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NONLINEAR MEAN FIELD THEORY FOR NUCLEAR MATTER AND SURFACE

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ABSTRACT

Nuclear matter properties are studied in a nonlinear relativistic mean field theory. By determining the parameters of the model from bulk properties of symmetric nuclear matter, we successfully account for other key properties of nuclei, such as the compression modulus, spin-orbit coupling, surface energy, and diffuseness of the nuclear surface. The energy dependence of the nucleon-nucleus optical model is predicted reasonably well. In this work, we stress the nonrelativistic limit of the theory and show that most of the above results can be obtained, to a good approximation, analytically. The strength of the required nonlinear term is consistent with that derived using a new version of the chiral mean field theory in which the vector mass as well as the nucleon mass is generated by the sigma field.

I. INTRODUCTION

It has been known for some time that important gross properties of nuclear matter, such as saturation, can be concisely described by a relativistic field theory model.[1)]

That a relativistic theory, with static interactions, can also account for the spin-orbit coupling and the energy dependence of the optical potential was observed by DUERR and TELLER.[2)] Subsequently, MILLER and GREEN[3)] showed that such a model can also describe the matter distribution in closed shell nuclei. The static Yukawa Interaction theory was cast into a field theory problem by WALECKA.[4)]

The difficulty with the pure Yukawa type relativistic theory considered by MILLER and GREEN, or for that matter, by WALECKA, is that in the Hartree approximation, where most of the relevant and interesting calculations have been done, the model does not have a saturating nonrelativistic limit. That is, in a Yukawa type field theory, saturation is a purely relativistic phenomenon.

On the other hand, a nonrelativistic approach to the study of nuclear phenomena has been quite successful. Of course relativistic effects are important for some nuclear properties, such as spin-orbit splitting. Still, it seems unreasonable to ascribe such a fundamental property as nuclear saturation to purely relativistic effects.

The relativistic field theory model of nuclear matter saturation proposed by SCHIFF[1)] and extended by BOGUTA and BODMER[5)] has as its basic ingredient not purely Yukawa type forces, but large many-body interactions. They are decisive in obtaining nuclear matter saturation. The disadvantage of this idea is that not only the many-body interactions are large, but the model predicts a wrong spin-orbit splitting.

Recently, one of us (J.B.)[6] considered a field theory model that retains the most important features of both of the above approaches. The nonlinear self interactions of the scalar field are important, but not dominant, and the model yields a non-relativistic saturating limit. In the present paper we will use this field theory model to relate some general properties of symmetric (N = Z) nuclear matter in the nonrelativistic limit of the model.

The advantage of having a nonrelativistic limit of the theory is twofold. First, we can assess the importance of relativistic effects. Second, the nonrelativistic approximation yields analytic expressions for the average surface properties such as diffuseness and surface energy, in terms of the elementary couplings of the model. Furthermore, the role of the nonlinear self-interactions of the scalar field σ can be explicitly studied and their effects on the binding energy estimated. By using the saturation condition and the field equations, it is possible to determine the necessary scalar and vector coupling strengths and the contribution of the self- interactions to the binding energy of nuclear matter as an explicit function of the average kinetic energy and the effective mass m^*.

In this way, it is easy to investigate the implied relations between such key properties as the nuclear matter compression modulus, the surface energy, effective mass, spin-orbit splitting, and the energy dependence of the nucleon-nucleus optical potential.

Although these different quantities can be well correlated using a particular strength of the nonlinear coupling, the existence of the latter does introduce into the theory an extra parameter, i.e. it makes it somewhat more phenomenological that the original version. However, one of us (J.B.) recently has suggested a new version of the chiral model[7] in which not only the nucleon mass but also the vector meson mass is generated by the σ field. This

assumption does not introduce any new parameters, but it leads to almost the same equations as the ones discussed in the present paper, with the strength of the nonlinear coupling no longer arbitrary, and in good agreement with the values derived here.

