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## NORMAL AND PION CONDENSED STATES IN NEUTRON STAR MATTER IN A RELATIVISTIC FIELD THEORY CONSTRAINED BY NUCLEAR BULK PROPERTIES II

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# Normal and Pion Condensed States in Neutron Star Matter in a Relativistic Field Theory Constrained by Nuclear Bulk Properties II

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Abstract

A relativistic field theory of isospin asymmetric matter and of neutron star matter in particular, which is compatible with nuclear matter bulk properties, was formulated in Part I of this series and treated in mean-field approximation. The self-consistent solutions include the $\sigma$, $\omega$, and $\rho$ fields for both the normal and pion-condensed states. The bulk properties of nuclear matter such as saturation density and energy, compressibility and asymmetry energy impose rather stringent restrictions on meson-nucleon coupling constants and the parameters of the nonlinear interactions in the $\sigma$-model. The numerical results include the equation of state, particle densities as functions of charge and baryon density and the composition of a neutron star. Based on these results, we discuss neutron star cooling for various equations of state.

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## I. Introduction

This paper presents the results for a relativistic field theory of isospin asymmetric baryon matter for densities around and above nuclear matter density but below the transition into the quark-gluon phase, which has been formulated elsewhere.[1)] The theory of dense nuclear matter is of interest both in nuclear physics with respect to heavy-ion collisions and in astrophysics concerning neutron stars and supernova explosions. The participating particles in this theory are the nucleons, the four mesons $\sigma$, $\omega_\mu$, $\rho_\mu$, and $\pi$ and $e^-$ and $\mu^-$ for $\beta$-stability. The normal as well as the pion-condensed states are studied. The main constraint on the parameters involved in this theory are the bulk properties of nuclear matter: saturation density $n_0 = 0.145\ \mathrm{fm}^{-3}$, a saturation energy of -15.96 MeV per nucleon, compressibility of 200-300 MeV, and asymmetry energy of 37 MeV. Also, the system is constrained by charge conservation and equilibrium with respect to its composition. The equation of state is extrapolated to higher density with the same set of parameters, which fits the saturation properties.

The paper is composed as follows: In section II, expressions for the energy density, pressure, and all field equations and constraints are formulated in mean-field approximation. They are derived from a Lagrangian with interacting mesons and nucleons and a nonlinear $\sigma$-model. In section III, various numerical results are given such as the equation of state, various particle densities, the chemical potential, the neutron/proton ratio, and the strength of the meson fields, all as functions of the baryon density. Section IV involves a scenario of a neutron star with mass and density distribution for several equations of state as well as a cooling estimate. Section V finally summarizes the results.

II. Energy Density and Self-Consistency Equations.

It is generally believed that at moderate density the interactions between nucleons can be represented by the exchange of mesons. Above a not very well defined critical density presumably a relativistic field theory of quarks and gluons takes over. In the low and intermediate density regime, the four mesons $\sigma$, $\omega_\mu$, $\pi$, and $\rho_\mu$ dominate the nucleon-nucleon interaction. Whereas the isospin zero particles $\sigma$ and $\omega_\mu$ are present at all densities and isospin states, the pseudoscalar $\pi$ occurs only about a certain critical density in the form of a pion-condensate, and the $\rho_\mu$ exist only in isospin-asymmetric matter according to its $\gamma_\mu$-coupling. A relativistic mean field theory based on $\sigma$ and $\omega_\mu$ only was introduced some time ago[2)] and has been extensively studied by Walecka et al.[3)] The repulsive $\omega_\mu$ and the attractive $\sigma$ together with nonlinear interactions of the $\sigma$ with itself[4)] can account for the saturation properties and the compressibility[5)] of symmetric nuclear matter, whereas the $\rho_\mu$ is essential for obtaining the asymmetry energy in asymmetric matter.

Since the choice of the Lagrangian and the resulting formalism has been discussed in part I of this paper, we will subsequently present only the essential results. The full Lagrangian $\mathcal{L}$ for the system of interacting nucleons and mesons is

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{Dirac} &+ \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\pi + \mathcal{L}_\rho \\ &+ g_\sigma\, \sigma\, (\overline{\psi}\psi) - g_\omega\, \omega_\mu\, (\overline{\psi}\, \gamma^\mu \psi) - U(\sigma) \\ &- g_\pi (\partial_\mu\, \underset{\sim}{\pi}) \cdot (\overline{\psi}\gamma_5\gamma^\mu \underset{\sim}{\tau}\psi) - g_\rho\, \underset{\sim}{\rho}_\mu \cdot \underset{\sim}{J}^\mu\end{aligned} \qquad (1)$$

where $J^\mu$ are the three total isospin 4-currents corresponding to $\mathcal{L}$. They are

$$\underset{\sim}{J}^{\mu} = \frac{1}{2}(\bar{\psi}\gamma^{\mu}\underset{\sim}{\tau}\,\psi) + \underset{\sim}{\pi} \times \partial^{\mu}\underset{\sim}{\pi} + \underset{\sim}{\rho}_{\nu} \times \underset{\sim}{\rho}^{\mu\nu}$$
$$+ g_{\pi}(\bar{\psi}\,\gamma_5\gamma^{\mu}\underset{\sim}{\tau} \times \underset{\sim}{\pi}\psi) + g_{\rho}(\underset{\sim}{\rho}^{\mu} \times \underset{\sim}{\pi}) \times \underset{\sim}{\pi} \tag{2}$$
$$+ 2g_{\rho}(\underset{\sim}{\rho}^{\mu} \times \underset{\sim}{\rho}^{\nu}) \times \underset{\sim}{\rho}_{\nu}$$

The free field Lagrangians are written in ref. 1). The attractive p-wave pseudovector $\pi$N-interaction is adopted, since the pseudoscalar coupling gives too large a repulsive s-wave interaction. The $\sigma$-self interaction $U(\sigma)$ is taken from ref. 1) and has the form

$$U(\sigma) = (\frac{1}{3}\,bm + \frac{1}{4}\,cg_{\sigma}\sigma)(g_{\sigma}\sigma)^3 \tag{3}$$

The $\rho_{\mu}$-meson has to be coupled to the total isospin current $J^{\mu}$ in order to avoid instabilities for simultaneously growing $\rho_{\mu}$ and $\pi$-fields.[6,7] The full (time and space dependent) solution of the Lagrangian $\mathcal{L}$ and the Dirac Equation following from it is intractable; hence a solution in mean-field (Hartree) approximation is sought. By replacing all the fields with their mean values with the coupling constants determined by nuclear matter properties rather than by hadron scattering, the Lagrangian becomes an effective many-body Lagrangian.

The $\rho_{\mu}$ is taken as a space-time constant in the third isospin direction. Another possibility would be a space-time dependent solution in the same isospin direction as the $\pi$-field. This solution, however, is energetically disfavored by the Lagrangian. The pion is the only meson, which has a space-time structure. This follows from the attractive p-wave interaction. It is a plane wave of the form

$$\pi^{\pm} = \frac{\bar{\pi}}{\sqrt{2}}\,e^{\pm ikx}\ , \quad \pi^{\circ} = 0$$

with $kx = k_0 t - \underset{\sim}{k}\,\underset{\sim}{x}$. The mean value of the pion field is given by the quantity $\bar{\pi}$. The nucleon source currents $\langle\bar{\psi}(x)\,\psi\,(x)\rangle$ and $\langle\bar{\psi}(x)\,\gamma_\mu \psi\,(x)\rangle$ are constant in infinite nuclear matter; therefore, the $\sigma$ and $\omega_\mu$-fields are constants as well. In reflection of a general theorem,[8] the space-like parts of the 4-vectors $\omega_\mu$ and $\rho_\mu$ vanish, and only the time-like components $\omega_0$ and $\rho_0$ survive.

With all the above replacements made, and after some algebra, the Dirac equation can be written as

$$[\gamma_\mu P^\mu - m^* + \gamma_\mu K^\mu \,(\tfrac{1}{2}\,\tau_3 + g_\pi\,\bar{\pi}\gamma_5\,\tau_2)]\;U(p) = 0 \tag{4}$$

with

$$P_\mu = p_\mu - g_\omega\,\omega_\mu \quad , \tag{5}$$

$$K_\mu = k_\mu - g_\rho\,\rho_\mu \quad , \tag{6}$$

$$m^* = m - g_\sigma\,\sigma \tag{7}$$

where the 8-component spinor $U(p)$ describes the nucleon wave function in a momentum eigenstate and

$$\psi_V(x) = U(p)\;e^{-ipx} \tag{8a}$$

and is related to the original Dirac field, $\psi$, of (1) by a local isospin rotation

$$\psi_V(x) = \exp\left(-\frac{i}{2}\,kx\,\tau_3\right)\psi\,(x) \tag{8b}$$

The Euler-Lagrange equations for the meson fields, derived from $\mathcal{L}$ are given by

$$m_\sigma^2 \sigma = g_\sigma \langle \bar{\psi}_V \psi_V \rangle - \frac{dU}{d\sigma} \tag{9a}$$

$$m_\omega^2 \omega_0 = g_\omega \langle \bar{\psi}_V \gamma_0 \psi_V \rangle \tag{9b}$$

$$[-K_\mu K^\mu - g_\rho^2 \rho_0^2 + m_\pi^2] \bar{\pi} = -g_\pi \langle \bar{\psi}_V \gamma_5 \gamma_\mu K^\mu \tau_2 \psi_V \rangle \tag{9c}$$

$$[m_\rho^2 + (g_\rho \bar{\pi})^2] \rho^0 = g_\rho J_3^0 \tag{9d}$$

$$J_3{}^\mu = \bar{\pi}^2 K^\mu + \langle \bar{\psi}_V \gamma^\mu (\tfrac{1}{2} \tau_3 + g_\pi \bar{\pi} \gamma_5 \tau_2) \psi_V \rangle \tag{9e}$$

where the brackets < > denote the expectation value of the nucleon currents in the ground state, i.e., the lowest energy eigenstates of (4). Explicit momentum space expressions for these can be obtained as in Ref. 1. The Fermi surface is discussed below. The equations (9c and 9d) have only one isospin component now. The resulting hadronic energy density is given by

$$\epsilon = -\bar{\mathcal{L}} + \langle \bar{\psi}_V \gamma_0 p_0 \psi_V \rangle + J_3^0 k_0 \tag{10}$$

where

$$\bar{\mathcal{L}} = \langle \mathcal{L} \rangle = -\frac{1}{2} m_\sigma^2 \sigma^2 - U(\sigma) + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} \bar{\pi}^2 (k_\mu k^\mu - m_\pi^2)$$
$$+ \frac{1}{2} m_\rho^2 \rho_0^2 - g_\rho \bar{\pi}^2 \rho_0 K_0 \tag{11}$$

$\bar{\mathcal{L}}$ is the ground state expectation value of the Lagrangian $\mathcal{L}$ after exploiting the Dirac equation (4). The leptonic energy density $\epsilon_{\text{lept}}$ coming from the Fermi seas of relativistic electrons and muons is given by

$$\epsilon_{\text{lept}} = 2 \int_0^{P_e} \frac{d^3p}{(2\pi)^3} \sqrt{p^2 + m_e^2} + 2 \int_0^{P_\mu} \frac{d^3p}{(2\pi)^3} \sqrt{p^2 + m_\mu^2} \tag{12}$$

$$P_e = \sqrt{\mu_e^2 - m_e^2}\ , \quad P_\mu = \sqrt{\mu_\mu^2 - m_\mu^2} \tag{13}$$

Beta equilibrium of the process $n \rightarrow p + e^- (\mu^-) + \bar{\nu}_e (\bar{\nu}_\mu)$ and (if pion condensation exists) $n(p) \rightarrow p(n) + \pi^-(\pi^+)$ requires the equality of the chemical potentials $\mu_e = \mu_\mu = \mu_\pi = -k_0$. Neutron excess results in a surplus of $\pi^-$ with negative pion energy $k_0$. In symmetric nuclear matter, $k_0$ is zero with equal numbers of positive and negative pions. The mean pion charge density $n_\pi$ is given by

$$n_\pi = \bar{\pi}^2 K_0 + g_\pi \bar{\pi} \langle \bar{\psi}_V \quad \gamma_0 \gamma_5 \tau_2 \psi_V \rangle \tag{14}$$

The Dirac equation has to be diagonalized in order to obtain the single particle energies. This results in a fourth order equation for the eigenvalues $p_0$.

$$D(P) = ((PP) - \varepsilon_0^2)^2 - (PK)^2 - (2g_\pi \bar{\pi})^2 [(PK)^2 - m^{*2} (KK)] = 0 \tag{15}$$

where

$$\varepsilon_0^2 = m^{*2} - \left[\frac{1}{4} + (g_\pi \bar{\pi})^2\right](KK) \quad . \tag{16}$$

This equation has four roots as functions of the 3-momentum $\underset{\sim}{P}$, $E_\pm(\underset{\sim}{P})$ for the particles and $\bar{E}_\pm(\underset{\sim}{P})$ for the anti-particles. Figure 1 gives an illustration. For $P_\perp = 0$, the two particle branches are displayed as function of $P_\parallel$, which is parallel to the k-axis. The Fermi seas of $E_\pm$ solutions are filled up to the Fermi energy. The eigenstates corresponding to the $E_\pm$ solutions should not be interpreted as protons and neutrons respectively but are mixtures of both. The Fermi surface has no reflection symmetry in the $P_\parallel$ axis except for the case $K_0 = 0$ of nuclear matter. For neutron matter, the lower (-) Fermi sea is predominantly occupied by neutrons. The isospin asymmetry is responsible for the asymmetry of the (-) branch, wheras a pion-condensate produces a gap between the (+) and (-) branch.

