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COMMENT ON WITTEN'S MASS HIERARCHY

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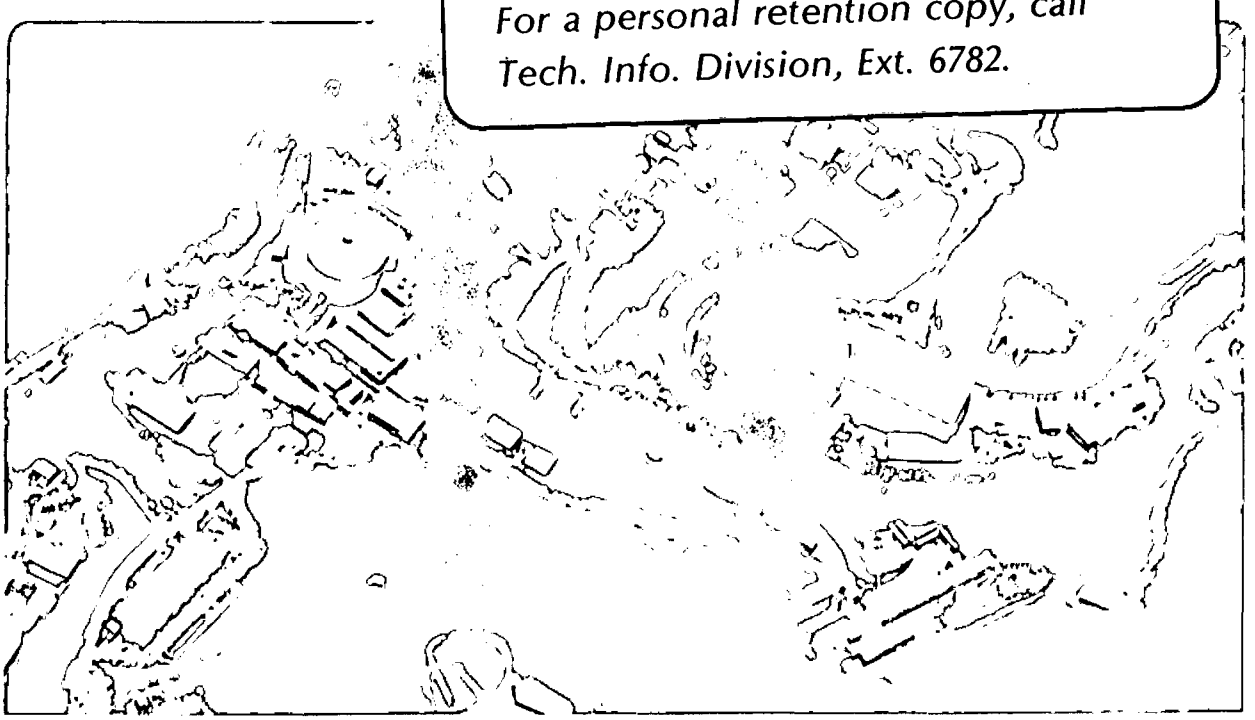

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COMMENT ON WITTEN'S MASS HIERARCHY*

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Abstract

It is pointed out that in a class of supersymmetric models introduced recently by Witten, the generation of a mass hierarchy of many orders of magnitude requires a delicate adjustment of coupling constants at each order of perturbation theory.

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Unification of strong and electroweak interactions requires that the simple unified group undergoes symmetry breaking at a scale at least twelve orders of magnitude above the scale at which weak symmetry breaking occurs. The introduction of such a mass hierarchy into gauge theories requires an exceedingly fine adjustment of parameters in the theory at each order in perturbation theory. This gauge hierarchy problem becomes more tractable when the gauge theory is made supersymmetric on scales above the weak breaking. For example, non-renormalization theorems in supersymmetric theories often remove the need for order by order tuning; and Higgs scalars needed for weak breaking may be made degenerate with fermions which are kept light by chiral invariance.

