LBL-14474
Preprint c.2

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

RECEIVED
LAWRENCE
BERKELEY LABORATORY
AUG 17 1982
LIBRARY AND
DOCUMENTS SECTION

EARTH SCIENCES DIVISION

To be published in Geotechnical Testing Journal

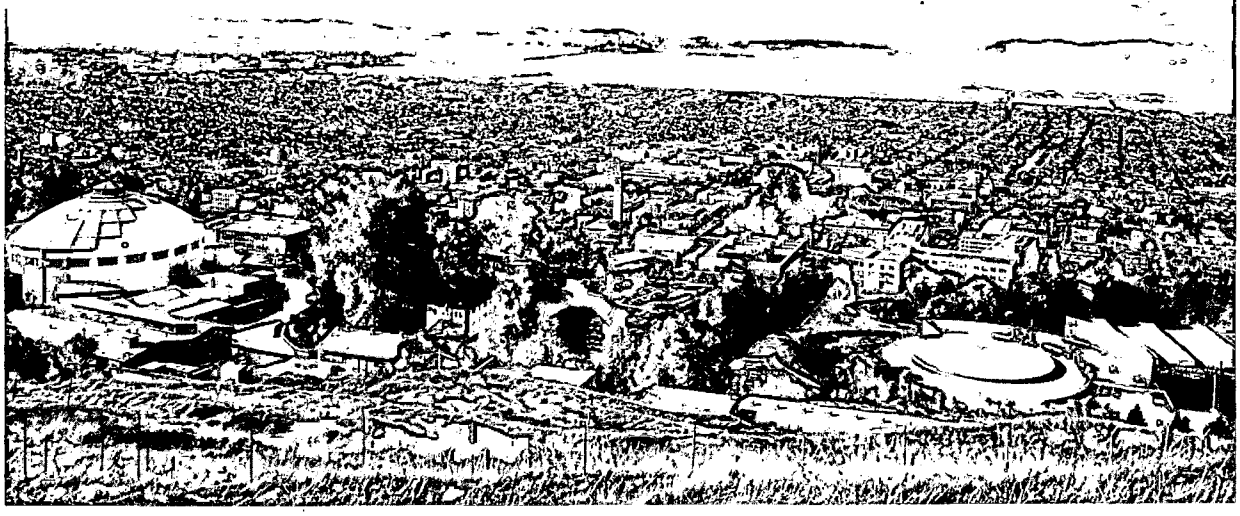
ELECTRICAL ANALOGY OF HYDRAULIC FLOW THROUGH ROCK FRACTURES

P.N. Sundaram and Douglas Frink

April 1982

TWO-WEEK LOAN COPY

This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 6782.



LBL-14474 c.2

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

To be published in Geotechnical Testing Journal
(American Society for Testing and Materials)

LBL-14474

ELECTRICAL ANALOGY OF HYDRAULIC FLOW THROUGH ROCK FRACTURES

P. N. Sundaram and Douglas Frink

Earth Sciences Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

April 1982

This work was supported by the Assistant Secretary for Nuclear Energy, Office of Waste Isolation of the U. S. Department of Energy under Contract DE-AC03-76SF00098. Funding is administered by the Office of Nuclear Waste Isolation at Battelle Memorial Institute.

ELECTRICAL ANALOGY OF HYDRAULIC FLOW THROUGH ROCK FRACTURES

P. N. Sundaram and Douglas Frink

ABSTRACT

Electrical analogy is used to study the influence of contact areas on the hydraulic flow through rock fractures. The investigation has shown that in the case of radial flow the location of contact zones, as well as the total contact area, influence the flow rate. It is also demonstrated that when injection tests are conducted in boreholes, where the fracture close to the borehole is partially plugged by loose particles or other debris, the flow rate may be substantially underestimated.

KEYWORDS: Electrical analogy, rock fractures, radial hydraulic flow, contact area, injection test.

A wide range of techniques is available for the study of fluid flow through porous and fractured media. Whatever the technique, a physical model must first be defined and the corresponding constitutive laws formulated [1]. The basic differential equations, as functions of space and time, are developed using laws of conservation and continuity. Solutions to these differential equations depend upon appropriate initial and boundary conditions. Much of the early work in fluid mechanics was accomplished through the use of rigorous mathematics for relatively simple initial and boundary conditions [2-3]. However, fluid flow problems which defy closed-form solutions have to be solved by alternative methods. These methods can be divided into two classes--numerical and analog. Numerical methods, in general, replace the

continuous functions of flow with discretized functions. Finite-difference methods [4], finite-element methods [5], and combinations thereof as well as boundary element methods [6], are the three main digital techniques currently available. These techniques require the use of high-speed digital computers and the cost includes the expense of software development, computation time and input preparation. With proper numerical procedures, errors resulting from the discretization can be minimized or eliminated.

On the other hand, analogs, according to Irmay [7] are "physical systems or mathematical models obeying partial differential equations with boundary conditions similar to those in the prototype..." Two systems are considered analogous if there is a one-to-one correspondence between different components and their behavior. Analogs could also be different scale representations of the same physical system, e.g., a sand-tank model which may be a scaled down version of an aquifer. However, analogs having different physical categories but obeying the same form of differential equations of continuity, are found to be more useful, e.g., the analogy between hydraulic and electrical flow.

There are two types of analog models: continuous and lumped. In the continuous model, the continuously distributed properties of the prototype are modeled by an equivalent medium which is also continuous in space. In the lumped system, the continuously distributed properties of the prototype are modeled by an arrangement of interconnected discrete systems of another medium. Portions of the lumped model thus represent corresponding portions of the prototype so that the model is an approximate representation of the prototype.

In general, the lumped model is convenient for representing anisotropy and heterogeneity. Time-dependent problems are usually solved by using lumped models.

Examples of analogs are electrical [8], thermal [9], optical [10] and membrane [11]. The analog techniques have tended to lose their popularity because of the improvement in digital techniques and the availability of fast digital computers. However, certain types of analog systems can be competitive with digital systems in terms of cost and convenience.

