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SOLUTION OF THE INFRA-RED PROBLEM*

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Abstract

An exact formula is given. It separates the complete electro-magnetic correction to a given coordinate-space Feynman function into a product of two factors. The first factor is a unitary operator that is expressed in closed form. It contains all contributions corresponding to the classical electro-magnetic radiation field. The second factor is expressed in terms of Feynman-like contributions with a modified coupling, and is free of infra-red divergences: it can be transformed into momentum space, and enjoys there the normal analytic properties. This formula provides a simple solution to the infra-red problem that maintains the physically and mathematically correct asymptotic properties in coordinate space.

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The well-known "infra-red catastrophe" in quantum field theory consists of the following fact: the electromagnetic corrections to the S-matrix are represented by integrals whose contributions from very soft photons often diverge. A way around this difficulty was indicated by Block and Nordsieck[1], who showed, in some simple cases, that these infra-red-divergent contributions cancel out of the expressions for the observable probabilities, provided the nonobservability of very soft photons is taken into account. The Block-Nordsieck observation has been generalized in a series of works that have culminated in the central work in this field, the paper of Yennie, Frautschi, and Suura[2]. This YFS paper gave lengthy arguments to support their contention that all of the infra-red-divergent contributions to the S-matrix can be collected into exponential factors that cancel out of the expressions for observable probabilities. However, at the end of a technical appendix to their paper YFS listed some of the difficulties with their arguments, and concluded that a rigorous proof of their conjecture would probably be prohibitively complicated. The difficulties with the YFS arguments are particularly serious when the S-matrix is evaluated at a singularity.

The YFS infra-red separation was used by Chung[3] to define an infra-red-finite S matrix: infra-red finiteness was (presumably) achieved by incorporating the YFS infra-red factor into coherent initial or final states. This infra-red-finite S-matrix was examined by Storrow[4], Kibble[5] and Zwanziger[6], who found that the pole singularity normally associated with a stable particle was converted by the effects of soft photons to a non pole form.

Such a change in the character of the S-matrix singularities could be as catastrophic as the infra-red-divergence itself. For the physical interpretation of the theory depends on the fact that stable particles propagate over macroscopic distances in an essentially classical way, and on the fact that any process involving an intermediate stable particle behaves essentially like a combination of two separate processes. These properties depend normally[7,8] on the pole character of the one-particle exchange singularity, and on the factorization of the residue of that pole. Any change of these analyticity properties would jeopardize the ability of the theory to accomodate stable (or nearly stable) charged particles.

These difficulties can be overcome by a new approach to the infra-red-divergence problem that separates out the classical electromagnetic effects already in coordinate space. For terminological convenience, and greater generality, we consider not QED itself but rather the electromagnetic corrections to a given (perhaps non-electromagnetic) process represented by a Feynman diagram D. Let $F^D(x)$ be the coordinate-space Feynman function corresponding to D. Here $x=(x_1,\ldots,x_n)$ specifies the spacetime locations of the n vertices of the spacetime diagram D(x) that is topologically equivalent to D.

Suppose, first, that the diagram D has no charged external lines. Then $F^D(x)$ is transformed by the inclusion of all electromagnetic interactions with D to the operator-function

$$\hat{F}^D_{op}(x) = U(L(x))\tilde{F}^D_{opr}(x). \tag{1}$$

Here U(L(x)) is a unitary operator in photon space that depends on the set L(x) of spacetime polygons formed by the charged lines of D(x). The explicit form of U(L(x)) is

$$U(L(x)) = e^{\langle a^*\cdot J(L(x))\rangle} e^{-\langle a\cdot J^*(L(x))\rangle} \times e^{i\Phi(L(x))-\frac{1}{2}\langle J^*(L(x))\cdot J(L(x))\rangle}, \tag{2}$$

where a*(k) and a(k) are photon creation and annihilation operators,

$$\langle f\cdot g\rangle = \int\frac{d^4k}{(2\pi)^4}\, 2\pi\delta^+(k) f_\mu(k)(-g^{\mu\nu})g_\nu(k), \tag{3}$$

$$J_\mu(L(x),k) = -ie\int_{L(x)} dx'_\mu e^{ikx'}, \tag{4}$$

and

$$\Phi(L(x)) = \frac{(-ie)^2}{8\pi}\int\int_{L(x)L(x)} dx'\cdot dx''\ \delta((x'-x'')^2). \tag{5}$$

The (Coulomb) phase $\Phi(L(x))$ is the classical action associated with the motion of a charged particle around the closed loops L(x). The remaining part of U(L(x)) acting on the photon vacuum creates the coherent photon state that corresponds to the classical electromagnetic field radiated by the motion of a charged particle around the closed loops L(x).