A promising mechanism for generating mass hierarchies in supersymmetric theories was invented by Witten [1]. The idea is to write a superpotential of the O'Raifeartaigh type [2] with mass parameter M and coupling constants λ_i in which the vacuum expectation value (vev) of a field (X) is not determined at the tree level. Radiative corrections will determine this vev. The effective potential, radiatively corrected to include all leading logarithms, has the form [3]

$$V_{eff} = f[\lambda_i(t), M_i(t)]e^{4t} = V_0\left(1 + F[\lambda_i(0)]t + O(t^2)\right) \qquad (1)$$

where $\lambda_i(t)$ $[M_i(t)]$ are renormalization group improved coupling constants (masses) evaluated at scale $t = \ln \frac{X}{M}$. In the original model of Witten $M \sim O(10^3 \text{GeV})$, while in the geometric hierarchy models [4] $M \sim O(10^{12} \text{GeV})$. For $F[\lambda_i(0)] > 0$, the effective potential is minimized for $X = 0$ [5]. For $F[\lambda_i(0)] < 0$, the minimum occurs when $t = t_0 > 0$ such that $F[\lambda_i(t_0)] = 0$. If $\lambda_i(0)$ are chosen such that

renormalization group scaling towards the ultraviolet allows the coupling constant trajectory to pass a point where F vanishes, then we expect an exponential hierarchy for $\frac{X}{M}$ to result (10^{14} for Ref. 1, and 10^7 for Ref. 4). We will demonstrate that for a mass hierarchy to be this large, the dimensionless couplings $\lambda_i(0)$ must usually be precisely chosen; they must be very close to an infrared fixed point of their renormalization group equations. We will illustrate this in a simple SU(5) model, discuss the degree of fine tuning, and consider possible ways of overcoming this problem.

We consider an SU(5) model with two adjoints (A,Y), a singlet X, three matter generations of $\bar{5} + 10$, and a Higgs $5 + \bar{5}$. The superpotential is

$$W = \lambda_1 \mathrm{Tr} YA^2 + \lambda_2 X(\mathrm{Tr}A^2 - M^2) + W_M \tag{2}$$

where W_M contains the usual Yukawa interactions for generating matter fermion masses, and M is taken to be $O(10^3\,\mathrm{GeV})$. The vev of X is undetermined at tree level. The effective potential is

$$V_{eff} = \frac{\lambda_1^2 \lambda_2^2 M^4}{\lambda_1^2 + 30\lambda_2^2} \left(1 + \frac{3}{4\pi^2} \frac{\lambda_2^2}{\lambda_1^2 + 30\lambda_2^2} (29\lambda_1^2 - 50e^2) \ln \frac{X}{\mu} + O(t^2) \right) \tag{3}$$

and hence $F(t) \propto 29\lambda_1^2(t) - 50e^2(t)$. The potential is minimized $[F(0) < 0]$ for $29\lambda_1^2(t_0) = 50e^2(t_0)$ with $t_0 = \ln \frac{X}{M}$ and SU(5) breaking to SU(3) × SU(2) × U(1) at the large energy scale X. As we are interested only in the hierarchy X/M, we will not be troubled by being unable to realistically break weak interactions, or by other

phenomenological disasters such as light color octet fields. We will, however, demand realistic couplings above M. If we assume the Higgs triplets will somehow become superheavy in the broken phase, the SU(3) β function vanishes at one loop in the region between M and X. Hence the SU(5) gauge coupling $\alpha(t_0)$ is about 1/10. Scaling strong and weak couplings of the broken theory gives unification at $t_0 = 37$.

We must now investigate the renormalization group equations for $\alpha_i = \frac{\lambda_i^2}{4\pi}$ $(i = 1,2)$ to see under what conditions such a hierarchy can occur. Using variables $u_i = \frac{\alpha_i}{\alpha}$ and

$$s = \frac{1}{b} \ln \left(1 + \frac{\alpha(t_0)}{2\pi} b\, t \right) \tag{4}$$

gives [3]

$$\frac{d\alpha}{dt} = \frac{b\alpha^2}{2\pi} = \frac{\alpha^2}{\pi}$$

$$\frac{du_1}{ds} = u_1 (21\, u_1 + 8u_2 - 32)$$

$$\frac{du_2}{ds} = u_2 \left(\frac{84}{5} u_1 + 56\, u_2 - 22 \right) \tag{5}$$

and F = 0 for $u_1 = 50/29$. The physical perturbative domain is $0 \leq \alpha_i(t) < 1$. It is convenient to consider starting at scale X with $u_1 = 50/29$ and $0 \leq u_2 < 10$, and running the renormalization group equations in reverse to find values of u_1 and u_2 at scale M, that is after scaling a distance of 37 in t. The results of such an analysis are shown in Figure 1. It is quite clear that $u_1(0)$ must be chosen to be rather close to the fixed point at (32/21, 0). In fact, to obtain a hierarchy of t = 37, the $u_i(0)$ must be chosen in a region about the fixed point of area $\sim 10^{-4}$, that is a fraction 10^{-6} of the