The integrals in the expression are two dimensional in $P_\perp$ and $P_\parallel$ and are performed numerically (where the momentum vector $\underset{\sim}{K}$ of the pion defines the parallel direction).

For the calculation of the energy density seven quantities must be determined at a given charge and baryon density: the four meson fields $\sigma$, $\omega_0$, $\rho_0$, and $\bar{\pi}$, the pion frequency and wave numbers $k_0$ and $\underset{\sim}{k}$, and the Fermi energy $E_F$. These require seven independent equations for their determination, the four Euler-Lagrange equations and three other ones

$$J_3^0 = \bar{\pi}^2 K^0 + \langle \bar{\psi}_V \, \gamma^0 \, (\tfrac{1}{2} \tau_3 + g_\pi \bar{\pi} \, \gamma_5 \, \tau_2) \, \psi_V \rangle = n_q - \tfrac{1}{2} n_B + n_e + n_\mu \tag{17}$$

$$\underset{\sim}{J}_3 = \bar{\pi}^2 \underset{\sim}{K} + \langle \bar{\psi}_V \, \underset{\sim}{\gamma} \, (\tfrac{1}{2} \tau_3 + g_\pi \bar{\pi} \, \gamma_5 \, \tau_2) \, \psi_V \rangle = 0 \tag{18}$$

$$2 \sum_\pm \int \frac{d^3p}{(2\pi)^3} \, \theta[E_F - E_\pm(\underset{\sim}{P})] = \langle \bar{\psi}_V \, \gamma_0 \, \psi_V \rangle = n_B \tag{19}$$

where $n_q$ is the total electric charge density, and $n_B$, $n_e$, $n_\mu$ are number densities for baryons, electrons, and muons. In the case of symmetric nuclear matter, the charge density is $n_B/2$, whereas in neutral neutron matter it is zero. The first two of the above equations are the time- and space-like component of the conserved third component of the isospin 4-current and follow from the equilibrium conditions

$$\frac{\partial \varepsilon}{\partial \underset{\sim}{k}} = \frac{\partial \varepsilon}{\partial k_0} = 0$$

that pions should condense in the lowest energy mode.

The third equation is simply the constraint for given baryon density, whereas the first equation is the constraint for given charge density. In the case of neutron matter in a neutron star, the charge density must be zero; otherwise the Coulomb interaction would overcome gravity.

When the seven above-mentioned quantities have been determined for given charge and baryon density, the energy density as well as the pressure given by[1]

$$P = \overline{\mathcal{L}} + E_F n_B - 2 \sum_{\pm} \int \frac{d^3p}{(2\pi)^3} E_{\pm}(\underset{\sim}{P})\, \theta_{\pm} \tag{20}$$

can be evaluated.

## III. Equation of State

In the preceding paragraph the essential formulas for the determination of the equation of state were given. The input quantities are the baryon and charge density and, of course, the coupling constants of the mesons. Since the Lagrangian is essentially a many-body Lagrangian because of the mean field approximation and the nonlinear $\sigma$-model potential $U(\sigma)$, it is reasonable to determine these quantities from nuclear matter properties. A critique of this method is given in ref. 9. The known bulk properties of nuclear matter are the binding energy of -15.96 MeV per particle at a saturation density $n_0$ = 0.145 fm$^{-3}$, a compressibility of 200-300 MeV, and an asymmetry energy of 37 MeV. The quantities to be determined are the ratios of coupling constants and masses $g_\sigma/m_\sigma$, $g_\omega/m_\omega$, and $g_\rho/m_\rho$ (only the ratios affect the energy), the $\pi$-N coupling constant $g_\pi$, and the parameters b and c of the potential $U(\sigma)$. The entire asymmetry energy comes from the difference in proton and neutron fermi energies and the $\rho$ meson; then the equation

$$-\frac{1}{2} m_\rho^2 \rho_0^2 + \langle\bar{\psi}_V \gamma_0 k_0 \tau_3 \frac{1}{2} \psi_V\rangle + \langle\bar{\psi}_V \gamma_0 p_0 \psi_V\rangle = \text{const } n_B + 37 \text{ MeV } (n_n - n_p)^2 n_B^{-1} \tag{21}$$

solved for nuclear density $n_0$ determines $g_\rho/m_\rho$ = 2.61 fm$^{-1}$. The three remaining bulk properties of nuclear matter cannot determine $g_\sigma/m_\sigma$, $g_\omega/m_\omega$, b, and c uniquely, thus leaving some liberty in the choice of these. We exploit this freedom by choosing parameters that yield a different behavior of the equation of state in the unknown high density region above saturation. Two sets of parameters, which yield a compressibility K = 280 MeV at the saturation point but which are soft (I) or stiff (II) at high density, are

$g_\pi = 1.13$ fm $\quad g_\sigma/m_\sigma = 1.26$ fm $\quad g_\omega/m_\omega = 0.98$ fm $\quad b = -0.734 \quad c = 6.856$ set I

$g_\pi = 1.22$ fm $\quad g_\sigma/m_\sigma = 3.14$ fm $\quad g_\omega/m_\omega = 2.31$ fm $\quad b = 0.004 \quad c = 0.008$ set II

In the normal state ($\bar{\pi} = 0$) the nuclear matter properties do not determine the $\pi$-N coupling constant $g_\pi$. The p-wave scattering length gives a $g_\pi$ = 1.41 fm. However, repulsive short-range correlations, $\Delta$-resonance admixtures to nucleons and finite $\pi$N vertex cutoffs tend to reduce the effective $\pi$-N coupling constant. Consistent with the view that our Lagrangian is an effective one whose coupling constants should be fixed by known nuclear properties, $g_\pi$ must not exceed 1.13 fm for set I and 1.22 fm for set II so that the critical density for condensation lies above the normal density, $n_0$. We take these upper limits in the subsequent calculations. This means that, within the frame of this theory, the pion condensate is as strong as compatible with bulk nuclear matter properties. All parameters are assumed density independent. There is no reliable experimental evidence for or against this assumption.*

A main difference between both equations of state is the effective mass m*. For small values of b and c corresponding to a small contribution of three- and four-body forces to the total density, m* is smaller and more density-dependent than for large values of b and c. The net contribution of many-body forces at nuclear density is given by

$$\frac{U(\sigma)}{n_0} = \begin{matrix} -36 \text{ MeV} & \text{set I} \\ +12 \text{ MeV} & \text{set II} \end{matrix} \qquad (22)$$

which actually is quite large for the softer equation of state.

Given a set of parameters $g_\sigma/m_\sigma$, $g_\omega/m_\omega$, $g_\rho/m_\rho$, $g_\pi$, b, and c, we proceed to solve the above-mentioned seven equations self-consistently. There are only five independent variables, which have to be determined in a

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*However, there is theoretical evidence in favor of it. See Chin in ref. 3.

numerical iteration procedure, namely $\sigma$, $\bar{\pi}$, $k$, $k_0$, and $E_F$. The two fields $\omega_0$ and $\rho_0$ are already determined by the baryon density (from eq. 9b) and by the charge densities (from eq. 17 and 9d).

The results for the energy per particle, the pion condensation energy $E_c/N$, and the asymmetry energy $E_\rho/N$ from the $\rho$-meson are given in fig. 2 and 3 for both the stiff and the soft equation of state. In symmetric nuclear matter, there is no $\rho$-field and the chemical potentials $\mu_\pi = \mu_e = \mu_\mu = -k_0$ are zero. The equation of state is calculated for a baryon density $n_B$ up to ~5 $n_0$. Above this somewhat arbitrary limit the foundations of the present theory become arguable, especially with regard to the transition into quark matter. Since the $\rho$-meson, which favors isospin symmetry, plays a decisive role in accounting for the asymmetry energy of nuclear matter, we consider its inclusion in a theory of neutron star matter to be essential. The proton population in dense neutron star matter is of course enhanced by the $\rho$-meson. This in turn enhances the pion condensate through the requirement of charge neutrality. Neutron matter has a larger energy per particle than nuclear matter; approximately half the difference is the excess neutron fermi energy and the rest comes from the $\rho$-meson. For neither parameter set is neutron matter self-bound, although the energy has a minimum at $n_B \sim 0.55\ n_0$ for the soft equation of state. Were it not for the $\rho$ meson, however, both equations of state would yield a self-bound state at $n_B \sim 0.6$–$0.8\ n_0$ (not shown).

The pion condensation energy is larger for the soft equation of state in spite of the smaller coupling constant. It also is larger for neutron matter than for nuclear matter, since the neutron excess drives the process $n \rightarrow p + \pi^-$. The $\pi$-N coupling constants are chosen such that the condensate threshold starts at saturation energy in nuclear matter. In neutron matter, the threshold is also around $n_B \sim n_0$ or a little above. The behavior of the condensate is quite different in the two cases. For the stiffer equation of state and in nuclear matter it exists only for densities $1.0\ n_0 \leq n_B \leq 3.9\ n_0$ with a very small condensation energy of ~0.5 MeV. For the soft equation of state both for nuclear and neutron matter, the condensation energy is larger and continuously growing with the density. The $\rho$ field encourages pion condensation, the condensation energy being about twice as large for $\rho \neq 0$ as compared with $\rho = 0$. The condensation energy in chiral models[10,11] depends very much on the details of the $\pi$N-interaction and ranges from ~0 to ~80 MeV at ~4 $n_0$.

The asymmetry energy $E_\rho/N$ has the same qualitative behavior for both versions of the equation of state; it increases monotonically in the normal state and tends towards a saturation in the pion condensed phase. This can be understood from the trend of the $\rho$-field and the chemical potential $\mu_\pi$; see fig. 4a,b. The chemical potential is quite sensitive to the pion condensate. Without it, it grows monotonically, while the neutron Fermi energy is balanced by the combined electron and proton energy. In the presence of a pion condensate, much of the negative charge of the Fermi-sea of electrons and muons is taken over by $\pi^-$, thus limiting the growth of the chemical potential. At $\rho_B \sim 5\ \rho_0$, the pion charge density $n_\pi$ is six to seven times as large as the total lepton density $n_e + n_\mu$. The results for the soft equation of state are not shown in fig. 4, because they differ only 10-20% from those for the stiff one. Without the $\rho$-field, the chemical potential has generally the same behavior (not shown), except that it is generally smaller for the reason that mostly the quantity $K_0 = k_0 - g_\rho \rho$ determines the time component of the isospin current $\mathcal{J}_3^0$, which is given by baryon and charge densities (eq. 17). For $\rho = 0$ also $k_0 = -\mu_\pi$ has to be smaller to yield the same $K_0$.

The $\rho$-field has in general a behavior similar to that of the chemical potential, fig. 4b. Without a $\pi$-field it increases first linearly then less than linearly, when the lepton density starts to contribute substantially to the current $\mathcal{J}_3^0$, as seen from eq. (17) and eq. (9d). With a $\langle\pi\rangle$-field, its growth is limited from the $\rho$ self-consistency equation (9d).

The $\rho$ contribution to the single particle energy is approximately proportional to the $\rho$-field and repulsive. At $n_B \sim 5\rho_0$ the repulsion is ~30 MeV with a $\pi$-field and a $\rho$-field of 0.05 $fm^{-1}$ and ~45 MeV without a $\pi$-field and a $\rho$-field of 0.1 $fm^{-1}$. The main source of the asymmetry energy results from the $\rho$-N coupling and not from the $\rho$-field energy $-1/2\, m_\rho^2 \rho_0^2$, which is attractive and yields only 15-30% of the $\rho$-N contribution. The $\rho$-field as well as the asymmetry energy $E_\rho/N$ are rather similar for both equations of state.

The $\rho$-field has also an important influence on the proton/neutron ratio $n_p/n_n$, since its repulsive energy favors isospin symmetry. Figure 5 displays the proton/neutron ratio for all combinations of $\rho$ and $\pi$ fields for neutron matter. The presence of a $\rho$ field enhances the proton density, which, in turn, enlarges the pion condensate through the charge neutrality condition. These effects can be quite substantial at densities around 5 $n_0$, where in the presence of a pion condensate more than a third of all baryons are protons. It should be kept in mind that for an interacting nucleon system the chemical potentials $\mu_n$ and $\mu_p$ are not dependent on the respective densities in a unique way, though still $\mu_n = \mu_p + \mu_e$ holds true.

Again, there is no substantial difference between the results for the soft and the stiff equation of state, the proton/neutron ratio for the latter one is generally a little larger. These results indicate that the matter in the center of a neutron star, where densities around ~5 $n_0$ and even larger can be expected, is far from pure or nearly pure neutron matter and contains a substantial amount of protons.

