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The Clutching of Two Colliding Nuclei*

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Abstract

The macroscopic model of nuclear coalescence and reseparation is extended to noncentral collisions. The relevant degrees of freedom are the following ones: asymmetry, fragment separation, neck size, and the total amount of sliding that has taken place until the two nuclei clutched. It is shown that, within certain limits, centrifugal force to a good approximation can be traded for electric repulsion via an "angular momentum fraction". The value of the angular momentum fraction predicted by the theory is consistent with the one used to obtain the best fit to data.

1. Introduction

The macroscopic model of nuclear coalescence and reseparation [1] describes, in the first place, head-on collisions, although it has been generalized schematically to include angular momentum. In the present work the treatment of ref. [1] is extended to noncentral collisions by taking into account the relative angular momentum and the rotation of the individual nuclei, following the discussion of ref. [2].

The equations of motion remain sufficiently simple after the inclusion of the angular momenta so that approximate solutions in closed form may still be found in several cases of interest. In particular, the theory provides the normal modes of motion around the saddle points of the potential energy surface. The modes already present in the description of central collisions are unaffected. There is, however, one further normal mode emerging in connection with the inclusion of angular momenta. The characteristic time for this latter mode is connected to the transition from sliding to rolling, often referred to as the "clutching" of the colliding nuclei.

The predictions of the extra push model [1] have recently been made the basis of a simple algebraic theory for the energy-dependence of the fusion cross section [3]. Recent experimental data [4] have been fit in terms of three parameters, the effective fissility, $(Z^2/A)_{eff}$, beyond which an extra energy over the interaction barrier (extra push) is needed to make the colliding nuclei fuse, the initial rate of increase of the extra push, a, and an "angular momentum fraction", f, the concept of which is based on the idea that centrifugal force can be effectively traded for electrostatic repulsion. The physics connected with the parameters $(Z^2/A)_{eff}$ and a is discussed in refs. [3], [5], and [6]. Here I am going to concentrate on the question of the angular momentum fraction f. In particular, I will present a simplified

treatment of the relative angular momentum and the rotation of the individual nuclei to examine whether the assumption of trading centrifugal force against electrostatic repulsion is justified. The results show that (at least in a reasonable range of colliding systems and for not too strong rotation) the above approximation can indeed be made. Furthermore, a theoretical value for the angular momentum fraction f is obtained, which will be compared to the one used to fit the data.

I do not attempt to be self-contained in the present report; since I will elaborate on one particular aspect of the extra push model [1], it appears to be convenient to use the general framework and notation developed earlier. The reader is referred to ref. [1] for details.

In Section 2 the treatment of the angular momenta will be described, while in Section 3 I will present some instructive results of the model. In Section 4, I will draw a few general conclusions.

2. Extension of the Extra Push Model for Noncentral Collisions

There are three relevants degrees of freedom required for the description of head-on collisions: the asymmetry, Δ, the fragment separation, ρ, and the neck size, α. The precise meaning of these depends on the shape parametrization used to describe the system. I will use the parametrization and the variables which pertain to two spherical nuclei connected by a conical neck and are defined and discussed in ref. [1]:

$$\Delta = \frac{R_1-R_2}{R_1+R_2} \quad , \quad \rho = \frac{r}{R_1+R_2} \quad , \quad \alpha = \left(\frac{\sin\theta}{\sin\theta_{max}}\right)^2 \tag{1}$$

with $\sin\theta_{max} = (R_1 - R_2)/r$ referring to a fully open window. As in ref. [1], $\alpha = 1/2$ will be taken to be the boundary between the "dinuclear" (necked-in shapes) and "mononuclear" regimes. The above shape-parametrization is illustrated in Fig. 1.

The main interest of the present work resides in the clutching of the colliding nuclei. This is expected to happen in the relatively early stages of the process, with the window less than half open ($\alpha < 1/2$). Therefore, in what follows, attention will be focused on the dinuclear regime. This results in simplifications in the solution of the equations of motion.

In addition to the shape of the system the relative rotation and the self-rotation of the two nuclei has to be characterized. These three rotational degrees of freedom have been introduced in ref. [2] as the angle of rotation, θ, of the straight line joining the centers of the colliding nuclei in some fixed inertial frame of reference, and the angles θ_1 and θ_2, describing the orientations of both nuclei relative to the same reference frame (see Fig. 1).

The equations of motion are derived from the Lagrangian and the dissipation function, which contain the dynamical ingredients of the theory [1]: a macroscopic potential energy in terms of Swiatecki's very elegant and useful simple parametrization, a macroscopic dissipation function in the form of the Wall-plus-Window formula, and inertias appropriate to two noncommunicating pieces (i.e. a reduced mass for the relative motion and $(2/5)M_i R_i^2$ for the moments of inertia of the individual nuclei, with M_i being their masses and R_i their radii).

