therefore possible to turn the mathematical procedure into a practical method. The key lies in employing the grand canonical approximation separately for each of the inclusive factors in (6). The grand canonical approximation is accurate for one-fragment distributions as long as the fragment considered is only a small part of the system. This condition is reasonably well fulfilled for most of the factors in the product (6), although it is substantially violated for the last few factors. Below we describe how this idea is implemented and discuss the quantitative validity of the approximation.

The replacement of the exact inclusive probability $\hat{p}(i_k|f_{k+1})$ by its grand canonical equivalent is carried out along the lines of refs. [1,2], except for a few important modifications designed to ensure that absolute conservation laws are respected. The relevant partition function is given by

$$Z_k = \sum_f \exp\{-\beta_k(E_f - \mu_k A_f - \nu_k T_f)\} \tag{7}$$

where $T_f = A_f/2 - Z_f$ is the isospin projection of the final state f. The three Lagrange multipliers β_k, μ_k, ν_k are determined by the constraints

$$\langle A_f\rangle = A_k,\ \langle Z_f\rangle = Z_k,\ \langle E_f\rangle = E_k \tag{8}$$

with $A_k Z_k E_k = i_k$.*) It is convenient to write

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*Instead of the total CM energy $M_k c^2 = (E_k^2 - P_k^2 c^2)^{1/2}$, it is often convenient to specify the available energy per nucleon $\varepsilon_k \equiv (M_k c^2 - (A_k - Z_k)\, m_n c^2 - Z_k m_p c^2)/A_k - B$, where $B \approx 8$ MeV is the average binding energy per nucleon.

$$\ln Z_k = A_k \sum_\alpha \omega_\alpha^k$$

$$= A_k \sum_\alpha \chi \frac{4\pi}{3} r_o^3 \cdot \left(\frac{2\pi M_\alpha}{h^2 \beta_k}\right)^{3/2} \cdot \zeta_\alpha \cdot e^{-\beta_k (M_\alpha c^2 - \mu_k A_\alpha - \nu_k T_\alpha)} \tag{9}$$

As explained in refs. [1,2], $\zeta_\alpha(\beta)$ is the intrinsic partition function for the species α, and the dimensionless parameter χ expresses the disassembly volume in units of $\frac{4\pi}{3} r_o^3 A_k$.

In deriving the form of ω_α^k, nonrelativistic kinematics has been used for convenience. However, in order to ensure exact relativistic energy conservation, the temperature parameter in the maxwellian momentum distribution has been increased to

$$\tau_k' = \tau_k \left(1 + \frac{5}{2} \frac{\tau_k}{M_\alpha c^2} \left(1 + \frac{39}{32} \frac{\tau_k}{M_\alpha c^2}\right)\right) \tag{10}$$

where $\tau_k = 1/\beta_k$ is the ensemble temperature determined from (8). This construction guarantees that the average relativistic kinetic energy $t = E - Mc^2$ equals the prescribed value through second order in the small quantity $\tau/Mc^2 \approx \varepsilon/mc^2$ at each stage in the fragmentation sequence; overall four-momentum conservation in each event is of course automatically ensured. (For more details see ref. [3].)

In the partition function (7) we exclude events containing fragments larger than the actual source i_k. This truncation guarantees that baryon number and charge are conserved although the relative multiplicity of large fragments will still be somewhat overestimated. [In this truncated grand canonical scheme fragments nearly as large as the source are still assumed to have a Poisson multiplicity distribution while in reality there can be at most one such fragment in each event.]

In order to illustrate the accuracy of this approximation, we display in fig. 1 the calculated mean multiplicities of fragments with baryon number A, for a selected case having $A_0 = 40$, $Z_0 = 20$, $\varepsilon_0 = 40$ MeV. For comparison the figure includes the grand canonical approximation $n_\alpha = A_0\omega_\alpha^0$. While not perfect, the agreement is found to be satisfactory on the whole.

