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IN ENERGETIC NUCLEAR COLLISIONS

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Statistical Simulation of Complete Events in Energetic Nuclear Collisions

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Abstract

A statistical model is developed for the generation of complete multi-fragment events in medium-energy nuclear collisions. Based on simple geometrical considerations, the collision system is divided into a few (participant/spectator) sources that disassemble independently. The sufficiently excited sources quickly explode into pions, nucleons, and composite, possibly particle unstable, nuclei. The different final states compete according to their microcanonical weight. The less excited sources, and the unstable explosion products, deexcite by sequential light-particle emission. The model has been implemented as a Monte Carlo computer code that is sufficiently efficient to permit generation of large event samples. The analysis of such multi-fragment events is addressed and some illustrative applications are discussed.

1. Introduction

In recent years the theoretical and experimental activity in medium and high energy nuclear collisions has increased rapidly. Previous theoretical studies (as well as earlier experiments) mostly focused on inclusive observables. In addition to statistical calculations based on the grand canonical approximation[1-3], several dynamical models [e.g., ref. [4]] have been put forward; they enjoy considerable (and comparable) success in reproducing certain features of inclusive data in the relativistic regime. Recently a new theory, based on nonequilibrium statistical mechanics, has been developed[5]].

However, it is now possible to detect electronically practically all charged fragments emerging from a collision, and thus good-quality, nearly exclusive data on multifragmentation processes can be obtained[6]. This fact

calls on theory to address exclusive quantities. Therefore, we have undertaken to develop a model for calculating complete multifragment events in medium-energy nuclear collisions. (We have in mind an energy range from a few tens of MeV to energies where particle creation becomes important.) The goal is to establish a reference model that invokes as few assumptions as possible about the specific dynamics of the collision process. Such a model may find several applications: i) it can provide the theoretical background against which subsequent, more refined and specific calculations can be judged, ii) it can be of help in analyzing the data, in particular in the search for peculiar structures in the individual event patterns, and iii) it can be of value in attempts to assess the bias introduced in the data by the acceptance criteria associated with a particular detection system.

The model developed can be briefly described as follows. The collision system is divided into a few subsystems, sources, each of which is assumed to disassemble in a statistical manner. For the subdivision we use the participant-spectator geometry supplemented with a prescription to share energy and momentum among subsystems. In this way we define one participant source and up to two spectator sources, each characterized by its number of nucleons, charge, and total four-momentum. Those sources that have an excitation energy sufficient for complete disassembly into free nucleons are said to be above the disassembly threshold. They are assumed to "explode" quickly into a number of pions, nucleons, and composite nuclei that are generally excited and particle unstable. Sources below the disassembly threshold are assumed to deexcite by sequential light particle evaporation; the same is assumed for the particle unstable explosion products.

In Section 2 the procedure for partitioning the system into separate sources is described. We introduce two parameters governing the sharing of

energy and momentum among the participants and spectators. By varying these parameters many different physical scenarios can be encompassed in the model.

The explosion of a sufficiently excited source is assumed to populate the accessible multifragment channels according to their statistical weight, as obtained by summing over the available phase space. This picture has been used to study the one-fragment inclusive quantities in medium-energy nuclear collisions by employing the grand canonical approximation[7,8]. However, the focus of the present study is on exclusive quantities, and the conservation laws must be obeyed event by event. We have therefore developed an aproximate microcanonical treatment of the disassembly process[9]. In Section 3 we describe the details of this treatment.

The evaporation process is treated in a way rather similar to that of ref. [8] and is briefly summarized in Section 4.

In Section 5 we turn to the use of the model. Several illustrative applications are made to cases under experimental study, and some discussion of the global analysis of multifragment events is given.

Section 6 contains our concluding remarks. Finally, in Appendix A we give a brief discussion of Lorentz invariance in the grand canonical approximation, while in Appendix B the details of the macroscopic mass formula used for heavier nuclei are given.

2. Division into independent sources

As discussed above, the collision system is divided into a number of sources, which are assumed to disassemble independently. For this task we invoke simple geometrical concepts that have proven their value for nuclear collisions at relativistic energies. Thus, the collision system $A_0 + B_0$ (where A_0 denotes the projectile and B_0 the target) is split into three

sources A,B,C where A and B are the projectile and target "spectators", respectively, and C is the "participant" system. Such a tripartition represents the minimal dynamical input that can be expected to have some correspondence with the data. In order to characterize the three sources we proceed as follows.

The partition of the nucleons is determined on the basis of the standard straight-trajectory clean-cut prescription associated with the nuclear fireball model[2]. Furthermore, we assume that the neutron-to-proton ratios in the two spectator sources are as in the two original nuclei. We thus take

$$
\begin{aligned}
A &= [a] \quad , \quad Z_A = \left[\frac{Z_{A_o}}{A_o}\, a + \frac{1}{2}\right] \quad , \quad N_A = \left[\frac{N_{A_o}}{A_o}\, a + \frac{1}{2}\right] \\
B &= [b] \quad , \quad Z_B = \left[\frac{Z_{B_o}}{B_o}\, b + \frac{1}{2}\right] \quad , \quad N_B = \left[\frac{N_{B_o}}{B_o}\, b + \frac{1}{2}\right] \\
Z_C &= Z_{A_o} + Z_{B_o} - Z_A - Z_B \\
N_C &= N_{A_o} + N_{B_o} - N_A - N_B
\end{aligned}
\tag{2.1}
$$

We have introduced the convenient notation

$$A = Z_A + N_A \quad , \quad B = Z_B + N_B \quad , \quad C = Z_C + N_C \tag{2.2}$$

for the total number of nucleons in the sources A,B,C respectively. Furthermore, the real numbers a and b denote the approximate mean number of nucleons in the two spectators as given by the expressions in ref. [2]. This tripartition is illustrated in fig. 1 for the two cases Ca + Ca and Ne + U. The brackets denote rounding down to the nearest integer. We note that for asymmetric systems the smaller spectator source vanishes when the impact parameter s is sufficiently small (approximately when $s < |R_{B_o} - R_{A_o}|$). Furthermore, for a nearly head-on collision of two equal nuclei, both

spectator sources vanish. The possible presence of empty sources poses no formal or practical problems and need not be considered separately in the further development.

Next we partition the momentum among the sources. This is most conveniently done in the overall CM frame in which $\vec{P}_{A_o} + \vec{P}_{B_o} = 0$. The momenta of the three sources are parametrized in the form

$$\vec{P}_A = (1 - y)\,\frac{A}{A_o}\,\vec{P}_{A_o}$$

$$\vec{P}_B = (1 - y)\,\frac{B}{B_o}\,\vec{P}_{B_o} \tag{2.3}$$

$$\vec{P}_C = -\vec{P}_A - \vec{P}_B$$

The parameter $y \in [0,1]$ governs the reduction of the motion of the spectator sources. When y vanishes, the spectators continue with their initial momentum per nucleon. In the other extreme, when y is unity, all three sources move with the same velocity. The parameter y defined in this way is somewhat analogous to the inelasticity parameter used to characterize two-body collisions.

Finally, the generated excitation energy Q, equal to the loss of translational energy implied by (2.3) must be partitioned. In the standard fireball model all the heat Q goes into the participant source while the spectators remain cold. In order to achieve a more realistic description, we employ the parameter $x \in [0,1]$ governing the leakage of heat into the spectators. We thus assume that the invariant source masses are given by

$$M_A c^2 = A\,(mc^2 + xq) + V_A$$

$$M_B c^2 = B\,(mc^2 + xq) + V_B \tag{2.4}$$

$$M_C c^2 = C\,(mc^2 + [1 + \frac{A+B}{C}(1 - x)]\,q) + V_C$$

Here $q = Q/(A_o + B_o)$ is the excitation energy per nucleon. It has been assumed that the two spectators are excited in proportion to their mass ratio: $Q_A : Q_B \equiv Aq:Bq = A:B$. The quantities V_A, V_B, and V_C are the ground state mass excesses of the nuclei (A,Z_A), (B,Z_B), and (C,Z_C), respectively, taken from ref. [10] for mass numbers ≤ 16 and calculated with a macroscopic formula (see Appendix B) otherwise.

When x vanishes there is no leakage of heat into the spectators; the participant source C takes up all the generated excitation energy. Conversely, when x is unity, the excitation per baryon, and hence the temperature, is the same in all three sources, corresponding to perfect heat sharing. Thus, the combination $x = y = 0$ corresponds to the standard fireball prescription, while $x = y = 1$ corresponds to a complete equilibrium of the collision system (except that the system is treated as three separate sources so there is an (usually unimportant) upper limit on the possible fragment size; see eq. (3.20)).

The relations (2.1) and (2.3) automatically conserve baryon number, charge, and momentum. To ensure energy conservation we must demand

$$E_A + E_B + E_C = E_{A_o} + E_{B_o} \tag{2.5}$$

where the source energies are

$$E_A = P_A{}^2c^2 + M_A{}^2c^4$$

$$E_B = P_B{}^2c^2 + M_B{}^2c^4 \tag{2.6}$$

$$E_C = P_C{}^2c^2 + M_C{}^2c^4$$

Thus, the generated heat Q is determined by the relation (2.5). [The value of Q is easily determined by iteration: starting from $Q^{(0)} = 0$, an improved approximation is given by $Q^{(k+1)} = Q^{(k)} + E_A + E_B - E_A^{(k)} - E_B^{(k)} - E_C^{(k)}$.]

The partition of the collision system depends on the parameters x and y, which are physically related to the efficiency with which the nuclear system transports energy and momentum, respectively. They are expected to depend on both the beam energy and the geometry of the collision. It is outside the scope of the present paper to attempt a calculation of these quantities from first principles, and they are simply considered as impact parameter and energy dependent model parameters. For the impact parameter dependence we employ the forms

$$\begin{aligned} x &= x_0 \left(1 - (s/s_{max})^2\right) \\ y &= y_0 \left(1 - (s/s_{max})^2\right) \end{aligned} \tag{2.7}$$

where s is the impact parameter and $s_{max} = R_{A_0} + R_{B_0}$ is the maximum impact parameter leading to a reaction under the present assumptions. Some information on the energy dependence is contained in recent data on target spectator velocities in Ne + Au reactions[11]. On this basis we take

$$\begin{aligned} x_0 &= 0.2 \\ y_0 &= e^{-\sqrt{\varepsilon_{beam}/\varepsilon_0}} \qquad , \ \varepsilon_0 = 125 \text{ MeV} \end{aligned} \tag{2.8}$$

where ε_{beam} is the laboratory kinetic energy per nucleon of the beam. These values yield a reasonable correspondence with the data and thus are useful for generating quasi-realistic event sets.

3. Explosion

In this chapter we describe how the "explosion" of a sufficiently excited source is treated. First, some useful notation pertaining to multi-fragment events is introduced. Then, using this formalism, the exclusive microcanonical distribution function describing the system right after the explosion is factorized into inclusive distributions. Finally, these are approximated by their grand canonical equivalents.