One of the most interesting aspects in the present calculation are the strength of the pion field $\bar{\pi}$ and related quantities like the spin-isospin density and the pion charge density $n_\pi$. The pion field $\bar{\pi}$ is important for the cooling scenario of neutron stars. Figure 6 shows a monotonic increase of the $\bar{\pi}$ field for neutron matter. The results for both equations of state are very similar. For symmetric nuclear matter, however, the form of the equation of state makes an enormous difference. Whereas a smooth increase of the $\bar{\pi}$ field accompanies the soft equation of state, the stiff one limits the $\bar{\pi}$ field to the region below 3.9 $n_0$ for a coupling constant $g_\pi$ = 1.22 fm. The p-wave $\pi N$ attraction is for this nonchiral theory below a certain effective nucleon mass m* not strong enough to support pion condensation in nuclear matter. In contrast, for neutron matter, the larger driving force associated with the difference of the neutron and proton chemical potentials maintains the condensate. The class of solutions for the $\pi^\pm$ fields do not result in oscillations of the baryon density $\langle\bar{\psi}_V\gamma_0\psi_V\rangle$ unlike the case of a neutral $\pi^\circ$ condensate. However, the spin-isospin density $\langle\bar{\psi}_V \gamma_5 \gamma_3 \tau_2 \psi_V\rangle$ has pronounced spatial oscillations. It is nonrelativistically given by $\langle\bar{\psi}_V \gamma_5 \gamma_3 \tau_2 \psi_V\rangle \sim n(p\downarrow) + n(n\uparrow) - n(p\uparrow) - n(n\downarrow)$ with the nucleon spin pointing in z-direction. The baryon density $\langle\bar{\psi}_V \gamma_0 \psi_V\rangle = n(p\uparrow) + n(n\uparrow) + n(p\downarrow) + n(n\downarrow)$ is constant. In the condensate phase, the quantity $R_{SI} = \langle\bar{\psi}_V \gamma_5 \gamma_3 \tau_2 \psi_V\rangle/\langle\bar{\psi}\, \gamma_0\, \psi_V\rangle$ measures the magnitude of the spin-isospin oscillations. The quantity $R_{SI}$ is given by eq. (9c)

$$R_{SI} = -\frac{k^2+m_\pi^2-k_0^2 + 2g_\rho\rho_0K_0}{g_\pi k\, n_B}\,\bar{\pi} \qquad (23)$$

Unlike symmetric nuclear matter, the $\rho$ field contributes to the spin-isospin amplitude as well. Figure 7 displays the quantity $R_{SI}$. As for the $\bar{\pi}$ field, the differences in $R_{SI}$ between the soft and the stiff equation of state are small for neutron matter and substantial for symmetric nuclear matter, where $R_{SI} \neq 0$ only in a limited density range. In spite of the large spin-isospin oscillations, the equation of state for nuclear matter is only moderately softened by pion condensation (more for the soft, less for the stiff version) up to a density of ~4 $n_0$, which is at the upper limit of what is obtainable in high-energy heavy-ion collisions.

Figure 8 shows the pressure for nuclear matter. The pressure is reduced to ~2/3 of its value without pion condensation for the soft version and nearly not affected for the stiff version. The effects on the hydrodynamics of nuclear collisions is limited in both cases. Even if attempts to determine the equation of state from heavy-ion collisions should succeed, it is unlikely to establish or refute the existence of a pion condensate from these results, since the difference between soft and stiff versions, both allowed by nuclear matter bulk properties, is substantially larger than the difference between the condensate and the normal solution. It is remarkable that the equation of state for the stiff version is nearly identical for the normal and pion condensed state with a <1 MeV difference in nuclear matter, and yet the spin-isospin oscillations reach as much as ~0.6 of the baryon density. It is possible that the pion condensate has a substantially larger impact on pion radiation and production than on the equations of state.

The particle densities for protons, neutrons, electrons, muons, and pions are displayed in Fig. 9 for neutron matter. The difference between the soft and stiff equations of state is not important. The pion density is defined as the charge density $n_\pi$ of the $\pi^-$ condensate. For the other (neutral) mesons; $\sigma,\omega,\rho$; the field strength rather than the particle density is the meaningful quantity.

## IV. Neutron Star Structure with Various Equations of State

The composition of a neutron star is of considerable interest for astrophysics (pulsar phenomenon and star development) and nuclear physics (dense nuclear matter). It offers a unique possibility to get--albeit rather indirectly—information about stable baryon matter at supernuclear densities. Starting from an assumed nucleon-nucleon force, one can construct equations of state and by inserting these into the Tolman-Oppenheimer-Volkoff (TOV) equations[12)] of general relativity to obtain mass, pressure, and density profiles of the star. These results include, for example, equations of state based on Reid, BJ, and tensor nucleon-nucleon interaction[13)], various variations of the Reid potential treated with Pandharipande's constrained variational technique[14)], and a mean-field theory including $\sigma$, $\omega$, and spin 2 $f^{\circ}$-meson[15)]. These models obtain maximum masses of 1.6-2.5 $M_{\odot}$, depending on the assumptions about nucleon-nucleon forces and coupling constants. The maximum masses, obtained with the present theory are within this range or lower, depending on the stiffness of the equation of state and on the existence of a pion condensate, respectively. A good review of neutron star structure and related problems can be found in ref. 16).

It is quite illustrative to have a look at the relation between the energy density $\epsilon$ and the pressure P; figure 10 displays the $P(\epsilon)$ curve for three cases. The soft equation of state comes nowhere close to the causality limit $P = \epsilon$, whereas the stiff one approaches this limit in the density range of interest for neutron star structure. All equations of state with a massive vectorboson approach the causality limit asymptotically. For the stiff version, the result is nearly identical for the normal and pion-condensed state. Again, above $n_B \sim 10\ n_0$ the validity of the present theory of baryon matter is questionable. It is interesting to see that the softest version, namely the soft equation of state with pion condensate, has a smaller or equal gradient to the curve $\log P = 4/3 \log \varepsilon$ in the density range $1.5\ n_o < n_B < 4\ n_o$. Such behavior can result in instabilities and oscillations of stars.

The presence of a pion condensate may lead to a considerable softening of the equation of state and the pressure, but not necessarily; again it depends on the qualities of the nonlinear $\sigma$ interaction. The difference in pressure for the stiff and the soft version can be as large as a factor 4.

The results for $P(\varepsilon)$ serve as input into the TOV equation for hydrostatic equilibrium

$$\frac{\partial P(r)}{\partial r} = - G \frac{[\varepsilon(r) + P(r)][M(r) + 4\pi r^3 P(r)]}{r^2[1 - 2GM(r)/r]} \tag{24}$$

Here $P(r)$ is the local pressure, $\varepsilon(r)$ the local energy density, and $M(r)$ the gravitational mass of a sphere of radius $r$

$$M(r) = \int_0^r \varepsilon(r')\ d^3r' \tag{25}$$

The gravitational constant is $G = 2.62 \times 10^{-40}\ \text{fm}^2$. Numerical integration of the TOV equation gives the profiles $M(r)$ and $P(r)$ for a given central energy density $\varepsilon_c(0)$. The main result of interest is the total mass at a given central density, since the mass is the only property of a neutron star on which there are some reasonably accurate data and is more strongly connected to the equation of state than other currently measurable properties. The other two measurable quantities, surface temperature and rotational velocity, depend on the initial conditions, on the age of the star, and only slightly on the structure of the baryonic matter. As can be seen from fig. 11, the total mass as function of the central energy density and the maximum mass depend strongly on the particular equation of state. The pion condensate together with the stiff equation of state does not affect the maximum mass of ~2 $M_\odot$ much; at the upper limit of the mass range the mass is very insensitive to the central density. For the soft equation of state, the pion condensate has a substantial effect, however; without a condensate a maximum mass of ~1.3 $M_\odot$ is achieved, with a condensate only one of ~0.9 $M_\odot$. The mass vs central density curve is still growing at a central energy density of ~8 $fm^{-4}$, which is assumed as a somewhat arbitrary upper limit of the density range of this theory corresponding to a baryon density of ~10 $fm^{-4}$. The determined masses for the pulsars Her X-1 and Vela X-1 are ~1.4 $M_\odot$[16]. This seems barely to allow the soft equation of state without pion condensation and appears to rule out the pion condensate with this equation of state. The experimentally determined masses, if they hold true, indicate that stiff versions of the equation of state, which describe the bulk properties of nuclear matter, can give the right mass of a neutron star together with a moderate central energy density of ~2 $fm^{-4}$.

A distinction between the pion-condensed and the normal phase cannot be made from the observed mass alone. The cooling behavior of a neutron star, especially that of the Crab pulsar, might clarify this question[17]. A pion condensate is able to speed up the URCA process and the cooling of the star afters its birth in a supernova explosion.

The URCA process is the $\beta$-decay of a neutron with interaction with a second neutron to conserve energy and momentum:

$$n + n \rightarrow n + p + e^- + \bar{\nu}_e \quad . \tag{26}$$

The cooling time of a neutron star from an initial temperature $T_i$ to a final temperature $T_f$ can be expressed by[17]

$$\Delta t\ (T_i \rightarrow T_f) = -\int_{T_i}^{T_f} \frac{C(T)dT}{L(T)} \tag{27}$$

where C(T) is the specific heat per unit volume and L(T) is the luminosity per unit volume. The heat capacity is dominated by the neutrons. On the other hand, the luminosity is dominated by the URCA process in the absence of the pion-condensate and by the pion-induced beta decay in the presence of the condensate.[17,18] We would like to compare the cooling times for different equations of state. Let us first regard the case without a pion condensate. Without nucleon superfluidity the URCA process dominates the cooling down to central temperatures of $\sim 10^7$ K, below which neutrino bremsstrahlung $n + n(p) \rightarrow n + n(p) + \nu + \bar{\nu}$ from the crust takes over. Since most of the luminosity is concentrated in the hottest central part, the central densities are taken for the calculation of the specific heat and the luminosity. The connection between cooling time and central temperature is given by integration of eq. (27)

$$\Delta t\ (\text{in seconds}) = \frac{C_o}{6L_o} \frac{1}{T_{9_f}^{6}} \quad 10^9 \tag{28}$$

where $C(T) = C_o\, T_9$ and $L(T) = L_o\, T_9^8$ and $T_{9_f}$ is the final temperature. The quantity $T_9$ means units of $10^9$ K. The initial temperature doesn't matter, since most of the cooling occurs shortly after the birth of the neutron star. Table I shows the results for the two equations of state with and without pion condensate. The standard cooling scenario[18] with $n_B = n_o$ and an effective nucleon mass $m^* = 0.8$ m yields a final temperature $0.32 \times 10^9$ K. Vastly different equations of state and central densities give similar results. In all cases the final central temperature lies below the observed upper limit of $0.85 \times 10^9$ K. It has to be kept in mind that the central temperature is connected to the surface temperature by model calculations of heat transport in the star and thus subject to further uncertainties. This particular central temperature corresponds to a surface temperature of $4.7 \times 10^6$ K.

Let us now focus on the case with pion condensation. The pion-condensed phase has a smaller specific heat than the normal phase because of a smaller number of electrons and muons. The luminosity, however, is greatly enhanced by the pion condensate. The most recent and complete calculation about pion cooling[18)] involves a chiral symmetric ansatz for the $\sigma$ and $\vec{\pi}$ fields. We adopt formula (53) of ref. 18) for the luminosity with $g_A = 1.36$ and $n_\pi/n_B = 1/2 \sin \theta$. A factor 4 has to be multiplied to this formula to account for the inverse reaction and the $\beta$ decay into muons.

To compare the stiff and soft equations of state, our model star is assumed to have a mass of $M \sim 0.75\ M_\odot$, since this is the maximum mass obtained with the soft version at $\epsilon_c = 8$ fm$^{-4}$. The equation (28) is slightly modified due to the different ($T^6$) temperature dependence of the pion cooling mechanism

$$\Delta t \text{ (in seconds)} = \frac{C_o}{4L_o} \frac{1}{T_{9_f}^4} \; 10^9 \tag{29}$$

The values of $\theta^2$ are 0.007 for the stiff and 0.86 for the soft version, since the pion condensate is roughly 10 times stronger at $\epsilon_c = 8$ fm$^{-4}$ than at $\epsilon_c = 1.13$ fm$^{-4}$. The final temperatures are a factor 30–100 below the ones without pion condensate. The model calculation of ref. [18)] would yield a cooling time of ~350 s. In the pion-condensed state, final temperatures and cooling times vary more with the equation of state than in the normal state;

these differences are of little importance, however, since they are not yet observable.

## V. Conclusion

Within the frame of a relativistic mean field theory of isospin symmetric and asymmetric matter, various aspects of the equation of state have been studied. No evidence for exotic behavior of the nuclear equation of state was found, no second minima and kinks due to pion condensation and the like. The pion condensate softens the equation of state, but so does the $\sigma$ potential $U(\sigma)$ to an even greater extent. Unfortunately, the particle spectra from heavy-ion collisions do not provide so far a useful tool for an even approximate determination of the equation of state. Together with previous calculations, i.e.[10,11], it is obvious that different models give rather different equations of state for $n_B > 2n_0$, even though they all agree at $n_B = n_0$.

Concerning neutron matter and the "experimental" situation for neutron stars, it looks somewhat better. The mass of ~1.4 $M_\odot$ seems to rule out certain soft equations of state, if they are in a pion-condensed state. The influence of the pion condensate on the composition of the star depends very much on the basic $U(\sigma)$ interaction and ranges from nearly none to substantial. The stiff equation of state allows a maximum mass of ~2 $M_\odot$ and at ~1.4 $M_\odot$ yields a moderately strong pion condensate at $n_B \sim 2.7\ n_0$ central baryon density. The central density is rather sensitive to the precise equation of state, however, and can go up to ~10 $n_0$ for softer versions. The $\rho$ meson is essential for a consistent theory of neutron star matter, since it contributes substantially to the symmetry energy in asymmetric nuclear matter. Its presence stiffens the equation of state and enhances the pion condensate.