Electrical Analogy of Hydraulic Flow

One of the popular analogs used for fluid flow through porous media is the electrical analog. The analogy stems from the similarity between Darcy's law for fluid flow through porous or fractured media [12] and Ohm's law for flow of electricity in a conducting medium. Symbolically, Darcy's law may be expressed as

$$\{v\} = - [K] \{\nabla h\} \tag{1}$$

where v is the velocity vector, h is the total head or potential and k is the hydraulic permeability matrix. For flow of current in a conducting medium,

$$I = V/R \tag{2}$$

where I is the current flow in amperes (analogous to the flow-rate); R is the electrical resistance and V is the voltage. Table 1 describes the parameters in the conduction analogies. The equation of continuity for steady-state conditions may be given as

$$a_x \frac{\partial^2 \varphi}{\partial x^2} + a_y \frac{\partial^2 \varphi}{\partial y^2} + a_z \frac{\partial^2 \varphi}{\partial z^2} = 0 \tag{3}$$

TABLE 1. Analogy Between Fluid and Electrical Conduction

	Description	Fluid	Electricity
1.	Potential	Total Head(h)	Voltage (V)
2.	Storage	Storativity (s)	Charge (Q)
3.	Conductivity	Hydraulic conductivity or permeability(K)	Electrical conductivity (σ)
4.	Flow rate	Flow Rate (Q)	Current (I)
5.	Gradient	$\bar{\nabla}h$	$\bar{\nabla}V$
6.	Conduction model	Darcy's law	Ohm's law
7.	Continuity	$\nabla^2 h = 0$	$\nabla^2 v = 0$

where ϕ is either the hydraulic or electric potential. The equation is commonly known as the Laplace Equation.

For flow through isotropic and homogeneous medium, the Laplace equation is simplified as

$$\nabla^2 \phi = 0. \tag{4}$$

Thus, solution to Eq. (4) is simply a function of the space coordinates and the boundary conditions. This equation also describes the field about a magnet, the displacement of a rubber membrane, and the thermal field in a conducting medium, for steady state conditions. Similar analogies exist between transient flow problems and electrical flow but will not be discussed in this paper.

Existing electric analogs for fluid flow are classified as (1) conducting sheet, (2) conductive-solid, (3) liquid analogs, and 4) lumped-element analogs. In general, conductive-sheet analogs are used for solving one- and two-dimensional flow problems. Conductive solid analogs present practical difficulties in that potential cannot be measured at interior points and, thus, are restricted to the cases where only the distribution on the exterior surfaces is needed. The principal sources of error in liquid analogs are from polarization, surface tension, and nonuniformity of electrolyte used as the conducting liquid [13]. Lumped-element analogs are comprised of electrical resistors, usually in square or rectangular arrays. Anisotropy and heterogeneity can be easily introduced by using resistors of varying magnitudes. Lumped-element circuits are useful for three-dimensional flow studies. Time-dependent problems can be studied by including capacitor elements.

Conductive sheet analogs are usually made of conducting papers or metallic sheets that conduct electricity. In 1948, an electro-sensitive recording paper, called Teledeltos paper, was introduced by the Western Union Telegraph Company. There are different types of Teledeltos available, each with different characteristics. The type normally used for analog simulation is made by adding carbon black to the paper-pulp during the paper-manufacturing stage and has no aluminum lacquer backing. The paper is relatively insensitive to changes in temperature. However, it has an inherent electrical anisotropy on the order of 10%, i.e., the longitudinal resistance of a roll of this paper is generally 90% of the transverse resistance. The paper may be easily damaged by probes used to measure the potential, can absorb moisture, and is easily

affected by bending or the formation of wrinkles [14]. In the present studies, we have used Teledeltos paper for studying steady-state flow in rock fractures.

Flow Through Rock Fractures:

The physical model for flow through rock fractures is the viscous flow through parallel and smooth-walled plates, usually termed the "parallel plate" model [15]. However, rock fractures are not smooth faced and they make contacts at a finite number of points which vary in size, geometry, number, and distribution, depending upon the stress regime and the type of rock mineral and weathering. No rigorous theory exists to take into account the presence of these contact areas which tend to obstruct and decrease the flow rate. Numerical methods such as finite-element or integrated finite-difference may be used to get an idea of the influence of contact area on the flow rate [16]. However, similar solutions can be obtained by electrical analog techniques which are comparatively cheap and fast. Statistical models are also being developed [17-18] to characterize the macro-roughness of the surfaces as well as the contact asperities. Laboratory studies on flow through fractured specimens of rock have been performed by several investigators [16, 19, 20] but only limited data are currently available to check the validity of numerical and statistical models.

Very few examples of use of electrical analogy to solve problems of water flow through rock fractures exist [21-22]. The following sections of this paper discuss the use of electrical analogs for a qualitative study of certain aspects of hydraulic flow in rock fractures. Two problems are investigated,

(1) effect of contact area on flow-rate, and (2) effect of plugging of fractures close to a borehole on flow rate. In each case, it is assumed that the flow is laminar and obeys Darcy's law.

Since the usual mode of flow for field and laboratory tests is radial, this mode is considered for the present study. Rigorous mathematical solutions are difficult to obtain for flow through fractures in which the walls make contact at finite points. If a fracture is subjected to effective stress changes, then there is an inevitable change in contact area. But the actual magnitude of change is a function not only of the physical characteristics of the fracture but also the stress-history of the fracture. The average spacing between the fracture surfaces changes with the change in area of contact. The effect of change in fracture aperture on flow rate was not considered in our investigations.

Experimental Arrangement

Figures 1 and 2 show the experimental arrangement. The Teledeltos paper was cut to the required shape and dimensions to represent the plan area of the fracture. Care was taken to avoid sharp bends or creases in the paper, since these would alter resistance. The electrodes were made of nichrome alloy wire 0.25 mm in diameter. One electrode was bent to the circular shape of the periphery of the conducting paper and the other was fitted around the hole in the center that simulates the borehole through the fracture. They were held in position with staples. To ensure complete contact with the conducting paper, silver conducting paint, with a suitable thinner (Acetone toluol), was applied