available perturbative domain. Why has this happened? All couplings are evolving logarithmically; so why should an exponential hierarchy not occur more easily? The answer is very simple: the numerical coefficients appearing in the equations for $\dot{u}_i$ are large, typically O(25). Hence one typically scales a distance of $s \sim 1/25$ rather than of order unity for a unit change of u_i. Such a small s produces a hierarchy $M/X \sim O(15)$, clearly insufficient for grand unified models. For SU(N) these coefficients are typically $O(N^2)$, so the situation is worse in theories with $N > 5$. It should be noted that $u_i(0)$ cannot both be made too small since the origin is ultraviolet attractive and the effective potential is unstable to this order.

There are two simple ways to alleviate this problem. Further chiral representations may be added to increase b sufficiently that a large enough t results even from a very small s (equation 4). Alternatively, additional terms may be added to the superpotential to increase the number of couplings and change the renormalization group equations.

Naively increasing b results in the fixed point moving to the right of the line $u_1 = 50/29$. The requirement $F[\lambda(0)] < 0$ forces $u_i(0)$ away from the fixed point decreasing the hierarchy. Even worse, $u_2(0)$ must be well away from zero to avoid the trajectory being attracted to the origin, and since $\frac{du_2}{ds} \supset 56u_2^2$, $u_2(t_0)$ will leave the perturbative domain. Of course, naively increasing b is not realistic; in order that the SU(3) coupling remains perturbative at scale X, the additional fields must be superheavy in the broken phase. This requires more couplings and leads to the second possibility.

In any realistic model with $M \sim O(10^3\text{GeV})$ X and/or Y must couple to additional fields in order to give some of their components large

masses (for example the color triplets of the Higgs 5 and $\bar{5}$: H and $\bar{H}$). Witten [1] added a term $\bar{H}YH$, which unfortunately gave an undesirable vacuum. The correct weak breaking can be obtained in an SU(5) model by incorporating 75 representations [6]. In all such models, the relevant coupling constant space is increased in dimension, and more couplings participate in the renormalization group equations and in the condition for the minimum of the potential, $F[\lambda_i(t_0)] = 0$. Such cases rapidly become difficult to analyze. However, to demonstrate that the basic argument given above is not destroyed, we will discuss a simple example.

The additional fields are two 5's (G and G') and two $\bar{5}$'s ($\bar{G}$ and $\bar{G}'$). The following term is added to the superpotential of equation 1:

$$\lambda_3 X G\bar{G} + mG\bar{G}' + \bar{m}\bar{G}G' \quad . \tag{6}$$

The masses m and $\bar{m}$ are O(M), and made sufficiently large to prevent the new fields from acquiring vev's. The coefficient of t in the effective potential is now

$$F \propto u_2(29u_1 + 100u_3 - 50) + \frac{10}{3} u_1 u_3 \quad , \tag{7}$$

and the renormalization group equations are

$$\begin{aligned}
\frac{d\alpha}{dt} &= \frac{2\alpha^2}{\pi} \\
\frac{du_1}{ds} &= u_1(21u_1 + 8u_2 - 34) \\
\frac{du_2}{ds} &= u_2\left(\frac{84}{5} u_1 + 56u_2 + 20u_3 - 24\right) \\
\frac{du_3}{ds} &= u_3\left(48u_2 + 28u_3 - 6\right) \quad .
\end{aligned} \tag{8}$$

The fixed point structure is now more complex, as is the surface F = 0. If $\alpha(t_0) = 1/10$ and $t_0 = 37$, s is reduced to 0.3 because b has increased. The fixed point at (34/21, 0, 0) is now a saddle point. Choosing $u_1(0)$ near 34/21, $u_2(0)$ near 0 and any $u_3(0)$ will result in a large t, since $\frac{du_1}{ds}$ and $\frac{du_2}{ds}$ are very small. It is necessary to tune two parameters as before. The third coupling $u_3(0)$ need not be tuned because u_3 does not appear in $\frac{du_1}{ds}$.

The coupling constants λ_i and α receive logarithmically divergent renormalization from wavefunction renormalization. This ensures that the u_i are unstable under renormalization, and consequently they must be finely tuned at each order in perturbation theory.

In this paper we have pointed out that inverse mass hierarchies of the Witten type in supersymmetric gauge theories are naturally of order ten. In order to generate a useful hierarchy, coupling constants must be finely tuned order by order in perturbation theory. In the simplest SU(5) model they must be chosen to occupy a fraction of order 10^{-6} of the available space.