The present experimental upper limit of the surface temperature of $4.7 \times 10^6$ K corresponding to a core temperature of $0.85\times10^9$ K, cannot discriminate between cooling mechanisms, let alone between equations of state, even though they yield rather different central densities. A lowering of this limit by a factor 4, however, would rule out all cooling scenarios without pions but would not be able to determine the strength of the condensate nor the equation of state.

In summary, the presently known quantities of a neutron star allow the conclusion that probably a stiff version of the equations of state is present together with a moderate central baryon density $n_B \sim 2\text{-}4\ n_0$. Whether a pion condensate exists or not is yet to be determined from further temperature measurements.

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Table 1. Final central temperature $T_f$ at an age of 924 yrs (Crab pulsar) and cooling time $\Delta t$ to the observability limit $0.85 \times 10^9$K of the central temperature are given for the stiff and the soft equation of state in the normal and the pion-condensed state. The mass M is determined by the maximum baryon density $n_B = 10\ n_o$ for the soft equation of state. The central energy density $\epsilon_c$, baryon density $n_B$, as well as the heat capacity C and the luminosity L, are shown.

| | | | $\epsilon_c$ $fm^{-4}$ | $n_B$ $fm^{-4}$ | C $erg(cm^3K)^{-1}$ | L $erg(cm^3s)^{-1}$ | $T_{9_f}$ (924 yrs) K | $\Delta t$ |
|---|---|---|---|---|---|---|---|---|
| stiff | normal state | $M = 1.33\ M_\odot$ | 1.75 | 2.4 | $2.5\ 10^{20}$ | $4.2\ 10^{20}$ | 0.39 | 8.6 yrs |
| soft | | | 9 | 10 | $6.4\ 10^{20}$ | $4.3\ 10^{21}$ | 0.32 | 21 yrs |
| stiff | pion condensate | $M = 0.75\ M_\odot$ | 1.13 | 1.6 | $2.3\ 10^{20}$ | $0.6\ 10^{25}$ | 0.014 | 1800 s |
| soft | | | 8 | 10 | $5\ 10^{20}$ | $1.2\ 10^{27}$ | 0.004 | 200 s |

## Figure Captions

Fig. 1. The single particle energy $E_{\pm}$ for both fermi seas as a function of $p_{\parallel}$ with $p_{\perp} = 0$.

Fig. 2a. The energy per particle minus rest mass for nuclear ($n_q = 0$) and neutron ($n_q = n_B/2$) matter as a function of the baryon density $n_B$ for the soft equation of state in the normal state ($\pi = 0$).

Fig. 2b. The (negative) condensation energy per particle as a result of pion-condensation for nuclear and neutron matter as a function of the baryon density $n_B$ for the soft equation of state.

Fig. 2c. The (positive) asymmetry energy per particle as a result of the $\rho$-meson coupling for neutron matter as a function of baryon density $n_B$ both in the pion-condensed and normal state for the soft equation of state.

Fig. 3a. The same as fig. 2a for the stiff equation of state.

Fig. 3b. The same as fig. 2b for the stiff equation of state.

Fig. 3c. The same as fig. 2c for the stiff equation of state.

Fig. 4a. The chemical potential $\mu_e$ in neutron matter as a function of baryon density $n_B$ for the stiff equation of state both in the normal and pion-condensed phase.

Fig. 4b. The $\rho$-field as a function of baryon density $n_B$ in neutron matter for the stiff equation of state, both in the normal and pion-condensed phase.

Fig. 5. The proton/neutron ratio $n_p/n_n$ in neutron matter as a function of the baryon density $n_B$ for the stiff equation of state. The full curves correspond to the normal state, the dashed ones to the pion-condensed state. The results for $\rho = 0$ and $\rho \neq 0$ are shown.

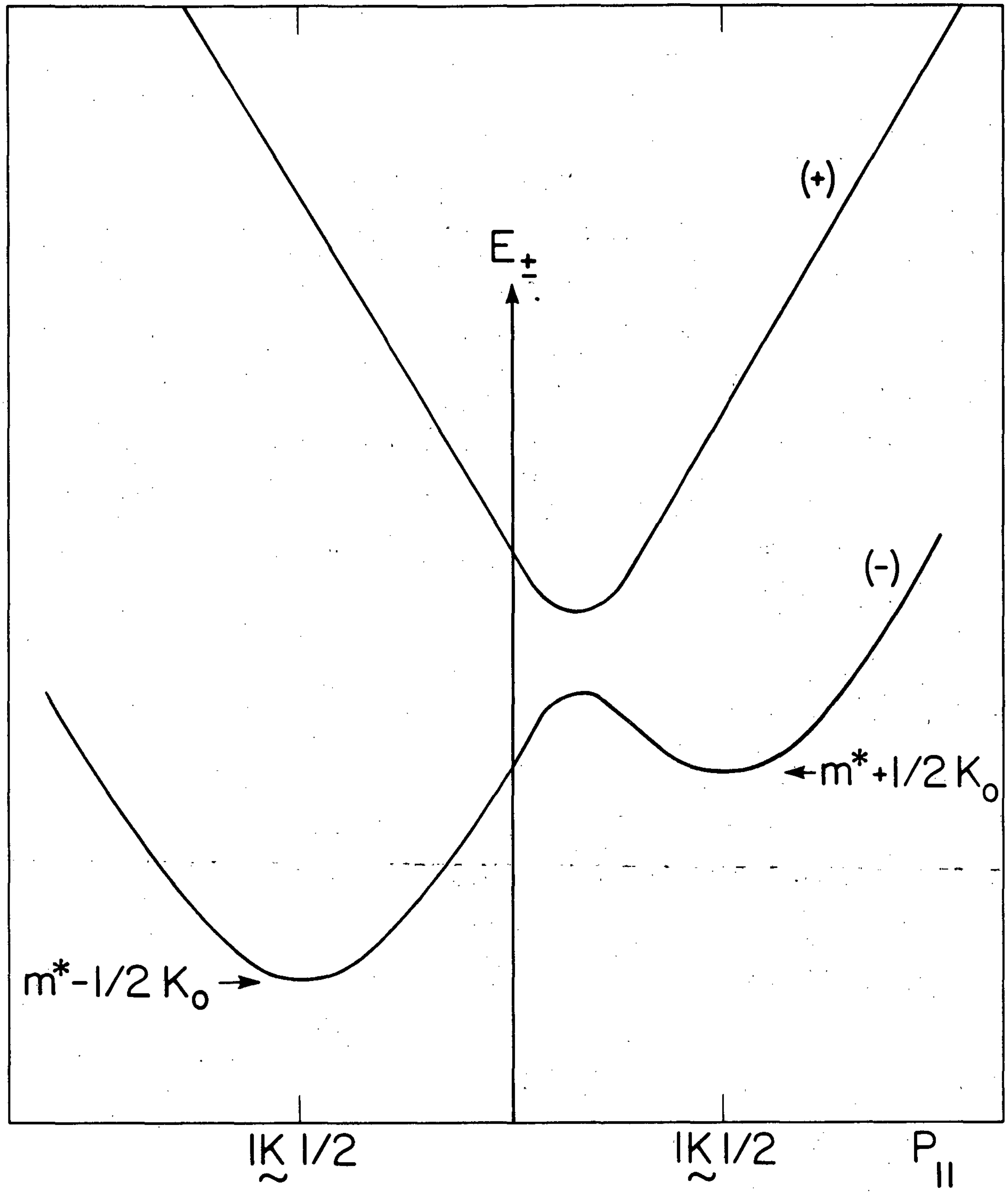


Fig. 1. The single particle energy $E_{\pm}$ for both fermi seas as a function of $p_{\parallel}$ with $p_{\perp} = 0$.

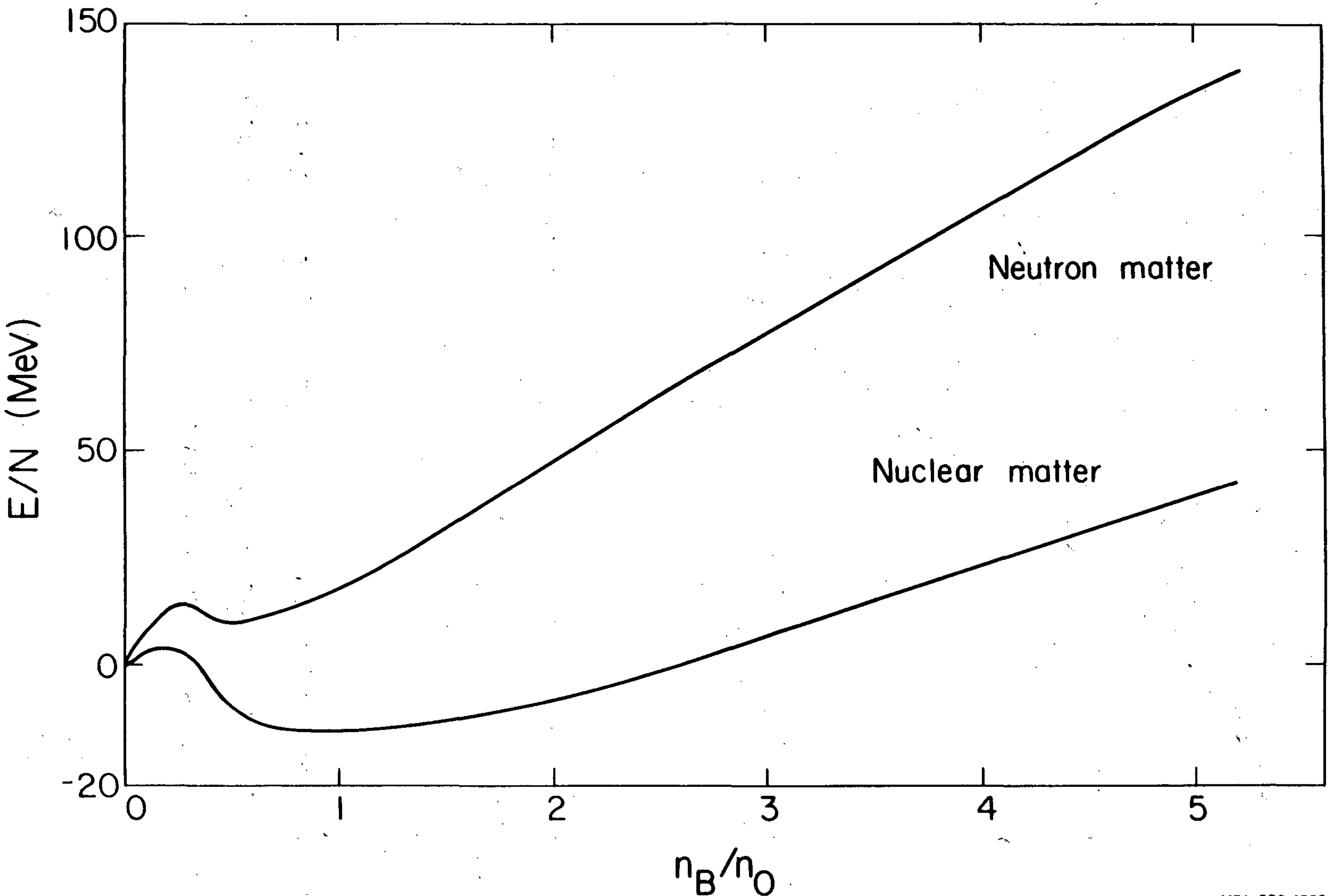


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Fig. 2a. The energy per particle minus rest mass for nuclear and neutron matter as a function of the baryon density $n_B$ for the soft equation of state in the normal state ($\overline{\pi}$ = 0).