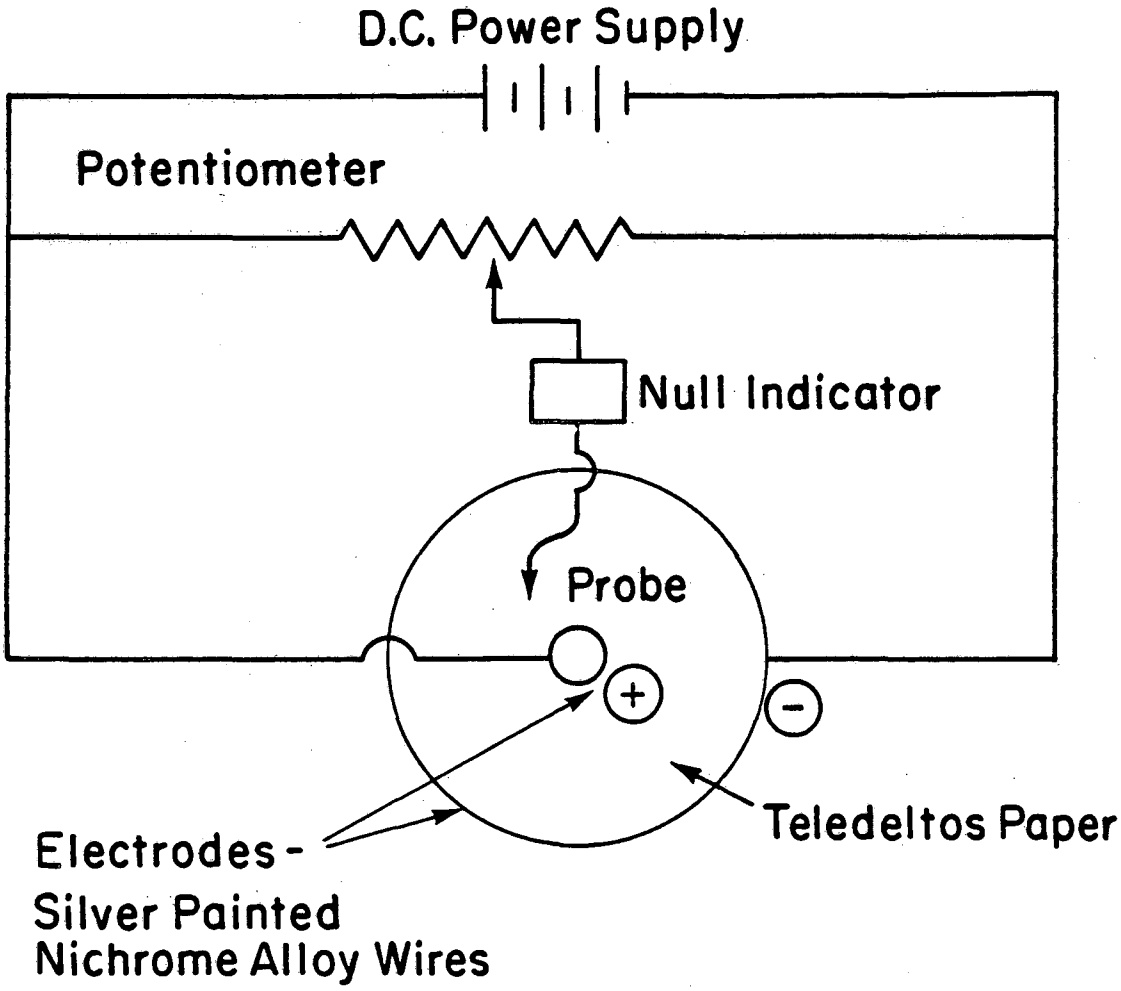


Figure 1. Schematic of electrical analog. [XBL 821-1635]

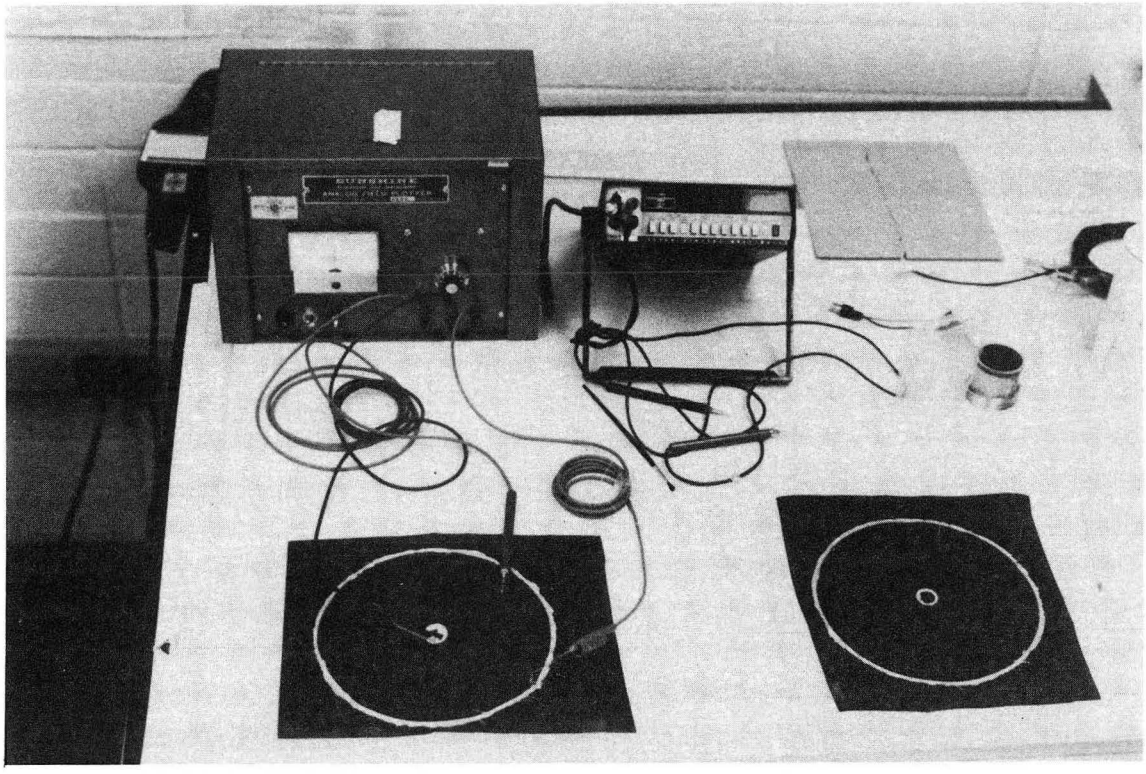


Figure 2. Experimental arrangement. XBB 822-1253

to the electrode wires with a soft camel hair brush. This treatment ensured very low voltage drop at the electrodes. At least two hours was allowed for the thinner to evaporate. The whole arrangement was clamped to a wooden drawing board.

Power was provided by a DC voltage regulator set to 12 volts. (The system could work on either AC or DC voltage.) The voltage regulator had a current rating of ± 10 micro-amperes full scale. The null-indicator was a high-sensitivity moving-coil DC micrometer with the zero reading at the center of the scale. The searching probe, or stylus, was composed of an insulated pencil-like brass rod and was used to locate points of the required potential on the Teledeltos paper. The probe was connected to the slider circuit of the voltage divider housed in the instrument box. The voltage divider was a multi-turn potentiometer varying between 0 and 100 percent of the total applied voltage. 0.1% accuracy was possible.

In experiments where the potential had to be measured at a point within the electrode boundaries, the probe was made to lightly contact the paper; in that position, the null indicator was operated until the needle pointed to zero. The micrometer reading gave the percentage of full potential at the location of the probe.

To draw lines of constant percentage of full potential (equipotential line) the micrometer was set at the required potential and the probe was moved until the pointer on the null indicator panel showed zero. The location was transferred to graph paper manually or with a pantograph. The pantograph

enables immediate transfer of points at very close intervals so that accurate mapping of equipotential lines is possible. In the present series of experiments, a precision digital multimeter completed the experimental set up. With the help of this multimeter, the resistance of the analog model between the electrodes was measured to ±1 Ω accuracy. To measure the total resistance of the model the power supply was disconnected and the multimeter used to measure the resistance directly.

