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DROPLET-MODEL PREDICTIONS OF CHARGE MOMENTS

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DROPLET MODEL PREDICTIONS OF CHARGE MOMENTS

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Abstract The Droplet Model expressions for calculating various moments of the nuclear charge distribution are given. There are contributions to the moments from the size and shape of the system, from the internal redistribution induced by the Coulomb repulsion, and from the diffuseness of the surface. A case is made for the use of diffuse charge distributions generated by convolution as an alternative to Fermi-functions.

INTRODUCTION

The Droplet Model was originally developed to provide a macroscopic description of the binding energy and spatial distribution of a saturating, two-component, leptodermous system,[1,2] and the model coefficients were later chosen to correspond to the values they would have in atomic nuclei.[3] Like the liquid drop model that preceded it, the Droplet Model nuclear binding energy predictions agree with the measured values to within a percent or less for the heavier nuclei (10 MeV out of 1000 MeV). Similar accuracy (or even better) seems to be obtainable in the description of the density distribution as well.

The Droplet Model only predicts the spatial extent (size) of the neutron and proton density distributions and the relatively small amount of internal redistribution induced by the Coulomb field. It must be supplemented by information about the shape and diffuseness of the distribution before comparison can be made with measured moments.

In the next sections the various contributions to the moments of the charge distribution are enumerated, with special emphasis on the convenience and utility associated with diffuse distributions generated by convolution.

DROPLET MODEL

The single most significant feature differentiating the Droplet Model from earlier developments in the macroscopic approach to nuclear properties is the neutron skin. Previously some measure of the nuclear size (the equivalent sharp radius, the 50% density radius, or the RMS radius) was taken to be proportional to $A^{1/3}$ (or an empirically determined polynomial in $A^{1/3}$). The new procedure, in terms of the Droplet Model, is to express the volume energy of the nucleus in terms of the bulk neutron and proton densities and the surface energy in terms of their effective sharp radii. Then the actual values of the densities and radii are determined by minimization. One result of this approach is the prediction of a neutron skin for neutron rich nuclei similar to the predictions of

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microscopic self-consistent calculations.[4] (Myers and Swiatecki recently reviewed the "Droplet Model" Theory of the Neutron Skin", and the work described here extends that discussion.[5]) Figure 1 serves to further illustrate this point.

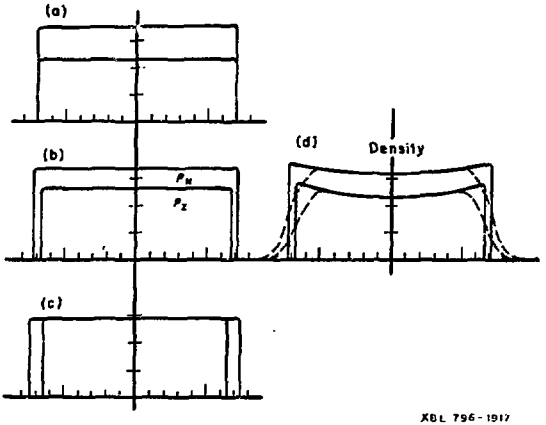

FIGURE 1

In this figure, schematic neutron and proton density distributions for ^{208}Pb are plotted vs the radial distance. The surfaces are drawn sharp (zero diffuseness) to help illustrate the points being made. Part (a) represents the strict "liquid drop model" in which $R_N = R_Z = R = r_o A^{1/3}$. Part (c) represents the opposite extreme where the central neutron and

proton densities are the same and all the excess neutrons are placed in the surface. Part (b) represents the actual (intermediate) situation predicted by the Droplet Model, where the neutrons are partially pushed into the surface by the volume symmetry energy. To the right of part (b), in part (d), the Droplet Model prediction for the central depression in the neutron and proton density distributions (caused by the Coulomb repulsion) is also shown. The diffuseness of the actual surfaces is given approximately by the dashed lines in part (d).