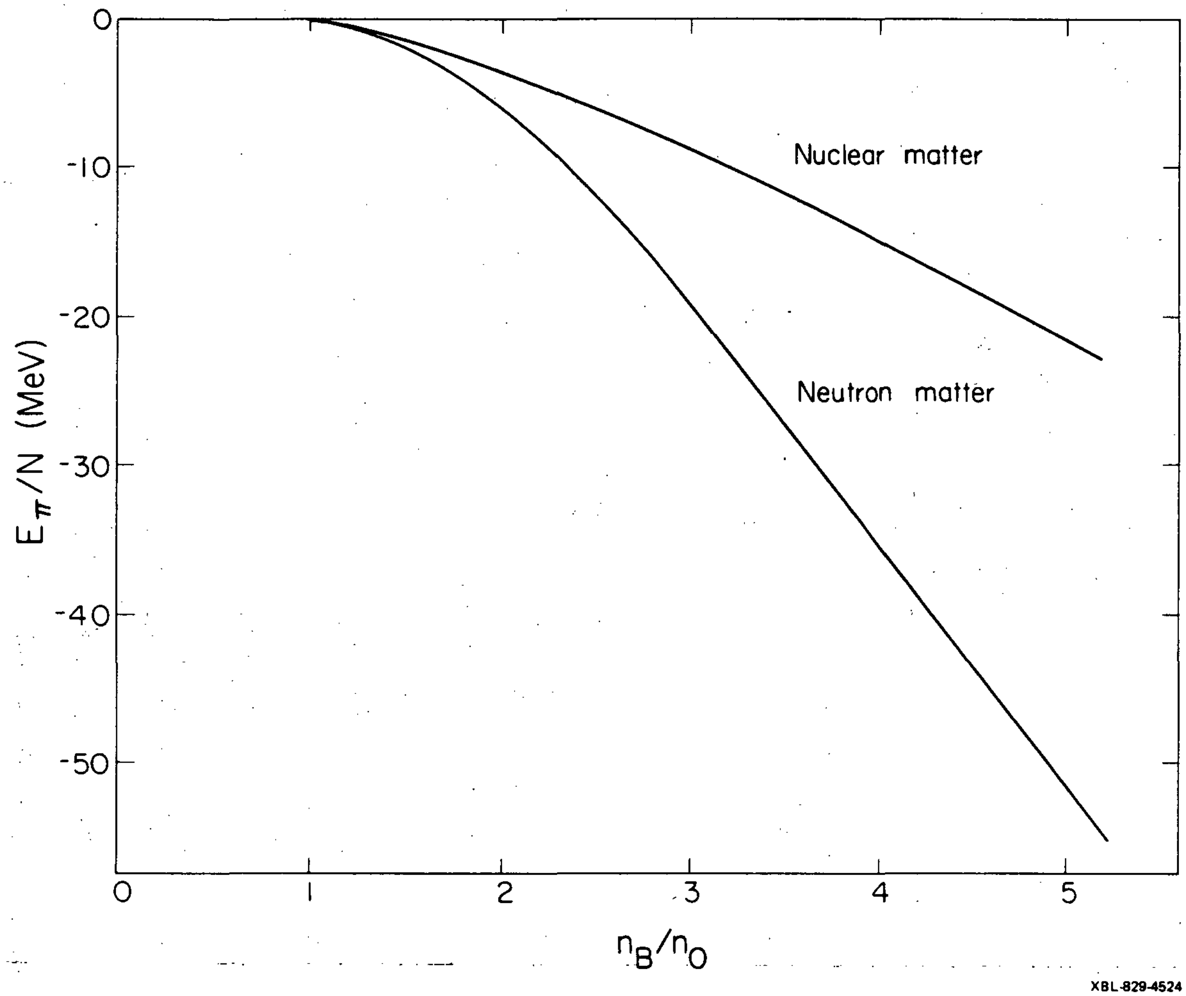


Fig. 2b. The (negative) condensation energy per particle as a result of pion-condensation for nuclear and neutron matter as a function of the baryon density $n_B$ for the soft equation of state.

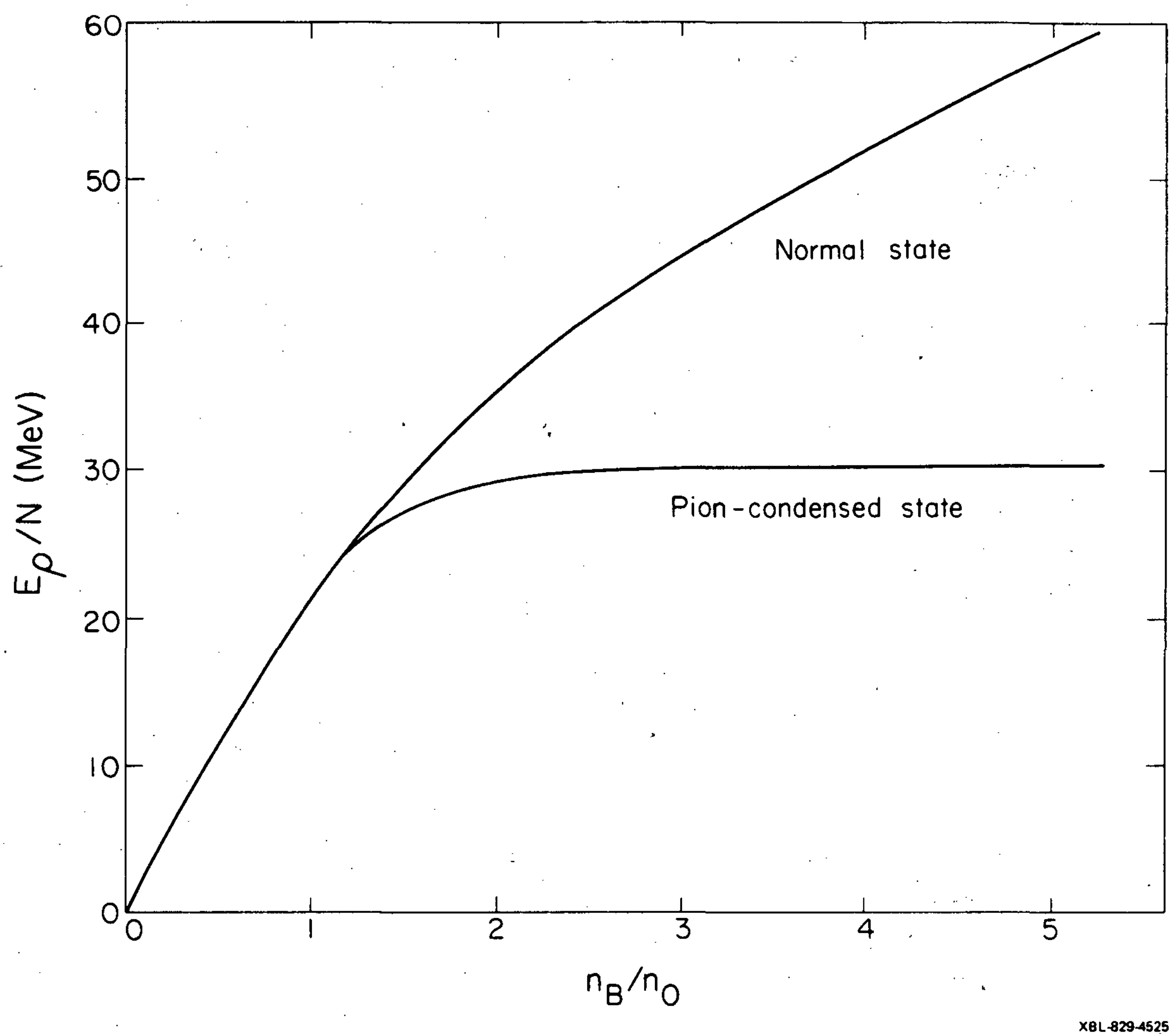


Fig. 2c. The (positive) asymmetry energy per particle as a result of the $\rho$-meson coupling for neutron matter as a function of baryon density $n_B$ both in the pion-condensed and normal state for the soft equation of state.

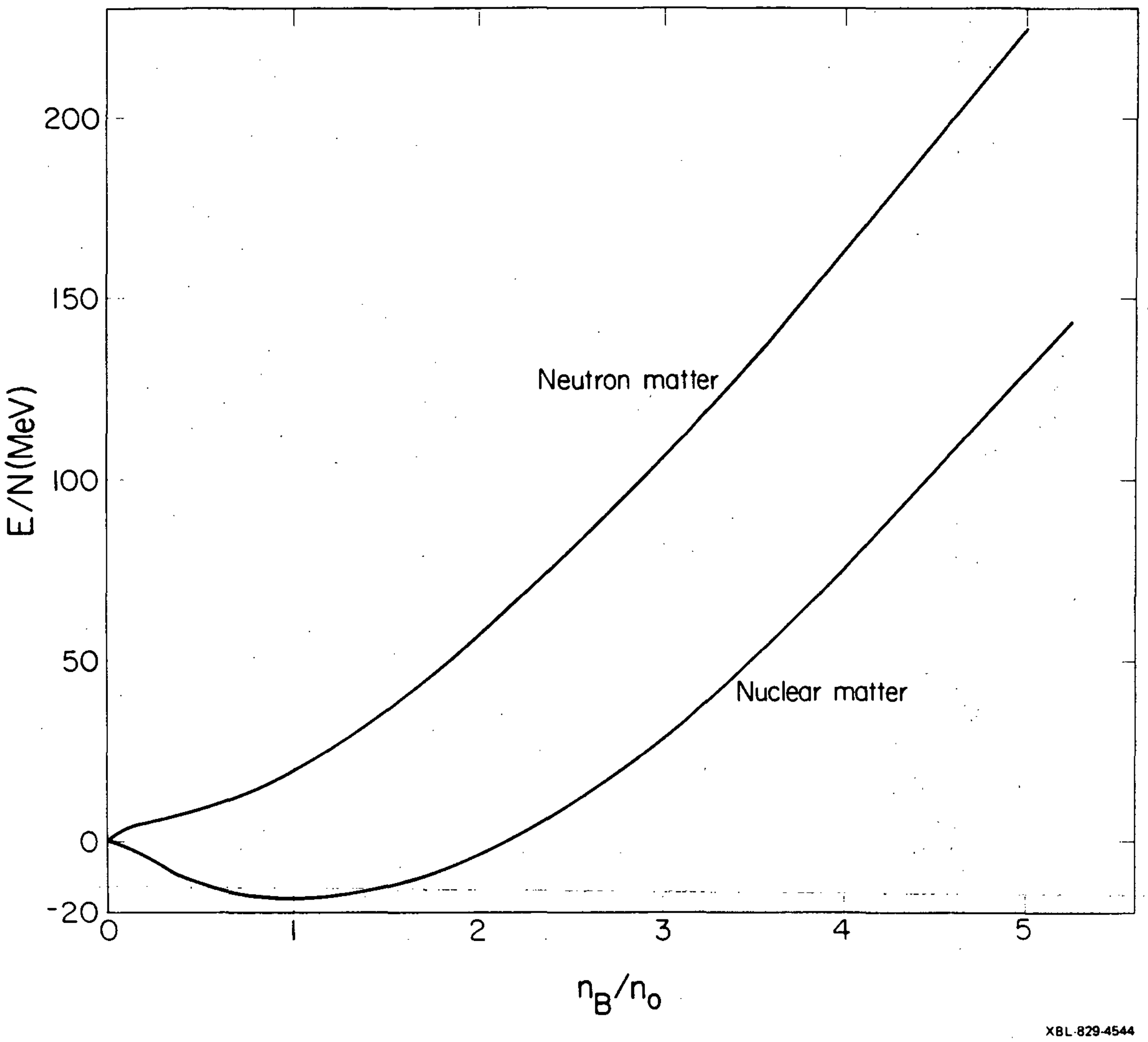


Fig. 3a. The same as fig. 2a for the stiff equation of state.

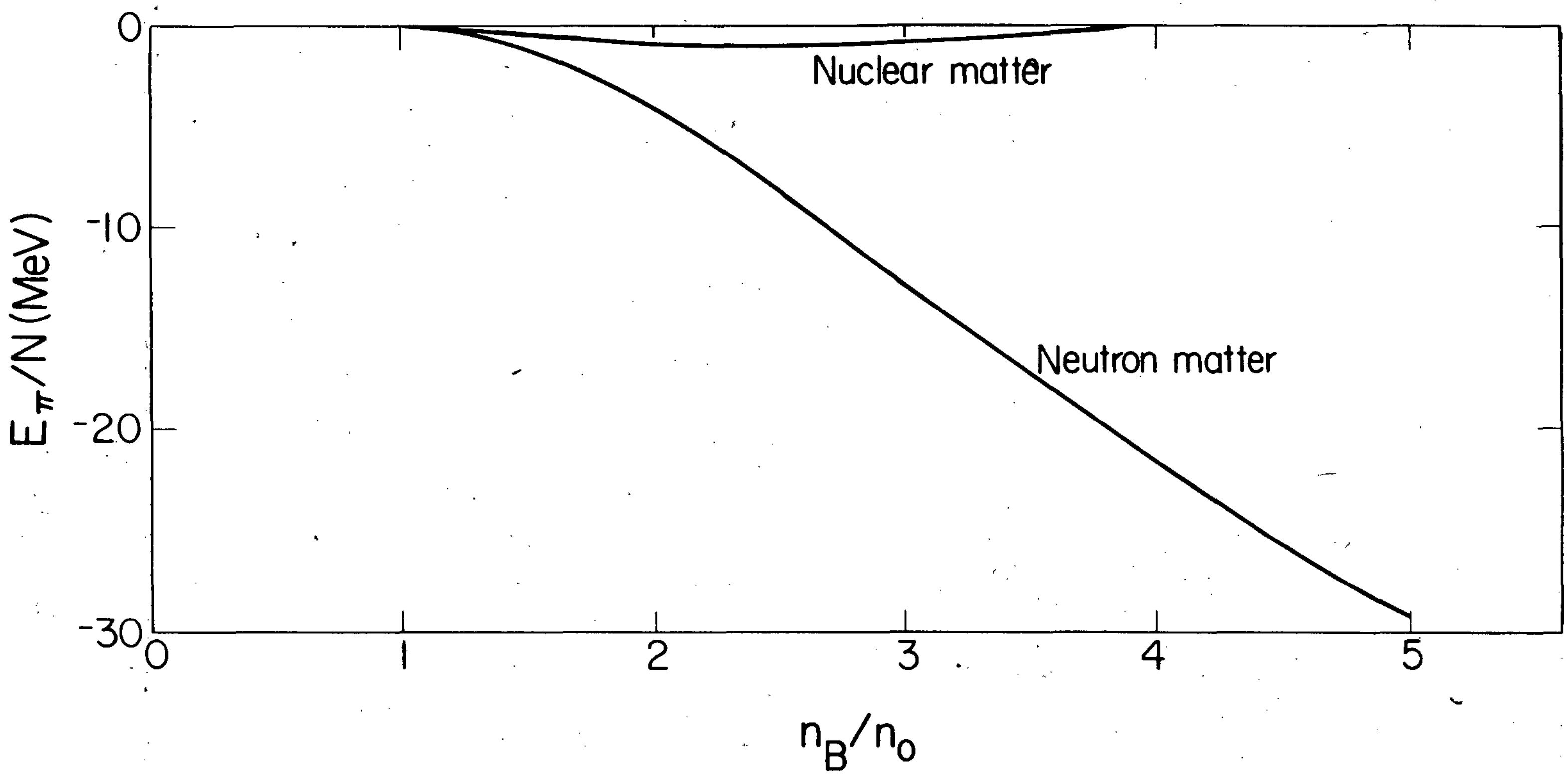


Fig. 3b. The same as fig. 2b for the stiff equation of state.