Effect of Contact Area on Flow Rate

The first step in investigating the effect of contact area on flow through a fracture was to analyze the potential distribution and resistance of a circular strip with a central hole. As shown in Figure 1, the inner electrode serves as the anode and the outer electrode as the cathode, thus duplicating divergent radial flow. This model is an analog of hydraulic flow from a borehole into a planar fracture (divergent flow mode). If we assume the contact area to be zero, i.e., the fracture planes do not make any contact, then the equipotential lines are clearly concentric circles. For this condition, the radial distance from the center, r, at which the potential is p, expressed as a fraction of the total potential, may be given as [23]

$$r = R_i e^{(1-p)\ \ln\left\{\frac{R_o}{R_i}\right\}} \tag{5}$$

where R_o is the outer radius and R_i is the radius of the borehole. In the experimental setup, R_o is 11.3 cm and R_i is 1.0 cm.

To check the electrical homogeneity of the conducting paper, the potential was measured at different radial distances at intervals of 1 cm. Table 2 gives the measured values along two orthogonal diameters. OA and OC form one diameter; OB and OD form the perpendicular diameter (see Fig. 4). The table includes the theoretical values for comparison. The maximum error occurs close to the outer periphery, and the average error is on the order of 10%. The theoretical distribution of potential along a radius is shown by Curve 1 on Figure 3. As shown on the figure, the measured points lie close to the theoretical line.

TABLE 2. Distribution of Potential in Divergent Radial Flow (Zero Contact Area).

Outer Radius: 11.3 cms, Inner Radius: 1.00 cm
Resistance of the Analog: 0.905 kΩ
Voltage, DC (average) = 12.07 volts

Radial Distance (cm)	% Potential at OA	OB	OC	OD	Theoretical Potential %
1.0	100.00	100.00	100.00	100.00	100.00
2.0	67.00	70.20	73.70	71.60	71.41
3.0	52.00	52.20	57.00	54.20	54.69
4.0	40.50	41.90	43.60	42.70	42.83
5.0	32.20	33.10	34.20	33.50	33.63
6.0	24.60	25.60	26.40	25.90	26.11
7.0	18.50	19.50	20.30	20.00	19.75
8.0	13.50	14.20	14.80	14.20	14.24
9.0	8.80	9.70	10.00	9.50	9.39
10.0	4.60	4.90	5.30	5.10	5.04
11.0	0.80	1.20	1.20	1.20	1.11

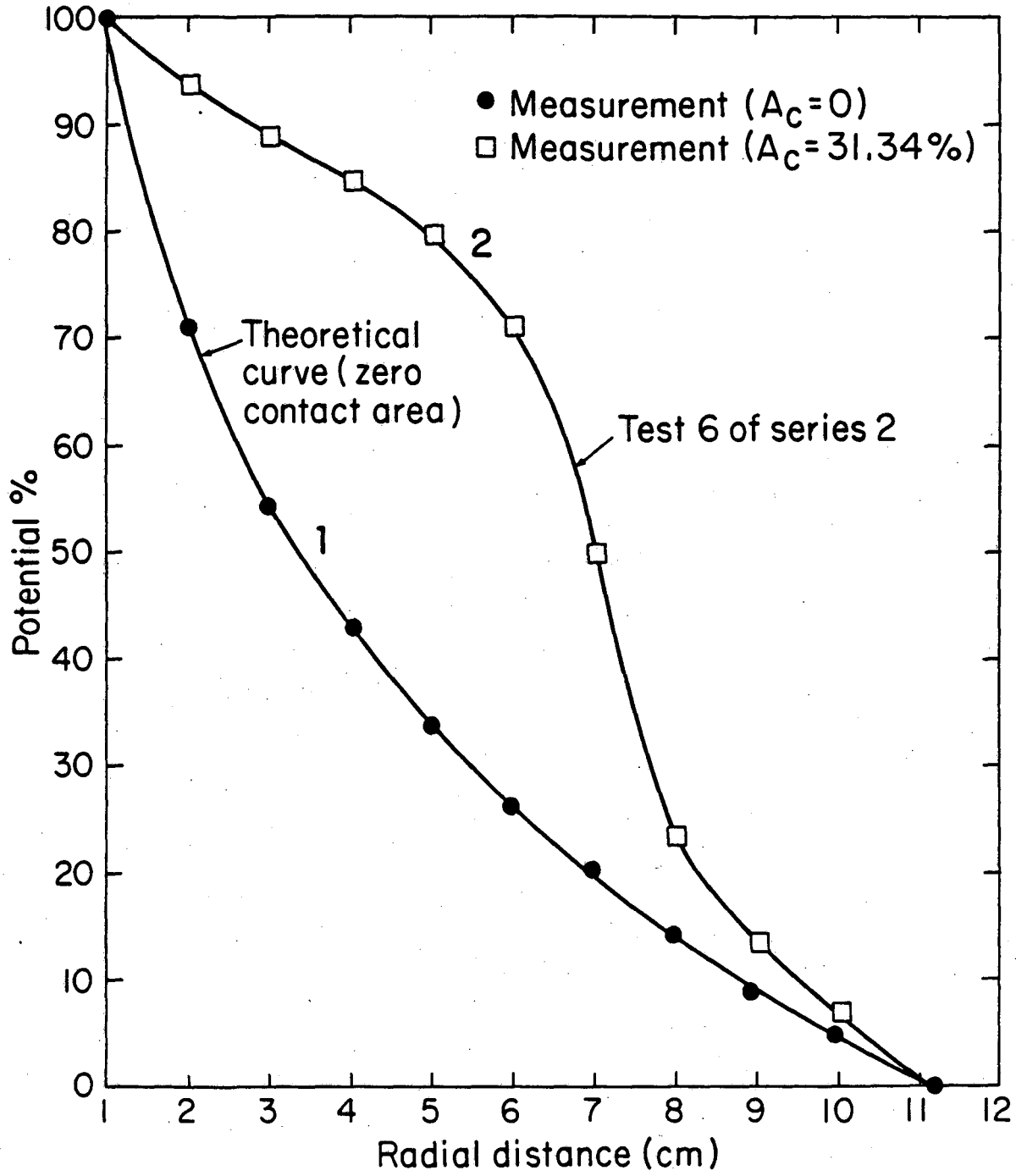


Figure 3. Potential distribution along a radius (divergent flow).