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FIGURE CAPTION

Fig. 1: Trajectories in the u_1 u_2 plane. The dashed lines are contours of fixed X/M. Choosing $u_1(0)$ and $u_2(0)$ along a contour results in the appropriate value of X/M. The vertical line corresponds to $F(t_0) = 0$ at $u_1 = 50/29$. The line A illustrates the case of evolution to a point outside the perturbative region. It is labeled by s.

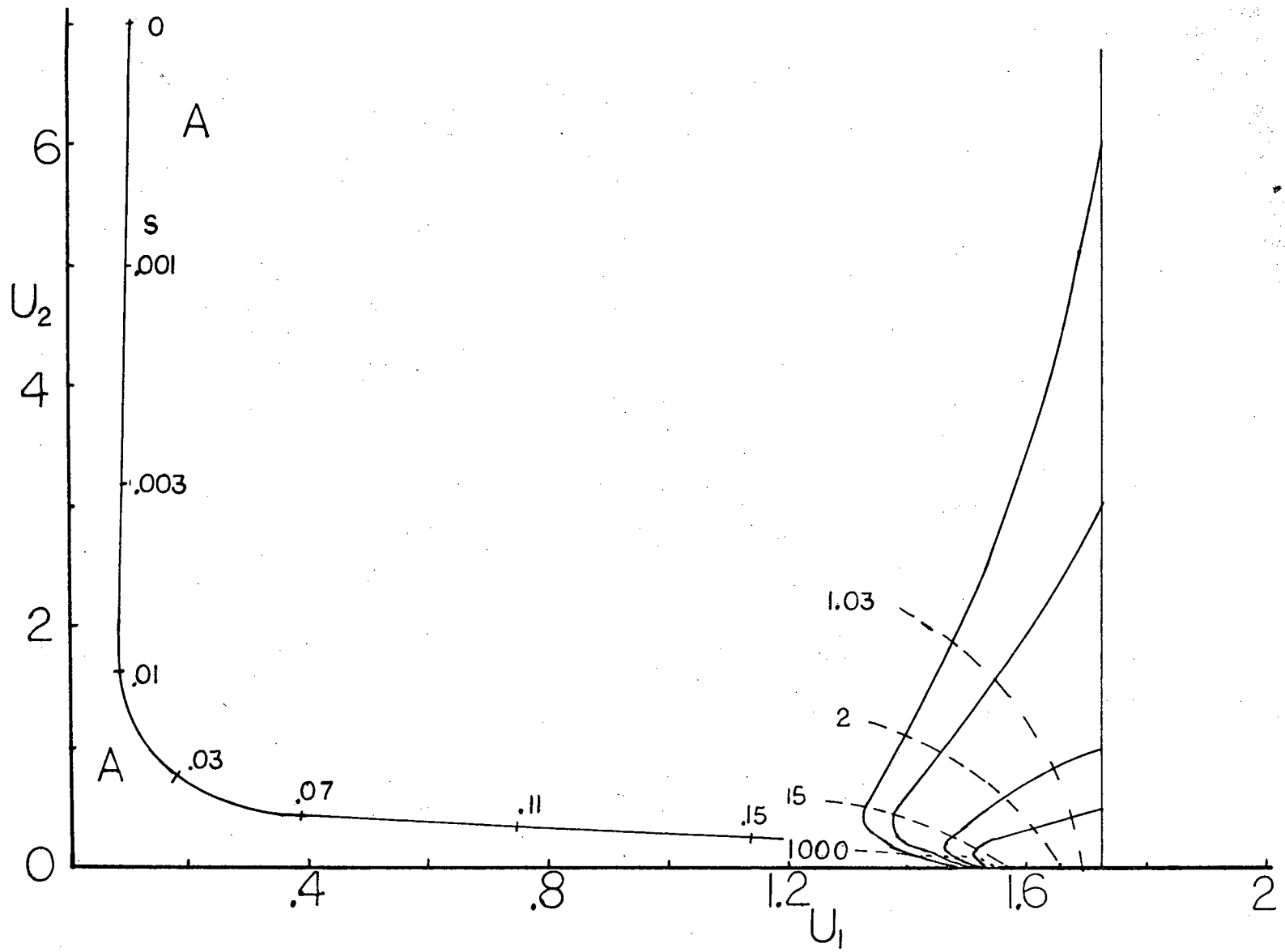

FIGURE 1

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