Like the fragment size, the fragment kinetic energy is unbounded in the grand canonical approximation. Therefore, in order to ensure energy conservation, we truncate the maxwellian momentum distribution at the kinematical limit. Thus, when a source with invariant mass M_k ejects a fragment with mass M the corresponding maximum momentum p_k is determined by

$$4M_k^2 p_k^2 = (M_k + M + M')(M_k + M - M')(M_k - M + M')(M_k - M - M')c^2 \qquad (11)$$

where the mass M' of the residue is calculated using a binding of≈8 MeV per baryon. In order to leave the mean kinetic energy unaffected by the truncation, the temperature parameter is increased appropriately. While this modification is small and inconsequential as long as the fragment is relatively small, it becomes essential towards the end of the fragmentation sequence. In particular, at the final stage, when the source splits into the last two fragments, the employed modification automatically yields the exact microcanonical momentum distribution. This limiting property greatly improves the accuracy of our method.

In order to illustrate our method with regard to the kinetic energies, let us disregard for simplicity all binding effects so that only nucleons are produced. Since the fragments are generated sequentially in our method it is possible to consider the energy distribution for the k'th nucleon generated where $k \in (1,A_0)$. For two samples of 1000 events corresponding to the total disassembly of a system with $A_0 = 40$, $Z_0 = 20$, $\varepsilon_0 = 40$ MeV, we display

in fig. 2 the mean kinetic energy $T_k = \langle E_k - mc^2 \rangle$ and the associated dispersion as functions of the sequence number k. We note that, apart from the statistical fluctuations arising from the finite sample size (1000), the mean kinetic energy does not depend on at which stage in the sequence the nucleon is generated. The average value of T_k is the overall average kinetic energy, which of course is guaranteed to equal the specified value $\varepsilon_0 = 40$ MeV.

The dispersions in T_k also exhibit statistical fluctuations around the appropriate value expected for a relativistic Boltzman gas of nucleons, as long as the respective sources are not too small. However, towards the end of the generation sequence (i.e. when $k \approx A_0$), the smallness of the sources gives rise to somewhat too large fluctuations, as would be expected. This feature is reflected as an increase of the kinetic energy dispersions. However, the effect is seen to be relatively modest, lifting the overall average dispersion from its grand canonical value of 31.5 MeV to around 32.6 MeV.

It is, of course, not easy to safely ascertain the quantitative accuracy of our calculational approximation since the exact solution, the microcanonical distribution, is not available. However, provided that the disassembling system is not too small, a reasonably accurate approximation to the one-fragment inclusive quantities is provided by the grand canonical distribution. Since this latter distribution is obtained on the basis of the total system, while our calculation uses (truncated) grand canonical distributions for successively smaller systems, a good correspondence between the two is not a priori guaranteed and need be tested explicitly.

The results discussed above, and similar analyses of other cases, lead us to conclude that the quantitative accuracy of the calculated one-fragment distributions is in fact satisfactory. In the statistical model, there are no correlations between the different fragments, apart from those imposed by the conservation laws. The same is true in our approximation method. Therefore, we expect that the generated multi-fragment distributions are equally satisfactory.

In conclusion then, we have developed an approximate procedure for generating complete multi-fragment events within a statistical model for nuclear disassembly. The method makes it possible to generate large samples (with sizes comparable to the corresponding data) of theoretical events without prohibitive computational requirements. While the method is rather general and hence of wide applicability, we especially expect it to be of value in the current exploration of nuclear collisions at medium and high energies.

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Figure Captions

Fig. 1. The mean multiplicity of nuclear fragments with a specified baryon number A, for the disassembly of a system characterized by A_0 = 40, Z_0 = 20, ε_0 = 40 MeV. The solid histogram is the result of the present model while the dashed histogram displays the corresponding grand canonical approximation. The calculations include the nuclear levels described in ref. [2].

Fig. 2. Total disassembly of a system characterized by A_0 = 40, Z_0 = 20, ε_0 = 40 MeV into nucleons. The mean kinetic energy $\langle t \rangle = \langle E - mc^2 \rangle$ and the associated dispersion σ_t are plotted as functions of the sequence number k (see text). The results for two samples with 1000 events each are shown. The exact overall values of $\langle t \rangle$ and σ_t (as given by the grand canonical approximation) are indicated by the horizontal lines while the arrow indicates the actual calculated overall σ_t.