3.1 The event set

An ideal exclusive measurement yields complete information on all fragments in the final state. An event f is then characterized by the multiplicity n_α of the various fragment species α:n,p,d,t,... together with their four-momenta:

$$f = \left\{ P_\alpha^{\ell_\alpha} \ , \ \ell_\alpha \in (1,n_\alpha) \right\}_\alpha \tag{3.1}$$

The n_α fragments of the species α are arbitrarily labeled by $\ell_\alpha \in (1,n_\alpha)$ and their four-momenta are denoted by $P_\alpha^{\ell_\alpha} = (\vec{P}_\alpha^{\ell_\alpha}, E_\alpha^{\ell_\alpha})$. Because of the identity of the fragments within a given species the actual labeling is without significance, i.e., f is invariant under arbitrary permutations of the labels ℓ_α.

The total multiplicity of the event f is given by $n_f = \sum_\alpha n_\alpha$. Furthermore, its total baryon number, charge, and four-momentum are, respectively,

$$\begin{aligned} A_f &= \sum_\alpha n_\alpha A_\alpha \\ Z_f &= \sum_\alpha n_\alpha Z_\alpha \\ P_f &= \sum_\alpha \sum_{\ell_\alpha=1}^{n_\alpha} P_\alpha^{\ell_\alpha} \end{aligned} \tag{3.2}$$

where A_α and Z_α denote the baryon number and charge characterizing the fragment species α. It is often convenient to use the brief notation $i[f]$ for $\{A_f Z_f P_f\}$ collectively.

The set of all such events, the event set $\mathcal{F} = \{f\}$, has certain notationally convenient algebraic properties. Most importantly, it is possible to define an <u>addition</u> in $\mathcal{F}$. Thus, for any two events $\tilde{f}, \bar{f} \in \mathcal{F}$ we have $f = \tilde{f} + \bar{f}$ iff $n_\alpha = \tilde{n}_\alpha + \bar{n}_\alpha \ \forall\alpha$ and $\left\{P_\alpha^{\ell_\alpha}, \ell_\alpha \in (1,n_\alpha)\right\} = \left\{\tilde{P}_\alpha^{\ell_\alpha}, \ell_\alpha \in (1,\tilde{n}_\alpha), \bar{P}_\alpha^{\ell_\alpha}, \ell_\alpha \in (1,\bar{n}_\alpha)\right\}$, i.e., the sum event f is obtained by simply extending the event $\tilde{f}$ by the event $\bar{f}$, as shown in Fig. 2. The event set $\mathcal{F}$ is an abelian semi-group with respect to addition and the null event, $f = 0$, which has $n_\alpha = 0 \ \forall\alpha$, is the <u>neutral element</u>. When $f = \tilde{f} + \bar{f}$ we may also write $\bar{f} = f - \tilde{f}$, which is often convenient. We note that the function $i[f]$ defined above is additive: $i[\tilde{f} + \bar{f}] = i[\tilde{f}] + i[\bar{f}]$.

It is also possible to introduce a partial ordering in the event set $\mathcal{F}$. We shall write $\tilde{f} \leq f$ (or equivalently, $f \geq \tilde{f}$), iff $\exists\, \bar{f} \in \mathcal{F}$: $f = \tilde{f} + \bar{f}$. In words: f encompasses $\tilde{f}$ iff $\tilde{f}$ can be extended to f. Obviously, this order relation is reflexive ($f \leq f \ \forall f$), transitive ($f \leq f' \wedge f' \leq f'' \Rightarrow f \leq f''$) and anti-symmetric ($f \leq f' \wedge f' \leq f \Rightarrow f = f'$), as it should be.

It is easy to see that, with respect to the two binary operations intersection, $\cap$, and union, $\cup$, acting on the sets (3.1) characterizing the events, $\mathcal{F}$ has the properties of a complete lattice, i.e., any non-empty subset $\{f,f',\ldots\}$ of $\mathcal{F}$ has a least upper bound and a greatest lower bound. These are given by

$$\begin{aligned} \sup\ \{f,f',\ldots\} &= f \cup f' \cup \ldots \\ \inf\ \{f,f',\ldots\} &= f \cap f' \cap \ldots \end{aligned} \tag{3.3}$$

respectively.

It is important to note the partition $\mathcal{F} = \bigcup_{i_0} \mathcal{F}_{i_0}$ of the event set, where the disjoint subsets $\mathcal{F}_{i_0}$ are defined by

$$\mathcal{F}_{i_0} = \{f \mid i[f] = i_0\} \tag{3.4}$$

For given initial conditions, characterized by the quantities $\{A_0 Z_0 P_0\} = i_0$, only events $f \in \mathcal{F}_{i_0}$ are physically accessible, due to the conservation of baryon number, charge, and four-momentum.

Events with unit total multiplicity are elementary objects in the event set. Any event f can be decomposed in terms of one-fragment events:

$$\forall f \; \exists \{f_k\} \; : \; f = \sum_{k=1}^{n_f} f_k \quad , \quad n_{f_k} = 1 \quad \forall k \tag{3.5}$$

This decomposition is unique (apart from permutations of the labels k).

The algebraic structure of the event set discused above finds useful application in deriving the factorization formula (3.14) on which the statistical generation of events is based.

In the discussion above an event f is defined in terms of the four-momenta of the fragments as they emerge from the explosion (see eq. (3.1)). It is important to recognize that the specification of such an f actually characterizes an entire class of different final states, f = {F}, each final state being of the form

$$F = \left\{ Q_\alpha^{\ell_\alpha} \, P_\alpha^{\ell_\alpha} \quad , \quad \ell_\alpha \in (1, n_\alpha) \right\}_\alpha \tag{3.6}$$

where the Lorentz vector $Q_\alpha^{\ell_\alpha}$ denotes the position of the fragment in space-time shortly after the explosion. In our statistical model it is assumed that the disassembly occurs at a definite time within a certain characteristic volume. The space-time information is then given by the spatial positions $\vec{R}_\alpha^{\ell_\alpha}$ of the fragments at the disassembly time. Since any

further interaction between the fragments after the explosion is neglected, all final states F differing only in the spatial configuration of their fragments at the time of disassembly emerge with the same set of four-momenta and thus belong to the same event f. In the statistical model all such final states are equally probable*, and the appropriate measure on the event set $\mathcal{F}$ can therefore be obtained by properly enumerating the different spatial fragment configurations in a given event f.

This task is generally complicated and we resort to the approximation introduced in ref. [7]: the integration over a given fragment's position is approximated by an effective volume:

$$\int d\vec{R}_\alpha^{\ell_\alpha} \rightarrow \chi\Omega_0 \tag{3.7}$$

Here Ω_0 is a suitable reference volume, usually equal to A_0/ρ_0 where A_0 is the number of baryons in the source and $\rho_0 \approx 0.17\ \mathrm{fm}^{-3}$ is the standard nuclear matter density. The model parameter χ, which is of the order of unity, controls the average effectively available volume and can be related approximately to the "break-up" density as discussed in ref. [8]. Thus it follows that the sum over final states F can be reduced to a sum over events (classes of final states) f:

$$\sum_F \cdot \rightarrow \sum_f (\chi\Omega_0)^{n_f} . \tag{3.8}$$

This defines the proper measure on $\mathcal{F}$.

---

*In our treatment we neglect conservation of the overall center-of-mass position and the total angular momentum, since these effects are expected to be rather small and their inclusion would complicate the treatment disproportionately.

3.2 Factorization

Consider now a disassembling system characterized by the quantities $\{A_0 Z_0 P_0\} = i_0$. In the statistical model all final states compatible with the conservation of these quantities are equally probable (ignoring the additional constraints associated with the center-of-mass position and the total angular momentum, cf. footnote on page 11). Therefore, the relative probability that the system disassembles into a final state belonging to a specified event f is given by

$$p(i_0|f) = (X\Omega_0)^{n_f}\ \delta(i_0 - i[f])/\mathcal{J}(i_0) \tag{3.9}$$

As discussed above, the volume factor expresses the statistical weight of the different spatial configurations of the n_f fragments in f. The normalization constant is determined from the requirement that p be normalized,

$$\mathcal{J}(i_0) = \sum_f (X\Omega_0)^{n_f}\ \delta(i_0 - i[f]) \tag{3.10}$$

This quantity is often referred to as the phase-space integral.

The distribution $p(i_0|f)$ pertains to the ideal situation where the specification of the event f is complete, corresponding to an exclusive measurement. Hence p is referred to as the exclusive distribution. When only partial specification of the event is made, as is most often the case in practice, the relevant quantity is the corresponding inclusive distribution $\hat{p}$. This distribution can be obtained from the exclusive distribution by integrating over the unspecified quantities.

In particular, when the partial specification is such that complete information is given for some of the fragments and none for the rest, the inclusive distribution is given by

$$\hat{p}(i_0|\tilde{f}) = \sum_{f \geq \tilde{f}} p(i_0|f) = \sum_{\bar{f}} p(i_0|\tilde{f} + \bar{f}) \tag{3.11}$$

Here the event $\tilde{f}$ characterizes the partial specification and the sum is over all events f that encompass $\tilde{f}$. [For example, if only one fragment is specified we have $\tilde{f} = f_1$, where f_1 is an elementary (i.e. one-fragment) event and the sum is over all f whose decomposition (3.5) into elementary events contain f_1 as a term.] The second equation in (3.11) follows by employing $\bar{f} = f - \tilde{f}$ as the independent variable.

It is possible to express inclusive distributions in terms of the phase-space integrals (3.10):

$$\begin{aligned} \hat{p}(i_0|\tilde{f}) &= \sum_{\bar{f}} p(i_0|\tilde{f} + \bar{f}) \\ &= \sum_{\bar{f}} (\chi\Omega_0)^{n_{\tilde{f}}+n_{\bar{f}}} \delta(i_0 - i[\tilde{f}] - i[\bar{f}])/\mathcal{J}(i_0) \\ &= (\chi\Omega_0)^{n_{\tilde{f}}} \mathcal{J}(i_0 - i[\tilde{f}])/\mathcal{J}(i_0) \end{aligned} \tag{3.12}$$

i.e. the reduced inclusive probability $\hat{p}(i_0|\tilde{f})/(\chi\,\Omega_0)^{n_{\tilde{f}}}$ for obtaining the partial event $\tilde{f}$ is equal to the complementary phase-space integral $\mathcal{J}(i_0 - i[\tilde{f}])$ divided by the total phase space integral $\mathcal{J}(i_0)$, as one would intuitively expect.

By combination of (3.9) and (3.12) it is possible to factorize $p(i_0|f)$ into simpler quantities. Thus, for any decomposition $f = \tilde{f} + \bar{f}$, we have

$$\begin{aligned} p(i_0|f) &= (\chi\Omega_0)^{n_f} \delta(i_0 - i[f])/\mathcal{J}(i_0) \\ &= (\chi\Omega_0)^{n_{\tilde{f}}+n_{\bar{f}}} \delta(i_0 - i[\tilde{f}] - i[\bar{f}])/\mathcal{J}(i_0) \\ &= (\chi\Omega_0)^{n_{\tilde{f}}} \mathcal{J}(i_0 - i[\tilde{f}])/\mathcal{J}(i_0)\cdot(\chi\Omega_0)^{n_{\bar{f}}} \delta((i_0 - i[\tilde{f}] - i[\bar{f}])/\mathcal{J}(i_0 - i[\tilde{f}]) \\ &= \hat{p}(i_0|\tilde{f})\cdot p(i_0 - i[\tilde{f}]|\bar{f}) \end{aligned} \tag{3.13}$$

This relation expresses the fact that the exclusive probability for obtaining the event f is equal to the inclusive probability for obtaining some specified part $\tilde{f} \leq f$ of the event f times the inclusive probability for also obtaining the remaining part of the event $\bar{f} = \tilde{f} - f$, given that $\tilde{f}$ has already been obtained.