In the discussion that follows a number of Droplet Model coefficients appear. One set of currently accepted values is[3]:

r_o = 1.18 fm, the nuclear radius constant,

$c_1 = \frac{3}{5}(e^2/r_o)$ = 0.7322 MeV, the Coulomb energy coefficient,

a_2 = 20.69 MeV, the surface energy coefficient,

J = 36.8 MeV, the symmetry energy coefficient,

Q = 17 MeV, the effective surface stiffness,

K = 240 MeV, the compressibility coefficient,

L = 100 MeV, the density symmetry coefficient.

The Size

The first step in constructing the Droplet Model nuclear density distributions is to calculate the size of the system (neutrons and protons together) from the expression

$$R = r_o A^{1/3} (1 + \bar{\varepsilon}) \quad , \tag{1}$$

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where r_o is related to the density of infinite (N = Z) nuclear matter by the expression

$$\rho_o = \left(\frac{4}{3}\pi r_o^3\right)^{-1} \quad , \tag{2}$$

and the small quantity $\bar{\varepsilon}$ is a measure of the dilatation (or squeezing) that occurs for finite nuclei. For a spherical nucleus the quantity R calculated in Eq. (1) is the equivalent sharp radius of the nuclear density distribution. For a deformed nucleus we can still use Eq. (1) with the understanding that the volume enclosed by the equivalent sharp surface is given by

$$V = \frac{4}{3}\pi R^3 \quad , \quad \text{and} \quad \bar{\rho} = A/V \quad , \tag{3}$$

where $\bar{\rho}$ is the average value of the density in the bulk.

The quantity $\bar{\varepsilon}$ in Eq. (1) can be calculated from the Droplet Model expression

$$\bar{\varepsilon} = \left(-2a_2 A^{-1/3} B_s + L\,\bar{\delta}^2 + c_1 Z^2 A^{-4/3} B_c\right)/K \quad , \tag{4}$$

where the first term represents squeezing by the surface tension, the second dilatation due to the neutron excess, and the third dilatation due to Coulomb repulsion. The compressibility K in the denominator acts to resist these changes. (The quantity B_s is the ratio of the surface area of a deformed nucleus to that of a sphere with equal volume, and B_c is the analogous term for the Coulomb energy.) The quantity $\bar{\delta}$ appearing in Eq. (4) is the bulk average of the local nuclear asymmetry $\delta = (\rho_n - \rho_z)/(\rho_n + \rho_z)$. It is related to the global nuclear asymmetry

$$I = (N-Z)/A \quad , \tag{5}$$

by the Droplet Model expression,

$$\overline{\delta} = \frac{I + \frac{3}{16}\,(c_1/Q)ZA^{-2/3}\,B_v}{1 + \frac{9}{4}\,(J/Q)\,A^{-1/3}\,B_s} \quad , \tag{6}$$

where B_v is another shape dependence related to the Coulomb energy.

Equation (6) is a little harder to analyze than Eq. (4) because of the denominator that should not be expanded (an essential feature of the Droplet Model). But we can expand it for pedagogical purposes, yielding

$$\overline{\delta} \approx I + \frac{3}{4}\left(\frac{1}{4}\,c_1 ZA^{-2/3}\,B_v - 3\,JIA^{-1/3}\,B_s\right)/Q \quad . \tag{7}$$

In this expression we see that the bulk nuclear asymmetry $\overline{\delta}$ is increased above the value I by the first term in parentheses (a Coulomb term that tries to expand the proton distribution leaving the neutrons behind). The second term in parentheses reduces the value of $\overline{\delta}$ (because the bulk symmetry energy tries to equate the neutron and proton densities, by pushing the excess neutrons into the surface) and the coefficient Q in the denominator resists both of these effects. The quantity $\overline{\delta}$ is usually smaller than I, and a neutron skin (of thickness t) is produced (except for some very light nuclei where $Z \geq N$), because of the geometric relationship,

$$t = \frac{2}{3}\,R\,(I - \overline{\delta})/B_s \quad . \tag{8}$$

Once Eqs. (1 and 8) have been used to calculate R and t, the separate effective sharp radii for the neutron and proton distributions can be calculated from the expressions,