In terms of the variables θ, θ_1, and θ_2 the equations of motion governing the time development of the angular momenta can be written in a simple form. Neglecting rolling friction they read [2]:

$$\dot{L} = -(\dot{L}_1 + \dot{L}_2) \tag{2}$$

$$\dot{L}_1 = \frac{1}{2}\rho\,\bar{v}\,\pi\,n^2\,[R_2(\dot{\theta} - \dot{\theta}_2) + R_1(\dot{\theta} - \dot{\theta}_1)]\,R_1 \tag{3}$$

$$\dot{L}_2 = \frac{1}{2}\rho\,\bar{v}\,\pi\,n^2\,[R_2(\dot{\theta} - \dot{\theta}_2) + R_1(\dot{\theta} - \dot{\theta}_1)]\,R_2 \tag{4}$$

where L is the relative angular momentum, L_1 and L_2 are the spins of the two nuclei with radii R_1 and R_2, respectively. The quantity n is the neck radius and $\frac{1}{2}\rho\,\bar{v} = \frac{27}{64\pi}\left(\frac{\pi}{3}\right)^{1/3}\frac{\hbar}{r_o^4}$ is the constant characterizing the one-sided flux of particles through a unit area of the window in a unit time. (We shall use the Lysekil values of the nuclear parameters, in particular $r_o = 1.2249$ fm [8].)

The equations (2)-(4) are easy to understand. Equation (2) expresses the conservation of angular momentum: if L_1 and L_2 are increased in the collision process by friction, L (and the centrifugal force) is bound to decrease accordingly. Equations (3) and (4) give the rate at which L_1 and L_2 are increased as a consequence of particle transfers through the window. Since the square brackets represent the relative tangential velocity of the colliding nuclei and πn^2 is the window area, $1/2\ \rho\bar{v}\pi n^2$ [ ] gives the tangential frictional force. Upon multiplying with the respective arm lengths R_1 and R_2, one obtains the torques responsible for the change of L_1 and L_2 in time. Thus, the right-hand sides of eqs. (3) and (4) describe sliding friction. Rolling friction is expected to be an order of magnitude smaller than the contribution from sliding [2] and (as mentioned before) has been neglected in the present work.

Note that the three degrees of freedom θ, θ_1, and θ_2 enter in the form of the linear combination $R_1(\theta - \theta_1) + R_2(\theta - \theta_2)$ in eqs. (3) and (4). As explained above, this variable is connected with sliding. Angular momentum conservation, on the other hand, manifests itself in the fact that the overall orientation of the system, θ, is a cyclic variable of the Lagrangian. The third rotational degree of freedom $R_1(\theta - \theta_1) - R_2(\theta - \theta_2)$ describes rolling and is absent from eqs. (3) and (4) because rolling

friction has been neglected. Under these conditions there is only one relevant rotational degree of freedom and only one more equation of motion than in Ref. [1]. In order to obtain the additional equation of motion from (2)-(4) L_1 and L_2 will be eliminated.

Using $\dot{\theta}_1 = \frac{L_1}{\mathcal{J}_1}$, $\dot{\theta}_2 = \frac{L_2}{\mathcal{J}_2}$ and $\dot{\theta} = \frac{L}{M_r(R_1+R_2)^2}$,

where $\mathcal{J}_i = \frac{2}{5} M_i R_i^2$ are the moments of inertia of the individual nuclei, and M_r is the reduced mass of the system, leads to

$$\dot{L} = -\frac{7}{4}\frac{\rho\bar{v}\pi n^2}{M_r}\left(L - \frac{5}{7}L_o\right) \tag{5}$$

where L_o is the initial orbital angular momentum of the system. Integrating eq. (5) yields

$$L = L_o\left[\frac{5}{7} + \frac{2}{7}\, e^{-\frac{7}{4}\frac{\rho\bar{v}\pi}{M_r}\int_o^t n^2(t')dt'}\right] \tag{6}$$

In addition to the "elegant variables" ν and σ, defined in Ref. [1] as

$$\nu \equiv \sqrt{\alpha} \quad , \quad \sigma \equiv \frac{\rho^2-1}{1-\Delta^2} \tag{7}$$

(and reducing to $\nu \rightarrow \frac{\text{neck radius}}{2\bar{R}}$, $\sigma = \frac{\text{tip separation}}{2\bar{R}}$ in the limit of a small neck with $\bar{R} = R_1R_2/(R_1+R_2)$) I will therefore use the new variable

$$\phi = \frac{5}{7} + \frac{2}{7}\, e^{-\frac{7}{4}\frac{\rho\bar{v}\pi}{M_r}\int_o^t n^2\,(t')\,dt'} \tag{8}$$

It is easy to show that $(\phi - 5/7)$ is proportional to the quantity in square brackets in eqs. (3) and (4), which is the relative tangential velocity of the two nuclei near contact. The quantity $\int_{t_0}^{t} (\phi - \frac{5}{7})\, dt$ represents therefore the total amount of sliding that has taken place since time t_0.

