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Reply to Comment by J.D. Bredehoeft and R.L. Cooley on, "A NOTE ON THE MEANING OF STORAGE COEFFICIENT"

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In their comment on our paper, "A Note on the Meaning of Storage Coefficient" [Narasimhan and Kanehiro, 1980], Bredehoeft and Cooley raise two issues: 1. that the transient flow equation includes, in general, drained as well as undrained conditions of loading and that it has been so demonstrated by Jacob; 2. that "fluid mass capacity", as defined by us is equivalent to Jacob's derivation and that our derivation differs from that of Jacob merely in style.

Of the two, the second issue is easy to resolve and will be considered first. We agree with Bredehoeft and Cooley that whatever the styles of derivation, the final expressions derived for specific storage will be fully equivalent or even identical, as long as one is consistent with conventions. In our paper we merely set out to assemble, in a single place, the different commonly used conventions so that comparisons can be readily made. The term "compressibility",as applied to a porous material, is expressed in the literature in three different ways according to three different conventions. Confusions can easily result when one does not carefully state the particular convention one is following. For example, the petroleum engineer uses "total compressibility" or "pore volume compressibility", the geotechnical engineer uses "coefficient of compressibility" and the hydrogeologist uses "compressibility". A clearer understanding of each other's convention could lead to improved dialogue among these groups.

Following their equation (16), Bredehoeft and Cooley also state, "..... ΔM is not solely a function of fluid pressure. More properly, $\Delta M = \Delta M(\rho[p], \sigma_e [p, \sigma_T])$". In this connection, we would like to reiterate our statement, ".....Before discussing the relative merits....., the assumptions ..... should be restated. First, and most important, it is assumed that the total stress remains constant. This implies that the above only applies directly to problems where fluid is injected or drained" [Narasimhan and Kanehiro, 1980, p. 428]. In view of this, our statement specifically states that ΔM is a function of fluid pressure, p, at a constant total stress. As we stated in the concluding paragraph of our paper, ".....it is obvious that the coefficient quantifying change in storage will vary as a function of stress. Such a dependence of S_s or S on σ' (effective stress) cannot be easily incorporated into a differential equation in which hydraulic head is the dependent variable" [Narasimhan and Kanehiro, 1980, p. 428]. We are aware that ΔM can be influenced by change in total stress. However, treating this dependence is not merely a question of algebraic manipulations as indicated in Bredehoeft and Cooley's equations (18) through (23). Herein indeed lies the fundamental cause of the first issue raised by them in regard to undrained conditions of loading. We shall show in the following paragraphs that Jacob's derivation, as incorporated in equation 12 of Bredehoeft and Cooley, has a physical contradiction built into it.

Let us now consider the first issue raised by Bredehoeft and Cooley. We now transpose and rewrite their equation 12 in the following form:

$$-\nabla.\rho\vec{v} + \rho\alpha \frac{\partial\sigma_T}{\partial t} = \varepsilon\rho \left(\beta + \frac{\alpha}{\varepsilon}\right) \frac{\partial p}{\partial t} \qquad (1)$$

This is the conventional form in which transient groundwater flow in a saturated medium is expressed. The left hand side represents the time-rate

of accumulation of mass of water per unit volume in an infinitesimal volume element. The right hand side expresses the same accumulation rate as an equivalent time-rate of change of average fluid pressure over the element. The term, $-\nabla.\rho\vec{v}$ on the left hand side represents the time-rate of mass accumulation controlled by the equation of motion, while the term, $\rho\alpha\partial\sigma_T/\partial t$ is like a "source"or a "sink" term and is a forcing function. The term $\partial p/\partial t$ on the right hand side, therefore, consists of two components, one due to the equation of motion and the other due to the source term. Thus, we may rewrite (1) as,

$$-\nabla.\rho\vec{v} + \rho\alpha\,\partial\sigma_T/\partial t$$

$$= \varepsilon\rho[\beta + \frac{\alpha}{\varepsilon}]\frac{\partial p}{\partial t}\Bigg|_{\substack{\text{equation}\\ \text{of}\\ \text{motion}}} + \varepsilon\rho[\beta + \frac{\alpha}{\varepsilon}]\frac{\partial p}{\partial t}\Bigg|_{\Delta\sigma_T} \qquad (2)$$