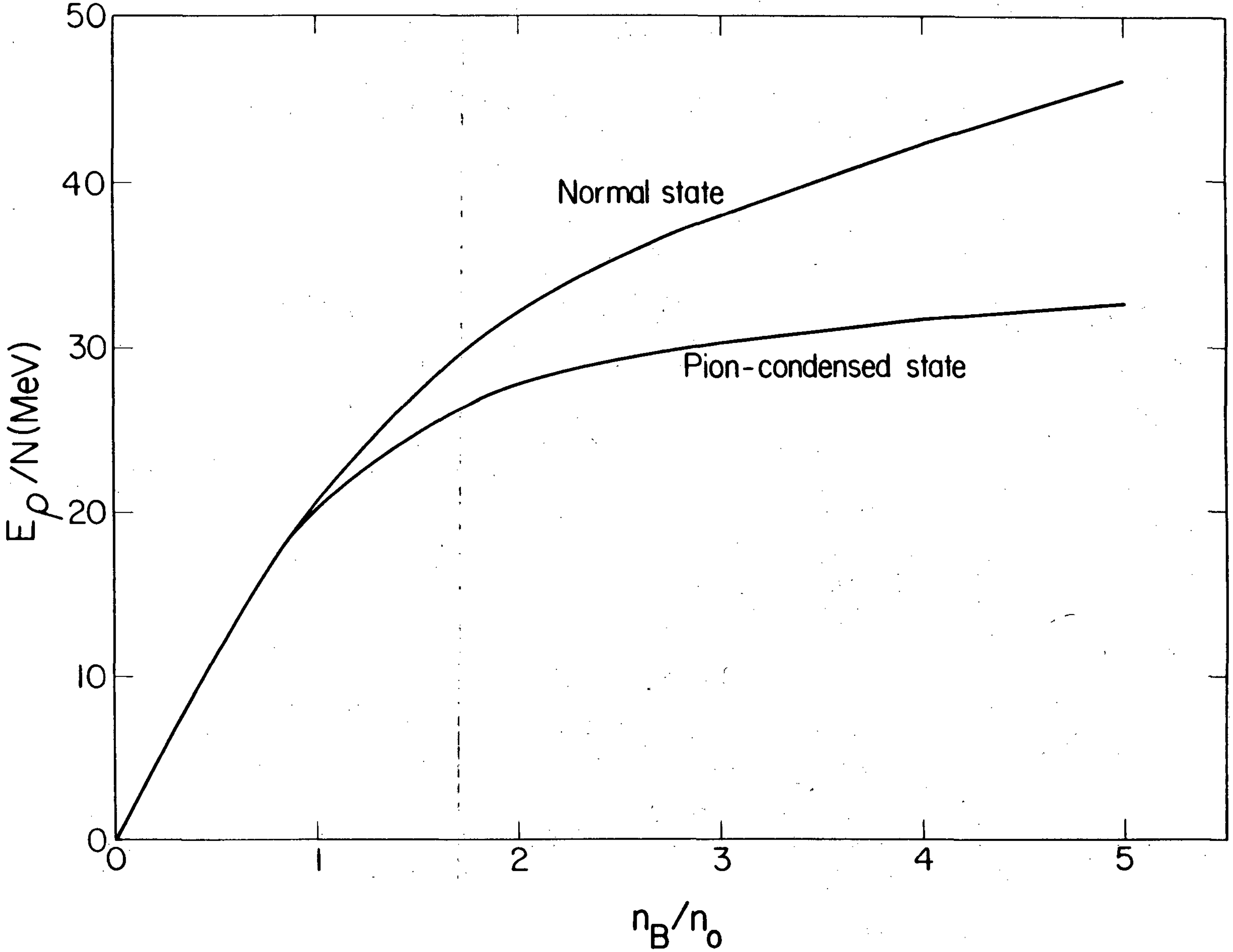


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Fig. 3c. The same as fig. 2c for the stiff equation of state.

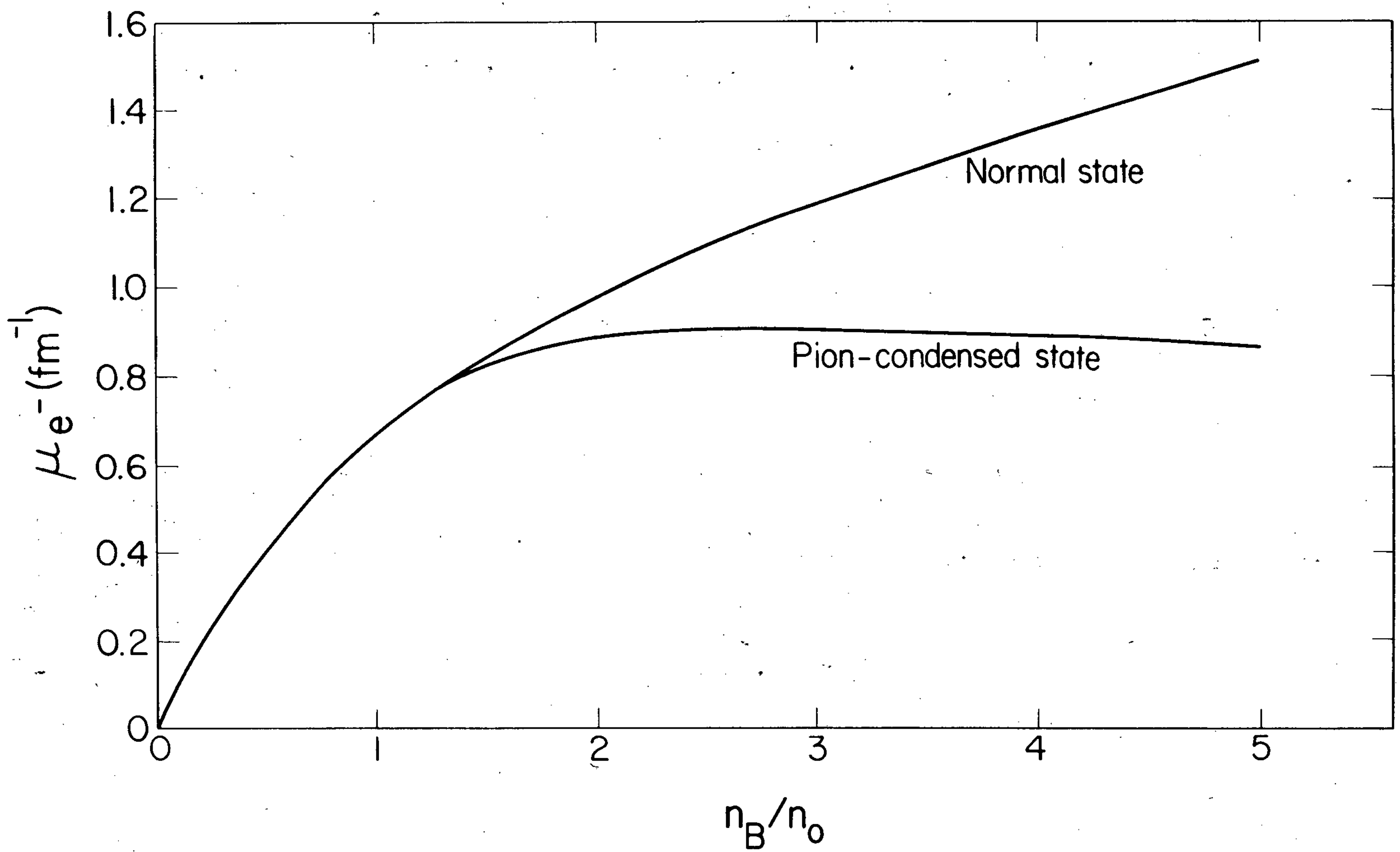


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Fig. 4a. The chemical potential $\mu_e$ in neutron matter as a function of baryon density $n_B$ for the stiff equation of state both in the normal and pion-condensed phase.

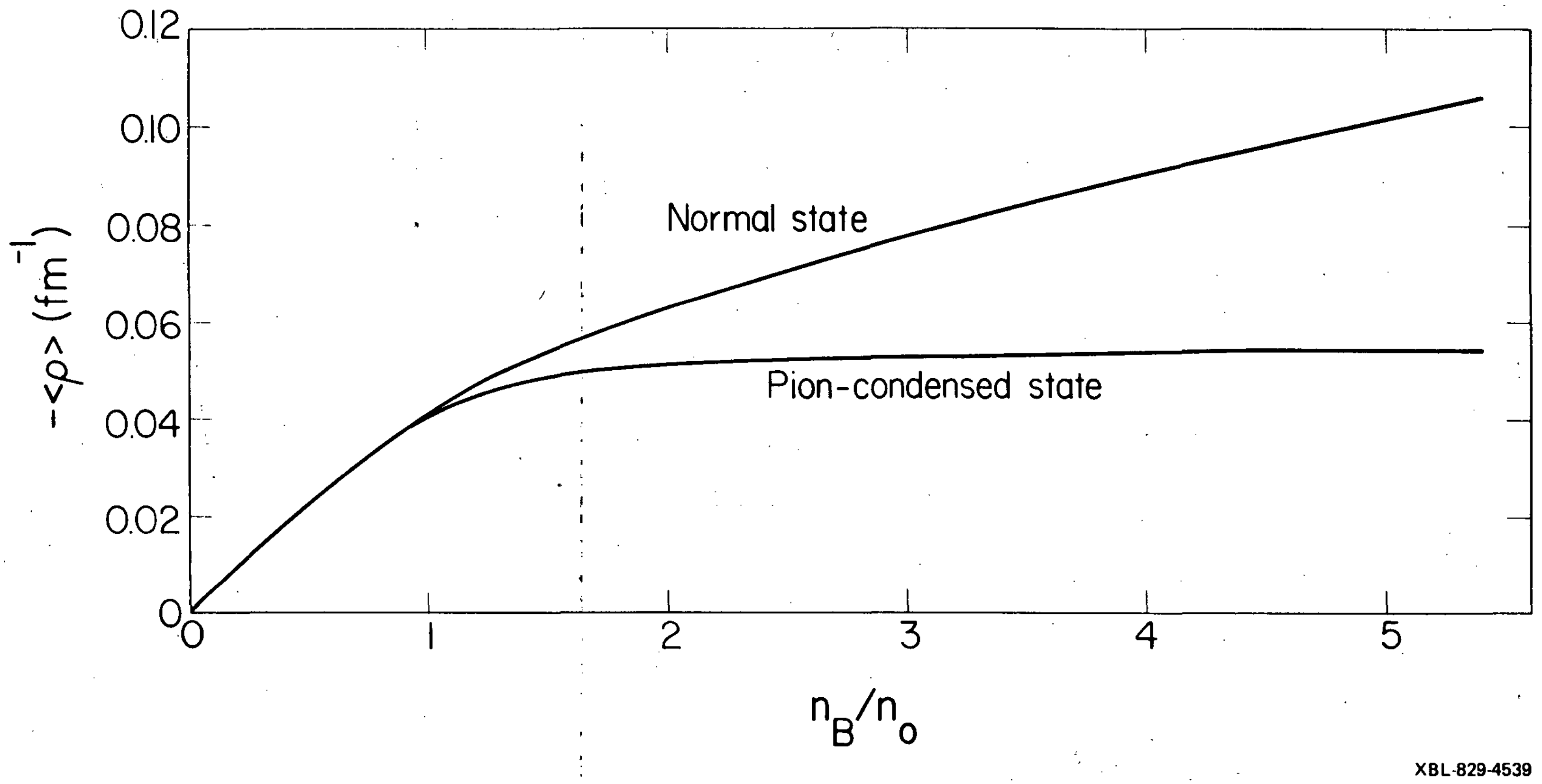


Fig. 4b. The $\rho$-field as a function of baryon density $n_B$ in neutron matter for the stiff equation of state, both in the normal and pion-condensed phase.