Following Ref. [1] then leads to the following generalized equations of motion for ν, σ, and ϕ in the dinuclear regime:

$$\mu\ddot{\sigma} + \nu^2\dot{\sigma} = X - \nu + Y\,\phi^2 \tag{9}$$

$$\dot{\nu} = \frac{2\nu - 3\nu^2 - \sigma}{4\nu(\sigma + \nu^2)} \tag{10}$$

$$\phi = \frac{5}{7} + \frac{2}{7}\, e^{-\frac{7}{2\mu}\int_0^{\tau} \nu^2 d\tau} \tag{11}$$

where $\mu = \dfrac{M_r}{2\pi(\rho\bar{v})^2\, R^4/\gamma}$ is the dimensionless reduced mass, τ is the time measured in the natural time unit $t_u = \rho\, \overline{v}R^2/\gamma$, X is the effective fissility of the system, and the rotational parameter Y is defined as

$$Y \equiv \frac{L_0^2}{4\pi\gamma M_r \bar{R}(R_1 + R_2)^3} \tag{12}$$

Now, for head-on collisions, the initial angular momentum L_0 is zero, and eqs. (9) and (10) reduce to the equations of motion treated in Ref. [1].

In the equations of motion (9)-(11) there are three parameters μ, X, and Y. Since the dependence of dynamical calculations on μ (at least for head-on collisions [1]) is relatively slight for many systems of interest, the time evolutions of dinuclear systems are expected to be approximately similar in practice,provided the effective fissilities X and the rotational parameters Y are the same. Moreover, on the basis of the idea that centrifugal force can

be effectively traded against Coulomb (electric) repulsion (outlined in refs. [1] and [3]) one expects a much stronger approximate "scaling rule": the quantity approximately determining the dynamical behavior of dinuclear systems should be a linear combination of the form X + fY, where f is the "angular momentum fraction", used in Ref. [3] to fit experimental data. Comparing to eq. (9) it is seen that f describes an average value of the quantity ϕ^2.

In order to obtain an approximate analytic solution to the equations of motion (9)-(11), I took the following simple steps (in close analogy with Ref. [1]):

1. In the vicinity of the saddle points of the potential energy surface, where $\phi \approx \bar{\phi} = 5/7$, the equations of motion have been linearized and their solution was written down in terms of the normal modes of motion.

2. The small time limit ($\tau \ll 1$) of the equations (9)-(11) has been investigated and a solution has been found in closed form. Note that the small time behavior of eqs. (9)-(11) is more complicated than that of the equations of motion in Ref. [1]. As a matter of fact, it was sufficient to assume $\sigma \ll 1$ and $\nu^2\dot{\sigma} \ll 1$ in Ref. [1] to arrive at an explicit solution, so that the resulting trajectory in configuration space could be used in the neighborhood of $\sigma = 0$ (i.e. $\rho = 1$) with confidence. In the present case I have only been able to obtain a solution in closed form in the small time limit, $\tau \ll 1$. I will refer to the results based on $\tau \ll 1$ as obtained in the small-τ approximation while $\sigma \ll 1$ and $\nu^2\dot{\sigma} \ll 1$ will be denoted the small-σ approximation.

3. The solutions A and B have been matched at an intermediate time τ_m. (The results are insensitive to the exact value of τ_m. In the calculations $\tau_m = 0.5$ has been used.) The solutions obtained this way

represent trajectories in the (ρ,α,Δ) configuration space. Some of the trajectories lead to the fusion of the system, others to reseparation. For heavy or strongly rotating systems bringing the two nuclei in contact (at rest) on the top of the interaction barrier does not automatically lead to fusion, but an "extra push" (an extra energy over the barrier) is necessary to form a compound nucleus. If, for a given X and Y, one wants the system to fuse, the dimensionless outward velocity at time $\tau = 0$ has to be $\dot{\sigma} < \dot{\sigma}_c$ or, equivalently, it is needed that the injection velocity $-\dot{\sigma} > -\dot{\sigma}_c$, where $-\dot{\sigma}_c$ is the critical injection velocity (at $\tau = 0$) that will bring the trajectory to rest at the saddle point of the potential energy surface. The trajectory, characterized by $-\dot{\sigma}_c$ separates the families of trajectories leading to fusion and reseparation, respectively, for the given X and Y.

3. Results

3.1. Normal modes of motion around the saddle points

The position of the conditional saddle points (where the potential energy is stationary with respect to small variations of ρ and α while the asymmetry Δ is frozen) in the presence of rotation is

$$\bar{\nu} = X + \frac{25}{49} Y$$
$$\bar{\sigma} = 2\bar{\nu} - 3\bar{\nu}^2 \qquad (13)$$

The number of normal modes of motion is increased by one, corresponding to the new variable ϕ. The normal modes already present in the description of head-on collisions are unaffected. The characteristic time, $\tilde{T}_{s1}$, for the additional normal mode, which describes the transition from sliding to rolling, is compared to the other characteristic times in the binary regime of

the system on Fig. 2. One can see that for reasonably large systems T_{s1} is smaller than the characteristic times of other modes. In other words the clutching mode represents the fastest time scale in these systems.

3.2. Comparison of the small-σ and small-τ approximations for Y = 0

The approximation based on matching the small-τ solution against the one around the saddle yields slightly different results for the case of no rotation from the one using the small-σ approximation. Since for $\tau \ll 1$ it follows that $\sigma \ll 1$ and $\nu^2\dot{\sigma} \ll 1$, but not the other way around, it is clear that the small-σ approximation is more precise than the small-τ approximation. In order to get an idea about the quality of the present approximation (small-τ), relative to the one used in Ref. [1] (small-σ), the critical injection velocities, $-\dot{\sigma}_c$, are compared in Fig. 3 as a function of the effective fissility, X, for the case Y = 0, where both approximations are feasible.