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$$R_n = R + \frac{Z}{A} t \quad ,$$
$$R_z = R - \frac{N}{A} t \quad , \tag{9}$$

The Shape

A simple, useful representation for the shape of the density distribution is in terms of Legendre polynomials,[6]

$$r = nR\,(1 + \alpha_2 P_2 + \alpha_4 P_4) \quad , \tag{10}$$

where volume normalization is insured by setting

$$n = (1 - \frac{1}{5}\alpha_2^2 - \frac{2}{105}\alpha_2^3 + \frac{2}{25}\alpha_2^4 - \frac{2}{35}\alpha_2^2\alpha_4 - \frac{1}{9}\alpha_4^2 \ldots). \tag{11}$$

In terms of this expansion the shape dependences above are

$$B_s = 1 + \frac{2}{5}\alpha_2^2 - \frac{4}{105}\alpha_2^3 - \frac{66}{175}\alpha_2^4 - \frac{4}{35}\alpha_2^2\alpha_4 + \alpha_4^2 \ldots$$
$$B_c = 1 + \frac{1}{5}\alpha_2^2 - \frac{4}{105}\alpha_2^3 + \frac{51}{245}\alpha_2^4 - \frac{6}{35}\alpha_2^2\alpha_4 - \frac{5}{27}\alpha_4^2 \ldots \tag{12}$$
$$B_v = 1 - \frac{1}{5}\alpha_2^2 - \frac{2}{105}\alpha_2^3 - \frac{253}{1225}\alpha_2^4 - \frac{4}{105}\alpha_2^2\alpha_4 + \frac{4}{9}\alpha_4^2 \ldots$$

The Redistribution

Once the Droplet Model size of the distribution has been determined, and some other source has been consulted for information about the shape (for example the recent Möller and Nix compilation of predicted masses and deformations[7,8]), we can proceed to the next level of refinement. The protons (and neutrons to some extent) are pushed toward the surface by the Coulomb repulsion and the small increase in binding that results is included in the Droplet Model energy expression. The corresponding change in the density is given by[2]

$$\tilde{\rho}_{\tilde{z}} = -\frac{1}{2}\,\rho_o \left(\frac{9}{2K} \mp \frac{1}{4J}\right) e\tilde{v} \quad , \tag{13}$$

where $\tilde{v}$ is the deviation of the Coulomb potential from its average value over the bulk.

For a spherical distribution,[2]

$$v = \frac{Ze}{R_z}\left[\frac{3}{2} - \frac{1}{2}\left(\frac{r}{R}\right)^2\right] ,$$

$$\bar{v} = \frac{6}{5}\frac{Ze}{R_z} , \qquad (14)$$

$$\tilde{v} = \frac{1}{2}\frac{Ze}{R_z}\left[\frac{3}{5} - \left(\frac{r}{R}\right)^2\right] ,$$

and for a distribution whose shape is given by Eq. (10),[6]

$$v = \frac{Ze}{R_z} n^2 \left[\frac{3}{2} - \frac{1}{2}\left(\frac{r}{R'}\right)^2\left(1 - \frac{6}{5}\alpha_2 P_2\right) + \frac{1}{3}\left(\frac{r}{R'}\right)^4 \alpha_4 P_4 \right.$$

$$\left. - \frac{3}{4}\sum_{k=0,2,4}(k-1)\left(\frac{r}{R'}\right)^k P_k\left(\alpha_2^2 \langle 22k\rangle + \alpha_2\alpha_4\langle 24k\rangle\right)\right] \qquad (15)$$

$$\bar{v} = \frac{6}{5}\frac{Ze}{R_z} B_c \quad \text{and} \quad \tilde{v} = v - \bar{v} ,$$

where $R' = nR_z$, and n is given by Eq. (11).