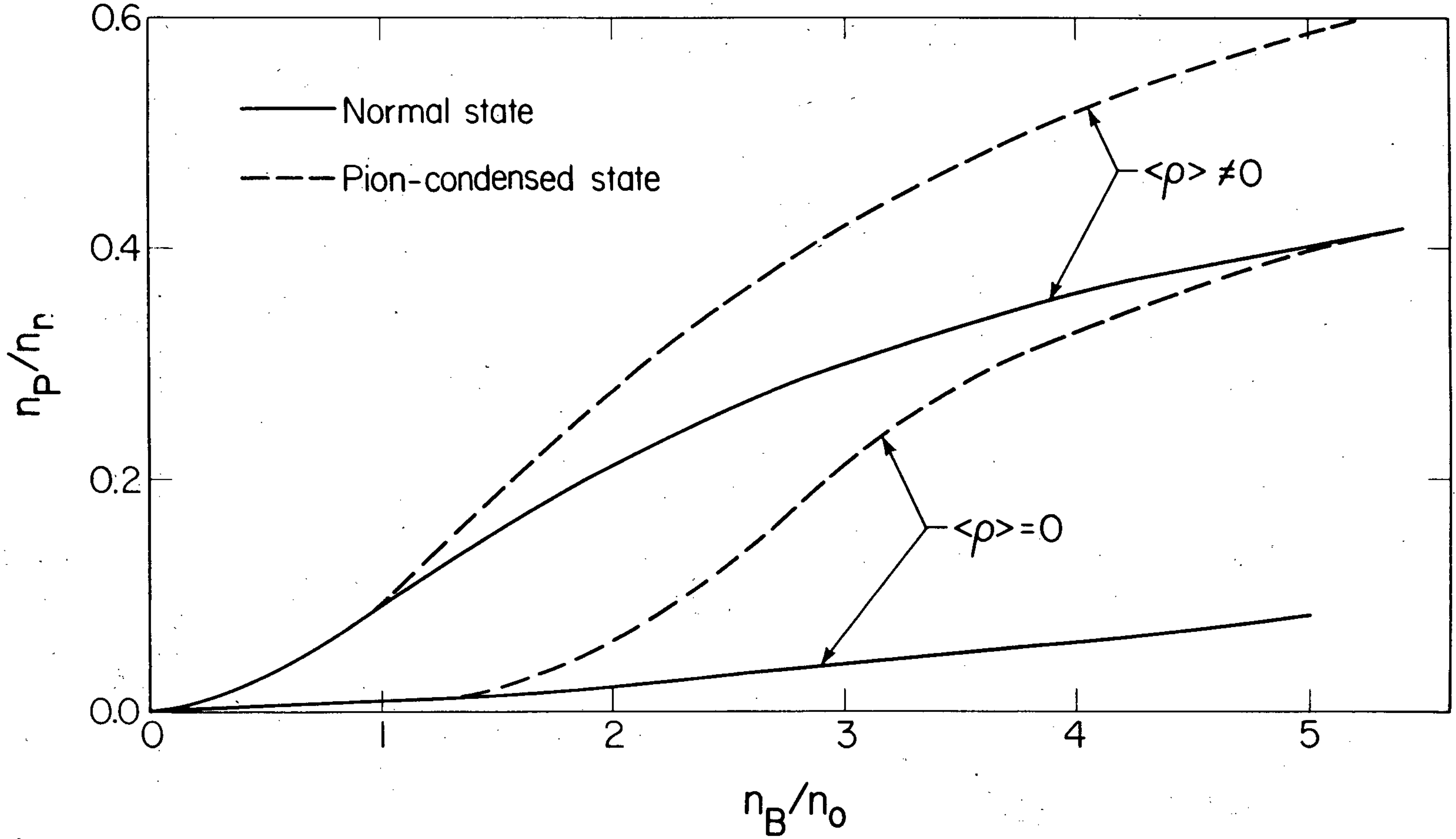


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Fig. 5. The proton/neutron ratio $n_p/n_n$ in neutron matter as a function of the baryon density $n_B$ for the stiff equation of state. The full curves correspond to the normal state, the dashed ones to the pion-condensed state. The results for $\rho = 0$ and $\rho \neq 0$ are shown.

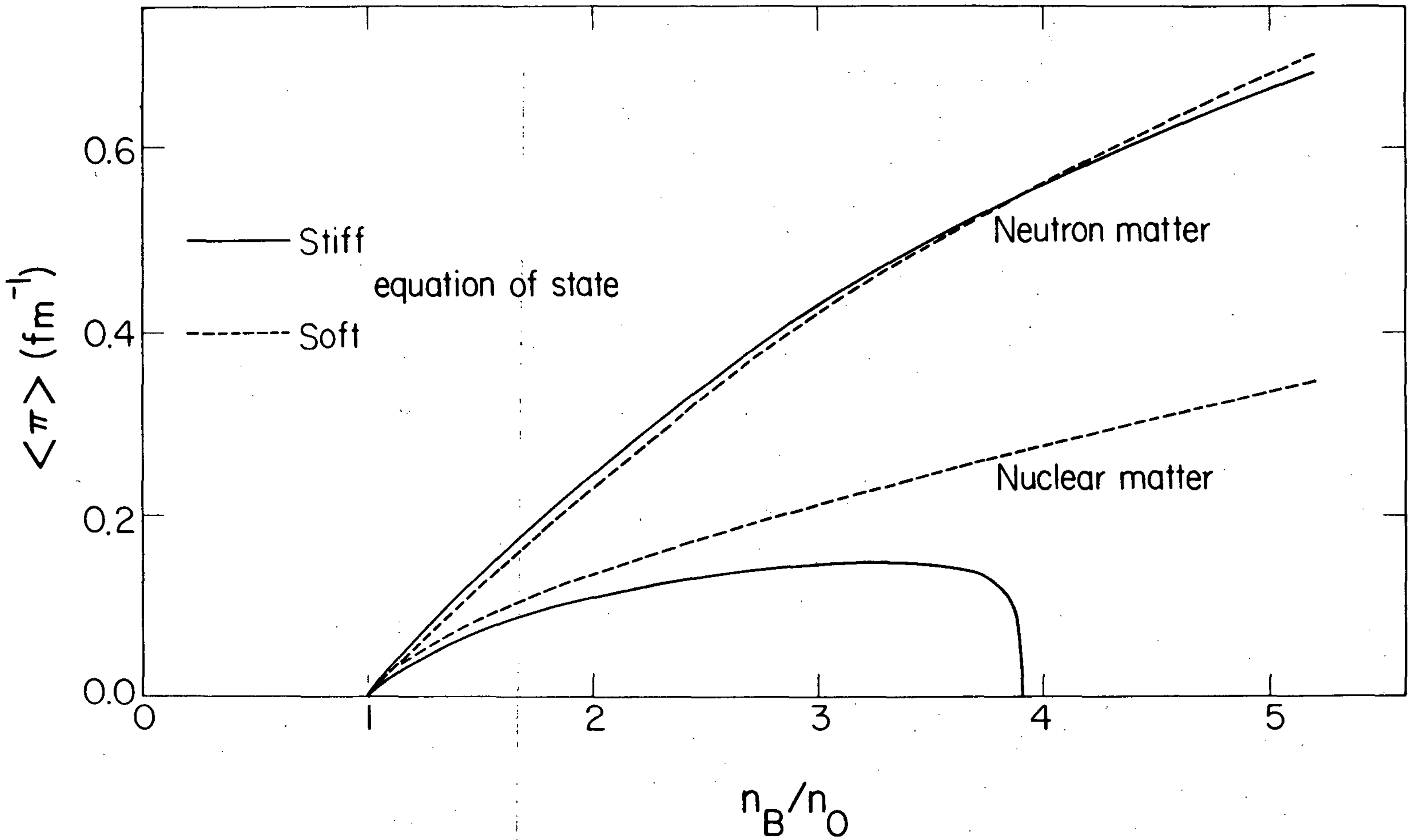


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Fig. 6. The pion field $\langle\pi\rangle$ in nuclear and neutron matter as a function of baryon density $n_B$. The full curves correspond to the stiff and the dashed ones to the soft equation of state.

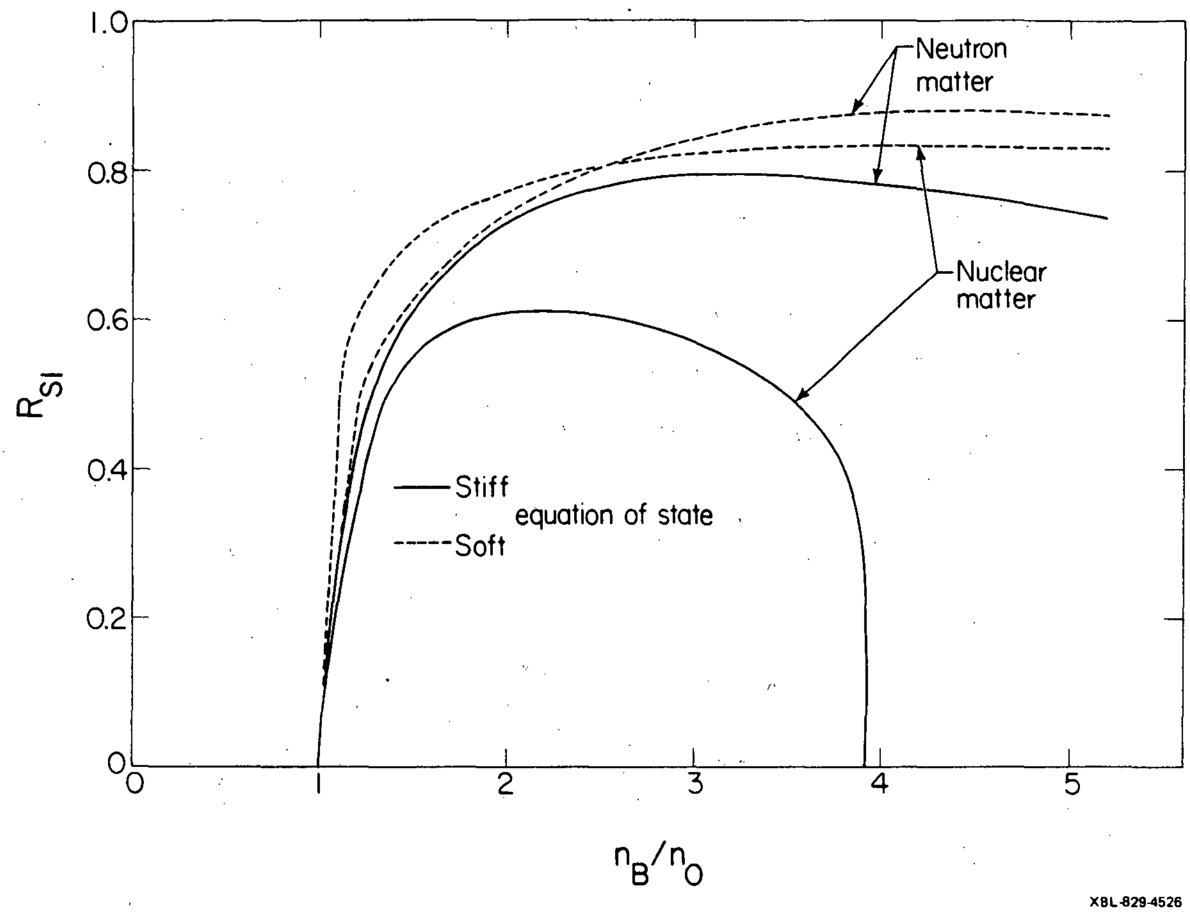


Fig. 7. The spin-isospin oscillation $R_{SI}$ for nuclear and neutron matter as a function of the baryon density $n_B$. The full curves depict the stiff and the dashed curves the soft equation of state.

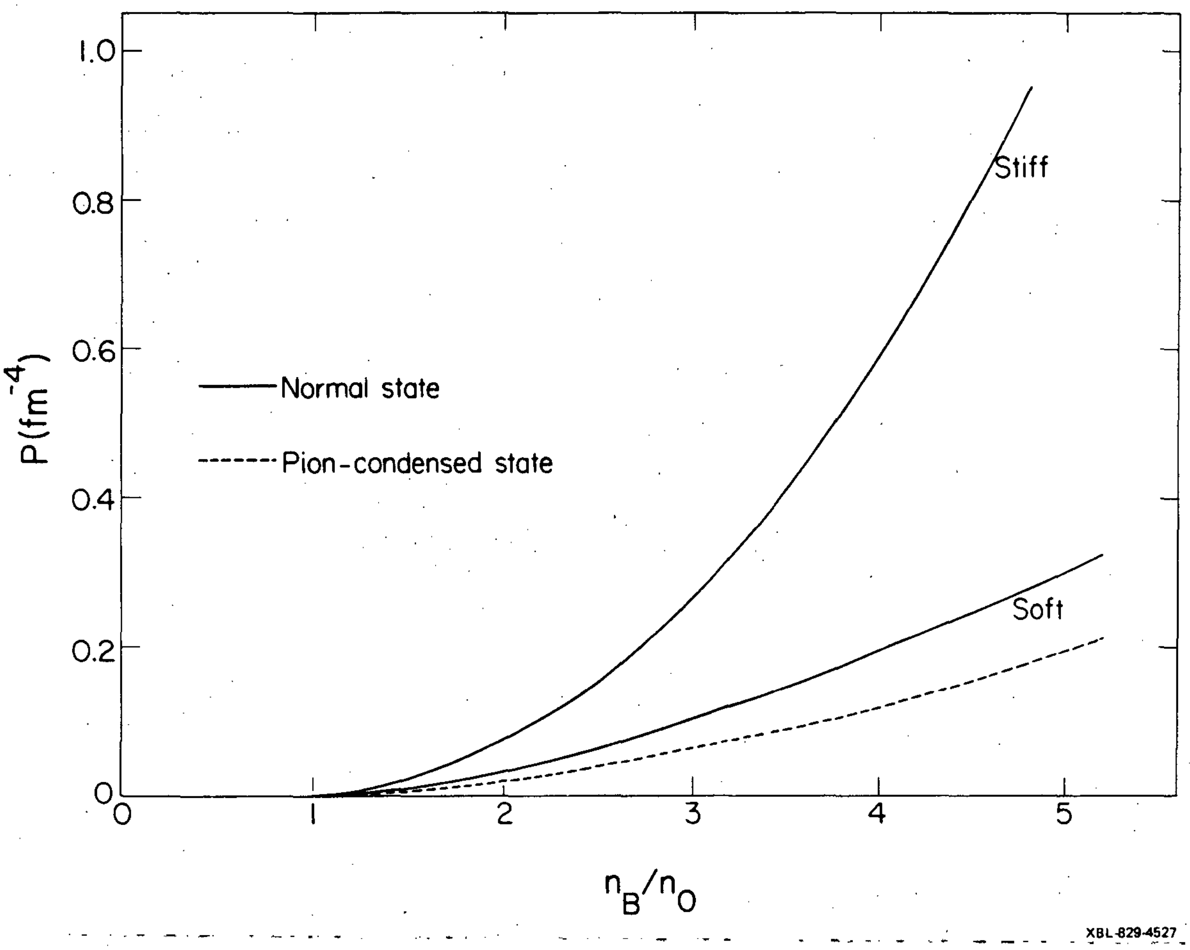


Fig. 8. The pressure as a function of the baryon density $n_B$ for nuclear matter and both equations of state. The full curves correspond to the normal state, the dashed one to pion-condensed state. For the stiff versions both curves are nearly identical.