One can see that the value of the critical fissility, above which an extra push becomes necessary $X_{eff\ thr} \approx 0.60$ in the present approximation, rather than 0.57, the value obtained if the small-σ solution is used to start off the trajectories.

On the other hand, the results differ only moderately if $|\dot{\sigma}_c|$ is not too large. In particular, in the interval $0.4 \leqslant X \leqslant 0.7$ where $|\dot{\sigma}_c|$ is small and therefore the model is expected to work best, the small-τ approximation yields somewhat smaller critical injection velocities than the small-σ approximation, the agreement is, however, satisfactory. Now we turn to the case $Y \neq 0$ in the small-τ approximation.

3.3. The critical injection velocity

Figure 4 displays the critical injection velocities, $-\dot{\sigma}_c$, for several values of the rotational parameter Y in the range of fissilities $0.4 \leqslant X \lesssim 0.7$.

Calculations have been performed for a large number of systems with different values of the asymmetry Δ. The curves have been drawn on the basis of the points representing symmetric systems. One observes that, for a given value of Y, the results for different Δ fall approximately on the same universal curve as long as $|\dot{\sigma}_c|$ is not too large. With increasing values of the critical injection velocities systematic deviations start to develop. I am going to focus attention on a relatively narrow strip around $\dot{\sigma}_c = 0$ (say $-0.4 \leqslant -\dot{\sigma}_c \leqslant 0.4$). In that region the critical injection velocity appears to be well represented by a more or less parallel family of curves, labeled by the rotational parameter Y. This confirms the validity of the approximate scaling (mentioned in Section 2) expected on the basis of the idea of trading centrifugal force against Coulomb repulsion.

To bring out this point clearly, on Fig. 5 I show the XY plane with locii of equal critical injection velocity, $-\dot{\sigma}_c$. The heavy contour line divides the plane into a region of automatic capture ($-\dot{\sigma}_c < 0$) and a region where an extra energy over the interaction barrier is needed to get the system to fuse ($-\dot{\sigma}_c > 0$). (Figure 5 is based on calculations with symmetric systems, $\Delta = 0$.) The scaling rule stated at the end of Section 2 manifests itself in the approximately linear and parallel nature of the contour lines connecting locii of equal $\dot{\sigma}_c$. Except for the lightest systems ($X \lesssim 0.15$) the contours are straight lines, the critical injection velocity is a function of $X + fY$ only:

$$-\dot{\sigma}_c = F(X + fY) \tag{14}$$

where the angular momentum fraction f can be obtained from Fig. 5 as the slope of the contour lines. In principle, f characterizes the function $F(X + fY)$ locally; in practice, however, with reasonable accuracy

$$f = 0.85 \pm 0.02 \tag{15}$$

This value is notably far from both f = 1, corresponding to a "mass-points" moment of inertia (and characterizing the system up to the moment of contact) and from the value corresponding to rigidly stuck spheres (f = 0.54 for a ratio of masses 208:50). On the other hand, it compares reasonably well to the value f = 3/4 ± 10%, used to obtain the best fit to experimental data in Ref. [3]. It also appears to be close to the mean of f = 1 and f = 5/7, the latter value characterizing two spheres rolling on each other without sliding.

4. Conclusion

In the present work I have suggested a simplified scheme to include angular momenta in the macroscopic model of nuclear coalescence and reseparation (extra-push theory) [1]. In the framework of the model I have been able to calculate the characteristic time for the clutching of the colliding nuclei and found clutching to happen fast compared to characteristic times of other normal modes of the system for not too light projectile-target combinations.

Within the model I have proved that centrifugal force can indeed be traded for electric repulsion via an "angular momentum fraction" to a good approximation at least in the region of fissilities and rotations close to the threshold, where an extra push first becomes necessary. Furthermore, the actual theoretical value of the "angular momentum fraction", deduced in the model, is in reasonable agreement with the value obtained from the best fit to experimental data.

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References

1. W.J. Swiatecki, LBL-10911 (1980); Physica Scripta 24 (1981) 113.
2. C.F. Tsang, Physica Scripta 10A (1974) 90.
3. W.J. Swiatecki, Nucl. Phys. A376 (1982) 275.
4. H. Sann, S. Bjørnholm, R. Bock, Y.T. Chu, A. Gobbi, E. Grosse, U. Lynen, E. Morenzoni, W. Muller, A. Olmi, D. Schwalm, and W. Wölfli, Proc. Int. Conf. on Nucl. Physics, LBL-11118;
   H. Sann, R. Bock, Y.T. Chu, A. Gobbi, A. Olmi, U. Lynen, W. Muller, S. Bjørnholm, and H. Esbensen, Phys. Rev. Lett. 47 (1981) 1248.
5. F. Beck, J. Blocki, and H. Feldmeier, Proc. of the "International Workshop on Gross Properties of Nuclei and Nuclear Excitations X", Hirschegg, 1982.
6. B. Sikora, J. Bisplinghoff, M. Blann, W. Scobel, M. Beckerman, F. Plasil, R.L. Ferguson, J. Birkelund, and W. Wilcke, Phys. Rev. C25 (1982) 686.
7. J. Randrup and W.J. Swiatecki, Ann. Phys. 124 (1980) 193.
8. W.D. Myers and W.J. Swiatecki, Ark. Fys. 36 (1967) 343.