The N-symbol brackets in Eq. (15) have the values

$\langle 220\rangle = \langle 22\rangle = 2/5$,

$\langle 222\rangle = \langle 224\rangle = \langle 242\rangle = 4/35$,

$\langle 244\rangle = 40/693$,

$\langle 240\rangle = \langle 24\rangle = 0$.

Then the change in the proton density can be written

$$\tilde{\rho}_z = \frac{1}{2}\rho_o C' f(r) , \qquad (16)$$

where

$$C' = \frac{1}{2}\left(\frac{9}{2K}\right) + \frac{1}{4J}\frac{Ze^2}{R_z} , \qquad (17)$$

$$\approx 0.0156\ ZA^{-1/3} ,$$

and for a spherical nucleus

$$f(r) = [3/5 - (r/R_z)^2] \quad . \tag{18}$$

For a deformed distribution the function $f(r,\theta)$ can be inferred from Eqs. (15).

Part (d) of Figure 1 shows how the redistribution creates a central depression in the proton density distribution (and a smaller one in the neutron distribution because of the change of sign in Eq. (13)). The upper part of Figure 2 is also meant to convey the idea that charge is moved outward from the center toward the surface. The lower part of Figure 2 shows how redistribution increases the charge on the ends of a

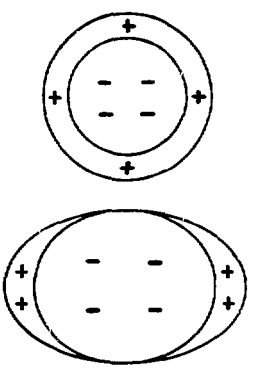

FIGURE 2

deformed nucleus, leading to an enhancement of the quadrupole moment.

The Diffuseness

Once the size, the shape, and the redistribution have been combined, the last step in the determination of the Droplet Model prediction for the density distribution is to add the diffuseness. The customary procedure of using Fermi functions is awkward and inconvenient (especially when the density is deformed and redistributed) and so we will use convolution to create the diffuseness. When a normalized, spherically symmetric, short-ranged function is folded into our sharp-surfaced distribution the resulting diffuse distribution still has the same volume integral (contains the same number of particles). What is most significant is the fact that the multipole moments of the distribution are unchanged and the radial moments (and moment of inertia) simply increase by a known constant.[9,10] So, we do not have to actually perform the convolution integral, unless we want to plot the density distribution itself.

A proposal for creating the diffuseness in this way was made by Helm,[11] in connection with the interpretation of elastic electron scattering measurements. More recent work along these same lines has been done by Friedrich and Voegler.[12,13] They find that "Helm Model" effective sharp radii extracted from experiment behave much as the Droplet Model says they should and that the variance of the gaussian folding function,

$$g(r) = (2\pi\sigma^2)^{-3/2} \exp(-r^2/2\sigma^2) \qquad (19)$$

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is nearly constant throughout the periodic table except for some evidence of shell structure. Using Figure 3, which is from their work,[13] we can deduce $\sigma = 0.95$ fm as a reasonable average value for use in the remainder of our calculations.

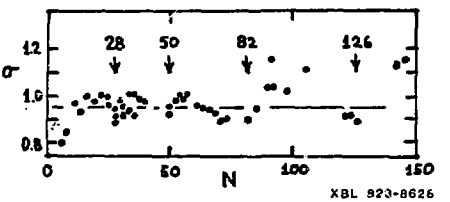

FIGURE 3

It should be mentioned in passing that $\sigma = b$ and that the geometrical relationships between the surface location C, the effective sharp radius R, the effective RMS radius Q, and the surface width b have been discussed by Süssmann[14] and other authors.[15] The advantages of the folding method have also received recent attention.[9,10,16]

Once the charge distribution has been determined various moments can be calculated for comparison with the measured values. This is relatively simple since the quantities of interest factorize into contributions from the size, the shape, the redistribution, and the diffuseness.