By repeated use of the above relation (3.13), it is possible to factorize the exclusive probability p into inclusive quantities. In particular, by decomposing the specified multi-fragment event f in terms of elementary events f_k, $p(i_o|f)$ can be factorized into one-fragment inclusive distributions. Thus, for $f = \Sigma f_k$,

$$p(i_o|f = \sum_{k=1}^{n_f} f_k) = \hat{p}(i_o|f_1)\cdot p(i_1|\sum_{k=2}^{n_f} f_k) = \ldots$$
$$= \prod_{k=1}^{n_f} \hat{p}(i_{k-1}|f_k) \cdot p(i_{n_f}|0) \tag{3.14}$$

Here we have defined $i_k \equiv i_{k-1} - i[f_k]$ for $k \in (1,n_f)$. The exclusive factor $p(i_{n_f}|0)$ vanishes unless the quantities specified by i_{n_f} all vanish, thus guaranteeing that the event f is in fact accessible by the disassembling system characterized by i_o.

3.3 Statistical event generation

The factorization (3.14) of the exclusive multi-fragment distribution p into inclusive one-fragment distributions is particularly convenient when one seeks to generate a statistical representation of p, i.e., a sample {f} of multi-fragment events that are statistically distributed in the event subset $\mathcal{F}_{i_o}$ according to the probability density $p(i_o|f)$. To accomplish this task, one may proceed as follows.

Each event f is considered as a sum of elementary one-fragment events, $f = \Sigma f_k$. To generate an event f, one first makes a random selection of the term f_1 on the basis of the inclusive probability distribution $\hat{p}(i_0|f_1)$. Once f_1 has been selected, the remaining part of the event is known to be characterized by the quantities $i_1 = i_0 - i[f_1]$. The next term f_2 is subsequently selected on the basis of $\hat{p}(i_1|f_2)$, and the further reduced residual event can be characterized. This procedure is iterated until no residual system remains. [That this is guaranteed to happen at some point follows from the fact that only actually accessible events are constructed by this procedure: In eq. (3.11) the inclusive probability $\hat{p}$ is nonvanishing only if in fact the specified event $\tilde{f}$ is part of an actually accessible event f, and thus $\tilde{f}$ has a counterpart such that $\tilde{f} + \bar{f} = f$. Therefore, the outlined procedure has unit efficiency (i.e., one need never abort the construction procedure and start over again).] In this way a single event f is constructed. By employing the procedure repeatedly, a sample $\{f\} \in \mathcal{F}_{i_0}$ of desired size can be generated.

The procedure described above is a mathematically valid way of generating a representative sample of the exact many-fragment distribution $p(i_0|f)$. However, it requires the exact one-fragment inclusive distributions, which are cumbersome to calculate, particularly when several excitable fragment species are included.

Some degree of approximation is therefore necessary. Fortunately, one-fragment distributions, which are the only ones required in the procedure, are much easier to approximate than more exclusive quantities. It is therefore possible to turn the mathematical procedure into a practical method. The key lies in employing the grand canonical approximation separately for each of the inclusive factors in (3.14). The grand canonical

approximation is accurate for one-fragment distributions as long as the fragment considered is only a small part of the system. This condition is reasonably well fulfilled for most of the factors in the product (3.14), although it is substantially violated for the last few factors. Below we describe how this idea is implemented and discuss the quantitative validity of the approximation.

3.4 Truncated grand canonical approximation

The replacement of each of the exact inclusive probabilities $\hat{p}(i_k|f_{k+1})$ by their respective grand canonical equivalents is carried out along the lines of refs. [7,8], except for a few important modifications designed to ensure that absolute conservation laws are respected.

Consider a source characterized by $i_k = \{A_k Z_k E_k^*\}$ where E_k^* is the invariant excitation energy of the source. [As discussed in Appendix A, the essential dependence on the four-momentum $P_k = (\vec{P}_k, E_k)$ is through the invariant combination $s_k = E_k^2 - \vec{P}_k^2 c^2 = (M_k c^2)^2$, or, equivalently, the excitation energy $E_k^* = M_k c^2 - (A_k - Z_k) m_n c^2 - Z_k m_p c^2$. The overall Lorentz boost into the desired frame of reference is elementary to perform after the explosion has been completed.] The corresponding partition function is

$$Z_k(\beta_k, \mu_k, \nu_k) = \sum_f (\chi \Omega_0)^{n_f} \exp \left\{ - \beta_k (E_f^* - \mu_k A_f - \nu_k T_f) \right\} \tag{3.15}$$

where $T_f = A_f/2 - Z_f$ is the total isospin projection of the event f. Since the fragments are treated as non-interacting, the partition function factorizes and one has

$$\ln Z_k = A_k \sum_\alpha \omega_\alpha^{(k)} \tag{3.16}$$

For the intensive quantities $\omega_\alpha^{(k)}$ we use the standard form[7,8]

$$\omega_\alpha^{(k)} = \chi\Omega_0 \left(\frac{2\pi M_\alpha}{h^2 \beta_k}\right)^{3/2} \zeta_\alpha^{(k)} \cdot e^{-\beta_k (V_\alpha - \mu_k A_\alpha - \nu_k T_\alpha)} \tag{3.17}$$

Here $\zeta_\alpha^{(k)}$ is the intrinsic partition function of the species α (see eq. 3.22) and V_α is its ground-state mass excess. (See Appendix B.)

The three Lagrange multipliers β_k, μ_k, ν_k are determined by the constraints on the ensemble averages

$$\langle E_f^* \rangle = E_k^* \quad , \quad \langle A_f \rangle = A_k \quad , \quad \langle Z_f \rangle = Z_k \tag{3.18}$$

This leads to the following three coupled equations

$$\begin{aligned} \varepsilon_k &= \sum_\alpha{}' \left(\bar{t}_\alpha^{(k)} + V_\alpha + \bar{\varepsilon}_\alpha^{(k)}\right) \omega_\alpha^{(k)} \\ 1 &= \sum_\alpha{}' A_\alpha \, \omega_\alpha^{(k)} \\ T_k/A_k &= \sum_\alpha{}' T_\alpha \, \omega_\alpha^{(k)} \end{aligned} \tag{3.19}$$

Here $\varepsilon_k = E_k^*/A_k$ is the excitation energy per baryon in the source $i_k = \{A_k Z_k E_k^*\}$. Furthermore, $\bar{t}_\alpha^{(k)}$ is the mean kinetic energy (see eq. (3.30)) and $\bar{\varepsilon}_\alpha^{(k)}$ the mean intrinsic excitation energy (see eq. (3.27)) for fragments of the species α. The primes on the summations in eq. (3.19) indicate the fact that we truncate these sums to observe exact baryon number and charge conservation as explained in the following.

3.4.1 Species truncation

In the grand canonical approximation the summation in (3.15) includes all fragment species. This yields a finite (though small) probability $\omega_\alpha^{(k)}$ for fragments α containing more nucleons than the source. To ensure that such fragments can never occur, we restrict the summation to fragments for which

$$Z_\alpha \leq Z_k \quad ; \quad A_\alpha - Z_\alpha \leq A_k - Z_k \quad , \quad 0 \leq A_\alpha < A_k \tag{3.20}$$

This truncation is indicated in the sum over species in (3.19) by the prime following the summation symbol. [The condition (3.20) is actually slightly stricter than necessary, since, at sufficient energy, the creation of negatively charged mesons might compensate for a nuclear fragment charge exceeding that of the source, and, at still higher energies (well beyond the region of present interest), the creation of antibaryons might compensate for fragments with an excessive number of baryons.]

The inclusive probability for the creation of a fragment of a given species α is

$$p_\alpha^{(k)} = \omega_\alpha^{(k)} / \sum_{\alpha'}{}' \; \omega_{\alpha'}^{(k)} \tag{3.21}$$

In the statistical event generation the species of the k'th fragment is then readily decided on the basis of the probabilities $\left\{p_\alpha^{(k)}\right\}_\alpha$.

3.4.2 Intrinsic excitation

After deciding the fragment species, the second step in the event generation is to decide on the amount of intrinsic excitation of that fragment. The intrinsic partition function is given by

$$\zeta_\alpha^{(k)} = \int d\varepsilon \; \rho_\alpha^{eff}(\varepsilon) \; e^{-\beta_k \varepsilon} \tag{3.22}$$

so the relative probability for a particular excitation energy is

$$p_\alpha^{(k)}(\varepsilon) = \rho_\alpha^{eff}(\varepsilon) \; e^{-\beta_k \varepsilon} / \zeta_\alpha^{(k)} \tag{3.23}$$

The intrinsic energy can then readily be decided at random according to the frequency function $p_\alpha^k(\varepsilon)$.

In the above relation, ρ^{eff} denotes the density of levels with a half-life sufficiently long to be included in the final phase space of the explosion. Following ref. [8], we take the lowest nuclear levels from ref. [10]. At higher excitation energy, and when A > 16, we use the Fermi gas form

$$\rho_\alpha(\varepsilon) = \frac{k_1}{A_\alpha^{5/3}} \exp\left\{2\sqrt{a_\alpha \varepsilon}\right\} \tag{3.24}$$

with the level density parameter given by

$$a_\alpha = \frac{A_\alpha}{8\ \text{MeV}}\left(1 - \frac{k_2}{A_\alpha^{1/3}}\right) \qquad (A_\alpha > 4) \tag{3.25}$$

and $k_1 = 0.2/\text{MeV}$, $k_2 = 0.8$ [ref. [8]]

The effective level density $\rho_\alpha^{eff}(\varepsilon)$ can be obtained from the level density $\rho_\alpha(\varepsilon)$ (3.24) by multiplying with the probability that a state with energy ε is sufficiently long lived. This probability is taken to be of a simple gaussian form

$$p(\Gamma < \Gamma_0) = \exp\left\{-\frac{(\varepsilon - B_{min})^2}{2\varepsilon_{cut}^2}\right\} \qquad (\varepsilon > B_{min}) \tag{3.26}$$

Here B_{min} is the lowest barrier energy, i.e., the barrier energy for the dominant evaporation mode (see Chapter 4). The cut-off parameter ε_{cut} is fairly uncertain. Two criteria have been used as guidance: 1) The specific experimental information on levels and widths in very light nuclei should be reasonably reproduced. This requirement dictates rough correspondence with the formula used in ref. [8] for A < 16. 2) At high temperature, the mean nuclear excitation energy

$$\bar{\varepsilon}_\alpha(k) = \int d\varepsilon\ \varepsilon\ \rho_\alpha^{eff}(\varepsilon)\ e^{-\beta_k \varepsilon} \tag{3.27}$$

should saturate at some constant energy per nucleon. We have, somewhat arbitrarily, demanded a saturation value of $\varepsilon_B = 8$ MeV per nucleon, which

dictates that $\varepsilon_{cut} \approx \varepsilon_B A^{1/2}$ for large A. On this basis we have adopted the form

$$\varepsilon_{cut}(A) = c \cdot \varepsilon_B \left[(A - 3)^{1/2} - 1\right] \qquad (A > 4) \tag{3.28}$$

The standard value c = 1 corresponds to $\Gamma_0 = 1$ MeV for the cut-off level width. The sensitivity to the specified value of Γ_0 can be examined by varying the parameter c.