Figure Captions

Fig. 1. Definition of the degrees of freedom employed. The nuclear configuration is parametrized by two spheres connected by a conical neck. The asymmetry is specified by Δ, the center separation by ρ, and the degree of window opening by α. The rotational degrees of freedom are introduced as the angles θ, θ_1, and θ_2 in a fixed inertial frame of reference. θ describes the overall orientation of the system, while θ_1 and θ_2 give the orientation of the individual nuclei.

Fig. 2. The characteristic times of the normal modes of motion around the saddle point in the dinuclear regime. The light curves depict the normal modes already present in the description of head-on collisions (the fission mode with a characteristic time T_f and the oscillating transverse mode with a period $2\pi T_{osc}$ and a characteristic damping time T_t) and are identical to the ones given in Ref. 1. The heavy curve represents the additional normal mode with characteristic time T_{sl}, connected with the transition from sliding to rolling.

Fig. 3. The critical injection velocity as a function of the effective fissility X for the case of no rotation (Y = 0) in the small-τ and small-σ approximations, described in the text.

Fig. 4. The critical injection velocity as a function of the effective fissility X for different values of the rotational parameter Y. Calculations have been performed for several values of the asymmetry Δ (different symbols), while the curves have been drawn on the basis of the points representing symmetric systems (Δ = 0).

Fig. 5. Contour lines of the function $-\dot{\sigma}_c = F(X,Y)$ in the (X,Y) plane. The curves are labeled with their critical injection velocity in units of $2\gamma/\rho\sqrt{R}$.