[XBL 821-1634]

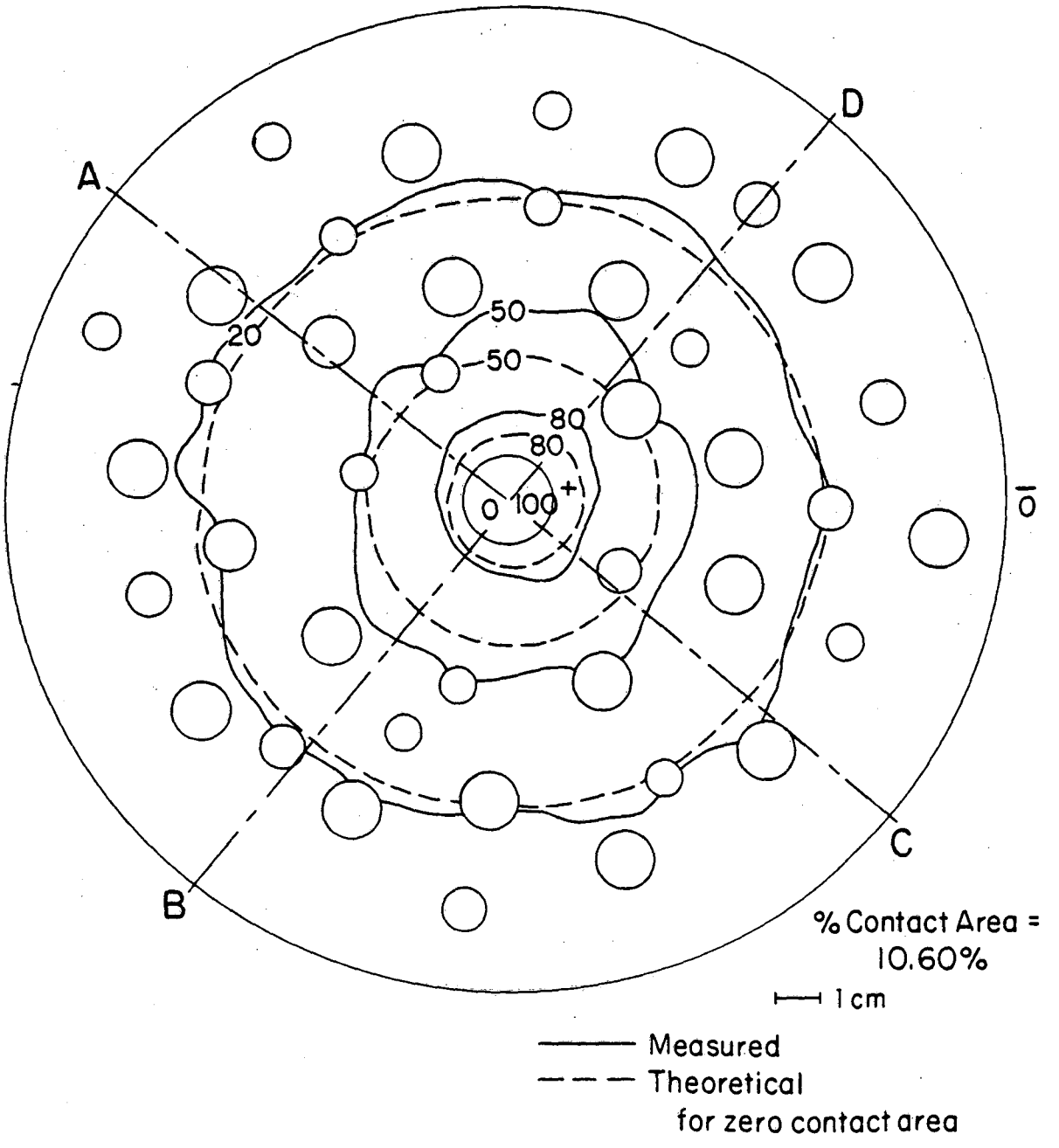


Figure 4. Example of analogy of radial flow in a fracture with randomly distributed contact areas. [XBL 821-1637]

In the next stage of the study, holes of various diameters were punched in the Teledeltos paper. The holes are analogous to the contact zones between the two surfaces of the fracture. Several tests were conducted with an increasing number of holes in each. For each test, the resistance of the analog was measured using the digital multimeter. Because the measured voltage fluctuated by ±5%, an average voltage was used to estimate the current I, using Ohm's law.* The current for the case of no holes was measured as I_o. Figure 4 shows a typical analog with randomly distributed holes (Test No. 5).

Table 3 is a summary of the results of Series 1 tests. The last column (I/I_o) is a measure of the flow rate expressed as a percentage of the flow for no holes (i.e., no contact between the two surfaces). Figure 5 graphically shows the percentage of flow rate, I/I_o, as a function of percentage of contact area, A_c/A_T, where A_c is the total contact area and A_T is the total area between the electrodes.

For the range of contact areas attempted in Series 1, a linear relationship may be fitted to the data points of the form

$$I/I_o = 1 - 2.0927\,(A_c/A_T), \qquad 0 < A_c/A_T < 0.30 \tag{6}$$

The above expression is sufficiently accurate if the contact areas are randomly distributed throughout the zone of radial divergent flow.

*If the voltage were constant, then the reciprocal of the resistance is proportional to the flow rate.

TABLE 3. Effect of Contact Area on Flow Rate (Divergent Flow Series 1 Tests)

Total area, A_t = 391.88 cm^2
Outer Radius, R_o = 11.3 cm
Inner Radius, R_i = 1.0 cm
Current, I_o = 0.013367 amps for A_c = 0.0

Test	No. of Holes and Diameter (cm)	Total Area of Holes, A_c (cm^2)	A_c/A_t (%)	Electrical Measurements			
				Resistance (Ohms)	Voltage (V)	Current (A)	I/I_o (%)
1	10 - 0.90	6.38	1.64	923	11.956	0.01295	96.88
2	20 - 0.90	12.76	3.28	960	11.960	0.01246	93.19
3	30 - 0.90	19.14	4.92	998	11.95	0.01198	89.59
4	40 - 0.90	25.52	7.56	1044	11.95	0.01145	85.63
5	20 - 0.90 20 - 1.35	41.53	10.60	1147	12.00	0.01046	78.26
6	40 - 1.35	57.55	14.69	1272	11.96	0.0094	70.40
7	55 - 1.35	79.13	20.20	1557	11.96	0.00768	57.44
8	82 - 1.35	117.98	30.1	2479	12.00	0.00482	36.10

Figure 6 shows the arrangement of holes used in Series 2 experiments in which the contact areas are rectangular. The number assigned to each rectangle shows the sequence in which they were cut. After each hole was cut, the resistance of the analog was measured and the current in amperes computed. The experimental data are summarized in Table 4 and the flow rates plotted on Figure 5 (square symbols). The data points do not fall on the line depicted by Eq. (6), but with the exception of the point for A_c/A_T = 31.34% the data from this series of tests also fall approximately along a straight line. The displacement of the A_c/A_T = 31.34% datum point from the trend of the other data may be explained by inspection of Figure 6. This point was obtained from Test 6 (see Table 4) in which flow from the borehole was restricted to a narrow channel along line OA.