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RADIAL MOMENTS

The most frequently measured quantity, the mean square radius $\langle r^2 \rangle$, can be calculated from the expression

$$\langle r^2 \rangle = \langle r^2 \rangle_u + \langle r^2 \rangle_r + \langle r^2 \rangle_d \quad , \tag{20}$$

where (the subscript Z, indicating protons, is dropped here since these expressions are completely general)

$$\langle r^2 \rangle_u = \frac{3}{5} R^2 \left(1 + \alpha_2^2 + \frac{5}{9} \alpha_4^2 + \ldots\right) \tag{21}$$

is the contribution from the size of the uniform distribution and its shape. The next term,

$$\langle r^2 \rangle_r = \frac{12}{175} C' R^2 \left(1 + \frac{14}{5} \alpha_2^2 + \frac{595}{108} \alpha_4^2 + \ldots\right) \tag{22}$$

is the contribution from the redistribution, and its shape dependence. The last term,

$$\langle r^2 \rangle_d = 3\sigma^2 \quad \text{or } 3b^2 \tag{23}$$

is the contribution from the diffuseness. It is interesting to note that the diffuseness correction has no shape dependence and that it is the same for all nuclei (so long as we assume that the diffuseness itself is a constant).

In order to assess the relative importance of the different terms we can consider the example of ^{238}U. The spherical Droplet Model value of R_z is 7.030 fm, and this becomes 7.027 fm when the effect of deformation (using the values $\alpha_2 = 0.139$ and $\alpha_4 = 0.065$ provided by Möller[8]) on the size of the distribution is included via the B's Eqs. (4, 6, and 8). The mean square radius of such a distribution is 29.627 (1+0.0218) fm^2, where the

first part comes from the size and the second from the shape. The redistribution contribution is 0.879 fm^2, and the diffuseness contribution is $3(0.95)^2 = 2.707$ fm^2, for a total of 33.860 fm^2. The predicted RMS radius is 5.819 fm, which is to be compared with the experimental value of 5.843 ± 0.012 fm.[17]

The three parts of Figure 4 show a broader comparison between measured and calculated values. These plots, which were made by Schmidt,[18] used somewhat different α's for calculating the mean square radius from those provided by Möller.[8] The upper plot shows the difference that remains when the measured RMS radii are compared to liquid drop model predictions (with deformation corrections included). The downward slope to the right for each isotopic sequence is removed when the Droplet Model is used, as can be seen in the second plot. At the bottom of this figure the same comparison is made but without corrections for deformation and redistribution.

The first few even moments of a uniform spherical distribution made diffuse by folding with a gaussian are

$$\langle r^2 \rangle = \frac{3}{5} R^2 \left(1 + 5\beta^2\right) ,$$

$$\langle r^4 \rangle = \frac{3}{7} R^4 \left(1 + 14\beta^2 + 35\beta^4\right) , \qquad (24)$$

$$\langle r^6 \rangle = \frac{3}{9} R^6 \left(1 + 27\beta^2 + 189\beta^4 + 315\beta^6\right) ,$$

where

$$\beta = \sigma/R .$$

To the best of my knowledge, the contributions to the higher moments from deformations and redistribution have not yet been worked out.