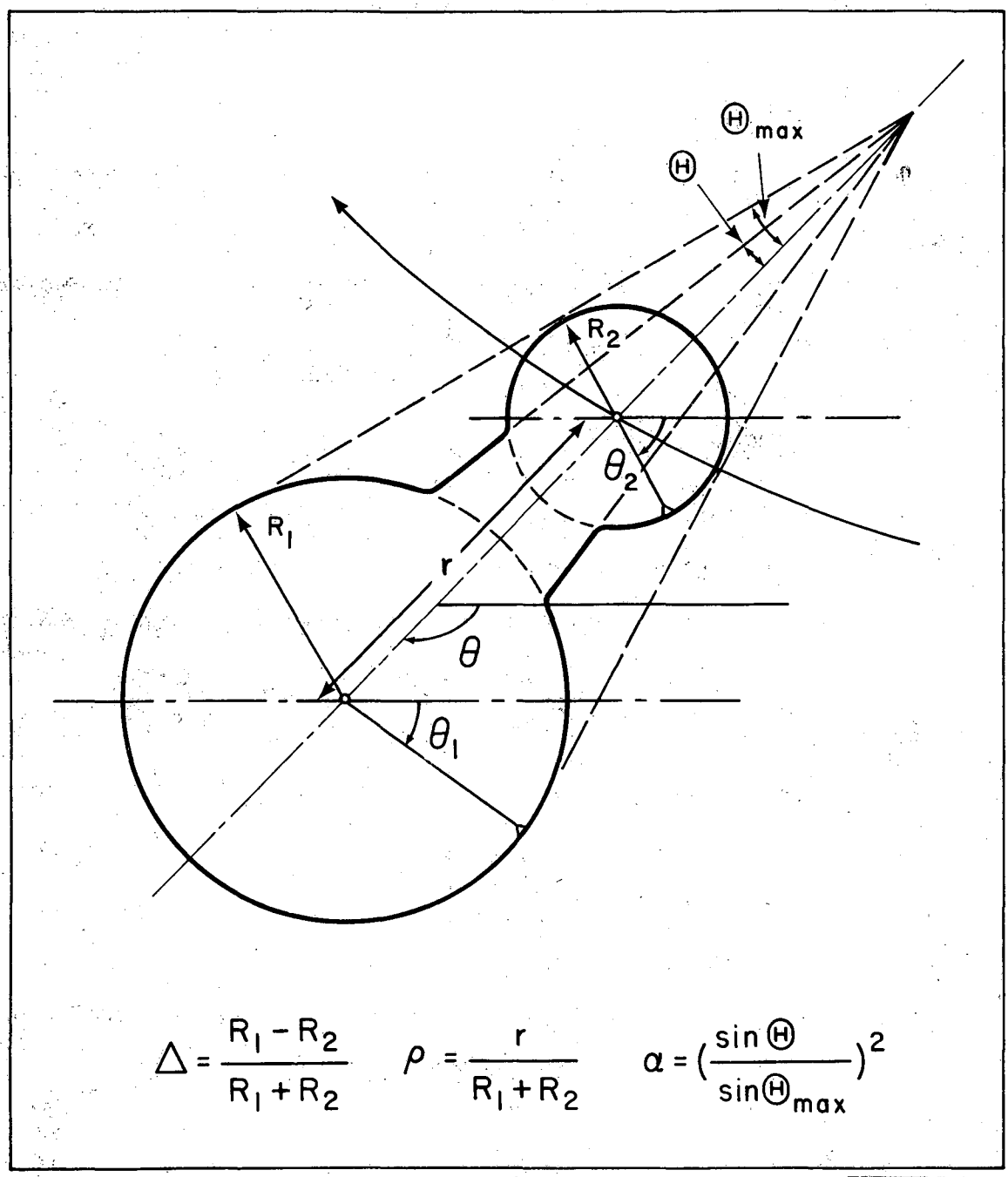

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Fig. 1

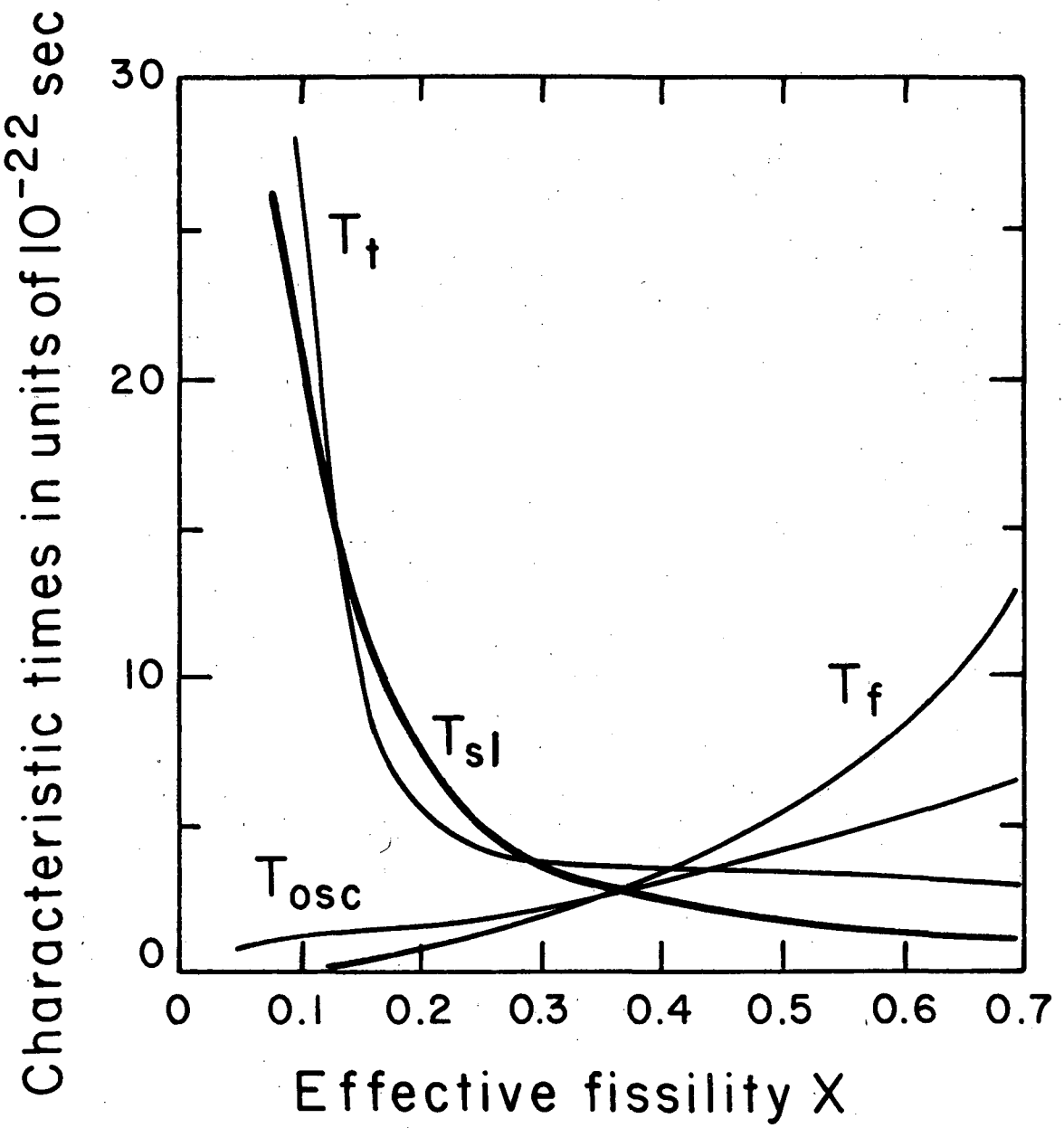

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Fig. 2