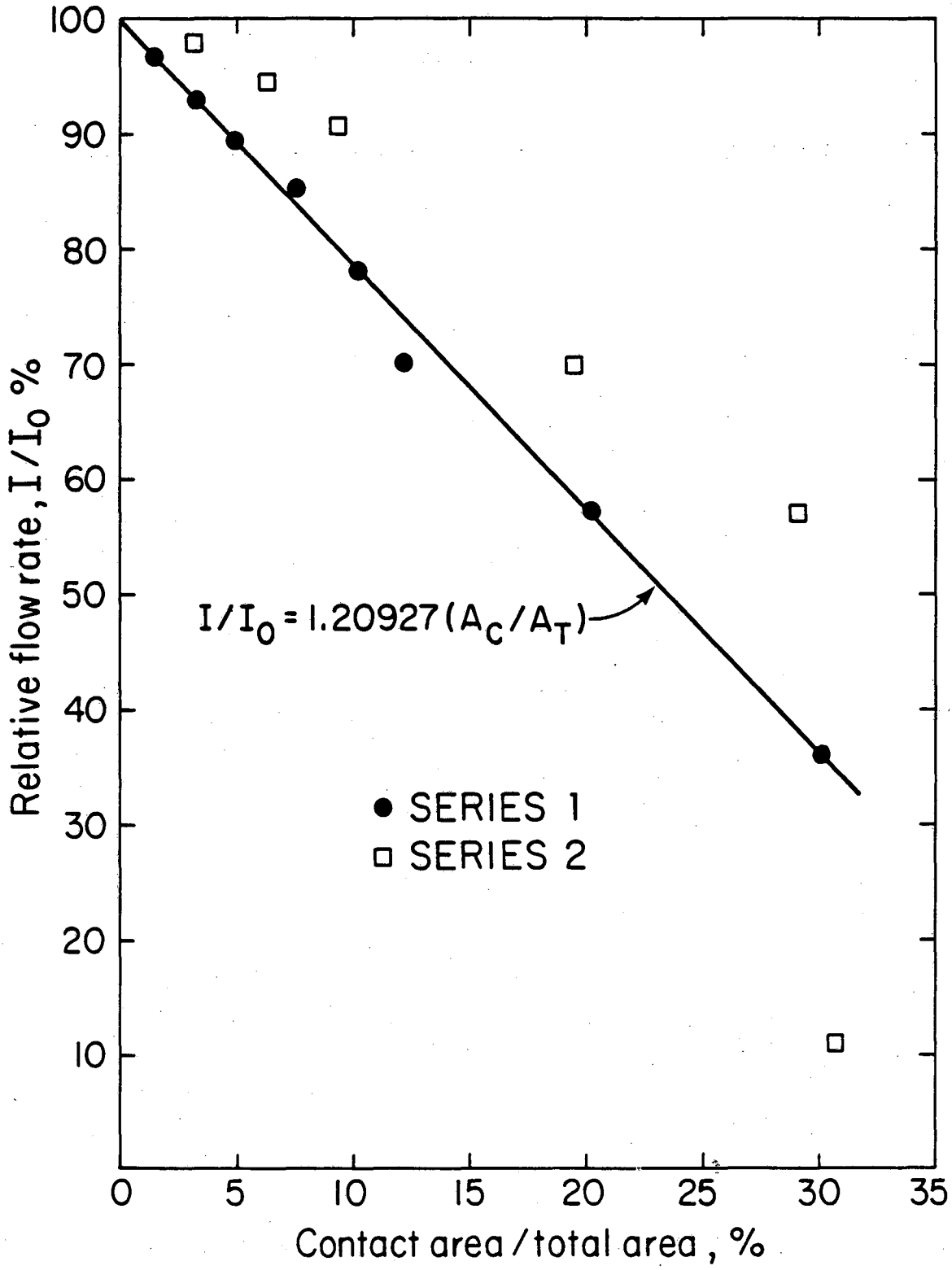


Figure 5. Relationship between flow rate and contact area. [XBL 821-1633]

TABLE 4. Variation of Flow Rate with Contact Area (Series 2 Tests)
Total Area, A_t = 398 cm^2, I_o = 0.012217 A.

Test	Hole No.	Total Area (cm^2)	Contact Area (%)	Electrical Measurements Voltage (V)	Resistance (Ohms)	Current I (A)	I/I_o %
1	#1	12.5	3.14	11.97	979.8	0.012217	97.57
2	#1, 2	25.0	6.28	11.97	1004.2	0.01192	94.13
3	#1, 2, 3	37.5	9.42	11.985	1041.9	0.01150	90.53
4	#1-6	75.0	18.84	11.97	1403.3	0.00853	69.81
5	#1-9	112.5	28.26	11.95	1726.6	0.00692	56.65
6	#1-14	124.75	31.34	12.05	9648	0.00125	10.22

It may be argued that the analog shown on Figure 6 is not representative of contact points in a natural fracture. However, the authors have observed in laboratory experiments on large sized samples (typically 0.76 m diam.) containing a natural fracture, that flow was not uniformly distributed over the surface area of the fracture but was concentrated in a narrow flow path and emerged as a jet from the periphery of the specimen. This could happen, for example, if the fracture is weathered and contains erosion channels.

The potential along line OA for Test 6 of Series 2 (A_c = 31.34%) is included in Figure 3 as Curve 2. In contrast to Curve 1 the case of zero contact area, Curve 2 suggests that most of the potential drop takes place somewhere in the zone between contact areas 2 and 9 (see Fig. 6).

The above studies indicate that in field pumping tests in natural fractures, actual water pressure distribution is strongly controlled by the distribution and location of contact areas.