II. NONLINEAR FIELD THEORY AND ITS APPROXIMATION.

The properties of symmetric nuclear matter can be described in terms of an isoscalar field σ and and isovector field ω_μ. The most general renormalizable Lagrangian is:

$$L = -\bar{\psi}\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + M\right)\psi - \frac{1}{2}\left(\frac{\partial\sigma}{\partial x_\mu}\right)^2 - V(\sigma) \,..$$

$$-\frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \frac{1}{2} M_V^2 \omega_\mu \omega_\mu + i\, g_V \bar{\psi} \gamma_\mu \psi \omega_\mu - g_S \bar{\psi} \psi \phi \tag{1}$$

where $$V(\sigma) = \frac{M_S^2 \sigma^2}{2} + \frac{b\sigma^3}{3} + \frac{c\sigma^4}{4} \tag{2}$$

$$F_{\mu\nu} = \frac{\partial\omega_\mu}{\partial x_\nu} - \frac{\partial\omega_\nu}{\partial x_\mu} \tag{3}$$

and M and M_S denote, respectively, nucleon mass and scalar meson mass.

Strictly speaking, the coefficient c must be positive, so that the theory will have a lower bound in energy. However, for densities less than about 5 ρ_0, the quartic contribution is small, and as a matter of practicality, it can be neglected in subsequent considerations of normal nuclear matter.

From the Lagrangian in Eq (1), it is seen that there are three distinct contributions to the energy density:

1.) Energy of the scalar field, which can be written as:

$$W_S = \frac{M_S^2 \sigma^2}{2\rho_V}\left(1 - \frac{\alpha g_S \sigma}{M}\right) \tag{4}$$

where σ is the amplitude of the scalar field $\sigma = \langle\underset{\sim}{\sigma}\rangle$ in the ground state.

2.) Energy of the Vector Field.

$$W_V = \frac{1}{2} (g_V^2/M_V^2) \rho_V \tag{5}$$

where g_V is the vector coupling constant and M_V is the mass of the vector meson. The exchange of the vector meson leads to the conventional short-range repulsive Yukawa interaction.

$$V_V = (g_V^2/4\pi) \; e^{-M_V r} / r \tag{6}$$

3.) The interaction energy of the nucleons with the scalar field.

$$W_I = (M^{*2} + \langle k^2 \rangle)^{1/2} - M \tag{7}$$

where $\langle k^2 \rangle$ denotes the average value of the square of the momentum due to Fermi motion. The effective mass M^* is related to the scalar field by

$$M^* = M + g_S \sigma . \tag{8}$$

It is convenient to express the effective mass in units of the free nucleon mass.

Thus we define:

$$m^* = M^*/M = (1 + g_S\sigma/M) \tag{9}$$

The single particle energy e(k) is given by:

$$e(k) = g_V\omega + (M^{*2} + k^2)^{1/2} \tag{10}$$

Solution of the field equations for ω gives:

$$\omega = (g_V/M_V^2) \rho_V \tag{11}$$

In the nonrelativistic approximation we find for the single particle energy (minus the nucleon rest energy):

$$e(\rho,k) = (g_V^2/M_V^2)\,\rho_V + (M^* - M) + (M/M^*)\,T(k) \tag{12}$$

where T(k) is the nonrelativistic kinetic energy.

This M^* is just the <u>conventional</u> effective mass defined by the momentum dependence of the nucleon-nucleus optical potential:

$$\frac{M}{M^*}(\rho_V, T) = \frac{\partial e\,(\rho,k)}{\partial T} = 1 + \frac{\partial U_{s.p.}}{\partial T}(\rho_V, T) \tag{13}$$

Here T denotes the kinetic energy.

In the nonrelativistic limit, the Fermi energy is given by:

$$e_F(\rho_V) = (g_V^2/M_V^2)\,\rho_V + (M^* - M) + (M/M^*)\,T_F \tag{14}$$

The energy per nucleon is, in the same approximation:

$$W(\rho_V) = \frac{1}{2}(g_V^2/M_V^2)\,\rho_V + \frac{V(\sigma)}{\rho_V} + (M^* - M) + \frac{3}{5}(M/M^*)\,T_F \tag{15}$$

We make use of the conditions (to be taken for <u>fixed</u> ρ_V):

$$\partial\{\rho_V W(\rho_V)\}/\partial\sigma = 0 \tag{16}$$

$$\partial\{\rho_V W(\rho_V)\}/\partial M^* = 0 \tag{17}$$

which follow from the field equations for σ and from Eq (8).