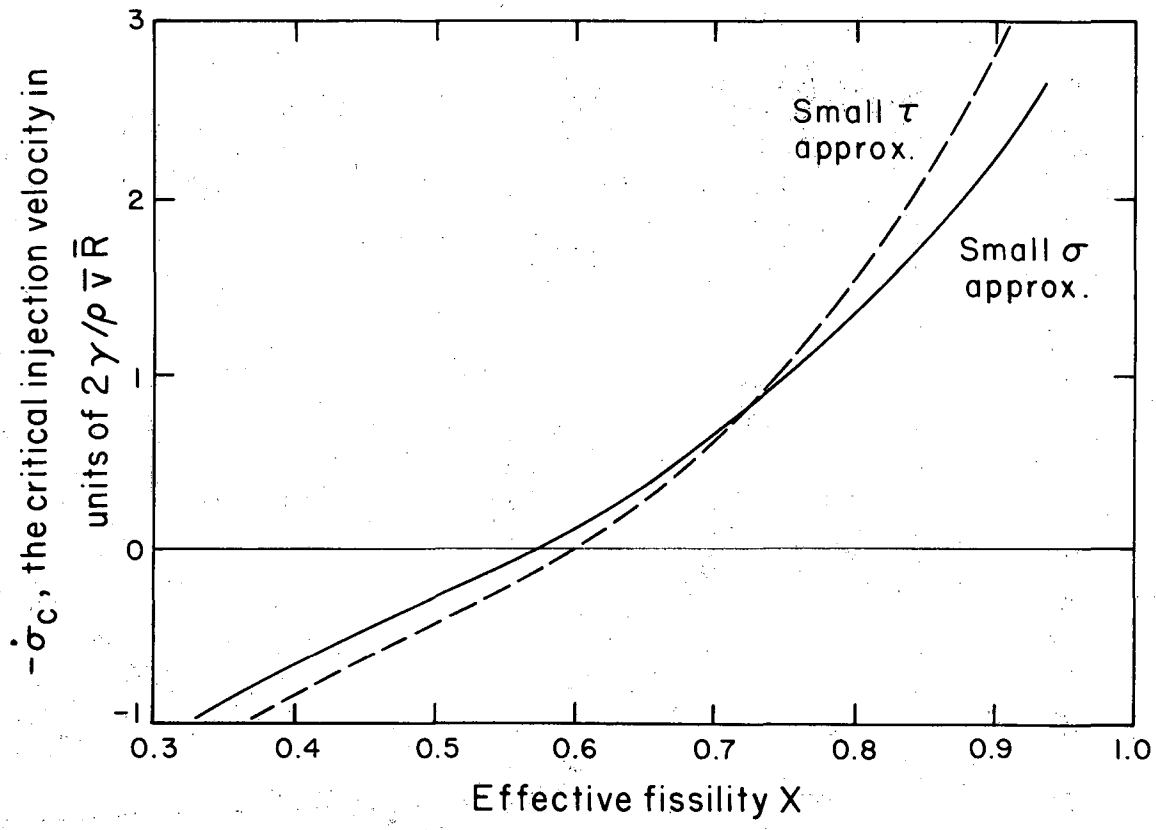

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Fig. 3

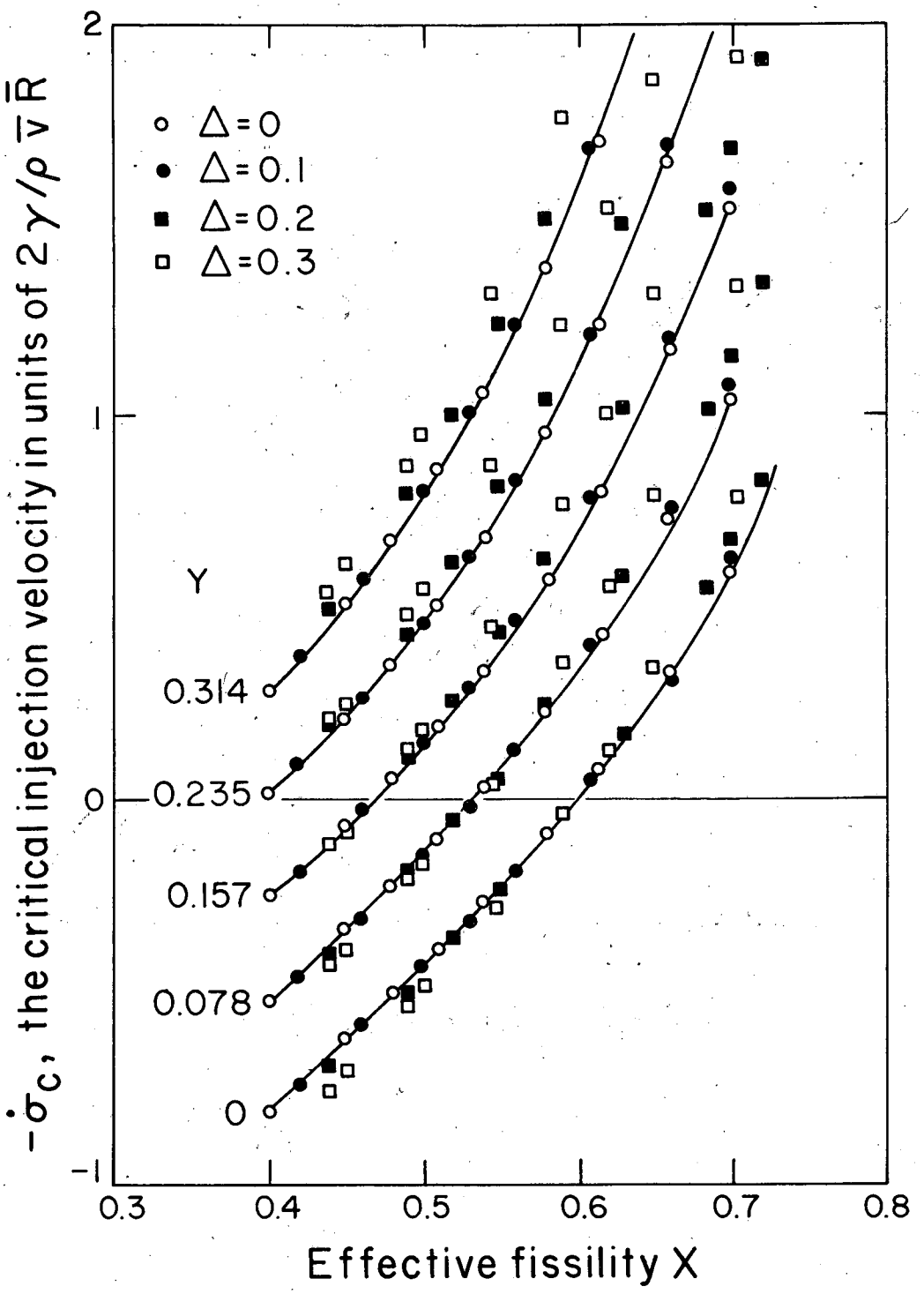