Now, since $\partial\sigma_T/\partial t$ is a forcing function, the second term on the left hand side is a priori prescribed. This forcing function, as a consequence of equation 15 of Bredehoeft and Cooley, namely,

$$\text{Tidal Efficiency, T.E.} = \frac{\Delta p}{\Delta\sigma_T} = \frac{\rho\alpha}{\rho\varepsilon[\beta + \frac{\alpha}{\varepsilon}]} \qquad (3)$$

gives rise to the component of $\partial p/\partial t$ due to total stress change $(\partial p/\partial t\Big|_{\Delta\sigma_T})$ on the right hand side of (2).

Now, if we are only interested in following the history of pore pressure changes over the volume element, then, (2) looks eminently adequate and logical.

However, a detailed evaluation of (2) leads to some serious questions. Note that the second term on the left hand side of (2) has dimensions of mass (of water) per unit volume per unit time. It is in the form of a conventional source or a sink term, and represents the addition of water to or abstraction of water from the volume element. In other words, one achieves the desired effect of generating the same magnitude of change in pore pressure within the volume element as caused by the total stress change, $\partial\sigma_T/\partial t$, by injecting an appropriate quantity of water into the element. Thus, although the magnitude of pore pressure change is exactly what one would desire, the physics of the phenomenon of interest is drastically changed. For, the pore pressure change caused by an external stress change is <u>unaccompanied</u> by any change in storage. Indeed, looking at (2), if $\partial\sigma_T/\partial t > 0$ (that is, an increase in compressive total stress), then $\partial p/\partial t \Big|_{\Delta\sigma_T} > 0$. Thus the presence of $\rho\alpha\partial\sigma_T/\partial t$ on the left hand side of (2) is equivalent to injecting an appropriate quantity of water into the system. In order to accomodate this injected quantity of water, <u>both</u> the porosity of the medium and the density of water would have to <u>increase</u> On the contrary, when a porous medium responds in an undrained fashion to an increase in the external stress, <u>porosity decreases</u> and density of water increases. It is clear, therefore, that equation 12 of Bredehoeft and Cooley, which is attributed to Jacob (1950), is physically inappropriate, although it appears superficially reasonable.

The expression for Tidal Efficiency [equation 15 of Bredehoeft and Cooley] has to be appreciated within proper constraints. This expression (3) simply states that the ratio of the pore pressure generated under undrained conditions to the causative change in total stress is equal to the ratio of the compressibility

of the porous medium to the specific storage. It is straightforward, then, to state

$$\Delta p \Big|_{\Delta\sigma_T} = (T.E.)\ \Delta\sigma_T \tag{4}$$

and express the pore pressure generated as a fraction of the change in total stress. But, if one were to cross multiply the terms in (4) and state,

$$\rho\varepsilon[\beta + \frac{\alpha}{\varepsilon}]\Delta p = \rho\alpha\ \Delta\sigma_T \tag{5}$$

then, one equates masses of water. In this case one has to exercise care as to the use of this equivalence in a particular physical situation. It is the implied use of this equivalence (5) in Jacob's derivation that has led to the physical inconsistency of generating pore pressure by injecting mass into the system. In order to generate pore pressure without the addition of mass (undrained response), one should actually use (4). But, since (2) is a mass balance equation, there arises a problem of dimensions in incorporating (4) into (2). To overcome this problem, one may divide through by $\varepsilon\rho[\beta + \alpha/\varepsilon]$ and express (2) as,

$$\frac{-\nabla.\rho\vec{v}}{\varepsilon\rho[\beta + \frac{\alpha}{\varepsilon}]} + (T.E)\ \frac{\partial\sigma_T}{\partial t} = \frac{\partial p}{\partial t}\Bigg|_{\substack{\text{equation}\\ \text{of}\\ \text{motion}}} + \frac{\partial p}{\partial t}\Bigg|_{\Delta\sigma_T} \tag{6}$$

In (6), we have simply added a <u>known</u> quantity, $[(T.E.)\partial\sigma_T/\partial t]$, to both sides of the equation. This trivial operation does not necessarily imply that the physics of undrained response and that of transient flow are related to each other. Furthermore, one has to exercise care to assure that the second term on the left hand side of (6) does not spuriously contribute to mass balance.