It is seen that the HUGENHOLTZ-VAN HOVE theorem[8,9])

$$e_F = \frac{d}{d\rho_V}(\rho_V W) = W + \frac{P}{\rho_V} \tag{18}$$

where P is the pressure, is satisfied identically.

At the saturation density $\rho_V = \rho_0$, $P = 0$ and

$$e_F = W \tag{19}$$

which is just the WEISSKOPF theorem.[9)]

For normal nuclear matter, both the average kinetic energy and the binding energy per nucleon are only a few percent of the nucleon rest mass. Thus in most of this paper, we will neglect all terms of order T^2. This is the standard nonrelativistic approximation. The validity of such an approximation depends critically on the value of the effective mass m^* at saturation, since the nonlinear contributions depend very sensitively on m^*. For the case when $m^* = 0.55$, which is the case in the pure Yukawa theory, the nonlinear self-interactions vanish, and the nonrelativistic approximation ceases to saturate nuclear matter; that is, the nonrelativistic approximation becomes extremely bad. On the other hand, the value of the effective mass deduced from the energy dependence of the nucleon-nucleus optical potential at low energies is $m^* = 0.7$. This is consistent with the results of recent calculations by FRIEDMAN and PANDHARIPANDE[10)] using the Fermi hypernetted chain approximation. The accuracy of the non- relativistic approximation, when m^* is chosen to be about 0.7, will be explicitly tested in Section V.

In addition, in some parts of this work, we replace the dependence of average kinetic energy by a linear one. This makes it possible to obtain simple analytic expressions for a variety of interesting nuclear properties. This approximation turns out to be fairly accurate, due to the smallness of the kinetic energy, relative to some of the other terms appearing in the theory.

Using the nonrelativistic approximation mentioned above, we have:

$$\langle p^2 \rangle = 2\, M\, T_0(\rho) = 2\, M\, T_0\, \hat{\rho}^{2/3} \tag{20}$$

where $\hat{\rho} = \rho/\rho_0$ (ρ is understood to refer to ρ_V), and T_0 is the average kinetic energy at normal density.

Replacing the $\rho^{2/3}$ dependence by ρ, we obtain:

$$\langle p^2 \rangle = 2\, M\, T_0\, \rho. \tag{21}$$

Thus, dropping terms of order T_F^2, we find:

$$W_I(\rho) = M \left\{ m^*(\rho) - 1 + \frac{T_0 \quad \hat{\rho}}{M \langle m^*(\rho) \rangle} \right\} \tag{22}$$

The energy of nuclear matter is a sum of three contributions:

$$W = W_V + W_\sigma + W_I \tag{23}$$

Next, using the condition:

$$\frac{dW}{d\sigma} = 0 \quad , \tag{24}$$

we can obtain an explicit expansion for σ in powers of ρ.

Up to order ρ^2, we obtain:

$$\sigma = - \frac{g_S\, \rho}{M_S^2} \left(1 - \frac{3}{2} \alpha\, B\, \hat{\rho} - \frac{T_0}{M} \hat{\rho} \right) \tag{25}$$

where B is a dimensionless parameter which will occur frequently in our treatment.

$$B = \frac{g_S^2\, \rho_0}{M_S^2\, M\, c^2} \tag{26}$$

This parameter is proportional to the square of the dimensionless coupling constant C_S used by WALECKA [4)]

$$B = \left(\frac{\hbar}{M\, c} \right)^3 \rho_0\, C_S^2 \tag{27}$$

In the last two equations, we include the appropriate powers of $\hbar$ and c.