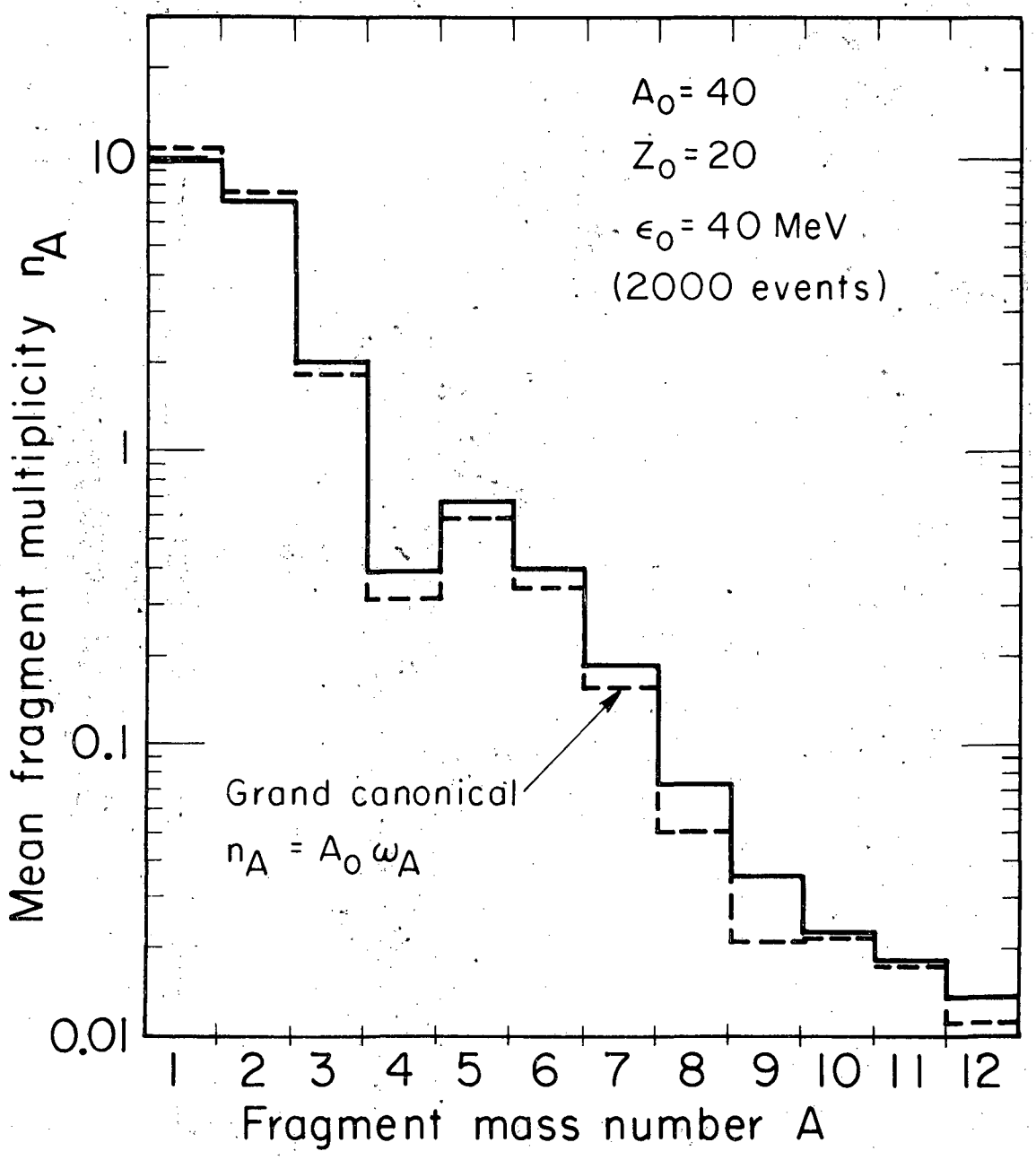

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Fig. 1

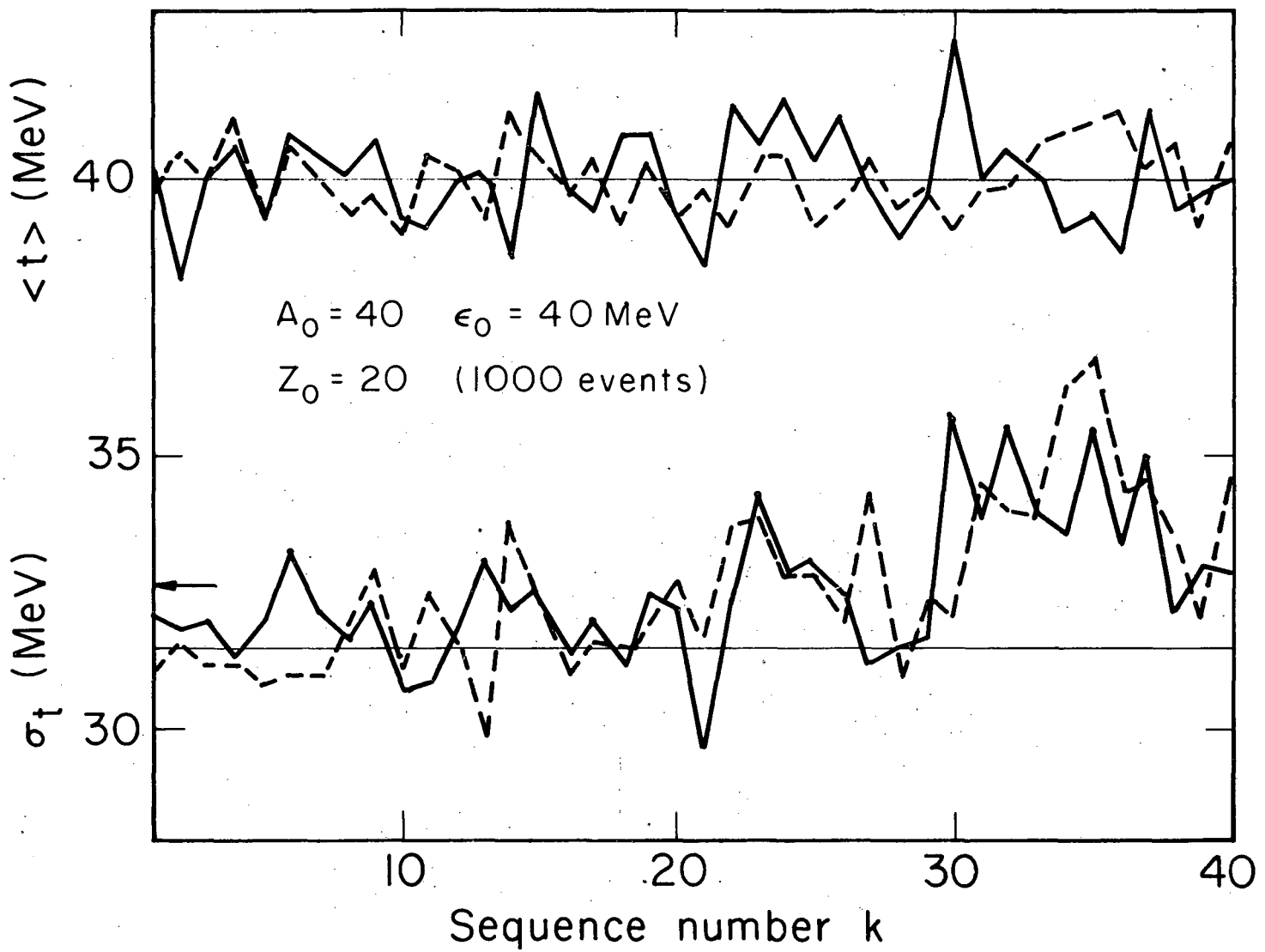

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Fig. 2

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