3.4.3 Relativistic effects

The third step in generating the k'th fragment is the determination of its momentum $\vec{p}_\alpha^{(k)}$. In the grand canonical approximation the fragment momentum has a relativistic Boltzmann distribution with the temperature $\tau_k = 1/\beta_k$. This gives rise to a spectral distribution involving the modified Bessel function $K_2(\beta_k M_\alpha)$. This is somewhat inconvenient, particularly since the relativistic fragment mass M_α is subject to statistical fluctuations (since it includes the excitation energy). Fortunately, at intermediate energies, where our interest is focused, relativistic effects are minor and we may circumvent the problem as follows.

We assume that the fragment momentum has a Maxwell distribution with some modified temperature τ'_k. The value of τ'_k is adjusted so as to ensure that the mean relativistic fragment kinetic energy $t_\alpha = E_\alpha - M_\alpha c^2$ equals the value corresponding to the exact relativistic distribution with the temperature τ_k. This is accomplished for

$$\tau'_k = \tau_k \left[1 + \frac{5}{2} \frac{\tau_k}{M_\alpha c^2} \left(1 + \frac{39}{32} \frac{\tau_k}{M_\alpha c^2}\right)\right] \tag{3.29}$$

through second order in the small quantity $\tau_k/M_\alpha c^2$. To the same order, the mean relativistic kinetic energy is

$$\bar{t}_\alpha^{(k)} = \frac{3}{2}\tau_k \left[1 + \frac{5}{4}\frac{\tau_k}{M_\alpha c^2}\left(1 + \frac{15}{16}\frac{\tau_k}{M_\alpha c^2}\right)\right] \tag{3.30}$$

This is the quantity entering in eq. (3.19). This approximate treatment conserves the relativistic energy but slightly distorts the spectral distributions; the effect on the second moment of the kinetic energy is of the order $(\tau_k/M_\alpha c^2)^2$ and unimportant at intermediate energies.

3.4.4 Momentum truncation

In the grand canonical approximation, the fragment momenta are also unbounded, leaving open the possibility of violating energy conservation. This problem is analogous to the unrestricted species summation discussed in Sec. 3.4.1. Although the problem is unimportant most of the time, it becomes essential when the source has only a few nucleons left. We remedy this unphysical aspect of the approximation in the following manner.

When a source with invariant mass M_k ejects a fragment with mass M_α the corresponding maximum fragment momentum p_o, in the source frame, is determined by

$$\begin{aligned} 4M_k^2\, p_o^2 = {} & (M_k + M_\alpha + M')(M_k + M_\alpha - M') \\ & (M_k - M_\alpha + M')(M_k - M_\alpha - M')\, c^2 \end{aligned} \tag{3.31}$$

where M' is the minimum mass of the residue. For the present purpose we use

$$M' = [(A_k - A_\alpha) - (Z_k - Z_\alpha)]\, m_n + [Z_k - Z_\alpha]\, m_p \tag{3.32}$$

i.e., we demand that the residue excitation be above the disassembly threshold. The maxwellian momentum distribution is therefore truncated at $p_\alpha^{(k)} = p_o$. In order that the mean kinetic energy be left unaffected by the truncation, the temperature parameter is increased appropriately. In

making this adjustment we use nonrelativistic kinematics since relativistic effects are at the corrective level already. We thus need to solve the equation

$$\frac{3}{2}\tau_k' = \int_0^{p_o} \frac{p_\alpha^2}{2M_\alpha} e^{-p_\alpha^2/2M_\alpha\tau_k''} p_\alpha^2 \, dp_\alpha \Big/ \int_0^{p_o} e^{-p_\alpha^2/2M_\alpha\tau_k''} p_\alpha^2 \, dp_\alpha \tag{3.33}$$

for the modified temperature τ_k''. This is quickly done by use of Newton's method.

As stated before, the modification is small and inconsequential as long as the fragment is relatively small. It becomes essential, however, towards the end of the generation sequence. In particular, at the final stage, when the source splits into the last two fragments, the modification automatically yields the exact microcanonical momentum distribution, which is a delta function at $p_\alpha = p_o$. [This corresponds formally to $\tau_k'' \to -\infty$.] This correct limiting property is instrumental in containing the accumulated error associated with our approximation procedure.

4. Evaporation

Subsystems with excitation energy below the disassembly threshold and metastable products of the explosion process are assumed to deexcite on a slower time scale via sequential light particle emission and, subsequently, gamma decay. This latter process, which does not effect the fragment yields and is expected to modify the fragment momenta only slightly, is ignored in the present treatment.

The deexcitation of heavier ($A \gtrsim 30$) nuclei by sequential light-particle emission has often been considered in the literature (e.g. ref.[12]), but, to our knowledge, there exists no reliable evaporation model that can be used both in the above mass region and for highly excited very light nuclei ($5 \leq A \lesssim 30$). Therefore, we have found it necessary to develop a simple universal treatment of the evaporation process.

Similarly to the explosion, we employ statistical ideas to describe the evaporation process. The physical picture is that of different light particles competing to be emitted at each stage of the deexcitation process according to the available phase space. In the actual calculation only the classically allowed emission of nucleons and alpha particles has been taken into account. The emission of other light particles, such as deuterons, is expected to be less favored and has been neglected for simplicity; it is, however, straightforward to include additional ejectile species in the model. Each evaporation mode i (= n, p, or α emission) is characterized by a separation energy

$$S_i = M_{ejectile} + M_{daughter} - M_{mother} \tag{4.1}$$

and a barrier

$$B_i = S_i + V_i \quad , \tag{4.2}$$

which represents the minimum excitation energy needed in the mother nucleus for the evaporation mode i to be allowed. The electrostatic contribution to the barrier is estimated by

$$V_i = \frac{Z_{ejectile} Z_{daughter} e^2}{R_{daughter} + d_i} \tag{4.3}$$

where $R_{daughter}$ is the radius of the daughter nucleus, $R_{daughter} = r_o A_{daughter}^{1/3}$ (with $r_o = 1.15$ fm), and the shifts d_i are 1.5 fm for the proton and 0.5 fm for the alpha particle, respectively[8]). For a given excitation energy ε_o in the mother nucleus and a given evaporation mode i the spectrum

$$f_i(\varepsilon) \sim (\varepsilon_{max_i} - \varepsilon)\, g(\varepsilon) \tag{4.4}$$

has been used in the daughter nucleus, where $\varepsilon_{max_i} = \varepsilon_o - S_i$ and $g(\varepsilon)$ is the degeneracy of the energy ε in the daughter (taken from ref. [10]) for $A \leq 16$ if available and put equal to $\rho(\varepsilon)\, d\varepsilon$ otherwise, with the level density $\rho(\varepsilon)$ (3.24)). The energy factor arises from the integration over the momentum of the ejectile. It is understood that

$$\varepsilon_o > B_{min} \tag{4.5}$$

is a necessary condition for particle decay; if the excitation energy of the mother is below the lowest barrier B_{min} the state is particle stable and will eventually decay to the ground state by gamma emission. Integrating (4.4) over energy and summing over the different evaporation modes yields the normalization and the probability according to which the evaporation mode is selected. The energy of the daughter nucleus is then picked according to the weight function (4.4). Finally, an ejection direction is chosen at random and the necessary Lorentz transformations are performed. The effect of the Coulomb repulsion has been neglected. One step in the evaporation sequence is illustrated in fig. 3.

This method of solving the evaporation problem yields in a convenient way full kinematic information on the emitted light particles and on the particle stable residues in any desired reference frame.

5. Some illustrative results

In the preceding, we have described the details of our model for multi-fragmentation. The model is based on calculating the available microcanonical phase space in the approximation given in Section 3. The validity of the approximation has been discussed in ref. [9]). The model has been implemented in the form of a Monte Carlo computer code (capable of running on any standard computer). Because of the care taken to use efficient methods, and in particular due to the factorization into grand canonical distributions, the code is rather fast. [On the CDC 7600 it typically takes 200-400 CP seconds to generate 1000 complete events.] Thus, it is practically possible to generate sufficiently large samples of events to permit quantitatively useful analyses. As mentioned earlier, we regard the model as a flexible tool which may find a number of applications. Here we will illustrate some of the possible uses by a few instructive examples. Specific comparisons with data, as well as more extensive theoretical studies, will be reported elsewhere.

As a first illustration, we display in Fig. 4 the mass distribution of four arbitrarily chosen events for the case of 120 MeV/n $^{40}Ar + {}^{208}Pb$, using a medium impact parameter $s = \frac{1}{2} s_{max}$. ($s_{max} = R_{A_0} + R_{B_0}$, the sum of the radii of the colliding nuclei.) The figure displays the multiplicity-histogram of fragments with masses up to 16, with heavier fragments indicated explicitly. The general character of the events is the same. The hatched parts of the nucleon and alpha yields result from the evaporation phase of the disassembly process; they are seen to constitute a major part of those yields. We note that in all cases there is one heavy fragment. This is the evaporation

residue of the target-like spectator source. Its mass is around 125, and it moves with a fairly low velocity in the forward direction.

In the following illustrations we consider two cases on which actual experiments have been/are being carried out: 395 MeV/n $^{40}Ca + ^{40}Ca$ and 795 MeV/n $^{20}Ne + ^{208}Pb$.

Figure 5 displays the mean total fragment multiplicity n as a function of the impact parameter s. The dispersion in this quantity is indicated by the bars. To illustrate the effect of the limitations of the detection system we also display an "observed" multiplicity that includes only charged fragments having a kinetic energy above 20 MeV in the laboratory. We note that this schematic cut introduces a substantial reduction in the multiplicity. In particular, for a central collision between Ne and Pb, nearly two-thirds of the fragments are eliminated! It is also noted that whether a cut is made or not, there is a fairly good correspondence between the impact parameter and the multiplicity. This result suports the use of the multiplicity as a rough indicator of the impact parameter.

When confronted with exclusive data we are faced with the task of reducing the wealth of information to a few instructive quantities. In cases when one is searching for evidence of a specific dynamical phenomenon, appropriate signals often suggest themselves. For example, in high-energy e^+e^- or $p\bar{p}$ collisions the basic quark structure of matter is signalled by the jet structure in the emission pattern. However, in nuclear collisions we are still at an early stage where we seek to gain a global impression of the events to obtain guidance for further studies.

In the global analysis of many-particle events it is often useful to employ observables that are insensitive to the specific form in which the emerging matter appears, i.e., whether it be in the form of elementary nucleons or composite nuclei of various species. This requirement can be

formulated as a demand for coalescence invariance. That is to say, if any composite fragment α_0 is split into two fragments α_1 and α_2 having the same velocities as α_0 and with masses $M_{\alpha_1} + M_{\alpha_2} = M_{\alpha_0}$, then the observable remains unaffected. Conversely, if two fragments α_1 and α_2 with similar velocities are lumped together as one fragment α_0 with the same velocity and a mass $M_{\alpha_0} = M_{\alpha_1} + M_{\alpha_2}$, then the observable remains unaffected. The advantage of using coalescence invariant observables is that the results become insensitive to possible shortcomings of the particular dynamical model as regards the description on how the emerging matter eventually condenses into physical fragments. This is especially relevant when using either cascade calculations, which yield a final state consisting only of elementary hadrons (nucleons and pions), or fluid dynamical models, which merely yield a structureless matter flow. It is readily seen that in order that an observable be coalescence invariant it must be additive in the four-momentum of the fragments, with weights that can be arbitrary functions of the fragment velocities. Although the present model takes proper account of composite fragments, in what follows we will use coalescence invariant global observables in order to facilitate comparisons with the dynamical calculations mentioned above.