The effective mass is then given by:

$$m^* = 1 + \frac{g_S \sigma}{M} = 1 - B\hat{\rho} + \frac{3}{2}\alpha B^2 \hat{\rho}^2 + \frac{T_0}{M} B \hat{\rho}^2 \tag{28}$$

We will use here:

$$B = 0.335 \quad . \tag{29}$$

For normal nuclear matter density, ($\rho_0 = 0.16\ \mathrm{fm}^{-3}$, $T_0 = 22$ MeV), this corresponds to

$$C_S = 26.0\ B^{1/2} = 15.0 \quad . \tag{30}$$

(One of us previously obtained $C_S = 15.6$ in a nonlinear mean field theory and in section IV, we will obtain $C_S = 15.85$, without making the approximations made in this section.)

This gives for the effective mass as function of density:

$$m^*(\hat{\rho}) = 1 - 0.335\ \hat{\rho} + 0.17\ \alpha\ \hat{\rho}^2 + 0.008\ \hat{\rho}^2 \quad . \tag{31}$$

As we will see, the value of the coefficient α is 0.164.

Finally, using

$$|W_0| = \frac{1}{2}\alpha M B^2 + T_0 B \tag{32}$$

which is derived in the next section, we obtain $m^* = 0.70$, close to the empirical value.

III. CALCULATION OF NUCLEAR MATTER ENERGY.

We now calculate the contribution of the terms mentioned above to the energy of nuclear matter (up to order ρ^2).

We find for the contribution of the vector field:

$$W_V = \frac{1}{2} M B R_V \hat{\rho} \tag{33}$$

where

$$R_V = (g_V^2 \cdot M_S^2 / g_S^2 \cdot M_V^2) \quad . \tag{34}$$

The scalar field terms (quadratic and cubic) are given by:

$$W_\sigma^{(2)} = \frac{1}{2} M B \hat{\rho} - \frac{3}{2} \alpha M B^2 \hat{\rho}^2 - T_0 B \hat{\rho}^2 \tag{35}$$

$$W_\sigma^{(3)} = \frac{1}{2} \alpha M B^2 \hat{\rho}^2 \tag{36}$$

The σ^3 term behaves like a repulsive three body interaction. We will see that it contributes quite substantially to the energy of nuclear matter.

The interaction term contributes an amount:

$$W_I = \frac{1}{2} T_F \hat{\rho} - M B \hat{\rho} + \frac{3}{2} \alpha M B^2 \hat{\rho}^2 + 2 T_0 B \hat{\rho}^2 \tag{37}$$

The sum of all three terms is:

$$W(\hat{\rho}) = < \frac{1}{2} T_F - \frac{1}{2} M B (1-R_V)> \hat{\rho} + (\frac{1}{2} \alpha M B^2 + T_0 B) \hat{\rho}^2 \tag{38}$$

Since we require saturation at $\hat{\rho} = 1$, we can write Eq. (38) in the simple form:

$$W(\hat{\rho}) = |W_0| (- 2 \hat{\rho} + \hat{\rho}^2) \tag{39}$$

$$|W_0| = \frac{1}{2} \alpha M B^2 + T_0 B \tag{40}$$

and

$$R_V = 1 - (T_F^0 + 4 |W_0|)/(M B) \quad . \tag{41}$$

Using our values for these quantities

$$T_F^0 = 36 \text{ MeV} \tag{42}$$

$$|W_0| = 16 \text{ MeV} \tag{43}$$

$$Mc^2 = 938 \text{ MeV} \tag{44}$$

$$B = 0.335 \tag{45}$$

we find

$$R_V = 0.682 \tag{46}$$

Note that the terms proportional to MB cancel in the expression for W. Individually, these terms contribute more than 100 MeV per nucleon. It may also be of interest that in the approximation used here, (i.e., binding energy taken as a quadratic function of the density), the compression modulus of nuclear matter is uniquely specified by the binding energy.

$$\frac{d^2W}{d\hat{\rho}^2}(1) = \frac{1}{9} K = 2\,|W_0| \tag{47}$$

We have also considered the role of the ρ^3 terms in W. These depend on more detailed assumptions, e.g., the possible role of σ^4 in the scalar field energy. However, the effect of such $\hat{\rho}^3$ dependence on the energy almost cancels in the calculation of the compression modulus.