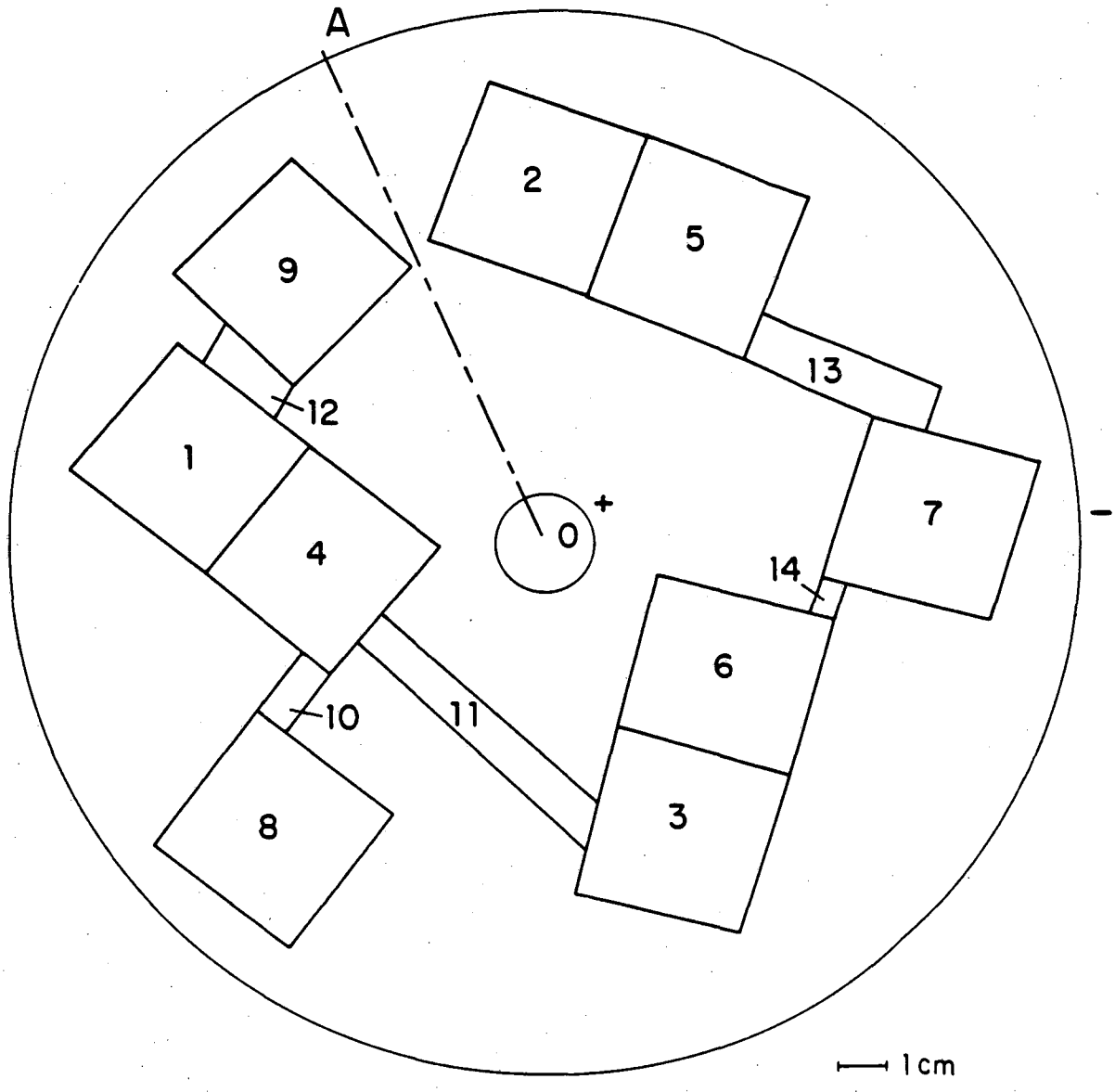


Figure 6. Analog for divergent radial flow (Series 2 tests). [XBL 821-1636]

Effect of Disturbance to a Natural Fracture by Drilling on Flow Rate

Field assessments of the hydraulic characteristics of natural fractures are frequently performed by drilling a hole intersecting the fracture and injecting water under a constant pressure between packers set in the borehole to straddle the fracture. However, the process of drilling may disturb the fracture close to the borehole. Specifically, there may be partial plugging of portions of the fracture by fine rock particles. If the fracture is open, it is possible for the particles to be transported away from the borehole. However, in tight fractures the particles may be permanently lodged close to the borehole. These particles reduce the area available for radial flow. It can be shown by a simple electrical analog experiment that the location of these particles is very important in estimating the flow characteristics of the natural fracture.

Figure 7 shows two analogs that simulate different locations of the contact or no-flow zones. In the first analog (Case A), three holes were cut close to the outer boundary. This resulted in flow rate of 93.6% of that for zero contact area. However, in the second analog (Case B) where three holes of the same geometry were closer to the inner hole, the flow rate was only 50% of the flow rate for zero contact area! This simple experiment suggests that the standard interpretation of data from tests may underestimate the flow rate in natural fractures.

The large drop in flow rate that results from partial plugging of the fracture close to the borehole is due to the very high hydraulic gradient occurring near the hole. Since the area available for flow is reduced close

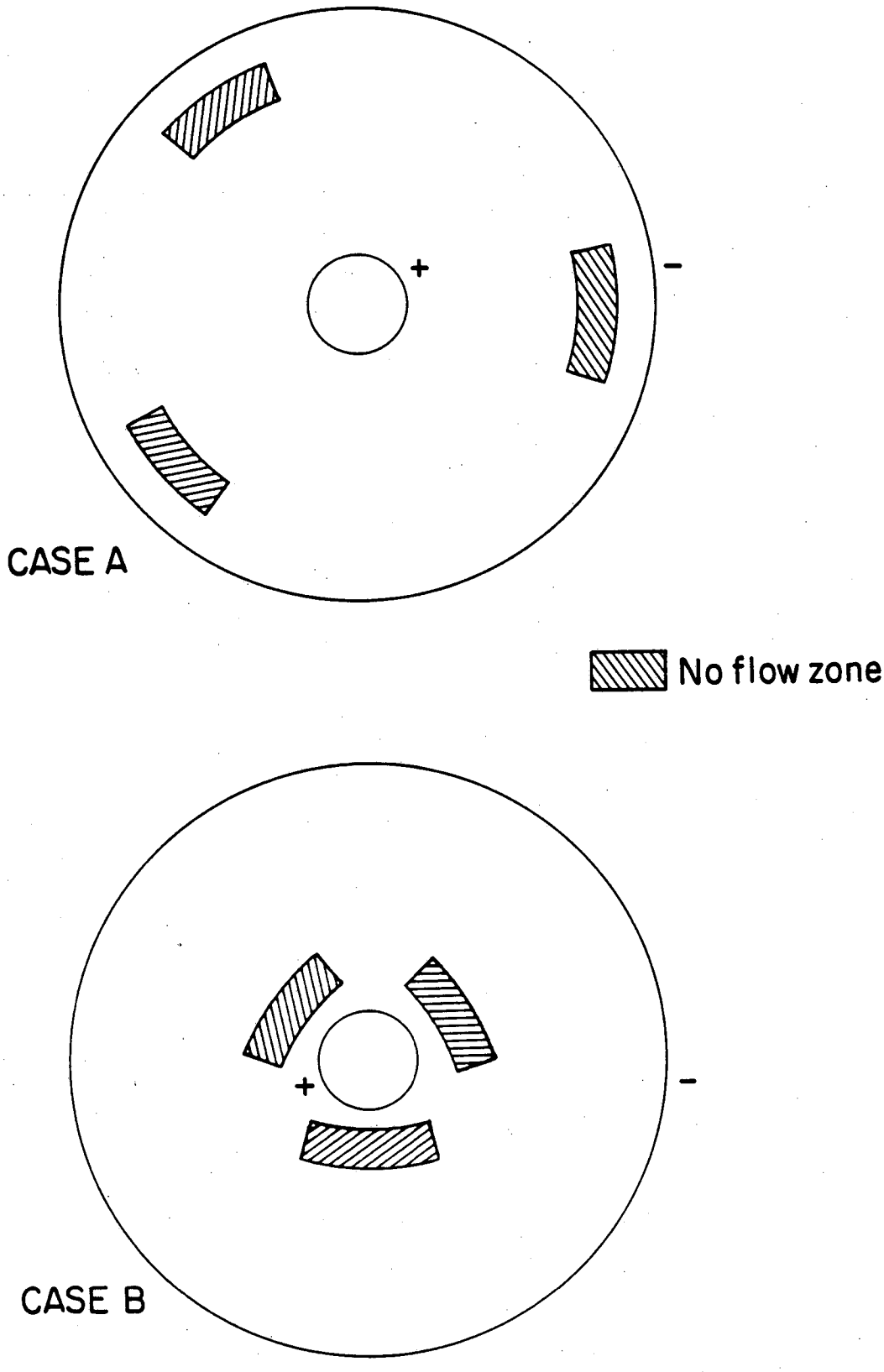


Figure 7. Analogs of radial flow in plugged fracture. [XBL 825-2228]