An example of such a coalescence invariant global observable is the Lorentz tensor

$$F^{\mu\nu} = \sum_k \frac{P_k^\mu P_k^\nu}{2M_k} \tag{5.1}$$

Here P_k is the four-momentum of the k'th fragment and M_k is its rest mass. That this quantity is indeed coalescence invariant follows from the fact that $P_k^\nu/M_k = (P_k^\nu/E_k)\,(E_k/M_k) = V_k^\nu \gamma_k$.

The spatial part of the above Lorentz tensor has been singled out in ref. [13] as especially useful for studying the global event structure. This "kinetic-flow" tensor can be written as

$$\overleftrightarrow{T}_f = \sum_{k=1}^{n_f} \frac{\overleftrightarrow{p_k p_k}}{2M_k} = \overleftrightarrow{t_1 t_1} + \overleftrightarrow{t_2 t_2} + \overleftrightarrow{t_3 t_3} \tag{5.2}$$

in the CM system of the n_f fragments included in the sum. For a given event f, the tensor $\overleftrightarrow{T}$ can be characterized by its three eigenvectors $\vec{t}_i$ labeled such that $t_1 \leq t_2 \leq t_3$. Thus it takes six independent quantities to specify $\overleftrightarrow{T}$ completely; for example, the overall size, the orientation, and the intrinsic shape of the equivalent ellipsoid.

The total (nonrelativistic) kinetic energy in the final state is given by

$$T_f = \mathrm{tr}\, \overleftrightarrow{T}_f = \sum_k \frac{p_k^2}{2M_k} = \sum_{i=1}^{3} t_i^2 \tag{5.3}$$

(in the CM frame, of course).

The major principal axis $\hat{3}$ has the direction (θ,φ). Here the angle θ between the beam and $\hat{3}$ is denoted the flow angle. The azimuthal angle φ can be arbitrarily put to zero, thus defining a "reaction plane" relative to which other directions can be specified. A third angle ψ is needed to specify the orientation of, e.g., the minor axis $\hat{1}$ in the plane perpendicular to $\hat{3}$. The three angles (θ,φ,ψ) are equivalent to the standard Euler angles needed to specify the orientation of a rigid body in space. [Note that for oblate-type events, where $t_1 \ll t_2 \approx t_3$, it may be advantageous to single out the minor principal axis 1 as a reference. In this case no sensible definition of a flow angle can be given. Hence, when studying "bounce-off" effects it may be useful to divide the events into prolate and oblate classes by e.g. $t_2^2 \gtrless t_1 t_3$.]

The remaining two quantities needed to characterize the kinetic tensor are associated with the intrinsic "shape" of the flow pattern. Since the kinetic tensor can be replaced by an equivalent ellipsoid it may seem natural to employ the standard (β,γ) parameters used to describe ellipsoidal-type deformations. However, since the definition of an equivalent ellipsoid (which has been used, e.g., in ref. [14]) is not unique, and since very large distortions may occur, this avenue appears less promising.

Instead, following the approach taken for the analysis of high-energy e^+e^- collisions,[15] we consider the following two shape parameters:

$$\begin{aligned} &\text{SPHERICITY:} \qquad S = \frac{3}{2}(1 - q_3) \\ &\text{COPLANARITY:} \qquad C = \frac{\sqrt{3}}{2}(q_2 - q_1) \end{aligned} \tag{5.4}$$

where $q_i \equiv t_i^2 / \sum_{i=1}^{3} t_i^2$ are the normalized eigenvalues of $\overleftrightarrow{T}$. A given event shape can then be represented in a Dalitz-type triangular plot, as illustrated in fig. 6. The equilateral triangle has the side length $2/\sqrt{3}$ and the distances from the representative point to the sides are the corresponding normalized eigenvalues q_i. If we impose the ordering $q_1 \leq q_2 \leq q_3$, only the lower left sixth of the domain is needed, as indicated by the dashed 30-60 triangle in the figure. It is convenient to introduce the above quantities S and C. As is clear from the figure, C measures the distance of the event from the hypotenuse and S is the distance along the hypotenuse (by convention a factor 3/2 has been introduced). It is therefore conventional to flip the triangle around so that S becomes the abscissa and C the ordinate. It should be evident from this discussion that the proper domain of such an SC-plot is bounded by the inverted 30-60 triangle and, furthermore, there is a natural relation between the horizontal and vertical scales in such a plot, namely the one implied by the above definitions (5.4).

As an illustration, we display in fig. 7 SC-plots for the two cases considered in fig. 5. In both cases only positively charged fragments with kinetic energies above 20 MeV have been accepted. The figure shows contours of equal probability in the triangular SC-plane. The impact parameters $s = (0.1, 0.3, 0.5) \times s_{max}$ have been added with the respective weight 1, 3, 5 in order to roughly simulate a selection of central collisions. Since these results are intended as illustrations only, no special effort has been made to obtain good statistics and only 200 events were generated for each impact parameter. (The limiting consideration here was not the cost of generation but rather the desire for convenient event storage and retrieval.) The indicated contours are therefore only rough approximations.

However, even with this relatively small event sample, a distinctive structure is apparent in the contour plots. In both cases, a two-peak structure emerges. In the case of Ne + Pb the two peaks have about the same height although the right-hand peak is less well defined. In the case of Ca + Ca the left-hand peak is notably higher than the right-hand peak. In both cases, the left-hand peaks arise predominantly from the impact parameter $s = 0.5\ s_{max}$. The smaller impact parameters, which carry a smaller weight, tend to have a larger sphericity, due to the smaller importance of the spectator sources.

It is noteworthy that preliminary data exhibit a somewhat similar structure [16]. It would be interesting to pursue the comparison in more detail when the data analysis is completed.

6. Concluding remarks

We have developed and implemented a statistical model for the generation of complete multi-fragment events in medium-energy nuclear collisions. In doing so, we have deliberately avoided, as far as possible, to invoke specific

assumptions about the collision dynamics. The model is intended to provide a useful framework for exploratory studies of these novel processes.

In order to characterize the disassembling sources, use was made of simple geometry. In addition, two parameters, denoted x and y, were introduced to describe the leakage of energy and parallel momentum from participants to spectators. In future studies, one might want to introduce a third parameter, say z, to describe the amount of transverse momentum given to the spectator sources. Such a parameter may be called for to describe a collective "bounce-off" of the two spectator sources. The introduction would be straightforward - in the present paper we have left it out in order to keep things as simple as possible.

In the present version of the model, we have considered only one participant and up to two spectator sources. This is the minimum number we expect to be of interest. However, the model readily submits to the introduction of more sources, should it appear warranted. For example, one might have projectile-like participants and target-like participants, hot and cold spectator matter, or a practically continuous set of sources as in the firestreak model, with due account taken of finite-particle number effects, of course. Thus the work reported in the present paper may ultimately find use in many different contexts involving the exclusive disassembly of excited nuclear matter.

We consider the quantities x and y and the numbers of sources to be the physical input parameters of our model. The parameters χ and c related to the extension of the sources in space and time (see eqs. (3.7) and (3.28), respectively) are also important, and different dynamical ideas may lead to different values of these quantities. The sensitivity of the results to these parameters has been discussed in a somewhat different context in ref. [8]).

For the sake of simplicity the standard values $\chi = c = 1$ have been used throughout the present paper.

A few further parameters, e.g. the radius parameter r_0, or the constants k_1 and k_2 in the level density (3.24), or the Coulomb shifts d_i in (4.3), are of the usual nuclear physics type: although they are not well known, sufficient information is available to fix them with reasonable accuracy. We do not consider these parameters essential for the present purpose. At the same time, we realize that e.g. Coulomb effects may play an important role, in particular for low-temperature sources [17]), and they should be taken into account (beyond the mean field treatment of ref. [8])) if a more refined description is attempted. In the present paper, however, we refrain from the introduction of further parameters and concentrate on the source characteristics as the main input of the model.

Clearly, it would be of interest to calculate the source characteristics in specific dynamical models. For example, one might employ intra-nuclear cascade or fluid dynamics at the initial collision stage and attempt to describe the outcome in terms of a few distinct sources. Alternatively, one might try to calculate the energy and momentum transport parameters x and y in a microscopic nuclear model with idealized geometry. Insofar as the character of the sources can be calculated on the basis of a nuclear model, the multi-fragmentation processes can be exploited to yield constraints on the nuclear energy-momentum transport properties.

The emphasis of the present paper is on the exposition of the model and its numerical implementation. The applications included are but a few illustrations of the possible uses of the model. We anticipate extended use of the model in the time ahead, partly for systematic theoretical studies and partly for aiding in the analysis of data. Results of these efforts will be reported as they emerge.

Appendix A: The grand canonical approximation

The macrostate of the system is specified by the (conserved) quantities $i_o = A_o Z_o \vec{P}_o E_o$. The phase space integral is the number of microstates F having $i[F] = i_o$,

$$\mathcal{J}(i_o) = \sum_F \delta(i[F] - i_o) = \sum_f (\chi\Omega_o)^{n_f} \delta(i[f] - i_o) \tag{A-1}$$

For $i \approx i_o$ we have

$$\mathcal{J}(i) \approx \mathcal{J}(i_o) e^{\lambda(i_o)\cdot(i-i_o)} \tag{A-2}$$

where

$$\lambda(i_o) \equiv \frac{\partial \ln \mathcal{J}(i)}{\partial i} \tag{A-3}$$

Therefore, the inclusive probability for a partial event with $i[\tilde{f}] << i_o$ is (cf. eq. (3.12))

$$\hat{p}(i_o|\tilde{f}) = (\chi\Omega_o)^{n_{\tilde{f}}} \mathcal{J}(i_o - i[\tilde{f}])/\mathcal{J}(i_o) \approx (\chi\Omega_o)^{n_{\tilde{f}}} e^{-\lambda(i_o)\cdot i[\tilde{f}]} \tag{A-4}$$

This is the grand canonical approximation.

In order to determine the quantity λ, use can be made of the identity

$$i_o = \sum_f i[f]\, p(i_o|f) = \sum_{f_1} \sum_{f \geq f_1} i[f_1] p(i_o|f) = \sum_{f_1} i[f_1] \hat{p}(i_o|f_1) \tag{A-5}$$

where f_1 is an elementary (i.e. one-fragment) event. Insertion of (A-4) for the inclusive distribution on the right-hand side then yields an equation for λ involving only elementary events.

$$i_o = \sum_{f_1} i[f_1]\, \chi\Omega_o\, e^{-\lambda(i_o)\cdot i[f_1]} \tag{A-6}$$

Due to Lorentz invariance the phase space integral of the system depends only on the total four-momentum through the combination $E_f^2 - \vec{P}_f^2 c^2 = (M_f c^2)^2$. Consequently, the Lagrange multipliers for energy and momentum are related:

$$\lambda_E = \frac{\partial \ln \mathcal{J}}{\partial E} = \frac{\partial (Mc^2)}{\partial E} \frac{\partial \ln \mathcal{J}}{\partial Mc^2} = \frac{E}{Mc^2} \beta$$

$$\vec{\lambda}_P = \frac{\partial \ln \mathcal{J}}{\partial \vec{P}} = \frac{\partial (Mc^2)}{\partial \vec{P}} \frac{\partial \ln \mathcal{J}}{\partial (Mc^2)} = -\frac{\vec{P}}{M} \beta \qquad \text{(A-7)}$$

where $\beta \equiv \partial \ln \mathcal{J} / \partial (Mc^2)$ and $\mathcal{J}(Mc^2)$ is the proper level density of the system. We note that $\lambda_E^2 - \vec{\lambda}_P^2 / c^2 = \beta^2$.