The quantity $\tilde{F}^D_{opr}(x)$ is likewise an operator in photon space that depends on x. It gives no infra-red difficulties: it can be Fourier transformed into momentum space ($x \to p$), and the dominant singularities of all (multi-) photon momentum-space matrix elements $\langle k'|\tilde{F}^D_{opr}(p)|k''\rangle$ of $\tilde{F}^D_{opr}(p)$ have the same character as the corresponding singularities in a theory with no massless particles. Moreover, the discontinuities around these singularities are infra-red finite.

Formula (1) is derived by separating the photon coupling into its "classical" and "quantum" parts: the coupling of a photon of momentum-energy k into a charged line i of D is expressed in the form

$$-ie\gamma_\mu = C_\mu(k,z_i) + Q_\mu(k,z_i), \tag{6}$$

where z_i is the spacetime difference between the two end points of the line segment i of D(x), and

$$C_\mu(k,z\ ) = -iez_\mu K(k\cdot z\ )^{-1}. \tag{7}$$

The operator $U(L(x))$ is the full contribution from all classical photons, which are those that couple into $L(x)$ only via the classical couplings. The operator $\tilde{F}^D_{opr}(x)$ is the full contribution from all quantum photons, which are those that couple into $L(x)$ with a quantum coupling on at least one end. The function $\tilde{F}^D_{opr}(x)$ is constructed as the sum of Feynman-like contributions from all the different ways that photon lines can be connected to D. The rules for computing $\tilde{F}^D_{opr}(x)$ are the same as the Feynman rules, except that one must use the coupling $Q_\mu(k,z_i)$ -- and sometimes $C_\mu(k,z_i)$ -- in place of $-ie\gamma_\mu$, and must use for the propagator of a photon with energy-momentum k emitted by a quantum coupling Q and absorbed by a classical coupling C the retarded propagator defined by replacing Feynman's denominator $k^2+i\varepsilon$ by $(k^o+i\varepsilon)^2-\vec{k}^2$. This modification of the propagator, together with the fact that the quantum coupling into a mass-shell line is linear in k, renders infra-red finite the functions $\langle k'|\tilde{F}_{opr}(p)|k''\rangle$ and their discontinuities.

The physical observables are the transition probabilities associated with spatially localized free-particle wave functions $\psi_i(x_i)$ of the initial and final particles of D. Folding these wave functions into $\tilde{F}^D_{op}(x)$ gives the transition operator-amplitude $T^D_{op}[\psi]$.

If there were no difficulties arising from massless particles then one could define the transition amplitudes for processes having charged-particle external lines by considering these processes to be subprocesses of a larger process D having no charged-particle external lines. The extraction could be effected in either one of two ways: (1), take the residues of the pole singularities associated with those internal lines of D that are external lines of the subprocesses, or (2) shift the wave packets of the external particles of $T^D_{op}[\psi]$ to

infinity in a way such that the full process represented by D is decomposed into subprocesses that occur in separate spacetime regions that are shifted to infinity in a way compatible with the idea that the intermediate charged particles travel like classical particles between these separated spacetime regions. The latter procedure is the physical one, but its equivalence to the pole-residue method is assured by the normal connection mentioned earlier between momentum-energy singularities and space-time asymptotic behavior.[7,8]

These two ways of deriving the transition amplitudes for processes with charged-particle external lines can be used also in the presence of massless photons. In the spacetime approach the external-particle wave functions $\psi\equiv\psi^\lambda$ are shifted to infinity as $\lambda\to\infty$ in a way such that the main contributions to the transition amplitude $T^D_{op}[\psi^\lambda]$ come only from points $x=(x_1,\ldots,x_n)$ near the point $\lambda X=(\lambda X_1,\ldots,\lambda X_n)$. That is, the localization of the wave packets ψ^λ_i render negligible (in an appropriate mathematical sense) the contributions to the amplitude from points x not close to λX.