Thus, including a ρ^3 term , we have (remember that W saturates at $\hat{\rho} = 1$):

$$W(\hat{\rho}) = W_2\,(-2\,\hat{\rho} + \hat{\rho}^2) + \frac{1}{2} W_3\,(-3\,\hat{\rho} + \hat{\rho}^3) \tag{48}$$

Thus

$$|W_0| = -W(1) = W_2 + W_3 \tag{49}$$

$$W''(1) = 2\,W_2 + 3\,W_3 \tag{50}$$

and

$$\frac{W''(1)}{|W_0|} = 2 + (W_3/|W_0|) \tag{51a}$$

provided $W_3 \ll |W_0|$, as seems to be the case.

We can write the expression for W_0 in a form which emphasizes the relative importance of the nonlinear term in the scalar field energy for nuclear

saturation. In the approximation made above, the contribution of the nonlinear terms to the nuclear matter energy is:

$$W^{NL} = |W_0| - T_0(1 - m^*) \tag{51b}$$

For T_0 = 22 MeV, we find that:

$$m^* = 0.7, \ |W_0| = 16 \text{ MeV}, \tag{52}$$

and

$$T_0 \ (1 - m^*) = 6.6 \text{ MeV} \tag{53}$$

Thus

$$W^{NL} = 9.4 \text{ MeV}. \tag{54}$$

Therefore the density dependence required by the nonlinear term seems to be <u>more</u> important for nuclear saturation than the momentum-dependence implied by the reduced value of the effective mass.

If we set $W^{NL} = 0$, which is the approximation made in the original mean field theory of WALECKA[4], then we find:

$$m^* = 1 - (|W_0|/T_0) \cong 0.3 \tag{55}$$

a very <u>large</u> momentum dependence, and contrary to experiment. In fact, in this case, terms of higher order in T_0 play an essential role, i.e., a nonrelativistic treatment is inadequate.

(A more accurate calculation given in the next section yields $m^* = 0.54$). It was already pointed out by WALECKA[4] that the original version of the mean field theory also leads to values considerably larger than empirical for the compression modulus.

A THOMAS-FERMI calculation for finite nuclei by BOGUTA and RAFELSKI[11] showed that the surface energy in this model is much too large.

IV. NONLINEAR TERM IN NUCLEAR MATTER - MORE ACCURATE CALCULATION.

In this section, we recalculate the contribution of the nonlinear term to the energy of nuclear matter more accurately.

From Eq. (4), we see that:

$$W^{NL} = -\frac{\alpha M_S^2 g_S \sigma^3}{2M} \tag{56}$$

(assuming no quartic or higher order terms in the field energy).

We do not make the approximation mentioned in the introduction; i.e., we do the full relativistic calculation for 3 dimensional nuclear matter.

Suppose we have saturation at a given density (i.e., average kinetic energy), binding energy, and effective mass.

There are three conditions which must be satisfied:

$$W(\rho_0) = -W_0 \tag{57}$$

$$\frac{dW}{d\rho}(\rho_0) = 0 \tag{58}$$

$$\frac{dW}{dm^*}(\rho_0) = 0 \quad . \tag{59}$$

Equation (59) is equivalent to the field equation for the scalar field amplitude. There are then three equations for ρ_0, W_0, m^* in terms of g_S, g_V, and α. These can be inverted and we obtain an explicit equation for W^{NL} in terms of T_0, W_0, and m^*.

$$W^{NL} = |W_0| + M < \frac{m^*(1-m^*)}{\varepsilon} + \varepsilon - 1 > \tag{60}$$

where

$$\varepsilon = < m^{*2} + (2T_0/M) >^{1/2} \quad . \tag{61}$$

In the nonrelativistic limit, i.e., dropping terms of order T_0^2, etc., the terms proportional to M cancel and we obtain:

$$W^{NL\ (Non\ Rel)} = |W_0| - \frac{T_0}{m^*}\left(\frac{1-m^*}{m^*} - 1 + \frac{2}{D}\right) \tag{62}$$

In Eq. (60), we have considered three dimensional matter, but in Eq. (62), the number of dimensions, denoted here by D, is left as an arbitrry parameter. Note that for the special case D = 2, and taking the limit $1 - m^* << 1$, we recover the result Eq. (53).