The inclusive probability for a fragment with energy e_1 and momentum $\vec{p}_1$ is then

$$\hat{p}(i_o|f_1) = X\Omega_o \, e^{-\lambda_E e_1 - \vec{\lambda}_P \cdot \vec{p}_1 - \lambda_A A_1 - \lambda_Z Z_1}$$

$$= X\Omega_o \, e^{-\beta (E_o e_1 - \vec{P}_o c \cdot \vec{p}_1 c)/Mc^2 - \ldots} \qquad \text{(A-8)}$$

$$= X\Omega_o \, e^{-\beta E_1 - \ldots}$$

Here we have used the fact that the invariant quantity $(E_o e_1 - \vec{P}_o c \cdot \vec{p}_1 c)/Mc^2$ is equal to the fragment energy E_1 as seen in the CM frame, where $\vec{P}_o$ vanishes.

Furthermore, the four constraint equations

$$E_o = \sum_{f_1} e_1 \, \hat{p}(i_o|f_1)$$

$$\vec{P}_o = \sum_{f_1} \vec{p}_1 \, \hat{p}(i_o|f_1) \qquad \text{(A-9)}$$

can be combined to a single equation for β:

$$M_o c^2 = (E_o^2 - \vec{P}_o^2 c^2)/M_o c^2$$

$$= \frac{1}{M_o c^2} \sum_{f_1} (E_o e_1 - \vec{P}_o c \cdot \vec{p}_1 c)\, \hat{p}(i_o | f_1) \qquad \text{(A-10)}$$

$$= \sum_{f_1} E_1 \, \chi \Omega_o \, e^{-\beta E_1 - \cdots}$$

where $(M_o c^2)^2 = E_o^2 - \vec{P}_o^2 c^2$.

Thus, Lorentz invariance reduces the number of Lagrange multipliers associated with the four-momentum to one single multiplier β associated with the rest energy of the system. Moreover, the standard form of the grand canonical approximation holds when the four-momenta are referred to the CM frame.

Appendix B: The macroscopic nuclear energy

For nuclei with $A > 16$ it is impractical to use the exact ground-state energy. Instead, we use the following macroscopic formula for the ground-state mass excess,

$$
\begin{aligned}
V_{A_Z}\,(\mathrm{MeV}) &= N\,V_n + Z\,V_H \\
&+ (-a_1 A + a_2 A^{2/3})(1 - I^2) + \left(\frac{c_3}{A^{1/3}} - \frac{c_4}{A}\right) Z^2 \\
&+ \begin{cases} \dfrac{12}{\sqrt{A}} - \dfrac{10}{A} & \text{(odd-odd)} \\[2ex] \dfrac{10}{A} & \text{(odd-A)} \\[2ex] -\dfrac{12}{\sqrt{A}} + \dfrac{10}{A} & \text{(even-even)} \end{cases} \\
&+ 30\left(|I| + \frac{1}{A}\ (\text{if } N = Z = \text{odd})\right) \\
&+ \frac{50}{A}
\end{aligned}
\tag{B-1}
$$

The true rest energy is given by $Mc^2 = V + 931.504\ \mathrm{MeV}\ A$.

Here $V_n = 8.07167$ MeV and $V_H = 7.28922$ are the mass excesses of the neutron and the hydrogen atom. (Whether or not the atomic binding energies should be considered is of no importance in the present context.)

In the second line we use the parameter values

$$
\begin{aligned}
a_1 &= 15.4941\ \mathrm{MeV}\ , \quad a_2 = 17.9439\ \mathrm{MeV} \\
c_3 &= 0.7053\ \mathrm{MeV}\ , \quad c_4 = 1.1533\ \mathrm{MeV}
\end{aligned}
\tag{B-2}
$$

The third line is the even-odd (pairing) term and the fourth line is the Wigner term[18].

Finally, in the last line we have added a phenomenological shift. This term is of little consequence for heavy nuclei but for very light nuclei it has the effect of reproducing the average trend of the masses. Due to this term, the formula can be used all the way down to $A \approx 4$ without systematic error (although we only use it down to $A = 17$).

It should be added, though, that in using the above macroscopic formula, shell effects have been neglected. In the present calculations, shell effects are expected to manifest themselves primarily at the last stages of the evaporation chains, when the nuclei have cooled sufficiently. The errors introduced in this way are unimportant at the present stage of development.

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Figure captions

Fig. 1. The mean number of nucleons in the various sources as functions of the impact parameter, for two cases of experimental interest: $^{40}Ca + ^{40}Ca$ and $^{20}Ne + ^{208}Pb$. The numbers a and b refer to the projectile and target spectator source, respectively, while c refers to the participant source.

Fig. 2. Illustration of event addition: The events f (containing six fragments) and f (containing four fragments) are added to form the event f (containing all ten fragments).

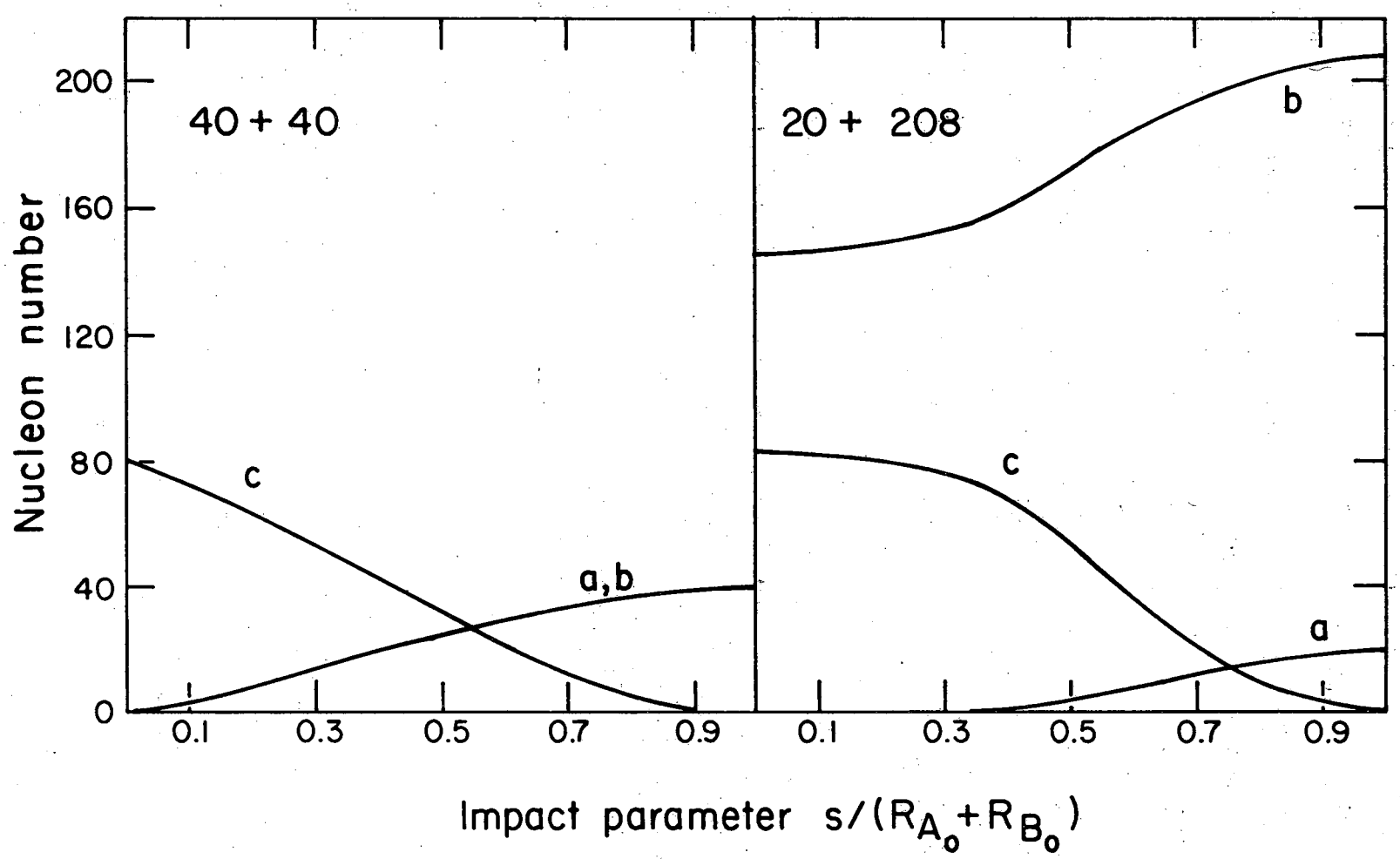
Fig. 3. Illustration of one step in the evaporation sequence. The total barrier B against emission of the particular type of ejectile is given by the separation energy S plus the Coulomb barrier V. The total available energy $\varepsilon_0 - S$ goes into kinetic energy KE of the final two-particle system and intrinsic excitation ε of the daughter nucleus. The curve indicates qualitatively the relative probability for reaching various excitations ε.

Fig. 4. The mass distribution of four generated events for the case of 120 MeV/n $^{40}Ar + ^{208}Pb$. In addition to the many light fragments there is one heavy fragment in each event: the evaporation residue of the target-like spectator source. The hatched parts of the nucleon and alpha yields result from the evaporation phase.

Fig. 5. The mean fragment multiplicity n as a function of impact parameter for two cases of experimental interest. The full curve is the total multiplicity while the dashed curve only includes charged fragments with a kinetic energy above 20 MeV in the laboratory, corresponding to a schematic detection bias.

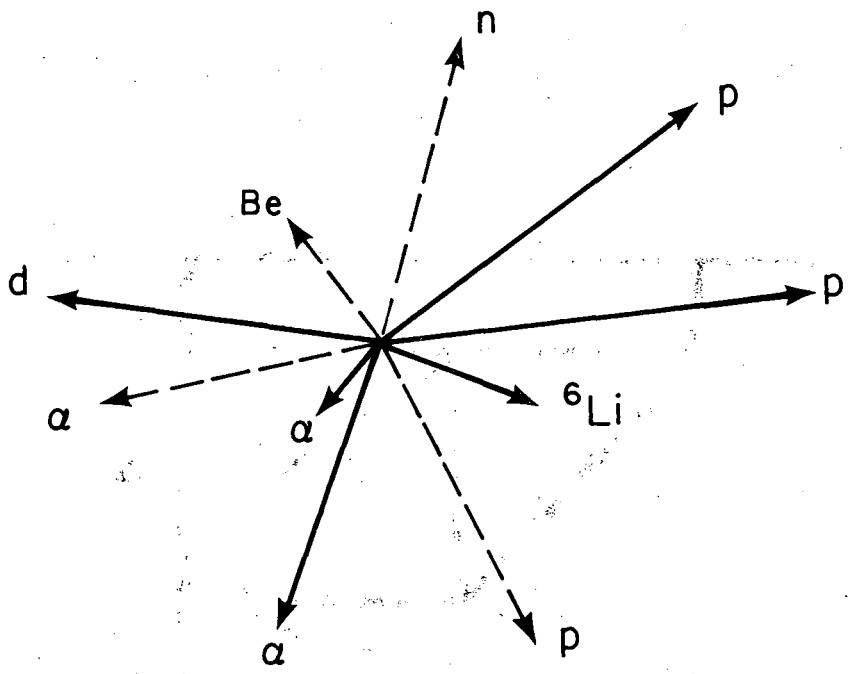
Fig. 6. The Dalitz-type triangular plot of the event shapes expressed in terms of sphericity and coplanarity, cf. text.