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Fig. 4

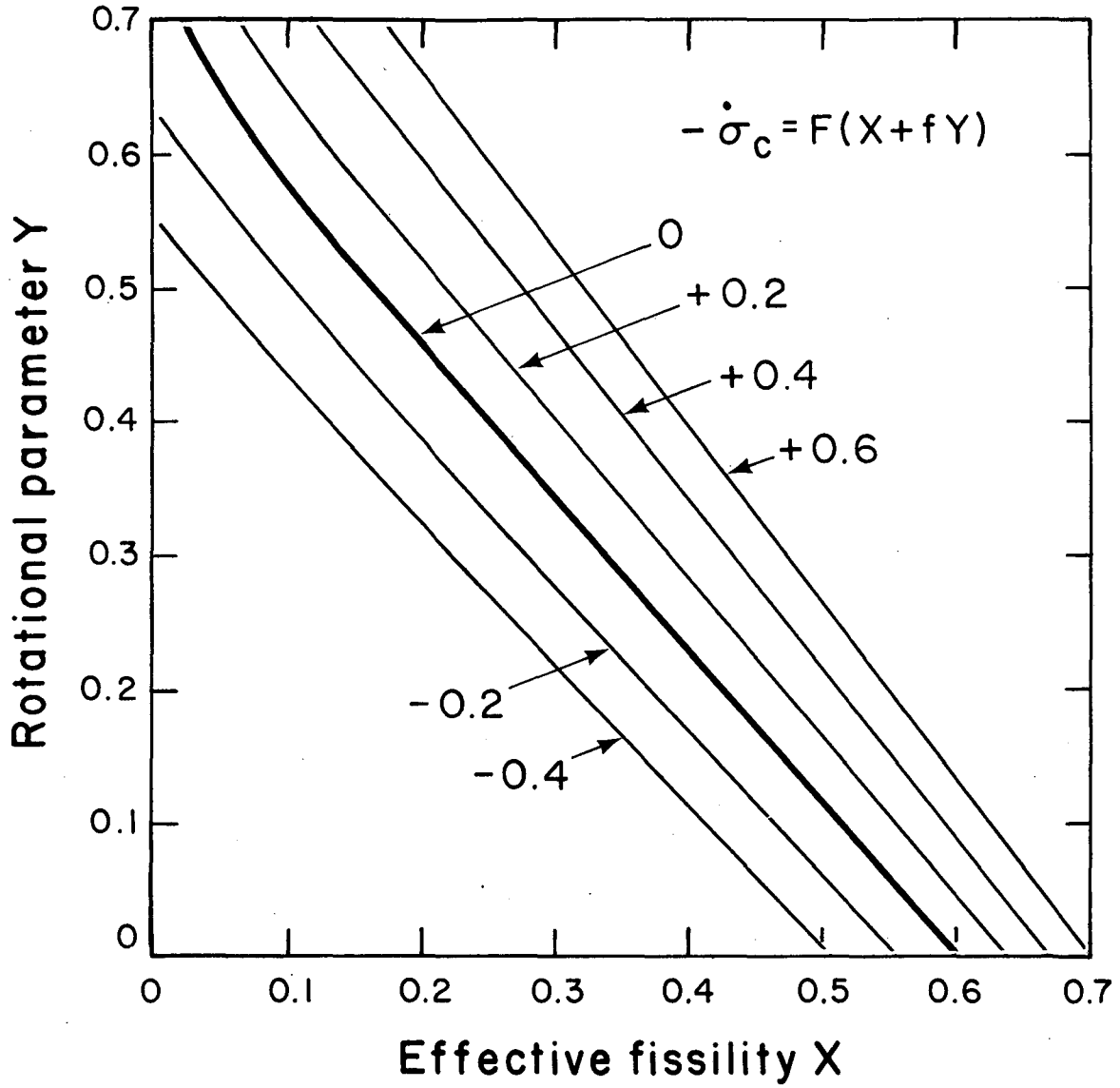

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Fig. 5

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