In the table below, we list the values of W^{NL} obtained using the three prescriptions mentioned above:

a.) Nonrelativistic in 2D,

b.) Nonrelativistic in 3D,

and c.) exact. $T_0 = 22$ MeV, $W_0 = -16$ MeV

W^{NL} in MeV.

$m^* = M^*/M$	a.) Simple Model Eq. (53)	b.) Non-Rel. Eq. (62)	c.) Exact Eq. (60)
0.5	5.0	-13.33	-7.45
0.6	7.2	3.77	6.26
0.7	9.4	13.01	14.12
0.8	11.6	18.28	18.80
0.9	13.8	21.42	21.65

As can be seen, for $m^* = 0.7$, the contribution of the nonlinear term is almost as large as the binding energy itself.

The simple model discussed in the previous sections gives qualitatively reasonable results, though it underestimates the magnitude of W^{NL} somewhat. The

latter can be traced to the difference in the density dependence of the kinetic energy, i.e, as $\rho^{2/3}$ vs ρ.

On the other hand, the value of the scalar coupling strength, $C_S = 15.85$, is only slightly larger than the value 15.0 deduced in the previous section. Also, the compression moduli are nearly the same. In particular, the quantity $\rho^2 (\partial^2 W/\partial\rho^2)_{\rho_0}$, turns out to be equal about 35.5 MeV, compared to 32 MeV in the simple model discussed above.

Finally, it should be noted that the nonlinear terms disappear for $m^* = 0.54$, which corresponds to the original WALECKA model. In this case, relativistic effects are quite important.

Indeed, there seems to be a <u>qualitative</u> difference between the results for $m^* \sim 0.54$ and $m^* \sim 0.7$, in spite of the relatively small difference in the effective mass itself.

V. THE LANDAU PARAMETER F_0

We can use the mean field theory to estimate the values of other key nuclear properties. In this section, we consider the Landau parameter $F_0(\rho)$ which is proportional to the volume integral of the spin and isospin independent part of the effective interaction,

The F_0 (ρ) is proportional to the direct matrix element of the effective interaction for zero momentum transfer, i.e., its volume integral.

$$F_0(\rho) = (dN/dE)\, V_{eff}\, (\rho,\, q = 0) \tag{63}$$

The density of single particle states dN/dE is given by:

$$dN/dE = \frac{3}{2}\, \rho_0\, m^* \,/\, T_F \tag{64}$$

and dependent on the local nucleon density.

In the analytic version of the mean field theory used here we obtain:

$$\tilde{V}_{eff}(\rho,q=0) = \frac{M\,c^2}{\rho_0}\, B\, \{\, (1-R_V) - (\, 3\, \alpha\, B + \frac{4\, T_0}{M\, c^2}\,)\, \hat{\rho}\, \} \tag{65}$$

Using an effective mass $m^* = 0.7$ and the ratio $R_V = 0.682$, as obtained in Sec IV, we find:

$$dN/dE = 0.00467 \tag{66}$$

and the following dependence for F_0:

$$F_0(\rho) = -\, 2.9 + 2.4\, \hat{\rho} \tag{67}$$

The limiting values of F_0 for $\rho = 0$ and ρ_0 are:

$$F_0^{ext} = F_0(0) = -\, 2.9 \tag{68}$$

and

$$F_0^{int} = F_0(\rho_0) = -\, 0.5 \quad . \tag{69}$$

This parameter was recently calculated by CELENZA, PONG, and SHAKIN[12] using a relativistic mean field theory which includes explicitly the small components in the Dirac wavefunctions of the nucleons.

Our limiting values for F_0 are essentially the same as theirs, although they obtain a slightly different density dependence.

The empirical values for the external and internal values of F_0, as deduced from nuclear spectra are -3.0 and -0.3, respectively.[12] Thus the mean field theory does very well in accounting for the values of F_0 both outside and inside the nucleus.