We shall now propose an alternate way in which the pore pressure generated due to undrained loading may be incorporated into the governing equation. For this purpose we shall express the equation for a small time step Δt, extending from an initial instant t_o to a final instant, $t_o + \Delta t$. To focus attention on the issues on hand, we will neglect consideration of sources and boundary conditions.

The Continuity Equation:

$$- \nabla.\rho\vec{v} = \rho\varepsilon[\beta + \frac{\alpha}{\varepsilon}] \frac{\Delta p}{\Delta t} \tag{7}$$

Conditions at the beginning of Δt (Initial Conditions):

$$p(t_o) = p_o \tag{8}$$

$$\sigma_T(t_o) = \sigma_{T,o} \tag{9}$$

Conditions at the end of Δt:

$$p\ (t_o + \Delta t) = p_o + \frac{\Delta p}{\Delta t}\Delta t + \frac{\Delta\sigma_T}{\Delta t}(T.E.)\ \Delta t \tag{10}$$

$$\sigma_T(t_o + \Delta t) = \sigma_{T,o} + \frac{\Delta\sigma_T}{\Delta t}\ \Delta t \tag{11}$$

What we have essentially done above is that we have incorporated the generated pore pressure into the statement of the transient problem without adding mass to the system. In order to achieve this we need to have additional information about the manner in which total stress changes in time over the volume element. When T.E. is less than unity and $\Delta\sigma_T > 0$, then, there will be a slight decrease in porosity and a slight increase in water density. These changes are automatically accounted for by the increase in pore pressure and effective stress at $[t_o + \Delta t]$. As shown by Narasimhan (1981), one could account for the dissipation of part of the pore pressure generated during Δt. But this is merely a detail not critical to our discussion. If, instead of using (10), we incorporate the change in total stress into the differential equation as in (1) or (2), then, for $\Delta\sigma_T > 0$, we should have an increase in porosity, which is inconsistent with an actual increase in effective stress for T.E. <1 and $\Delta\sigma_T > 0$.

The very fact that S_s is defined as, "..... the volume of water that a unit volume of the aquifer releases from storage because of expansion of water and compression of the grains under a unit decline in average head within the unit volume" [Domenico, 1972 p. 220], implies that, by definition, S_s relates to the drained phenomenon. It states that change in storage implies change in pore pressure and vice versa. If, indeed, there is change in pore pressure without an accompanying change in storage then S_s is clearly undefined for this process. Furthermore, it is only under drained conditions that an increase in the volume of water due to its expansion and a decrease in the volume of the voids can be added together to obtain the total volume of water drained. In the undrained case, the void volume and the volume of water either both increase or both decrease and the mass of water within the element remains unchanged.

In following Jacob's derivation verbatim Bredehoeft and Cooley have not recognized the fundamental differences that exist between drained and undrained conditions of loading. In our paper we attempted focus attention on these important differences in the form of a table [Table 1, Narasimhan and Kanehiro,1980].

The transient groundwater flow equation, by itself, describes the changes in pore fluid pressure caused by perturbations originating within the pores. That is, due to the movement of water caused by existing hydraulic gradients and due to injection or withdrawal of fluids, pore pressures are caused to be changed. In turn, the pore pressure changes cause effective stresses to change. The loading is "endogeneous" in nature. In contrast, undrained loading relates to changes in the loads acting on the boundaries of the groundwater system. These external load changes lead to changes in pore pressure with the amount of the mass of water in the element remaining unchanged. Undrained loading is therefore "exogenous" in nature. Once generated, these pore pressures may dissipate as governed by the transient groundwater equation provided that the magnitude of pore pressure generated is spatially variable. However, the mechanism of generation itself is a phenomenon different from the drained phenomenon. It is for this reason that the term $\Delta p \Big|_{\Delta\sigma_T}$ is included in the final condition (10) rather in the body of the main equation 7.

We believe that we attempted, in our paper, to focus attention on an important physical issue rather than confusing the same as Bredehoeft and Cooley suggest.

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