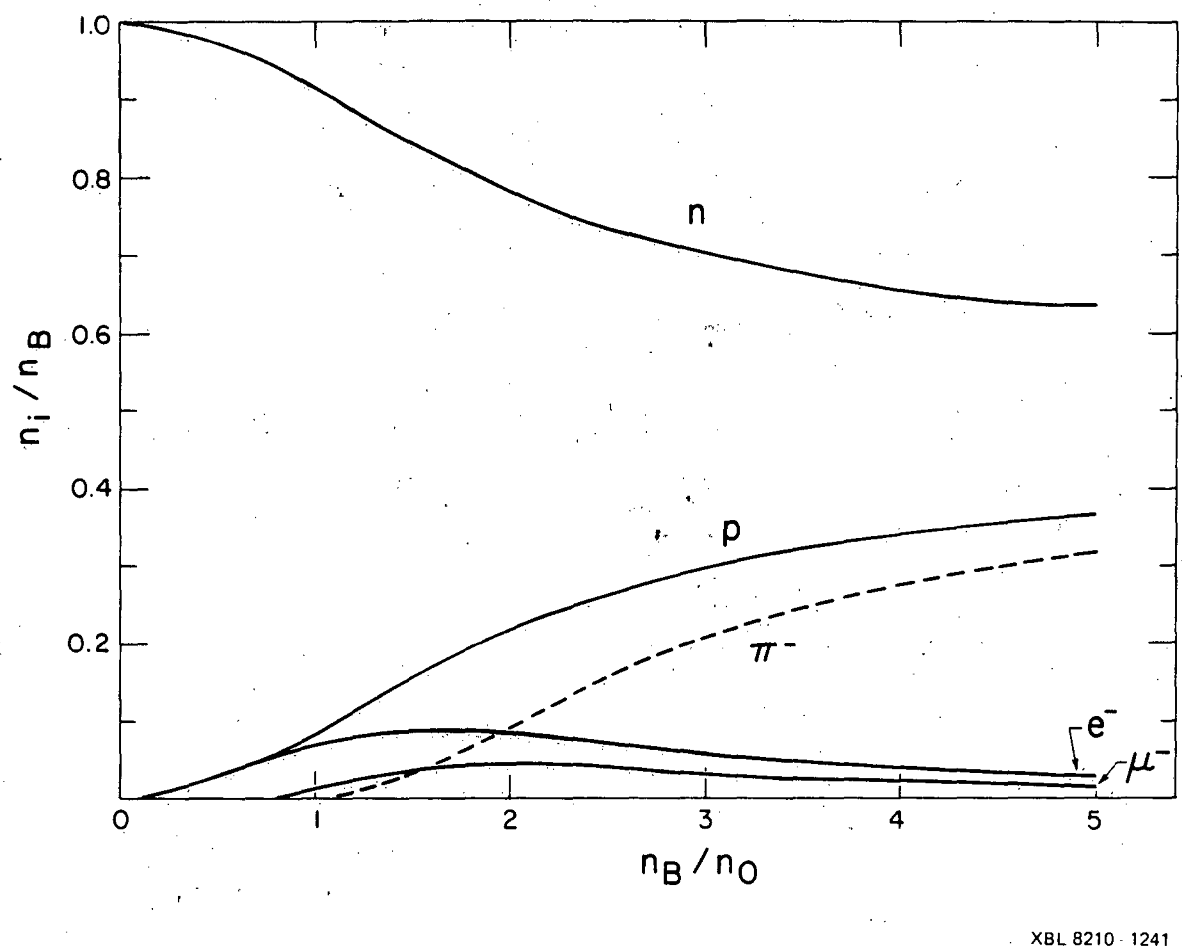


Fig.9 . The particle densities for protons, neutrons, electrons, muons, and pions in units of the baryon density for neutron matter as a function of the baryon density for the stiff equations of state.

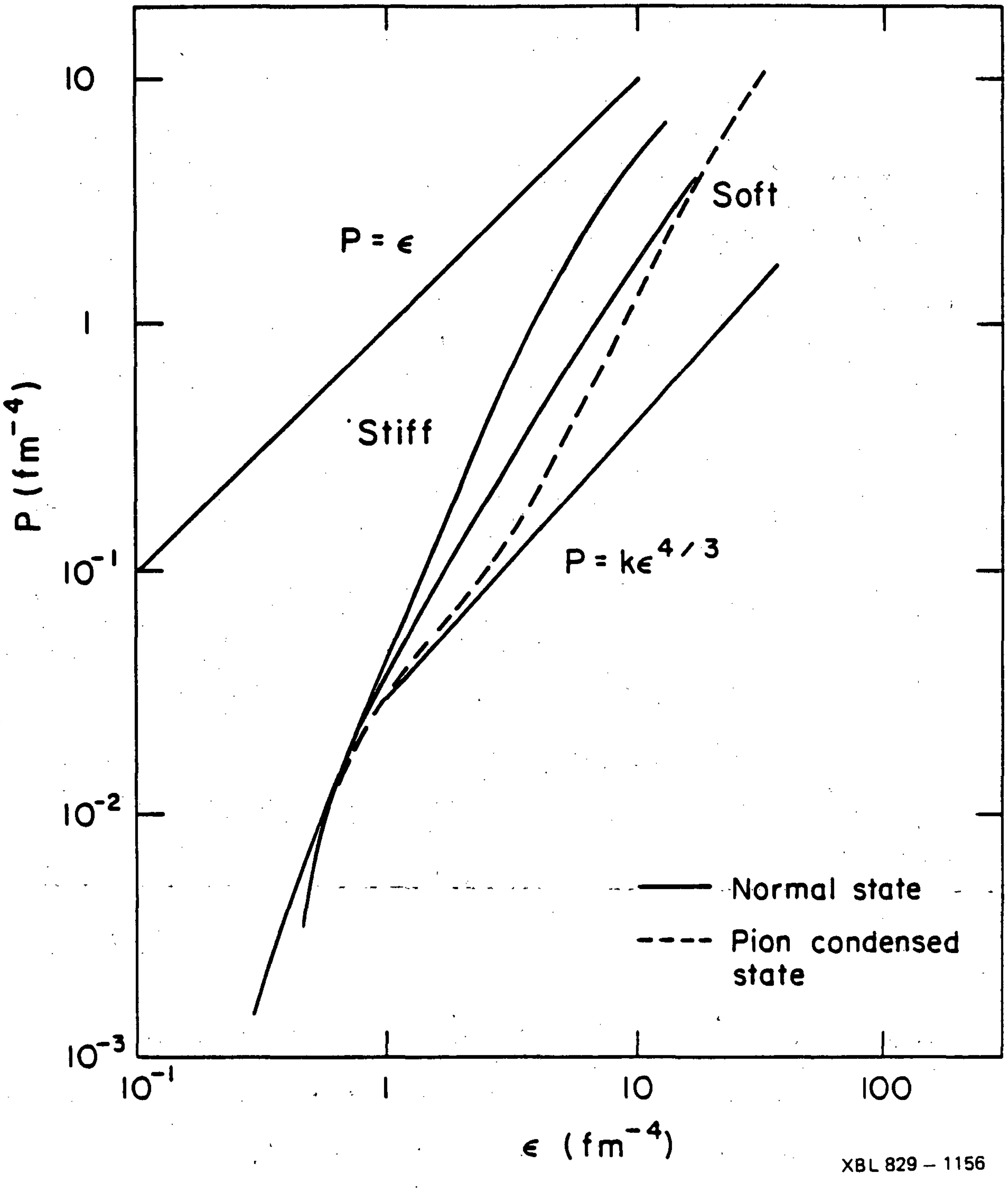


Fig. 10. The pressure as a function of energy density $\varepsilon$ for neutron matter and both equations of state. The full curves correspond to the normal state, the dashed one to the pion-condensed state. For the stiff version, both are very similar. The straight curve $P = \varepsilon$ shows the causality limit. The other straight full line corresponds to the relation $\log P = 4/3 \log \varepsilon + \text{const.}$

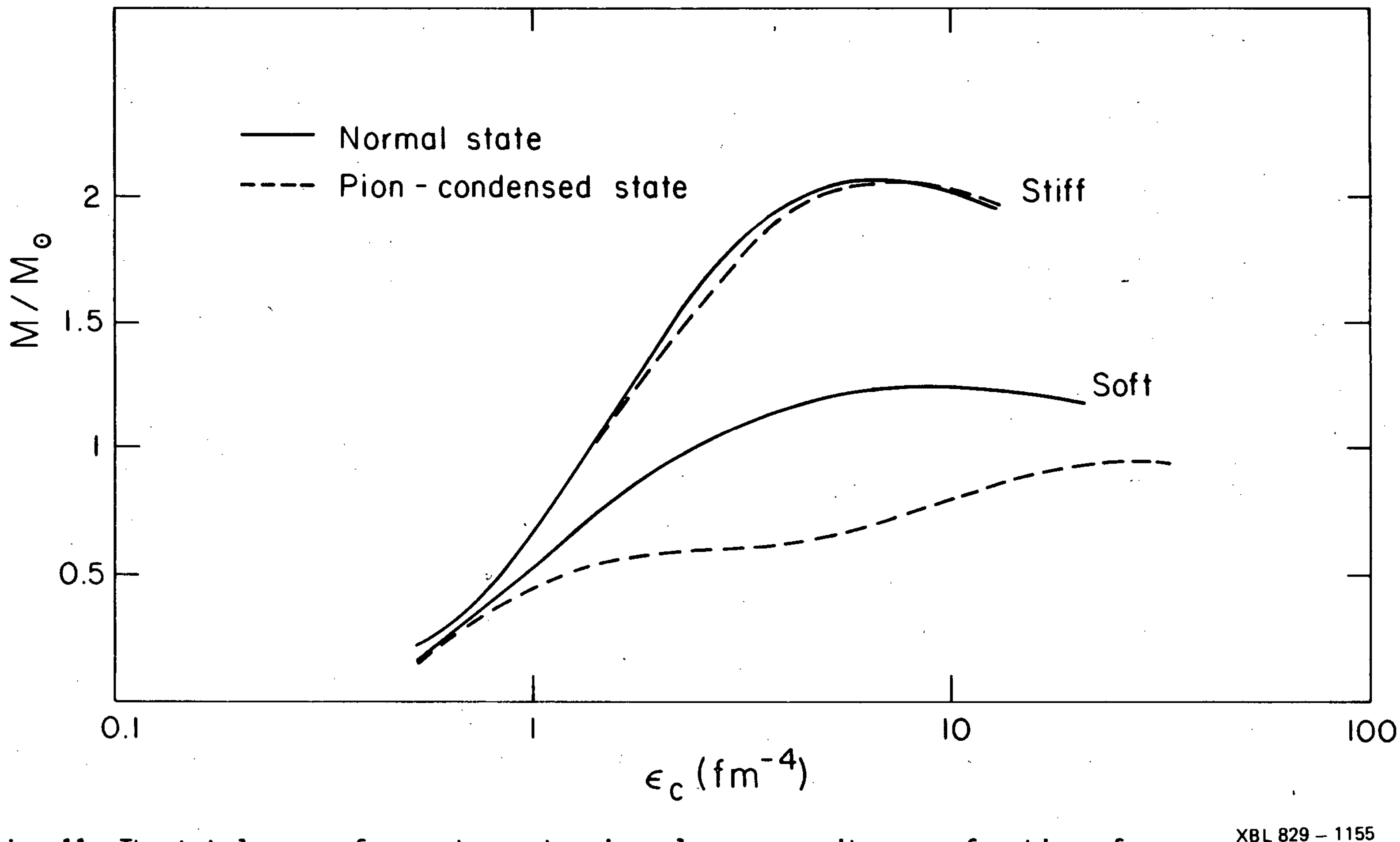


Fig. 11. The total mass of a neutron star in solar mass units as a function of the central energy density $\epsilon_C$ for both versions of the equation of state and for the normal and pion-condensed state. The relation between baryon density and energy density depends on the equation of state and is not given by a unique factor.

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