VI. SPIN-ORBIT POTENTIAL.

Here we make only a crude estimate of the single particle spin-orbit potential.

The standard mean field theory gives the following expression for the l·s interaction:

$$V_{l\cdot s} = \frac{1}{2}\left(\frac{\hbar}{M\,c}\right)^2 \left(g_V \frac{1}{r}\frac{dV_0}{dr} - g_S \frac{1}{r}\frac{d\sigma}{dr} \right) l\cdot s \tag{70}$$

Now the solution for the vector field gives:

$$V_0 = g_V \rho / M_V^{\,2} \quad . \tag{71}$$

The scalar and vector terms contribute approximately equally <u>and with the same sign</u> to the l·s interaction.

To order B, we obtain the very simple result:

$$V_{l\cdot s} = \frac{\hbar^2}{M} B \frac{1}{r}\frac{d\hat{\rho}}{dr}\, l\cdot s. \tag{72}$$

Using the values of the parameters deduced in previous sections, we find:

$$\frac{\hbar^2}{M} B = 15 \text{ MeV} \tag{73}$$

which agrees fairly well with the values quoted in the literature.

VII. NUCLEAR SURFACE.

Again, standard mean field theory gives a simple expression for the extra energy density due to spatial variation of the fields (which is a consequence of the inhomogeneity of the nucleon density). We have:

$$e(\rho,\nabla\rho) = e(\rho) + \frac{1}{2}(\nabla\sigma)^2 - \frac{1}{2}(\nabla V_0)^2 \quad . \tag{74}$$

Here $e(\rho)$ is the energy density for uniform matter at density ρ.

Next, we solve the field equations for σ and V_0. We must take into account the nonlinear term in the scalar field. However, we will assume that the density changes relatively slowly over a meson Compton wavelength. We find, approximately:

$$\sigma = \sigma_{NM}(\rho) \left\{ 1 + \frac{\nabla^2\rho}{\mu_S^2\,\rho} \right\} \tag{75}$$

and similarly for V_0. σ_{NM} denotes the value for constant ρ.

It turns out that the contributions of the extra terms essentially cancel. Thus, although the <u>fields</u> at the nuclear surface are modified by the density variation there, it is all right to use the local density approximation (i.e., at each point set the fields equal to the values for nuclear matter at <u>that</u> density), at least for calculation of energies.

We obtain for the energy density (to order B in the inhomogeneity term):

$$e(\rho,\nabla\rho) = \frac{1}{2} M\, B\, \lambda^2 (\nabla\rho)^2 + |W_0|(-2\hat{\rho} + \hat{\rho}^2) \tag{76}$$

where $$\lambda^2 = M_S^{-2} - M_V^{-2} \quad . \tag{77}$$

The nuclear surface energy is a sum of two terms:

First the contribution due to the inhomogeneity (the first term in Eq. 76) and second, the deficiency in binding due to the fact that the density at the nuclear surface is less than the nuclear matter value.

We find for the <u>total</u> surface energy (per nucleon):

$$W_S = \frac{1}{2} M B \lambda^2 A^{-1} \int (\nabla\rho)^2 d^3R + |W_0| \int \rho(1-\hat{\rho})^2 d^3R \tag{78}$$

This expression is of the standard form used in the calculation of nuclear surface properties.

Assuming a Fermi form for the density:

$$\rho = \rho_0 \int \langle 1 + \exp[(r-R)/a)] \rangle \tag{79}$$

and assuming $a \ll R$, the energy is minimized for the following value of the diffuseness parameter a:

$$a = \lambda \, [MB/6|W_0|]^{1/2} \tag{80}$$

and the surface energy per particle is:

$$W_S = 3 \, |W_0| a / R \tag{81}$$

Using our values of the parameters, and $M_S = 500$ MeV, $M_V = 780$ MeV, which leads to $\lambda = 0.30$ fm, we obtain:

$$a = 0.55 \text{ fm}, \qquad W_S = 20 \, A^{-1/3} \text{ MeV} \tag{82}$$

both in good agreement with empirical values.