This effective confinement of x to a neighborhood of λX entails that the contributions to the observable probabilities of classical photons with $k\varepsilon\Omega$ become negligible as the size of Ω tends to zero. In particular, let Ω be a small neighborhood of k=0 and let $U_\Omega(L(x))$ and $U^\Omega(L(x))$ represent the operators obtained from $U(L(x))$ by restricting the k integration to Ω, and to the complement of Ω, respectively. (The δ-function in (5) must be Fourier analyzed to effect this separation.) Then

$$U(L(x)) = U_\Omega(L(x))U^\Omega(L(x)) \tag{8}$$

$$= U_\Omega(L(\lambda X))U^\Omega(L(x)) \times U_\Omega(L(\lambda X))\left[U^{-1}_\Omega(L(\lambda X))U_\Omega(L(x))-1\right]U^\Omega(L(x)). \tag{9}$$

Suppose Ω is confined to the space corresponding to undetectable photons. Observable probabilities involve sums over all final states of the undetectable photons. Thus in the calculation of observable probabilities the unitary operator $U_\Omega(L(\lambda X))$ in (9) drops out. Hence in the contribution to observable probabilities coming from the first term in (9), and the corresponding contribution from $U^\dagger(L(x))$, the classical photons with $k\varepsilon\Omega$ have no effect at all. On the other hand, for sufficiently small Ω the contributions from the second term in (9) can be shown to be negligible due to the near equality of $U_\Omega(L(\lambda X))$ and $U_\Omega(L(x))$. The very soft classical photons have, therefore, essentially no effect on probabilities and the infra-red contributions arise only from $\tilde{F}^D_{opr}(p)$. The normal analytic structure and infra-red finiteness of the functions $\langle k'|\tilde{F}^D_{opr}(p)|k''\rangle$ and their discontinuities thus ensure that the observable probabilities enjoy the asymptotic behavior that corresponds to stable charged particles.

The same arguments show that if the initial $k\varepsilon\Omega$ photon states are Fock states then the main $k\varepsilon\Omega$ contribution to the asymptotic probabilities come from final states that are $U_\Omega(L(\lambda X))$ times Fock states. Thus to define amplitudes that contribute significantly to the asymptotic behavior of probabilities one should use these coherent final states. But then the operator $U_\Omega(L(\lambda X))$ in (9) is cancelled, and the earlier arguments now show that the very soft classical photons have negligible contribution to the amplitudes. Then the normal analytic structure of the functions $\langle k'|F_{opr}(p)|k''\rangle$ ensure that infra-red-finite amplitudes for processes with charged-particle external lines can be equivalently defined by the factorization of residues of charged-particle poles or by the factorization of asymptotic amplitudes. Details are given in references 9 and 10.

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REFERENCES

1. F. Block and A. Nordsieck, Phys. Rev. 52, 54 (1937).
2. D. Yennie, S. Frautschi, H. Suura, Annals of Physics (N.Y.) 13, 379 (1961).
3. V. Chung, Phys. Rev. 140B, 1110 (1965).
4. J.K. Storrow, Nuovo Cim. 54, 15 (1968).
5. T. Kibble, Phys. Rev. 173, 1527 (1968).
6. D. Zwanziger, Phys. Rev. 7D, 1026 (1973).
7. C. Chandler and H.P. Stapp, J. Math. Phys. 10, 826 (1969).
8. D. Iagolnitzer and H.P. Stapp, Comm. Math. Phys. 14, 15 (1969).
9. H.P. Stapp, Exact Solution of the Infra-red Problem, Lawrence Berkeley Laboratory Report LBL-13651.
10. T. Kawai and H.P. Stapp, Infra-red Finiteness of QED, Lawrence Berkeley Laboratory Report LBL-14724.

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