to the borehole, the flow rate is diminished considerably. However, away from the borehole the hydraulic gradient is comparatively small and for the same total area of plugging, the effect on flow rate is correspondingly small.

In linear flow the hydraulic gradient is constant and the effect of location of contact zones on the flow rate is negligible. Thus, in the case of linear flow, even though the actual gradient in flow through natural fractures may be affected by roughness and other physical characteristics of the fracture, the effect of man-made disturbances should be less pronounced than in the case of radial flow. Where conditions are favorable, two parallel rows of closely drilled holes may serve to generate one-dimensional flow in a single horizontal fracture. However, in practice, cost considerations may still dictate the use of conventional radial flow methods of performing injection tests.

Conclusions

The effectiveness of the electrical analog method as an economical research tool to study certain aspects of hydaulic flow through rock discontinuities has been demonstrated. The effect of contact area on flow rate has been studied using this technique. The analysis has shown that in the case of radial flow, the location of contact zones in addition to the total contact area, influences the flow rate. By a simple experiment, we have shown that it is possible to underestimate the flow rate in a fracture when injection tests are conducted in boreholes where the fracture close to the borehole is partially plugged by loose rock particles or other debris.

Acknowledgments

This work was supported by the Assistant Secretary for Nuclear Energy, Office of Waste Isolation of the U. S. Department of Energy under contract DE-AC03-76SF00098. Funding for this project was administered by the Office of Nuclear Waste Isolation at Battelle Memorial Institute. Mike Wigley of the Office of Nuclear Waste Isolation, Dai Watkins, and Laurel Egenberger of Lawrence Berkeley Laboratory, offered valuable suggestions. Discussions with Stanley Murgenski of Sunshine Scientific Instruments, suppliers of the Electric Analog device were very useful.

References

1. Prickett, T. A., "Modelling Techniques for groundwater evaluation," Advances in Hydroscience, Vol. 10, Academic Press, New York, 1975, pp. 1-143.

2. Polubarinova-Kochina, P. Y., Theory of Ground Water Movement, Princeton University Press, 1962, 613 pp.

3. Harr, M. E., Groundwater and Seepage, McGraw Hill, New York, 1962.

4. Rushton, K. R., and Redshaw, S. C., Seepage and Groundwater Flow, John Wiley & Sons, New York, 339 pp.

5. Zienkiewicz, O., Mayer, P. and Cheung, Y. K., "Solutions of anisotropic seepage by finite elements," Journal of the Engineering Mechanics Division, American Society of Civil Engineers, No. 92, EM1, pp. 111-120.

6. Ligget, J. A., "Location of Free Surface in porous Media," Journal of the Hydaulics Divison, American Society of Civil Engineers, No. 103, HY4, 1977, pp. 353-365.

7. Irmay, S., Theoretical Models of Flow Through Porous Media, Rilem Symposium on the Transfer of Water in Porous Media, Paris, April 7-10, 1964.

8. Stallman, R. W., "Analog Model Analyzer--For Steady State Flow, Operating Instructions," U. S. Geological Survey, Washington, D. C., 1959.

9. Javandel, I., and Witherspoon, P. A., "Use of thermal model to investigate the theory of transient flow to a partially penetrating well," Water Resources Research, Vol. 3, No. 2, 1967, pp. 591-597.

10. De Josselin De Jong, G., "Moire patterns of the membrane analogy for ground water movement applied to multiple fluid flow," J. GEOPHYS. RES., Vol. 66, no. 10, 1961, pp. 3625-3629.

11. Todd, D. K., "Membrane Models," Groundwater Hydrology, Wiley, New York, 1969, pp. 316-318.

12. Cedergren, H. R., Seepage, Drainage & Flownets, John Wiley & Sons, Second Edition, 1977, 534 p.

13. Einstein, P. A., "Factors limiting the accuracy of electrolytic plotting tanks," British Journal of Applied Physics, Vol. 2, 1951, pp. 49-55.

14. Karplus, W. J., Analog Simulation-Solution of Field Problems, McGraw Hill Book Co., 1958, 434 p.

15. Snow, D. T., "A Parallel plate model of fractured permeable media," Ph.D Thesis, Department of Civil Engineering, Univ. of California, Berkeley, California, 1965.

16. Iwai, K., "Fundamental Studies of Fluid Flow through a single fracture," Ph.D Thesis, Department of Civil Engineering, Univ. of California, Berkeley, California, 1976, 208 p.

17. Gangi, A. F., "Variation of whole and fractured porous rock permeability with confining pressure," International Journal of Rock Mechanics and Mining Science, Vol. 15, no. 5, 1978, pp. 249-257.

18. Tsang, Y. W., and Witherspoon, P. A., "Hydromechanical behavior of a deformable rock fracture subject to normal stress," Journal of Geophysical Research, Vol. 86, 1981, pp. 9287-9298.

19. Gale, J. E., "A numerical field and laboratory study of flow in rocks with deformable fractures," Ph.D. Thesis, Department of Civil Engineering, University of California, Berkeley, California, 1975.

20. Thorpe, R., Watkins, D. J., Ralph, W. E., Hsu, R., and Flexser, S. "Strength and Permeability Tests on Ultra-Large Stripa Granite Core," Report No. SAC-31, LBL-11203, Lawrence Berkeley Laboratory, Berkeley, California.

21. Caldwell, I. A., "The theoretical determination of the permeability tensor for jointed rocks," Percolation Through Fissured Rock, Symposium Proceedings, Stuttgart, 1972, T1-c.

22. Sharp, J. C., and Maini, Y. N. T., "Fundamental Considerations on the Hydraulic Characteristics of Joints in Rock," Percolation Through Fissured Rock, Symposium Proceedings, Stuttgart, 1972, T1-F.

23. Bernaix, J., "New Laboratory Methods for studying the mechanical properties of rocks," International Journal of Rock Mechanics and Mineral Science, Vol. 6, pp. 43-90.

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720