The surface energy of semi-infinite matter can be obtained more accurately by using the calculus of variations. As was first pointed out by SKYRME,[13] the more accurate (and self-consistent) ρ is then:

$$\rho \sim \tanh^2 c(r-R_1) \tag{83}$$

where R_1 differs slightly from R_0.

For the self-consistent density distribution, the surface energy is a fraction $8\sqrt{3}/15 = 0.924$ of the value for the Fermi distribution, which was derived above. Thus, use of the self-consistent density leads to a slight improvement in the calculation of the nuclear surface energy.

VIII. IMPLICATIONS OF CHIRAL SYMMETRY

The aim of relativistic quantum field theory is to be a comprehensive theory of nuclear matter and nuclear structure. We have shown that some aspects of nuclear structure are unified if treated from a relativistic quantum field point of view.

One of the most serious deficiencies of the present theory is that it cannot take into account pions in the linear realization of chiral symmetry. Indeed it is well known that the simple sigma model does not lead to saturation, but to a condensed state, as was first pointed out by LEE and WICK.[11] Also, this model leads to large attractive three body interactions, as was pointed out by BARSHAY and BROWN.[12]

Thus the nonlinear model discussed in the previous sections appears at first sight to be quite different from that expected from chiral symmetry. In fact, this need not be the case if one extends the relativistic theory to include the Higgs phenomenon. Recently, one of us (JB)[7] showed that if not only the nucleon mass but also the mass of the vector field is dynamically generated by the scalar field, then a normal nuclear matter equation of state is possible and the usual chiral symmetry results are preserved.

In this version of the chiral model, M_V^* is not a constant but is:

$$M_V^* = M_V + g_V \sigma \tag{84}$$

In order that both M_V^* and M_N^* vanish for the same value of σ, it is necessary that

$$g_V/M_V = g_S/M_S \tag{85}$$

When we do this, we can reproduce the results of the nonlinear theory with only minor modifications.

We find that the energy of nuclear matter $W(\hat{\rho})$ is still of the form specified by Eq. (38). In the theory discussed in the previous sections, α was

an arbitrary parameter, and we needed

$$\alpha = 0.164 \tag{86}$$

in order to fit the data.

In the modified chiral model, α is no longer arbitrary, but is given by

$$\alpha = 2\ R_V - 1 \tag{87}$$

where R_V is defined by Eq. (34).

For comparison, in the conventional chiral model, the nonlinear terms are also important, and the parameter α, defined by Eq. (4), is given by

$$\alpha = -\ 1 \tag{88}$$

Our value of $R_V = 0.682$, obtained in Sec IV, would imply that

$$\alpha = 0.36 \tag{89}$$

which is fairly close to the phenomenological value.

IX. CONCLUSIONS

We have seen that important nuclear matter properties can be successfully described by a nonlinear relativistic mean field theory, which has a meaningful nonrelativistic limit. The parameters of the model, such as g_S, g_V, and M_S, determined by fitting the bulk properties of nuclear matter and the nuclear surface, are consistent with their values deduced from analysis of nucleon-nucleon scattering.

The mean field theory considered in most of this work is phenomenological. Presumably, the vector field could correspond to the combined effects of the ω meson and multimeson exchanges involving both pions and rhos. As was pointed out by Brown,[14] the latter seems to be the dominant source of the short range nucleon-nucleon repulsion. The scalar field probably represents mainly multipion states. It is well known that scalar meson exchange simulates quite well the main effects of two-pion exchange proceeding mainly via excitation of the Δ isobar. What is indeed remarkable is that all of these simulations can correlate so many hitherto unrelated measurable bulk properties of nuclei.

Of course, the density dependent effects implied by the nonlinear term in the mean field theory do not <u>necessarily</u> require that there must be strong repulsive three body forces in nuclei. Some of this density dependence could result from higher order effects of tensor forces. Nevertheless, in the framework of the mean field theory, the results can be very naturally interpreted as a consequence of effective three body interactions.

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