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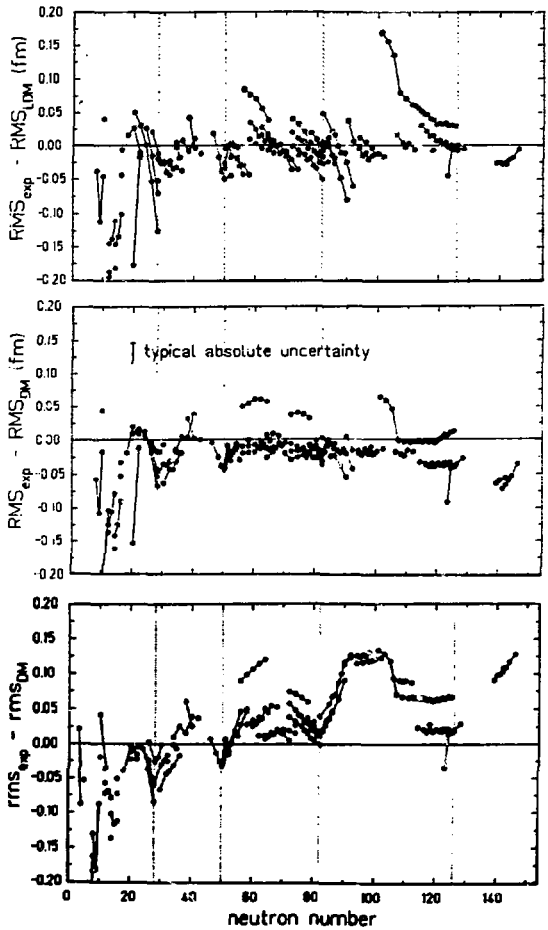

FIGURE 4

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MULTIPOLE MOMENTS

Using the definition,

$$Q_\ell = 2 \int r^\ell P_\ell \, d^3r \quad , \tag{25}$$

we can derive expressions for the multipole moments analogous to Eq. (20). Thus,

$$Q_\ell = Q_\ell^u + Q_\ell^r + Q_\ell^d \quad , \tag{26}$$

where Q_ℓ^u is the contribution from the uniform distribution, Q_ℓ^r the contribution from redistribution, and (as we mentioned before) Q_ℓ^d is the contribution from the diffuseness, which is identically zero. For the quadrupole moment,

$$Q_2^u = \frac{6}{5} ZR^2 \left(\alpha_2 + \frac{4}{7}\alpha_2^2 + \frac{8}{7}\alpha_2\alpha_4 + \frac{200}{693}\alpha_4^2 + \ldots\right), \tag{27}$$

$$Q_2^r = \frac{48}{175} c' ZR^2 (\alpha_2 + \ldots) \quad ,$$

and for the hexadecapole moment,

$$Q_4^u = \frac{2}{3} ZR^4 \left(\alpha_4 + \frac{54}{35}\alpha_2^2 + \frac{120}{170}\alpha_2\alpha_4 + \frac{486}{1001}\alpha_4^2 + \ldots\right), \tag{28}$$

$$Q_4^r = \frac{8}{245} c' ZR^4 (\alpha_4 + \ldots) \quad .$$

The reader is warned that these expressions may not be accurate enough if the αs are not sufficiently small. Another, more serious problem is that expressions like these are sometimes used to infer values of α_2 and α_4 (or related deformation parameters) from measured Q's under the assumption that $R \approx 1.2\ A^{1/3}$ fm. In the case of ^{238}U the Droplet Model (and measurements of $\langle r^2 \rangle$) favor a value of R_z that is about 6% smaller, which would increase α_2 by 12% and α_4 by 24%.

SUMMARY AND CONCLUSIONS

We have shown how the Droplet Model, when supplemented with information about the shape and diffuseness of the nucleus, can provide moments of the density distribution in good agreement with the measured values. The accuracy obtainable by this method seems to be about 0.01 fm (or 10 am), which corresponds to 0.2% in the RMS r^2 s for a heavy nucleus. This is almost as good as the 0.1% accuracy (1 MeV out of 1000 MeV) that can be obtained for the binding energy, when the macroscopic approach is supplemented by shell corrections.

Indeed, the one thing missing here is some sort of phenomenological estimate of the contribution to the density distribution from the *quantum* nature of the system. Since we were mainly concerned here with moments, it wasn't necessary to consider the small wiggles in the density distribution that have been observed in electron scattering. These wiggles have a wavelength corresponding to the Fermi energy and their sign and magnitude probably depend mainly on whether or not an s-state has just been filled. At present only Hartree-Fock calculations are capable, in principle, of describing such wiggles.[19]

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