Fig. 7. Contour plots in the sphericity-coplanarity plane of 200 "central" events for the cases 395 MeV Ca + Ca and 795 MeV Ne + Pb. The labels on the contours indicate the relative probabilities on a linear scale.



XBL 8210 - 1286

Fig. 1



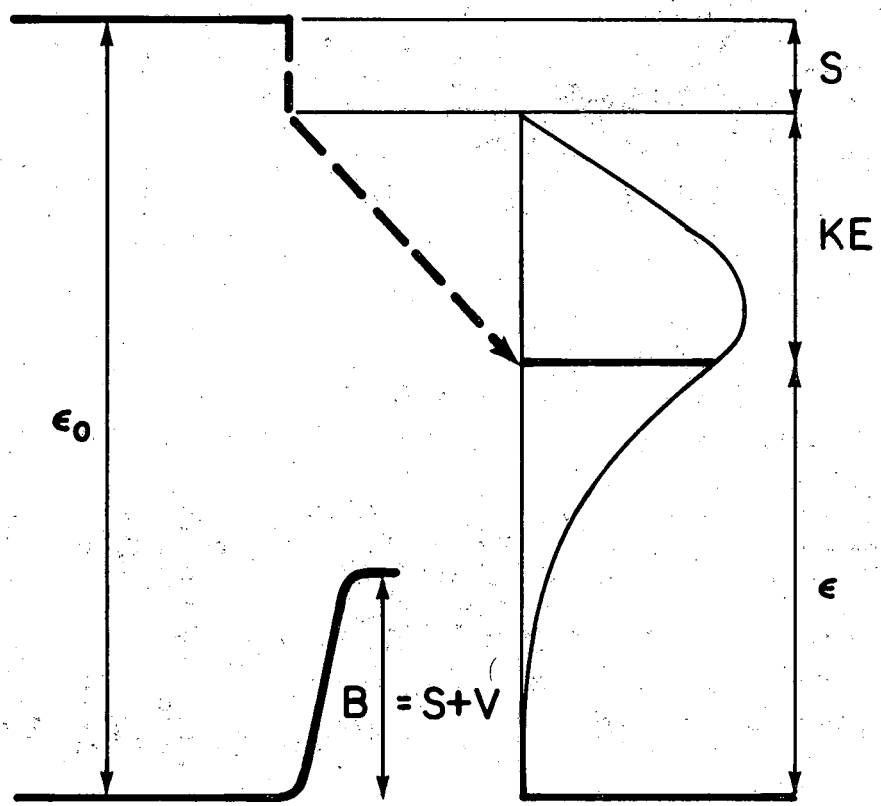
$$\tilde{f} = \left\{ \tilde{p}^1_p, \tilde{p}^2_p, \tilde{p}^1_d, \tilde{p}^1_\alpha, \tilde{p}^2_\alpha, \tilde{p}^1_{^6Li} \right\}$$

$$\bar{f} = \left\{ \bar{p}^1_n, \bar{p}^1_p, \bar{p}^1_\alpha, \bar{p}^1_{^9Be} \right\}$$

$$f = \tilde{f} + \bar{f} = \left\{ p^1_n, p^1_p, p^2_p, p^3_p, p^1_d, p^1_\alpha, p^2_\alpha, p^3_\alpha, p^1_{^6Li}, p^1_{^9Be} \right\}$$

XBL 8210 - 1288

Fig. 2



XBL 8210 - 1287

Fig. 3

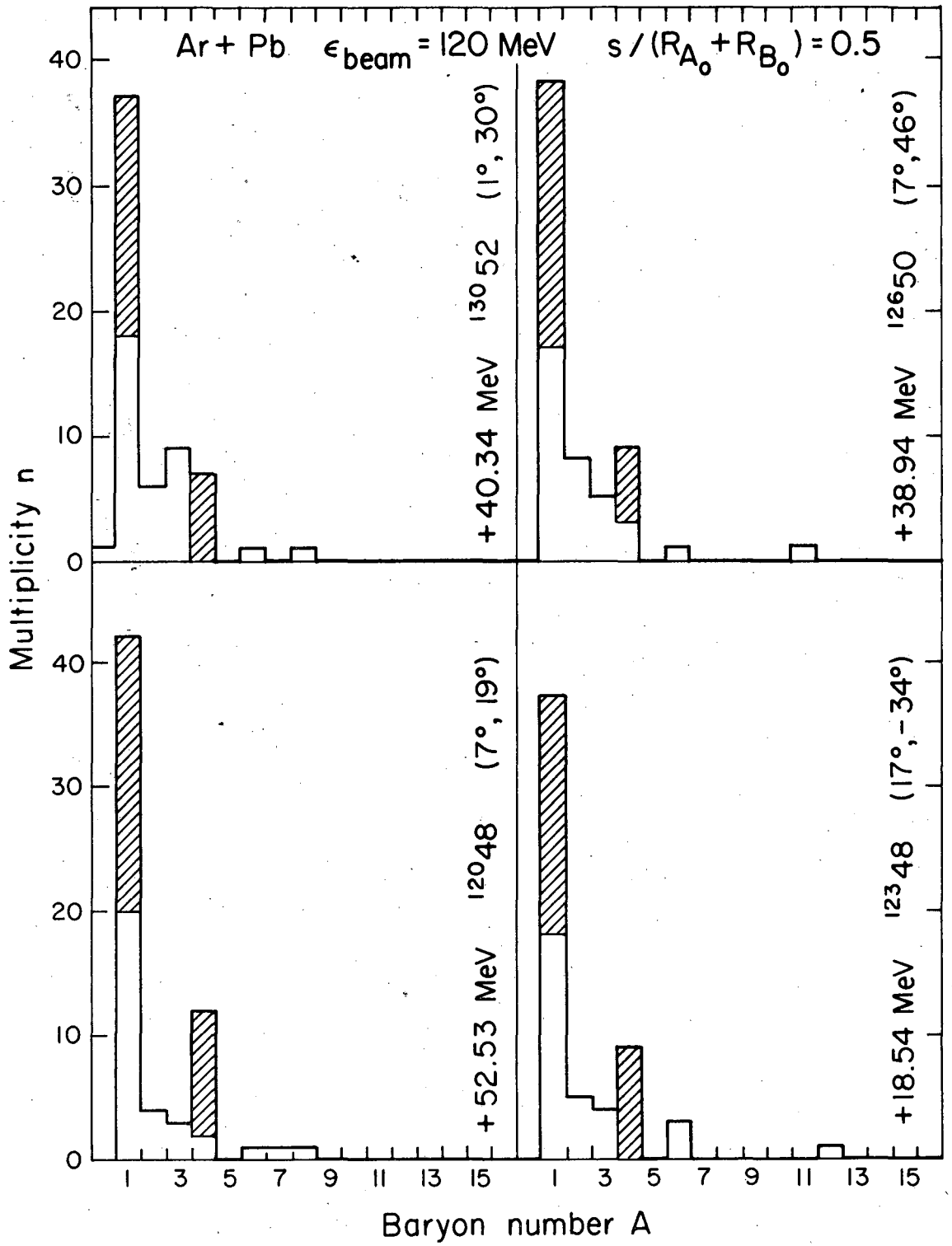
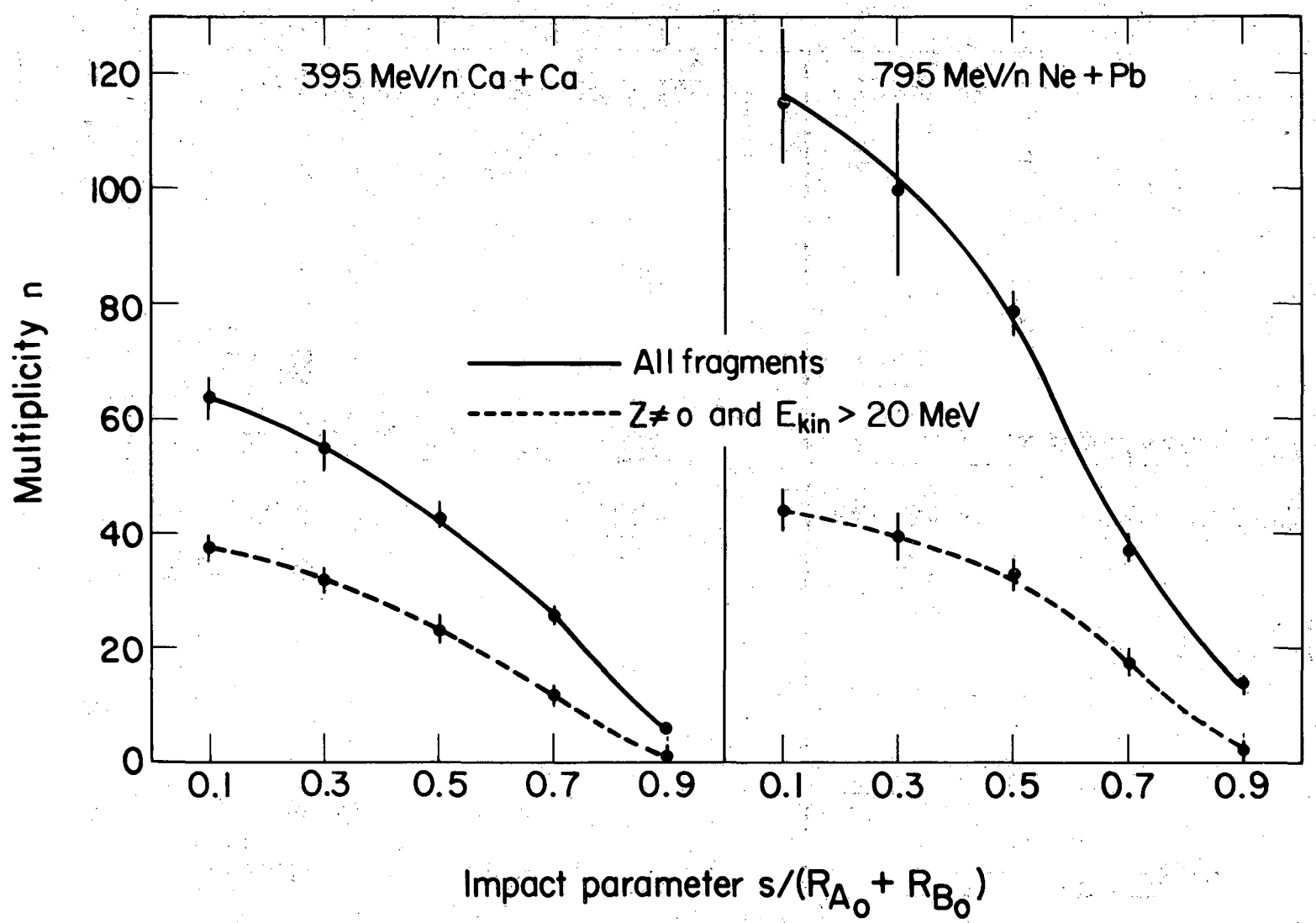
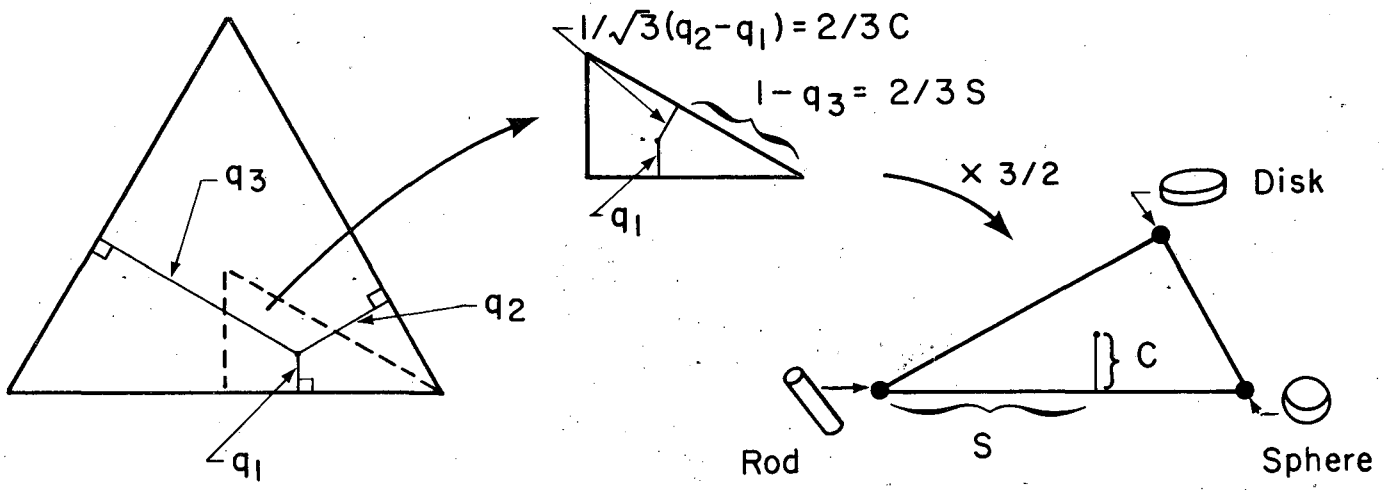

Ar + Pb ϵ_{beam} = 120 MeV s / (R_{A_0} + R_{B_0}) = 0.5
+40.34 MeV ^{130}52 (1°, 30°)
+38.94 MeV ^{126}50 (7°, 46°)
+52.53 MeV ^{120}48 (7°, 19°)
+18.54 MeV ^{123}48 (17°, -34°)
Multiplicity n
Baryon number A
0
10
20
30
40
1
3
5
7
9
11
13
15
XBL 828 – 1076

Fig. 4



XBL 829-4613

Fig. 5



XBL 8210 - 1247

Fig. 6

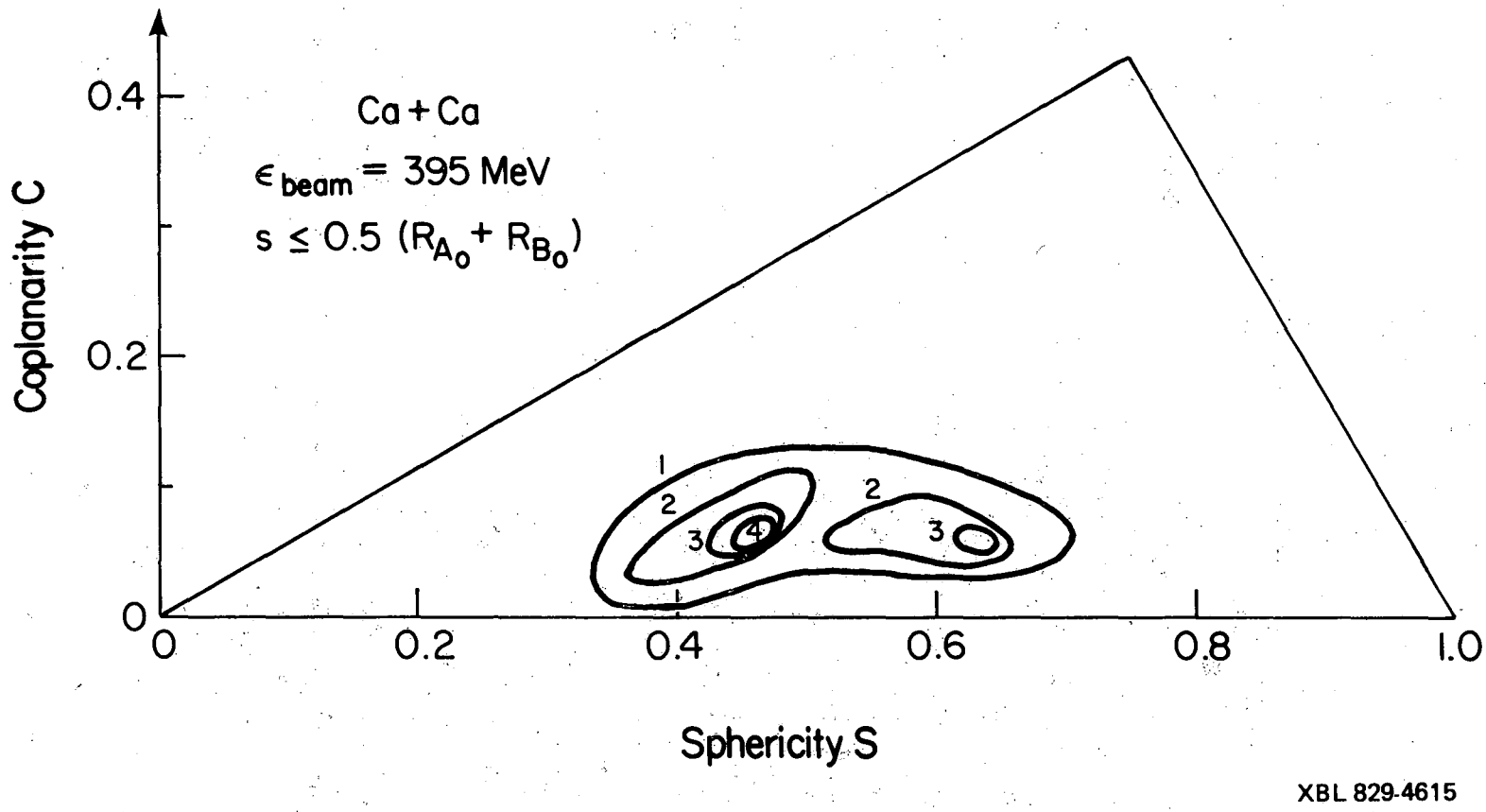
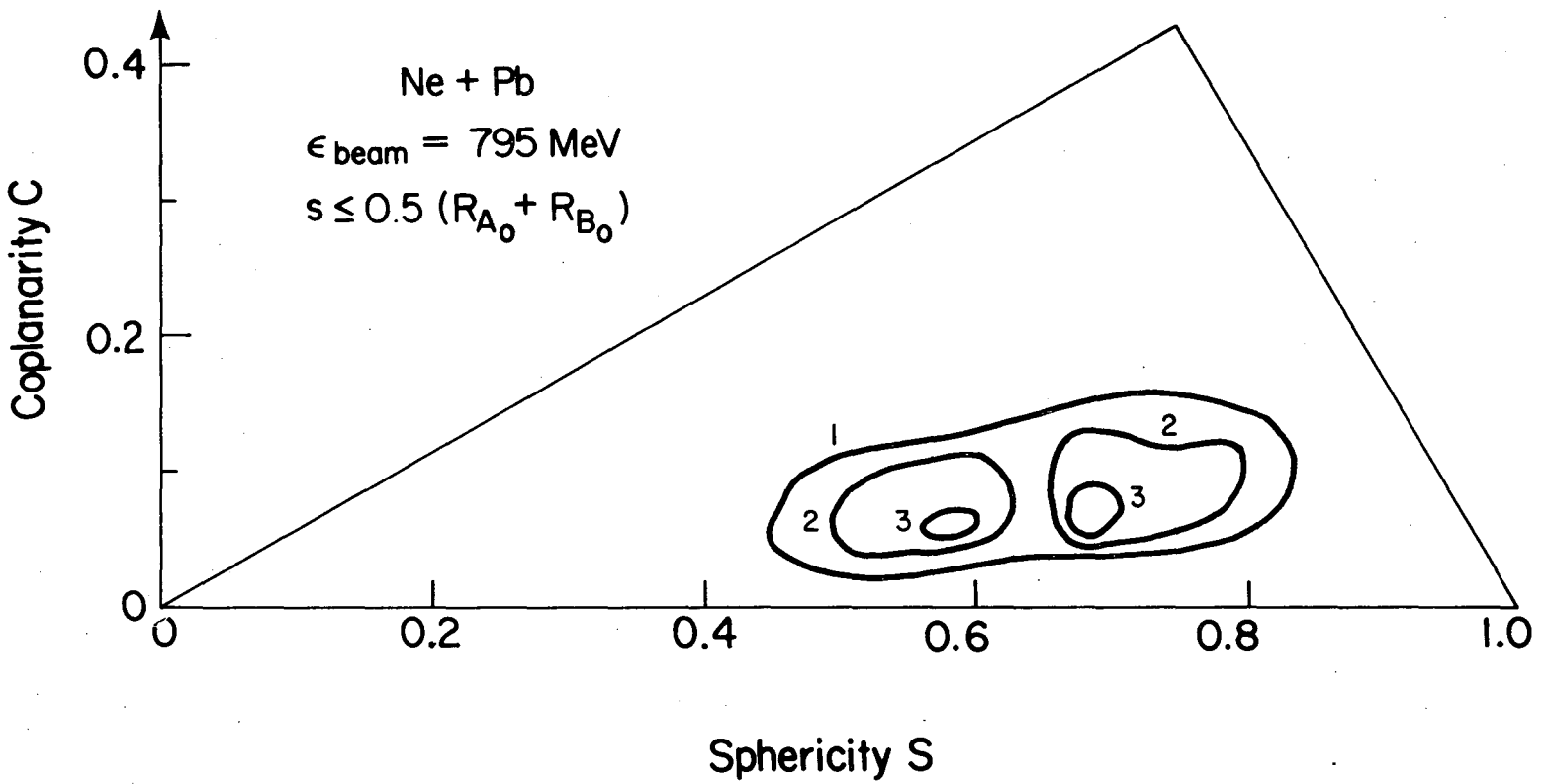

Ca + Ca
ϵ_{beam} = 395 MeV
$s \leq 0.5\ (R_{A_0} + R_{B_0})$
Coplanarity C
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1.0
Sphericity S
1
2
3
4
2
3
XBL 829-4615

Fig. 7a



XBL 829-4612

Fig. 7b

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